Geometric spin decay in confined mesoscopic structures

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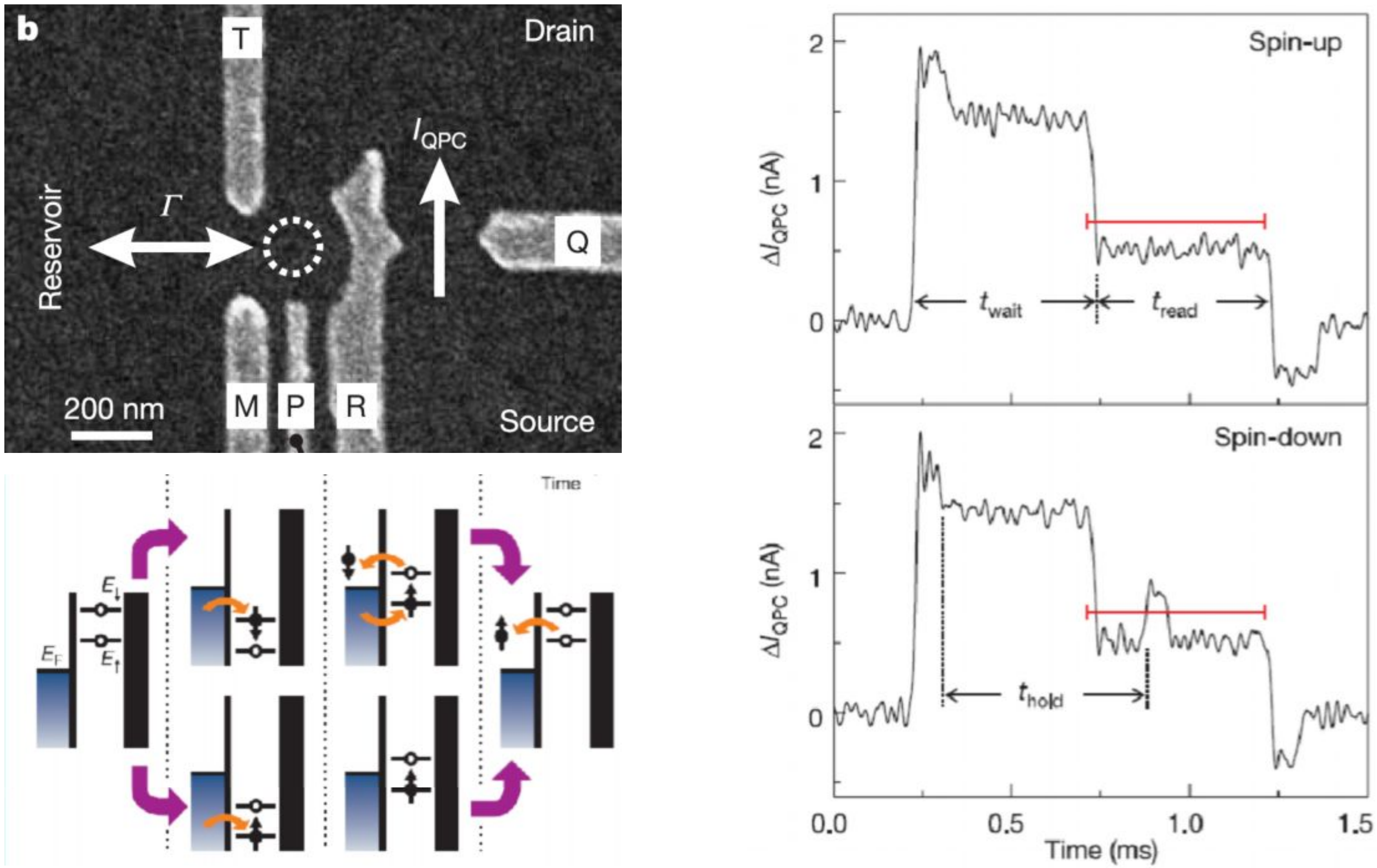
Baruch Horovitz	Ben Gurion

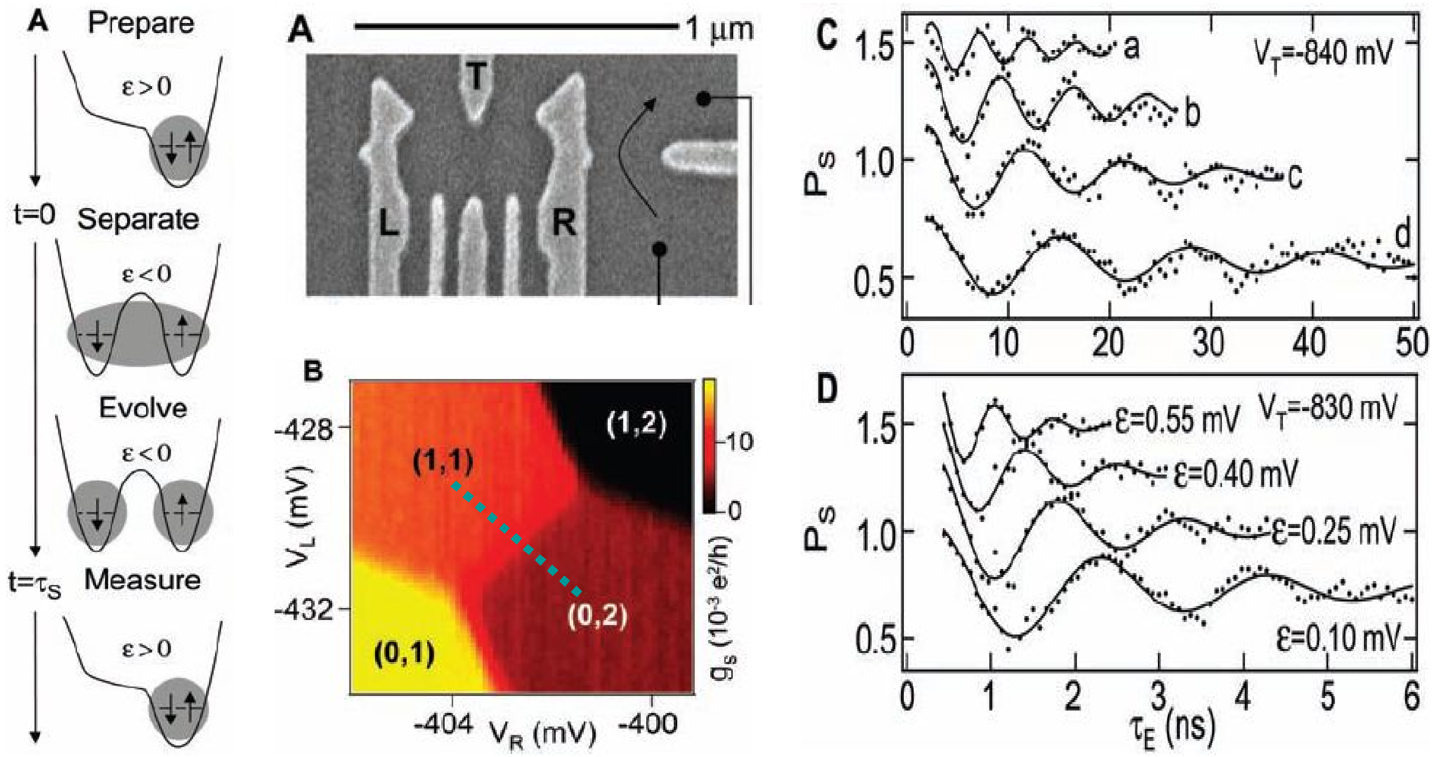
P. San-Jose, G.Z., A. Shnirman, and G. Schon, PRL 97, 076803 (2006)
P. San-Jose, G. Schön, A. Shnirman, and G.Z. PHYSICA E 40, 76 (2007)
P. San-Jose, B. Scharfenberger, G. Schön, A. Shnirman, and G.Z. PRB 77, 045305 (2008)

Motivation: Recent experiments to control spin



Single-shot read-out of an individual electron spin in a quantum dot
J. M. Elzerman, R. Hanson, *et al.* Nature **430**, 431 (2004).

Motivation



J. R. Petta, *et al.* Science **309** 2180 (2005)

For spin manipulation coherence is crucial…

Spin decay mechanisms in Q-bits

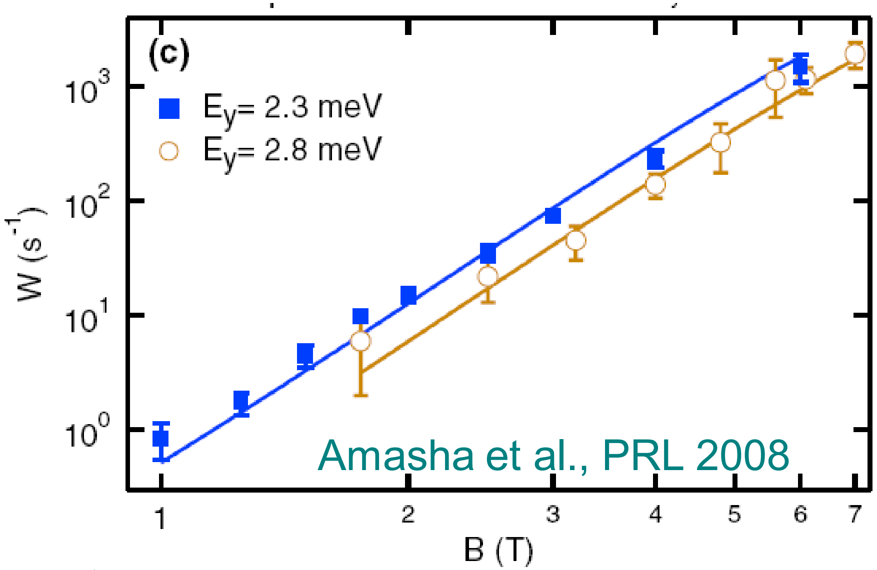
Nuclear spins $\approx 1\,\mathrm{mT} \quad (100\,\mathrm{ns})$: important but (almost) static

[Khaetskii, Loss, Glazman, 2002]

Piezoelectric phonons + spin-orbit coupling

[Khaetskii, Nazarov (2001); Golovach, Khaetskii, Loss (2004); Stano, Fabian (2005)]

– important for large B
– Scales as $1/T_1 \sim B^5$



Questions:

•What happens as $B \to 0$? ⇨ **Berry phase relaxation**

• Other sources of relaxation ? ⇨ **Ohmic fluctuations**

• Can there be relaxation at T=0 temperature ??? ⇨ **Yes ???**

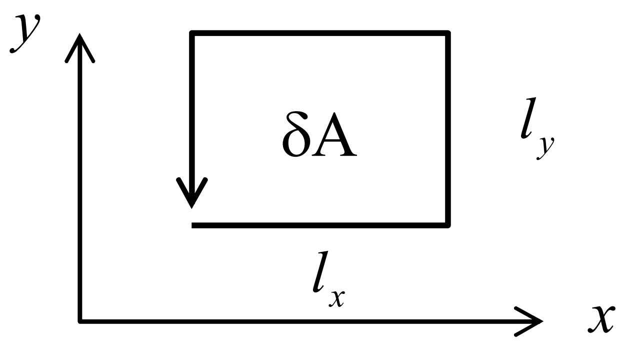
Geometrical relaxation (due to a Berry phase)

(with P. San-Jose, B. Scharfenberger, G. Schön, A. Shnirman)

Berry phase, unconfined electrons

Electron in 2DEG: $\boxed{H=\frac{\vec{p}^2}{2m}+\alpha m(v_y\sigma_x - v_x\sigma_y)}$ ← **Rashba coupling**

Move electron around:



$$v_x \neq 0 \rightarrow B_{\mathrm{eff},y} \neq 0 \Rightarrow U_x = e^{-i\, l_x 2\alpha m \sigma_y/2}$$

$$v_y \neq 0 \rightarrow B_{\mathrm{eff},x} \neq 0 \Rightarrow U_y = e^{i\, l_y 2\alpha m \sigma_x/2}$$

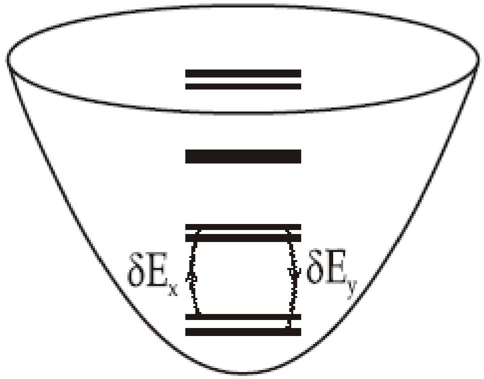
Rotation after cycle:

$$\mathrm{U} = \mathrm{U}_y^+ \mathrm{U}_x^+ \mathrm{U}_y \mathrm{U}_x \approx e^{i\,\delta\varphi\,\sigma_z/2}$$

$$\delta\varphi = 8\delta A / \lambda_{SO}^2$$

Rotation proportional
to directed area !

Confined electron



$$H_0 = \frac{\mathbf{p}^2}{2m^*} + V(\mathbf{r}) - \frac{g\mu_B}{2}\vec{B}\cdot\vec{\sigma} + H_{SO} + \delta V(t)$$

confinement in 2-deg

EM fluctuations

fluctuations:

Slow compared to level spacing

$$\boxed{\delta V(t) = -e\,\delta E_x(t)\cdot\hat{x} - e\,\delta E_y(t)\cdot\hat{y} + \ldots}$$

$$\delta V(t) = \sum_k X_k(t)\cdot\hat{O}_k$$

$$X_k(t) = \delta E_x, \delta E_y, \nabla_x \delta E_x, \ldots \qquad \hat{O}_k = x, y, x^2, \ldots$$

(phonons, charge fluctuations)

B=0 adiabatic approximation

SLOW classical fluctuations $\vec{X}(t) \quad (\leftrightarrow \delta E_{x,y}(t))$

$$\left|\Psi(t)\right\rangle \approx a_{\uparrow}(t)\left|\Phi_{\uparrow}(\vec{X}(t))\right\rangle + a_{\downarrow}(t)\left|\Phi_{\downarrow}(\vec{X}(t))\right\rangle$$

instantaneous ground doublet
(Kramers degenerate!)

$$\Rightarrow \quad \boxed{i\frac{da_{\sigma}}{dt} = H^{eff}_{\sigma\sigma'}(t)\,a_{\sigma'}} \qquad \boxed{H^{eff}_{\sigma\sigma'}(B=0) = -i\left\langle \frac{d\phi_{\sigma}(\vec{X})}{dt}\middle|\phi_{\sigma'}(\vec{X})\right\rangle}$$

Non-Abelian
Berry Phase !

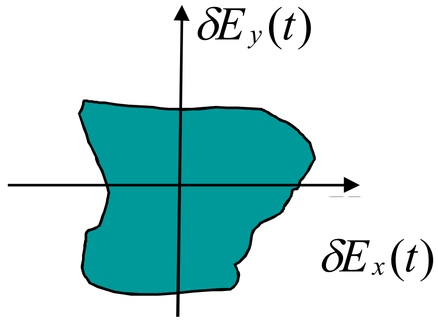
Perturbation theory

Electric fields only: $\vec{X} = \delta\vec{E}(t)$

$$\boxed{H_{eff}(t) = B_{eff}(t)\,\sigma^z}$$

$$B_{eff}(t) = \left(\frac{d\delta E_x}{dt}\delta E_y - \delta E_x \frac{d\delta E_y}{dt}\right) C \sim dA/dt$$

$$C = -i\,e^2 \sum_{n\neq\sigma} \frac{\hat{x}_{\uparrow,n}\hat{y}_{n,\uparrow}}{(\varepsilon_\sigma - \varepsilon_n)^2}$$



Fluctuations

⇓

random spin rotation proportional to the area

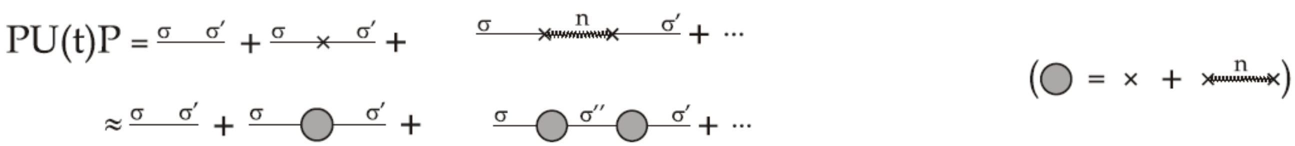
⇓

RELAXATION

E. Abrahams, Phys. Rev. 107, 491 1957 !

Adiabatic approximation: $B \neq 0$

Resummation of perturbation series for evolution operator in time-dependent field:



one 'photon'

Static two 'photons', Van Vleck cancellation

$$\boxed{\begin{aligned} H_{\text{eff}} = \quad & -\frac{1}{2} B_{\text{eff}}\, \tilde{\sigma}_z + X^\mu C^{(1)}_\mu \cdot \tilde{\sigma}_x + X^\mu X^\nu \vec{C}^{(2)}_{\mu\nu} \cdot \vec{\tilde{\sigma}} \\ & + \frac{1}{2} \left(\dot{X}^\mu X^\nu - X^\mu \dot{X}^\nu \right) C^{(3)}_{\mu\nu} \cdot \tilde{\sigma}_x \end{aligned}}$$

Berry phase !

$B \longrightarrow 0$

$$C^{(1)} \sim \frac{B}{\epsilon} \max\{\tilde{\alpha}, \tilde{\beta}\} \to 0$$

$$|\vec{C}^{(2)}| \sim \frac{B}{(\epsilon)^2} \max\{\tilde{\alpha}^2, \tilde{\beta}^2\} \to 0$$

$$C^{(3)} \sim \frac{1}{(\epsilon)^2} \max\{\tilde{\alpha}^2, \tilde{\beta}^2\} \to const.$$

All information in spectral function $\rho(\omega)$ of $\delta\vec{\mathrm{E}}(t)$

Berry phase term! $$1/T_1^{(3)} = 2|\vec{C}^{(3)}_{XY,a}|^2 S_{\dot{X}Y-X\dot{Y}}(B_{\text{eff}}) \,,$$

$$S_{\dot{X}Y-X\dot{Y}}(\omega) = \frac{\pi}{2}\int d\tilde{\omega}\, \frac{\tilde{\omega}^2 \varrho(\frac{\omega+\tilde{\omega}}{2})\varrho(\frac{\omega-\tilde{\omega}}{2})}{1-\cosh(\tilde{\omega}/2T)/\cosh(\omega/2T)}$$

Phonons	Ohmic fluctuations
$\rho_{ph}(\omega) = x_0 \lambda_{ph} \omega^3$	$\rho_{\Omega}(\omega) = \lambda_{\Omega}\omega$
$1/T_1^{(1)} \propto B^4 \max\{B, T\}$ $1/T_1^{(2)} \propto B^2 \max\{B^7, T^7\}$ $\boxed{1/T_1^{(3)} \propto \max\{B^9, T^9\}}$	$1/T_1^{(1)} \propto B^2 \max\{B, T\}$ $1/T_1^{(2)} \propto B^2 \max\{B^3, T^3\}$ $\boxed{1/T_1^{(3)} \propto \max\{B^5, T^5\}}$

Quantum fluctuations for parabolic confinement

Parabolic confinement:

$$V(\hat{\boldsymbol{r}}) = \tfrac{1}{2} m \omega_0^2 \hat{\boldsymbol{r}}^2$$

$$\mathcal{H}(t) = \frac{\hat{\boldsymbol{p}}^2}{2m} + V(\hat{\boldsymbol{r}}) + e\boldsymbol{E}(t)\hat{\boldsymbol{r}} + \mathcal{H}_{\mathrm{SO}}$$

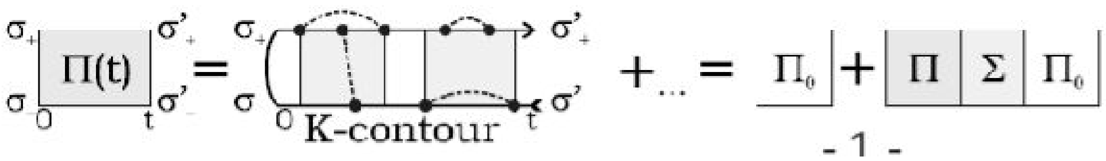
$$= \frac{\hat{\boldsymbol{p}}^2}{2m} + V[\hat{\boldsymbol{r}} - \boldsymbol{R}_C(t)] + \mathcal{H}_{\mathrm{SO}} + \mathcal{C}(t),$$

Electric field shifts parabola

$$\boldsymbol{R}_C(t) \equiv -e\boldsymbol{E}(t)/m\omega_0^2$$

Dyson equation :

$$\Pi(t,0) = \Pi^0(t,0) + \int_0^t dt_1 dt_2 \Pi^0(t,t_1)\Sigma(t_1,t_2)\Pi(t_2,0),$$

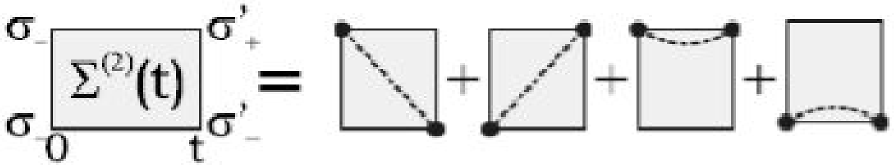


Equation of motion:

$$\dot{\tilde{\rho}}_D(t) = L_0\tilde{\rho}_D(t) + \int_0^t \Sigma(t-t')\tilde{\rho}_D(t') \qquad L_0\tilde{\rho}_D(t) = i[\tilde{\rho}_D(t), H_Z]$$

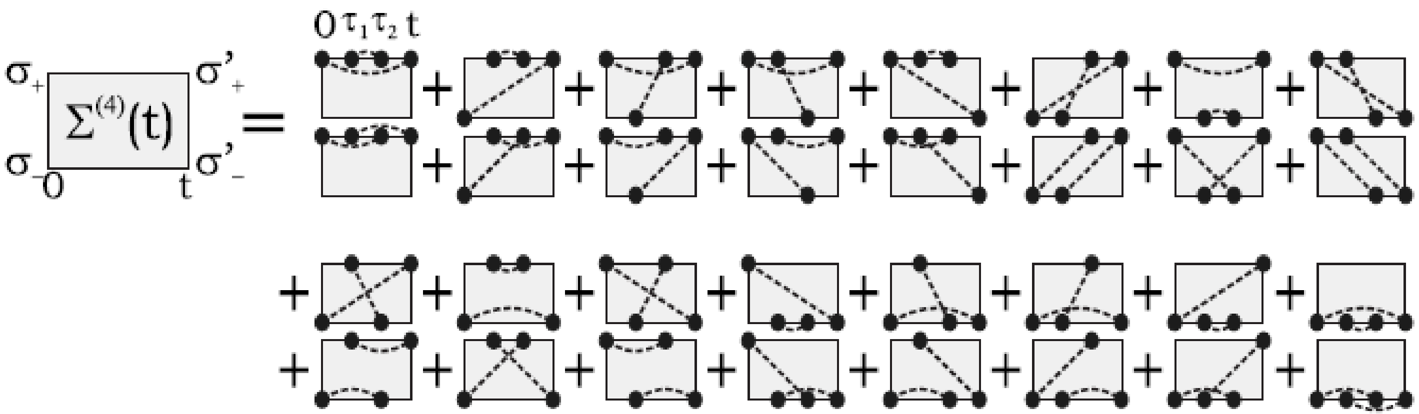
Diagrammatic perturbation theory

„Born approximation”



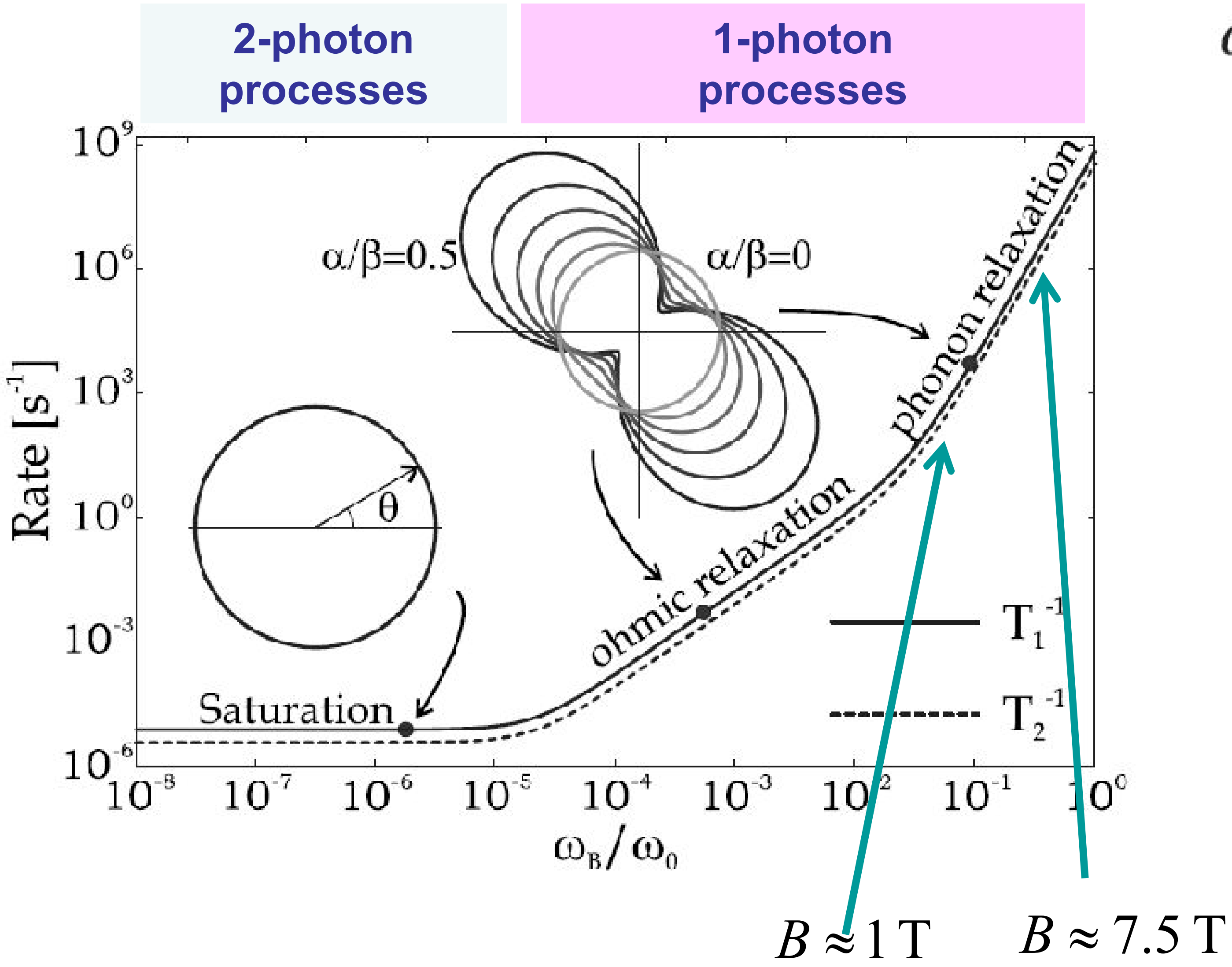
usual terms…

Higher order corrections



These contain Berry phase !!!

Angular dependence of relaxation (5K dot)



$\alpha/\beta = 4, \ \theta = 0,$

$l_{SO} = 3 \ \mu m$

$\lambda_\Omega = 10^{-3}$

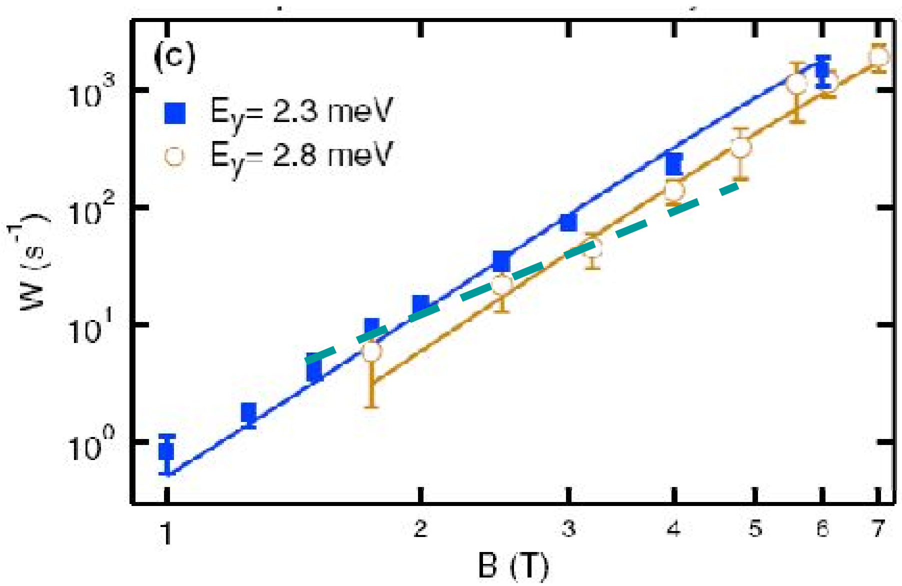
$\omega_0 = 5$ K

$T = 100$ mK

Experiments ?

• Ohmic environment ?

Amasha et al., PRL 2008



• Berry phase can be observed in large dots only…

• p-type quantum dots !

Gerardot et al, Nature 2008
Mircea Trif, Pascal Simon, Daniel Loss, arXiv:0902.2457

•Hyperfine coupling is not very important (nodes)
•Relaxation rate is second order in SO coupling

Spin Relaxation on a Ring

(with Pierre Le Doussal and Baruch Horovitz)

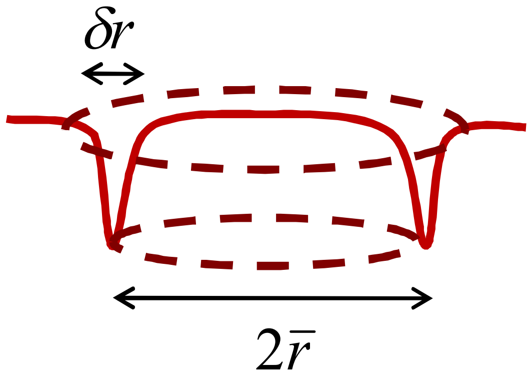
Confining the particle to a ring… (1)

Is there spin relaxation at T=0 for a confined electron?

Consider simpler problem: electron confined to ring and Coupled to Ohmic fluctuations

Hamiltonian:

$$\mathcal{H}_0 + \mathcal{H}'$$



Confinement in radial direction

$$\mathcal{H}_0 = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r}\right] + V_0(r) \, ,$$

Motion along the ring:

$$\mathcal{H}' = \frac{p_\theta^2}{2mr^2} + \alpha_0(\mathbf{S}_x p_y - \mathbf{S}_y p_x) + \beta_0(\mathbf{S}_x p_x - \mathbf{S}_y p_y)$$

Rashba **Dresselhaus**

Take limit : $\delta r \to 0, \quad \bar{r}$ finite

Confine motion to lowest radial eigenmode

Generate effective Hamiltonian

$$\mathcal{H}_1 = \frac{p_\theta^2}{2mr^2} + \frac{\alpha_0}{2r}\{\mathbf{S}_r^+, p_\theta\} - \frac{\beta_0}{2r}\{\mathbf{S}_\theta^-, p_\theta\}$$

$$\mathcal{H}_2 = i\alpha_0\hbar\mathbf{S}_\theta^+(\partial_r + \frac{1}{2r}) - i\beta_0\hbar\mathbf{S}_r^-(\partial_r + \frac{1}{2r})$$

$$\mathbf{S}_r^\pm \equiv \cos\theta\sigma_x \pm \sin\theta\sigma_y$$

$$\mathbf{S}_\theta^\pm \equiv \cos\theta\mathbf{S}_y \mp \sin\theta\mathbf{S}_x$$

Effective Hamiltonian:

$$\mathcal{H}_{ring} = \langle 0|\mathcal{H}_1|0\rangle - \sum_{n\neq 0}\frac{\langle 0|\mathcal{H}_2|n\rangle\langle n|\mathcal{H}_2|0\rangle}{E_n - E_0}.$$

Can compute analytically

$$\boxed{\mathcal{H}_{ring} = \frac{1}{2m\bar{r}^2}[p_\theta + \boldsymbol{h}(\theta)\cdot\mathbf{S}]^2}$$

Effective field: $\boldsymbol{h} \equiv (\alpha\cos\theta - \beta\sin\theta, \alpha\sin\theta - \beta\cos\theta)$

Conserved quantity: $Q \equiv p_\theta + \boldsymbol{h}(\theta)\cdot\mathbf{S}$

Including fluctuations

$$\boxed{\mathcal{H} = \mathcal{H}_{ring} + V(\theta, \xi)}$$

fluctuations

Equations of motion:

Position: $$\ddot{\theta} = -\frac{1}{m\bar{r}^2}\partial_\theta V(\theta, \xi)$$

⇨ Dynamics of θ decouples from spin!!!

Spin:

$$\Phi_t \equiv \Gamma_{\theta_0}(\theta_t - \theta_0)/2\pi$$

$$\frac{d\mathbf{S}}{dt} = \dot{\theta}\, \boldsymbol{h}(\theta) \times \mathbf{S} \Rightarrow \qquad \frac{d\mathbf{S}}{d\theta} = \boldsymbol{h}(\theta) \times \mathbf{S} \quad (\vec{h}(\theta) \approx h_0\, \hat{z})$$

„Geometrical” evolution

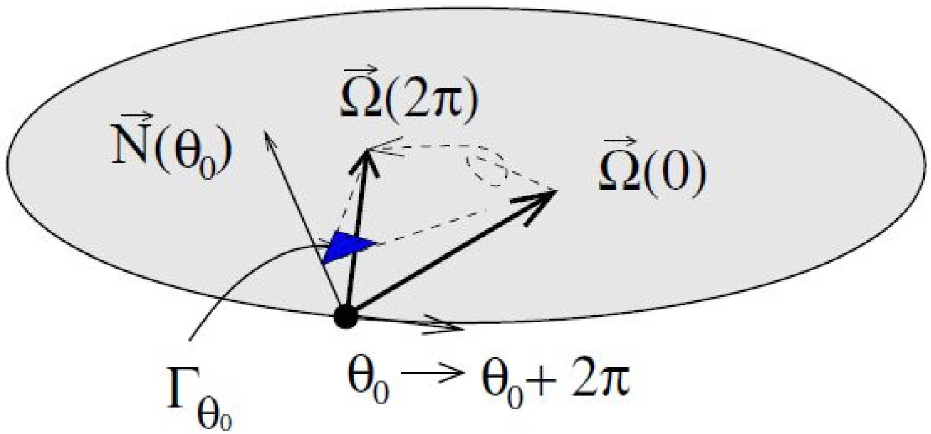
Spin evolution

$$\frac{d\mathbf{S}}{d\theta} = \boldsymbol{h}(\theta) \times \mathbf{S} \quad \Rightarrow \quad S_i(\theta) = R_{ij}(\theta, \theta_0) S_j$$

2π **rotations**

$$R_{2\pi}(\theta_0) = R(\theta_0 + 2\pi, \theta_0)$$

Makes rotation around special axis $\quad \mathbf{N}(\theta_0) = R_{2\pi}(\theta_0)\, \mathbf{N}(\theta_0)$



θ **randomized by** ξ $\Rightarrow$ **spin collapses to** $\mathbf{N}(\theta_0)$

Path integral for spin evolution operator

$$\langle \theta_f | e^{-it\mathcal{H}} | \theta_i \rangle = \sum_{m_\theta} \int_{\theta_i}^{\theta_f} \mathcal{D}\theta \, U(\theta_f + 2\pi m_\theta, \, \theta_i) \, e^{i \, S_P[\theta,\xi]}$$

$$U(\tilde{\theta}_f, \tilde{\theta}_i) = P_\theta \exp\left\{ -i \int_{\tilde{\theta}_i}^{\tilde{\theta}_f} h(\theta) \cdot \mathbf{S} \, d\theta \right\}$$

U **transforms coherent state** $|\mathbf{\Omega}_0\rangle$ **to** $|\mathbf{\Omega}(\theta)\rangle$

$$\mathbf{\Omega}(\theta) = R(\theta, \theta_0) \, \mathbf{\Omega}_0$$

$$\boxed{U(\theta, \theta_0) |\mathbf{\Omega}_0\rangle = e^{i\psi(\theta, \theta_0, \Omega_0)} |\mathbf{\Omega}(\theta)\rangle}$$

$$\psi = -S \int d\theta \; [\boldsymbol{A}(\mathbf{\Omega}) \cdot d\mathbf{\Omega}/d\theta + \boldsymbol{h} \cdot \mathbf{\Omega}] \qquad \boldsymbol{A} = \hat{z} \times \mathbf{\Omega}/(1 + \hat{z} \cdot \mathbf{\Omega})$$

$$\frac{d\mathbf{\Omega}}{d\theta} = \mathbf{h}(\theta) \times \mathbf{\Omega}$$

Expectation value of spin

$$\langle \mathbf{S}(t) \rangle_{\vec{\Omega}_0(\theta_0)} = \int_{\theta_0}^{\theta_t^+} \mathcal{D}\theta^+ \int_{\theta_0}^{\theta_t^-} \mathcal{D}\theta^- \, \langle e^{i\,(S_P[\theta^+,\,\xi^+] - S_P[\theta^-,\,\xi^-])} \rangle_{\xi^+,\xi^-} \, e^{i(\psi_+ - \psi_-)}$$
$$\langle \mathbf{\Omega}(\theta_t^-) | \mathbf{S} \, | \mathbf{\Omega}(\theta_t^+) \rangle \; \delta_{2\pi}(\theta_t^+ - \theta_t^-)$$

Initial spin direction

Semiclassical limit: $S \to \infty$ $\quad\Rightarrow\quad$ $\langle \mathbf{S}(t) \rangle = S \, \langle \mathbf{\Omega}(\theta_t) \rangle_{\theta,\xi}^{m_+ - m_- = 0}$

with $\quad \mathbf{\Omega}(\theta_t - \theta_0) = \mathbf{N}_0(\mathbf{N}_0\mathbf{\Omega}_0) + \sin(\Phi_t)\, \mathbf{N}_0 \times \mathbf{\Omega}_0$
$\quad + \cos(\Phi_t)\, (\mathbf{\Omega}_0 - \mathbf{N}_0(\mathbf{N}_0\mathbf{\Omega}_0))\,,$

Rotation axis

$\Phi_t \equiv \Gamma_{\theta_0}(\theta_t - \theta_0)/2\pi$

however $\sin(\Phi_t)$ **and** $\cos(\Phi_t)$

decay algebraically to zero , even at $T = 0$!!!

[Imaginary time proof: H. Spohn and W. Zwerger, J. Stat. Phys. 94, 1037 (1999)]

$\Rightarrow$ $\boxed{\langle \mathbf{S}(t) \rangle \longrightarrow S \;\; \mathbf{N}_0(\mathbf{N}_0\mathbf{\Omega}_0)}$

Conclusions

- **Spin relaxation is dominated by Berry phase relaxation as** $\mathrm{B} \to 0$

Relevant for large dots or p-type dots ?

- **Ohmic fluctuations can be important at low fields / temperatures**

- **Hamiltonian of an electron confined to a ring**

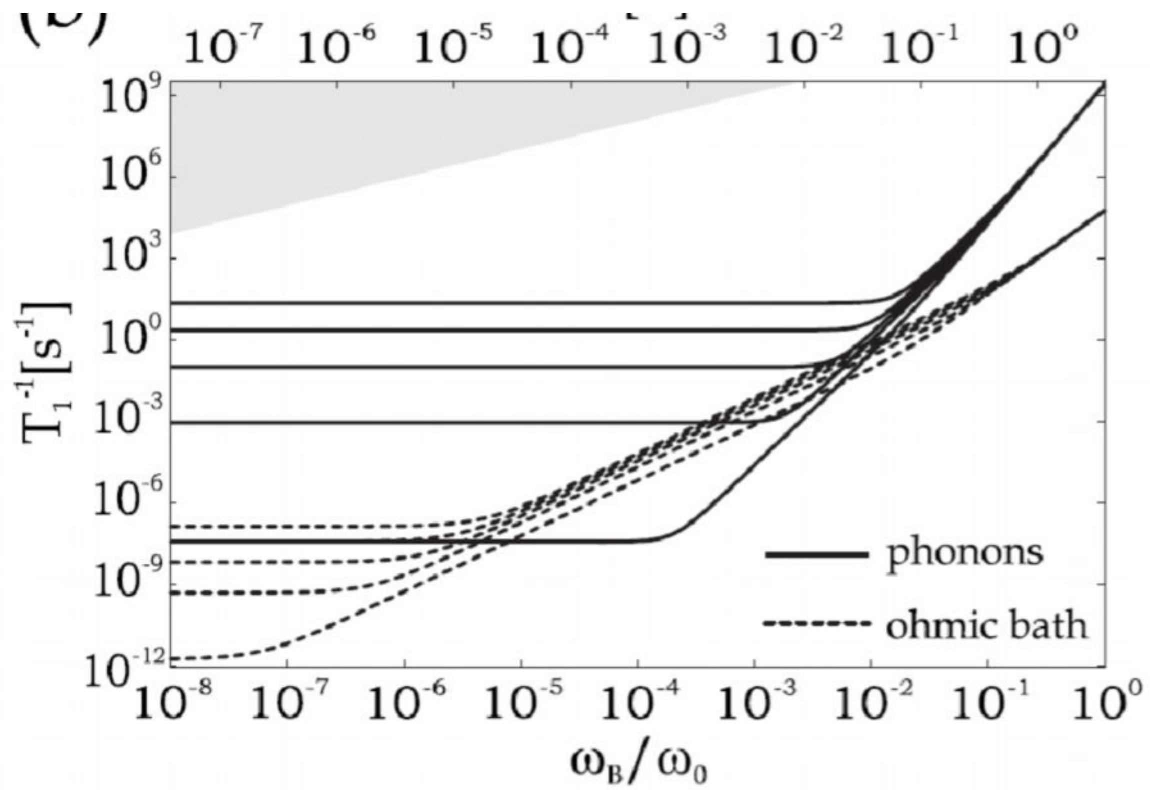
- **Algebraic spin decay on a ring at** $T = 0$ **?**

- **Real time correlations of** $\cos(\hat{\theta}(t))$ **on a ring**

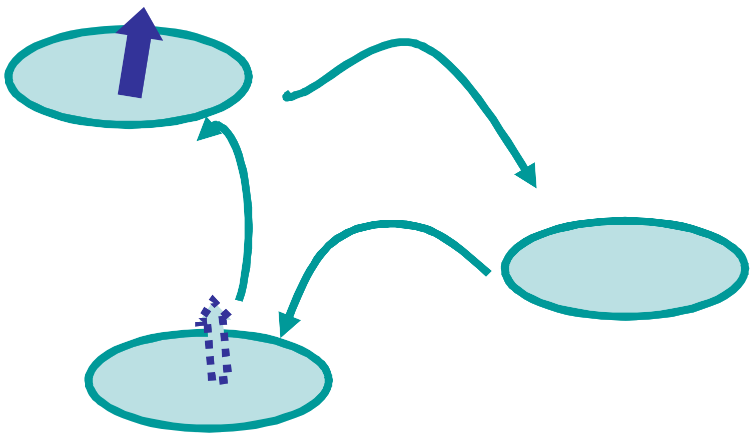
P. San-Jose, G.Z., A. Shnirman, and G. Schon, PRL 97, 076803 (2006)
P. San-Jose, G. Schön, A. Shnirman, and G.Z. PHYSICA E 40, 76 (2007)
P. San-Jose, B. Scharfenberger, G. Schön, A. Shnirman, and G.Z. PRB 77, 045305 (2008)

Results: Keldysh formalism, small dot

$l_{SO} = 3000$ nm $\qquad \lambda_{\Omega} = 10^{-4} \qquad x_0 = 36$ nm $(\omega_0 = 10$ K$)$



Spin manipulation…



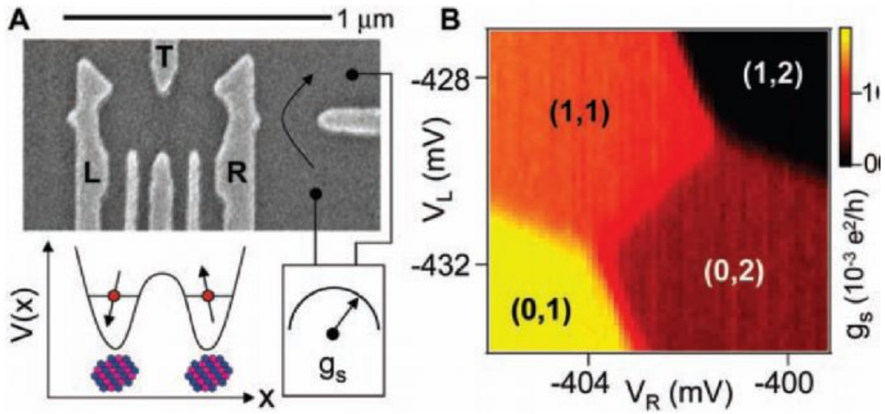
$$d \sim l_{SO}$$

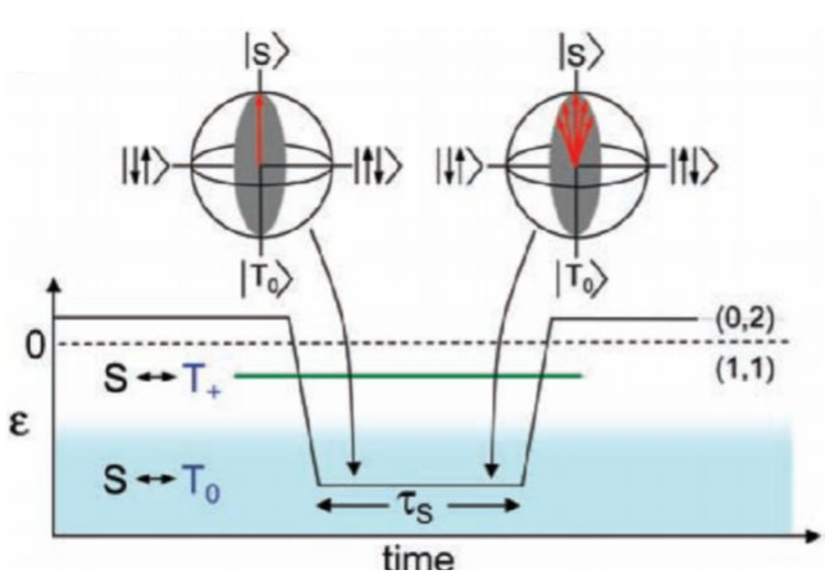
All-electric control of spin can be reached…

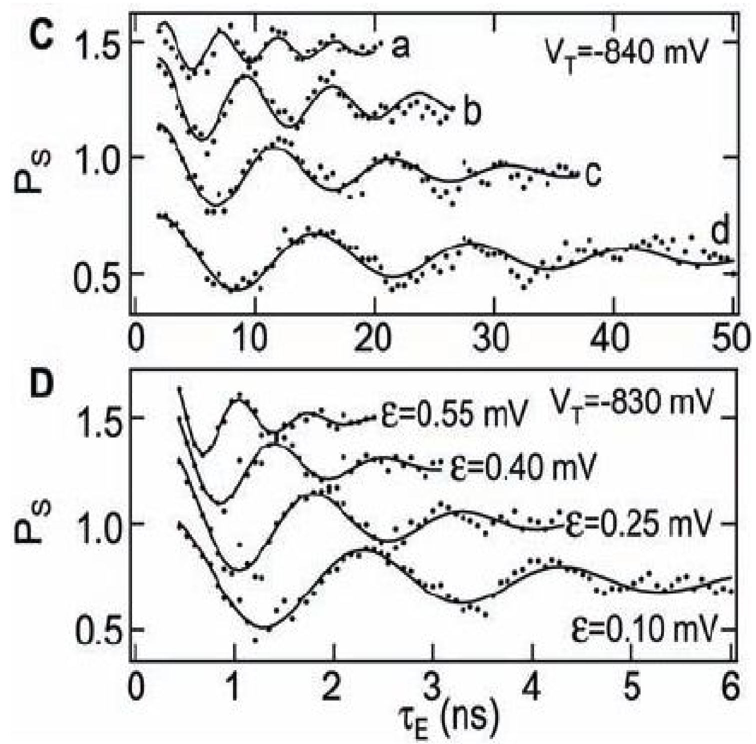
Motivation

Coherent Manipulation of Coupled Electron Spins in Semiconductor Quantum Dots

Petta et al., Science, 2005

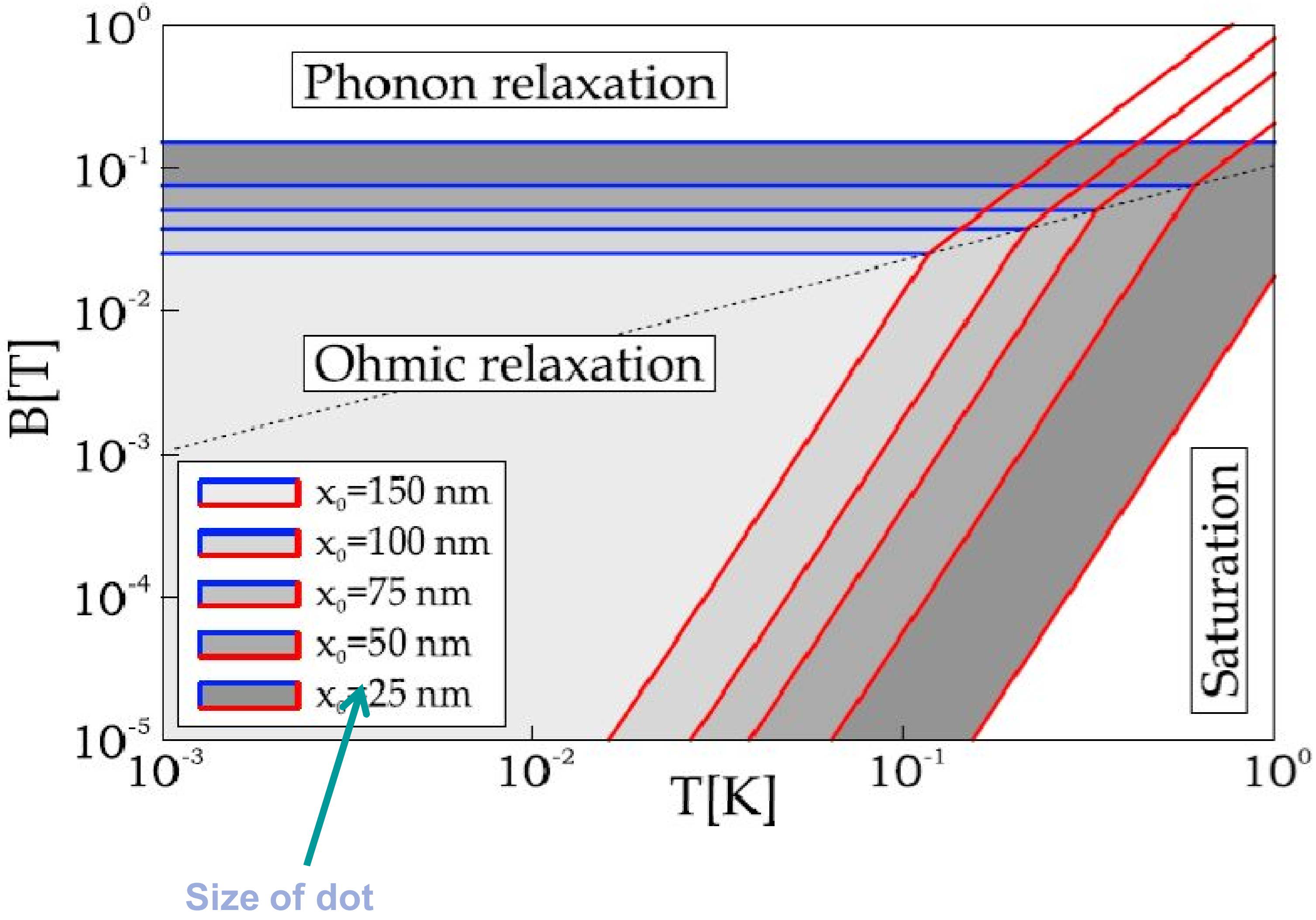






Pulse sequence to measure T_2*

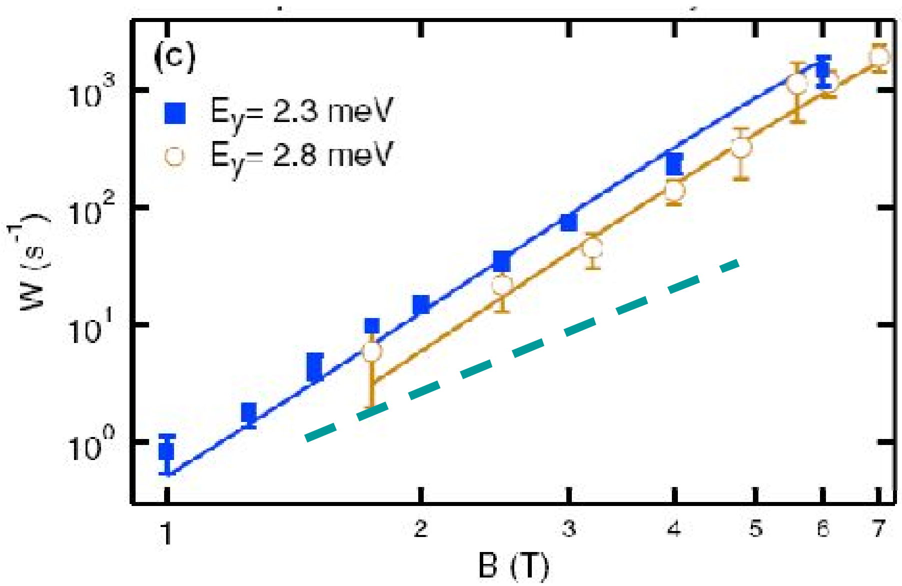
„Phase diagram"



Experiments ?

• Ohmic environment ?

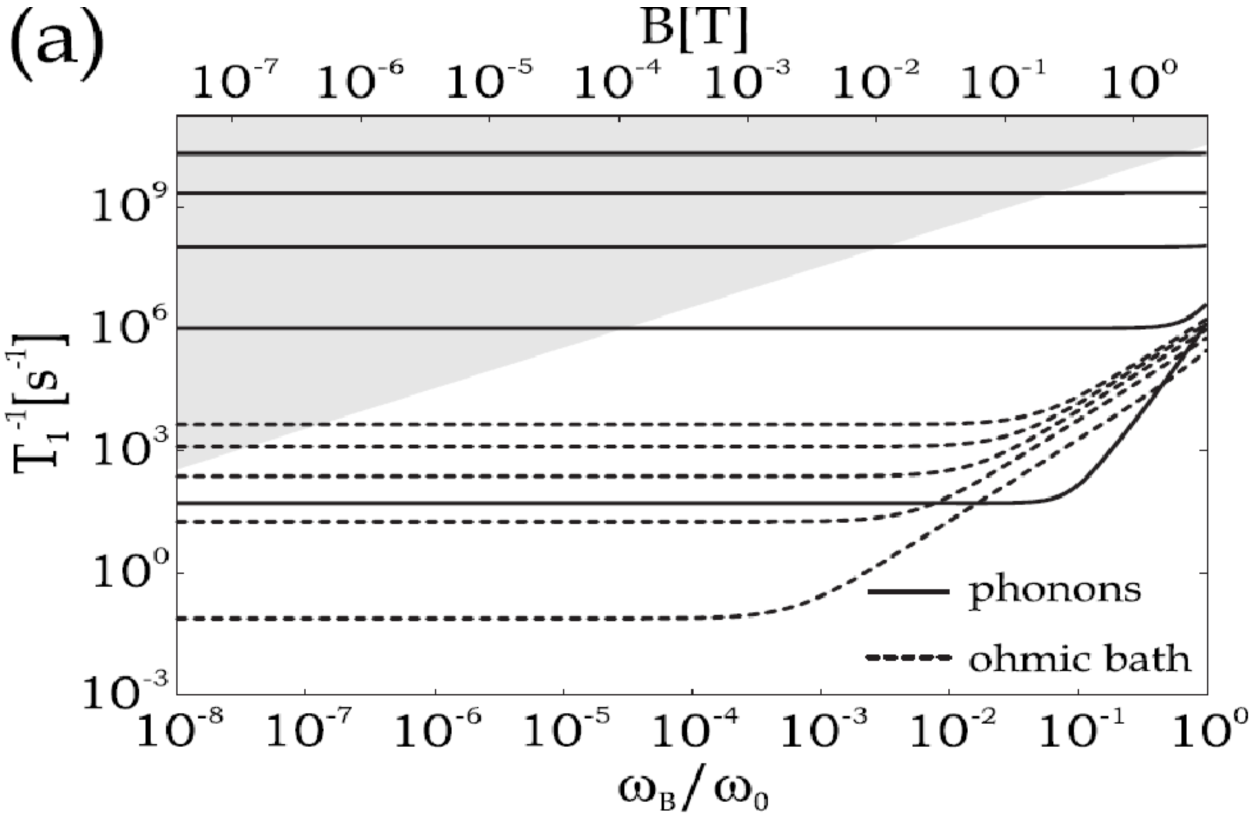
Amasha et al., PRL 2008



• Berry phase can be observed in large dots only…

Results: Keldysh formalism, large dot

$l_{SO} = 1500$ nm. $\lambda_{\Omega} = 5 \times 10^{-3}$ $x_0 = 115$ nm $(\omega_0 \approx 1$ K$)$



$T = 100$ mK, …, 900 mK