On frame-like gauge invariant formulation for mixed symmetry fields

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Outlook

1. Completely symmetric (spin-)tensors
   - Massless fields
   - Massive fields
   - Partially massless fields

2. Mixed symmetry (spin-)tensors
   - Massless fields
   - Massive fields
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Metric versus frame-like — bosons

- Metric formulation — symmetric double traceless tensor $\Phi_{\mu_1 \dots \mu_s}$
  Gauge transformations:

$$\delta\Phi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}, \qquad g^{\mu_1 \mu_2} \xi_{\mu_1 \mu_2 \dots \mu_{s-1}} = 0$$

- Frame-like formulation — symmetric traceless one form $\Phi_\mu{}^{a_1 \dots a_{s-1}}$ and auxiliary field $\Omega_\mu{}^{a_1 \dots a_{s-1},b}$. Gauge transformations:

$$\begin{aligned} \delta\Phi_\mu{}^{a_1 \dots a_{s-1}} &= \partial_\mu \xi^{a_1 \dots a_{s-1}} + \eta^{a_1 \dots a_{s-1}}{}_\mu, \qquad \eta^{(a_1 \dots a_{s-1},b)} = 0 \\ \delta\Omega_\mu{}^{a_1 \dots a_{s-1},b} &= \partial_\mu \eta^{a_1 \dots a_{s-1},b} \end{aligned}$$

  Massless Lagrangian:

$$\begin{aligned} \mathcal{L}_0 &= \left\{ {}^{\mu\nu}_{ab} \right\} [\Omega_\mu{}^{a(s-2),c} \Omega_\nu{}^{b(s-2),c} + \frac{1}{s-1} \Omega_\mu{}^{(s-1),a} \Omega_\nu{}^{(s-1),b}] - \\ &\quad - \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{a(s-2),b} \partial_\nu \Phi_\alpha{}^{c(s-2)} \end{aligned}$$

Metric versus frame-like — fermions

- Metric formulation — symmetric triple γ transverse spin-tensor $\Psi_{\mu_1 \dots \mu_s}$. Gauge transformations:

$$\delta \Psi_{\mu_1 \dots \mu_s} = \partial_{(\mu_1} \xi_{\mu_2 \dots \mu_s)}, \qquad \gamma^{\mu_1} \xi_{\mu_1 \dots \nu_{s-1}} = 0$$

- Frame-like formulation — symmetric γ transverse one form $\Psi_\mu{}^{a_1 \dots a_{s-1}}$. Gauge transformations and massless Lagrangian:

$$\delta \Psi_\mu{}^{a_1 \dots a_{s-1}} = \partial_\mu \xi^{a_1 \dots a_{s-1}} + \eta^{a_1 \dots a_{s-1}}{}_\mu, \qquad (\gamma \xi)_{a_2 \dots a_{s-1}} = 0$$

$$\mathcal{L}_0 = \frac{i}{2} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} [\bar{\Psi}_\mu{}^{(s-1)} \gamma^a \gamma^b \gamma^c \partial_\nu \Psi_\alpha{}^{(s-1)} - -6(s-1) \bar{\Psi}_\mu{}^{a(s-2)} \gamma^b \partial_\nu \Psi_\alpha{}^{c(s-2)}]$$

Gauge invariance for massive fields

- Bosons:

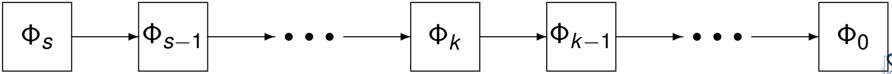
$$\mathcal{L} \sim \sum_{k=0}^{s} [\Omega_k \Omega_k + \Omega_k \partial \Phi_k + m(\Omega_k \Phi_{k-1} + \Omega_{k-1} \Phi_k) + m^2 \Phi_k \Phi_k$$

$$\delta\Phi \sim \partial\xi + \eta + m\xi, \qquad \delta\Omega \sim \partial\eta + m\eta + m^2\xi$$

- Fermions:

$$\mathcal{L} \sim \sum_{k=0}^{s} [i\bar{\Psi}_k \hat{\partial} \Psi_k + m(\bar{\Psi}_k \Psi_k + i\bar{\Psi}_k \gamma \Psi_{k-1})]$$

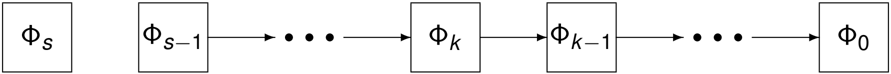
$$\delta\Psi_k \sim \partial_{(1}\xi_{k-1)} + m[i\gamma_{(1}\xi_{k-1)} + \xi_k + g_{(2}\xi_{k-2)}]$$



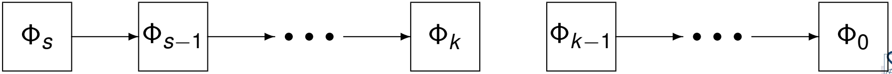
(Partially) massless limits

- Minkowski space

- Anti de Sitter space



- De Sitter space



Metric versus frame-like — bosons

- Metric formulation
  - Mixed symmetry tensor $\Phi_{\mu_1\dots\mu_k,\nu_1\dots\nu_l}$
  - Two gauge transformations: $\xi_{\mu_1\dots\mu_k,\nu_1\dots\nu_{l-1}}$ and $\zeta_{\mu_1\dots\mu_{k-1},\nu_1\dots\nu_l}$
  - Reducibility: $\chi_{\mu_1\dots\mu_{k-1},\nu_1\dots\nu_{l-1}}$
- Frame-like formulation
  - Traceless two forms:

$$\Phi_{\mu\nu}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1}}, \qquad \Omega_{\mu\nu}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1},c}$$

  - Gauge transformations:

$$\delta\Phi_{\mu\nu}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1}} = \partial_{[\mu}\xi_{\nu]}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1}} + \eta_{[\mu}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1}}{}_{\nu]}$$

$$\delta\Omega_{\mu\nu}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1},c} = \partial_{[\mu}\eta_{\nu]}{}^{a_1\dots a_{k-1},b_1\dots b_{l-1},c}$$

  - Lagrangian:

$$\mathcal{L}_0 \sim \Omega\wedge\Omega + \Omega\wedge\partial\wedge\Phi$$

Metric versus frame-like — fermions

- Metric formulation
  - Mixed symmetry spin-tensor $\Psi_{\mu_1\ldots\mu_k,\nu_1\ldots\nu_l}$
  - Two gauge transformations: $\xi_{\mu_1\ldots\mu_k,\nu_1\ldots\nu_{l-1}}$ and $\zeta_{\mu_1\ldots\mu_{k-1},\nu_1\ldots\nu_l}$
  - Reducibility: $\chi_{\mu_1\ldots\mu_{k-1},\nu_1\ldots\nu_{l-1}}$
- Frame-like formulation
  - γ-transverse two form:

$$\Psi_{\mu\nu}{}^{a_1\ldots a_{k-1},b_1\ldots b_{l-1}}$$

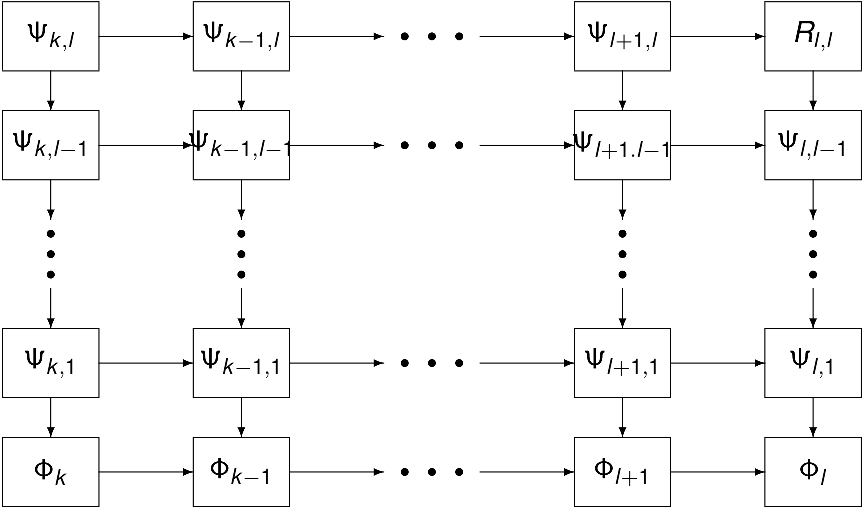
  - Gauge transformations:

$$\delta\Phi_{\mu\nu}{}^{a_1\ldots a_{k-1},b_1\ldots b_{l-1}} = \partial_{[\mu}\xi_{\nu]}{}^{a_1\ldots a_{k-1},b_1\ldots b_{l-1}} + \eta_{[\mu}{}^{a_1\ldots a_{k-1},b_1\ldots b_{l-1}}{}_{\nu]}$$

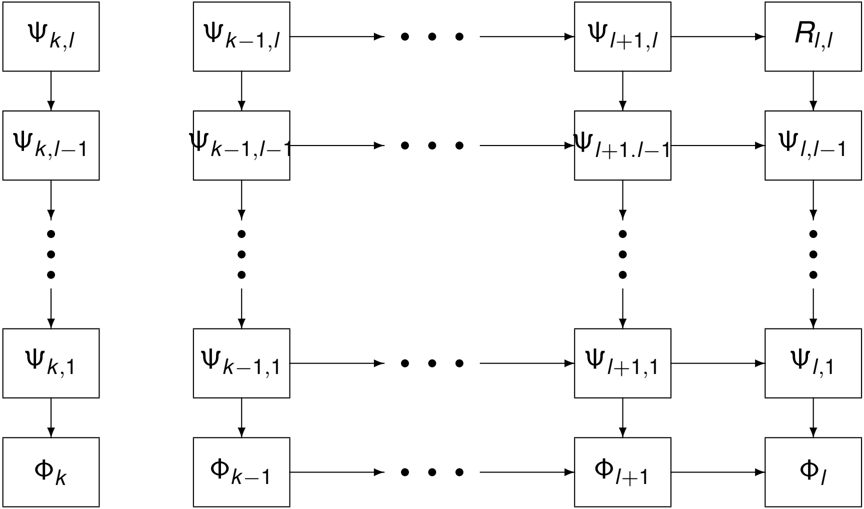
  - Lagrangian:

$$\mathcal{L}_0 \sim \bar{\Psi} \wedge \partial \wedge \Psi$$

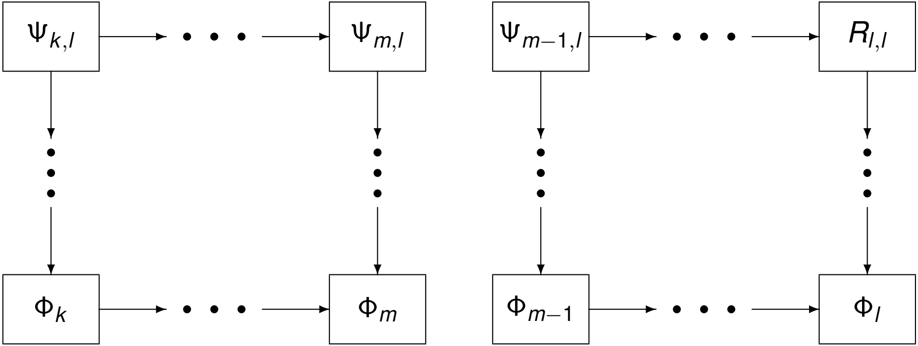
General massive theory



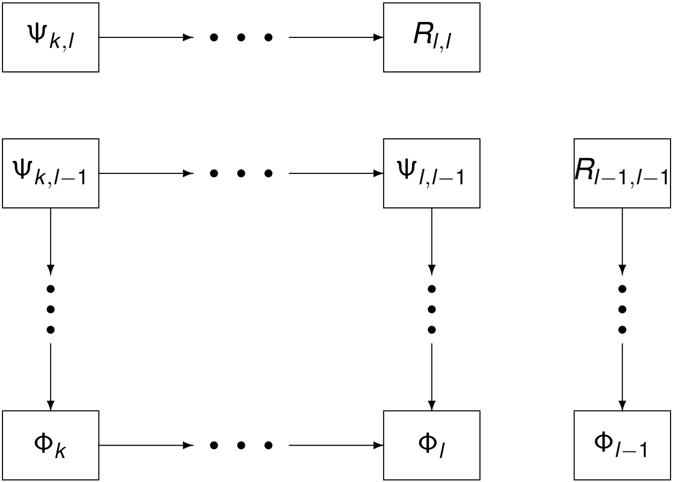
Unitary (partially) massless limit in anti de Sitter space

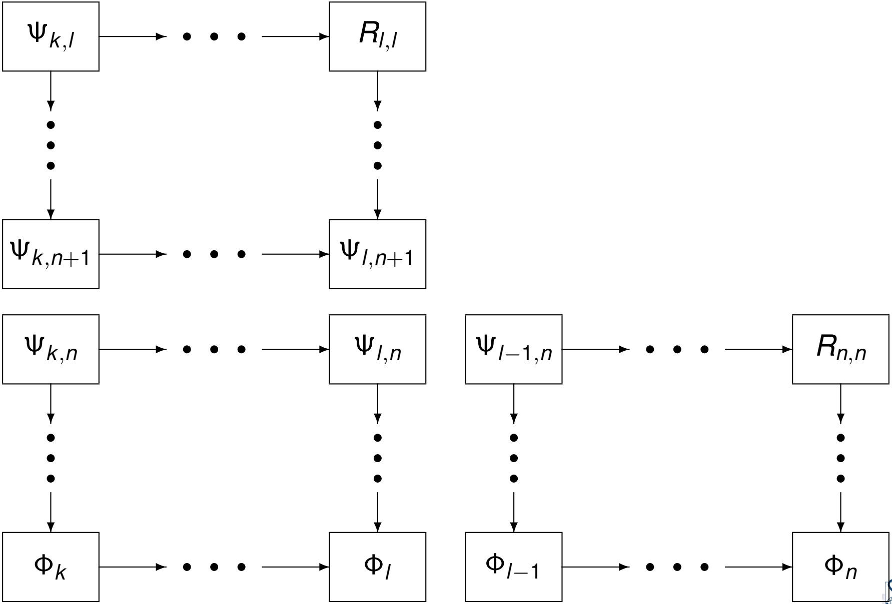


Partially massless limits in anti de Sitter space



(Partially) massless limit in de Sitter space




$\Psi_{k,l}$
$R_{l,l}$
$\Psi_{k,n+1}$
$\Psi_{l,n+1}$
$\Psi_{k,n}$
$\Psi_{l,n}$
$\Psi_{l-1,n}$
$R_{n,n}$
Φ_k
Φ_l
Φ_{l-1}
Φ_n

Conclusion

- Frame-like formalism provides an elegant and convenient framework for mixed symmetry (spin-)tensors as well as for completely symmetric (spin-)tensors.
- In all cases investigated gauge invariant formulation for massive fields works equally well both in flat Minkowski space as well as in (anti) de Sitter spaces and allows us investigate all possible massless and partially massless limits
- Main objective — search for the mechanism of spontaneous symmetry breaking deforming interacting massless theories in anti de Sitter space into massive ones in the Minkowski space

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- Main objective — search for the mechanism of spontaneous symmetry breaking deforming interacting massless theories in anti de Sitter space into massive ones in the Minkowski space