# Mass Dependence of Radiative Quark Energy Loss in QCD Matter

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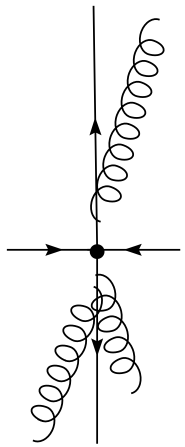
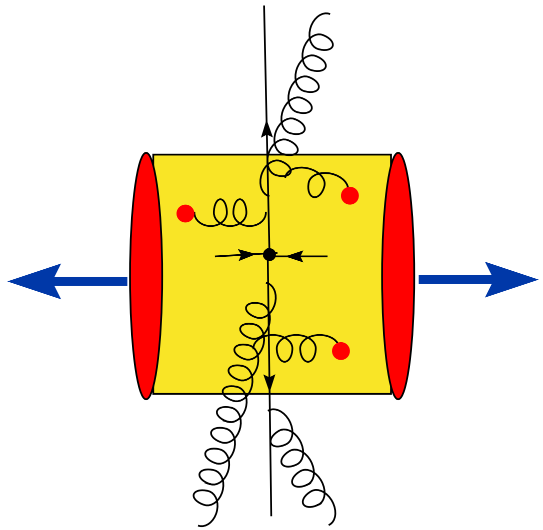
# Motivation

Gluon emission from fast partons produced in $NN$ and $AA$ collisions differs due to the final-state interaction effects in the QGP.



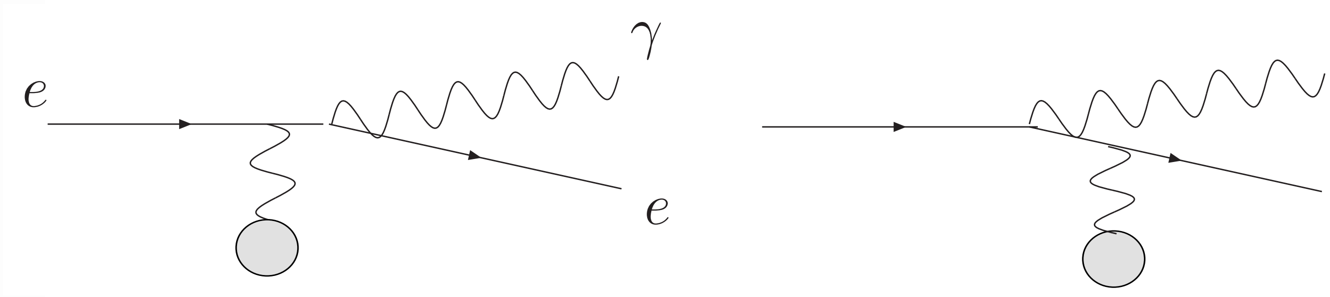
gluon emission
in NN–collision



gluon emission
in AA–collision

In vacuum $q \to qg$ splitting is suppressed at angles $\theta \lesssim M_q/E$. It was suggested [Dokshitzer and Kharzeev (2001)] that this "dead cone" effect should suppress the induced gluon emission for heavy quarks in the QGP in $AA$-collisions as well.
Calculations for expanding finite-size QGP [BGZ (1998,2007)] within the light-cone path integral approach (LCPI) [BGZ (1996,1997,1999)] give $\Delta E_b > \Delta E_c > \Delta E_u$ at sufficiently high energies ($\sim$ 100 GeV). The RHIC data on the electrons from charm and bottom quarks decays say that the energy loss of heavy quark may be similar to that for light quarks. But charm and bottom are not separated yet.

# Bethe-Heitler cross section and LCWF




$$L_f \sim 1/q_z \sim \frac{2E(1-x)}{m_e^2 x}$$

For $a \to bc$ $L_f \sim 2E x_b x_c/\epsilon^2$ with $\epsilon^2 = m_b^2 x_c + m_c^2 x_b - m_a^2 x_b x_c$. At $L_f \gg a$ ($a$ is the screening radius, in QED $a \sim r_B/Z^{1/3}$) the Bethe-Heitler cross section in QED and QCD can be written in terms of LCWF [Nikolaev, Piller, BGZ (1995)]

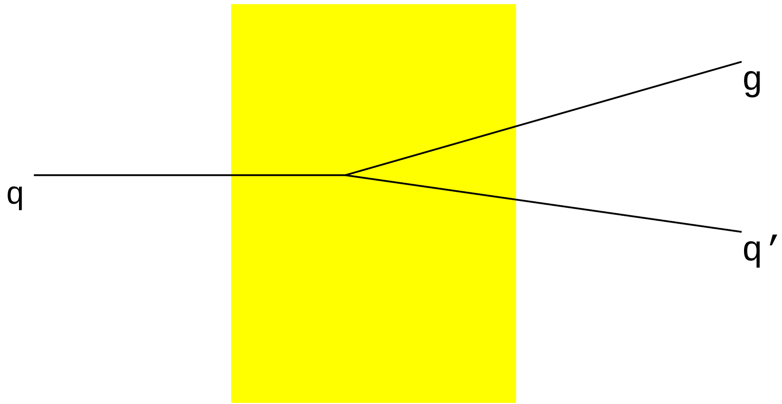
$$|a_{phys}\rangle = |a\rangle + \Psi_a^{bc}(\vec{\rho}, x)|bc\rangle \Rightarrow \hat{S}|a_{phys}\rangle = S_a\{|a_{phys}\rangle + [S_{\bar{a}bc} - 1]\Psi_a^{bc}(\vec{\rho}, x)|bc\rangle\}$$

for $a \to bc$ $$\frac{d\sigma^{BH}(x)}{dx} = \int d\vec{\rho}|\psi^*(\vec{\rho}, x)|^2 \sigma_{\bar{a}bc}(\rho, x)$$

$$\Rightarrow \quad d\sigma/dx \propto P_{ba}(x)/\epsilon^2$$

for non-flip part of $\sigma(e \to \gamma e) \Rightarrow$ $$\left(\frac{d\sigma}{dx}\right)^{BH}_{nf} \approx \frac{4\alpha^3 Z^2[4 - 4x + 2x^2]}{3m_e^2 x} \log(2am^e)$$

# Brief review of the LCPI approach




The $S$-matrix element of the $q \to gq'$ transition is written in standard form

$$\langle gq'|\hat{S}|q\rangle = -ig\int dy\bar{\psi}_{q'}(y)\gamma^{\mu}A^{*}_{\mu}(y)\psi_{q}(y)\,.$$

The quark wave function is written in the form
$\psi_i(y) = \exp[-iE_i(t-z)]\hat{u}_{\lambda}\phi_i(z,\vec{\rho})/\sqrt{2E_i}$, $\lambda$ is quark helicity, $\hat{u}_{\lambda}$ is the Dirac spinor operator. The gluon wave function is written in a similar way. The $z$-dependence of the transverse wave functions $\phi_i$ is governed by the two-dimensional Schrödinger equation

$$i\frac{\partial\phi_i(z,\vec{\rho})}{\partial z} = \left\{\frac{(\vec{p}-g\vec{G})^2+m_q^2}{2\mu_i} + g(G^0-G^3)\right\}\phi_i(z,\vec{\rho})\,,$$

$G$ is the external vector potential (the color indexes are omitted), $\mu_i = E_i$.

The transverse part $\vec{G}$ can be ignored for gauges with potential vanishing at large distances. Thus we have a Schrödinger equation with "time"-dependent potential $U = g(G^0 - G^3)$.
The $z$-evolution of $\phi_i$ can be written in terms of the Green's function as

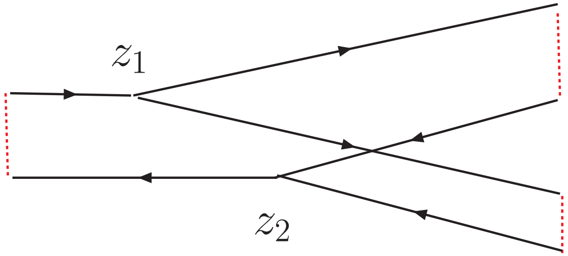
$$\phi_i(z_2, \vec{\rho}_2) = \int d\vec{\rho}_1 K_i(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1)\phi_i(z_1, \vec{\rho}_1)$$

$\Rightarrow$ One can write the cross section in terms of the initial ($z = z_i$) and final ($z = z_f$) transverse density matrices and the Green's functions $K$ and $K^*$.
The Green's functions are written in the path integral form

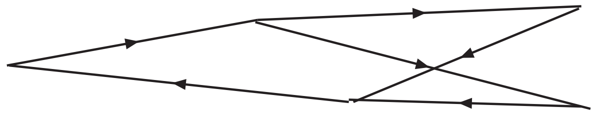
$$K_i(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1) = \int D\vec{\rho} \exp\left\{ i \int dz \left[ \frac{\mu_i (d\vec{\rho}/dz)^2}{2} - U(\vec{\rho}, z) \right] - \frac{i m_i^2 (z_2 - z_1)}{2\mu_i} \right\}$$

In calculation of the gluon spectrum $\propto \langle\langle |\langle q' g|M|q\rangle|^2 \rangle\rangle$ the averaging over the medium states $\langle\langle \ \rangle\rangle$ is performed before the path integration.

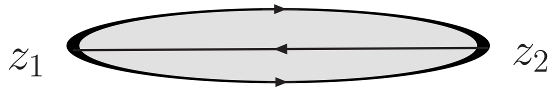
# Diagrammatic representation




$x, p_T - spectrum$



$x - spectrum$

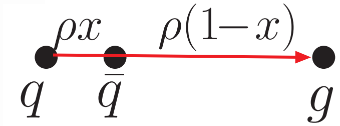



$x - spectrum$

The graph for the $x$-spectrum can be obtained from that for the $(x, p_T)$-spectrum with the help of the relation

$$\int d\vec{\rho}_2 K(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1) K^*(\vec{\rho}_2, z_2|\vec{\rho}_1', z_1) = \delta(\vec{\rho}_1 - \vec{\rho}_1') \,.$$

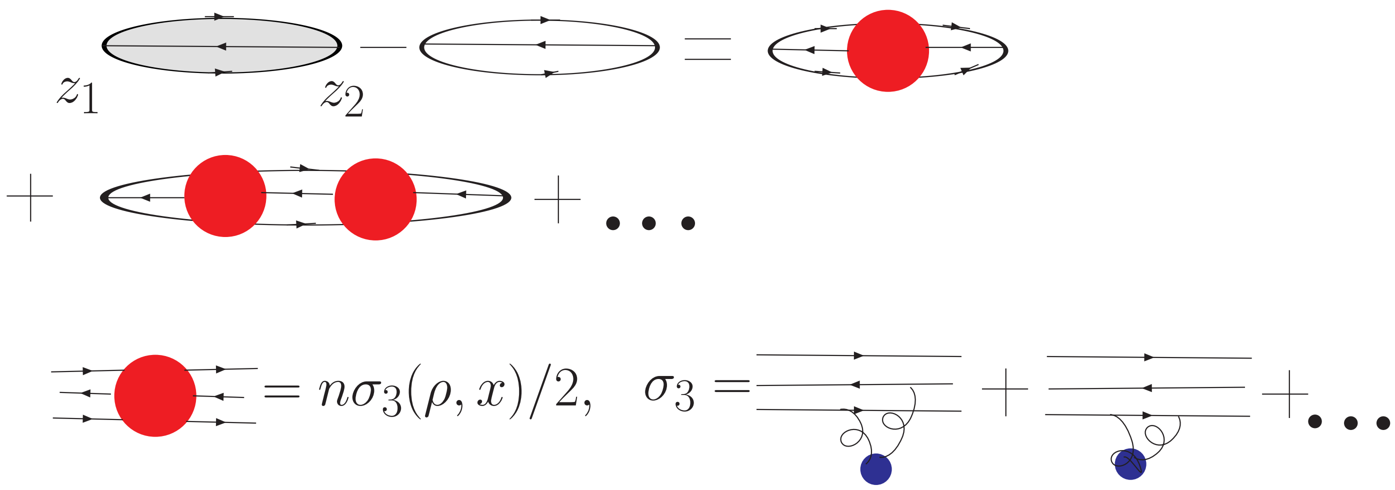
The path integration over the center of mass coordinates can be carried out analytically.
The only dynamical coordinate is $\vec{\rho} = \vec{\rho}_g - \vec{\rho}_q$




$\Rightarrow q\bar{q}g \approx |88\rangle$ at $x \to 0$, $q\bar{q}g \approx |\bar{3}3\rangle$ at $x \to 1$.

The LCPI formalism can be derived from the Feynman diagram technique. [ Aurenche, BGZ, Zaraket (2008)]

# The formulas for a static medium




The Hamiltonian for the dressed Green's function describing the internal dynamics of the $q\bar{q}g$ state reads

$$H = \frac{\vec{q}^{\,2}}{2\mu(x)} - i\frac{n(z)\sigma_3(\rho, x, z)}{2} + \frac{1}{L_f}$$

$\mu(x) = E_q x(1-x)$, $L_f = 2E_q x(1-x)/\epsilon^2$ is the gluon formation length in the Bethe-Heitler regime, $\epsilon^2 = [m_q^2 x^2 + m_g^2(1-x)]$, $\sigma_3 = \sigma_{q\bar{q}g}$. The three-body cross section can be written in terms of the dipole cross section $\sigma_2 = \sigma_{q\bar{q}}$

$$\sigma_3(\rho, x, z) = \frac{9}{8}[\sigma_2(\rho, z) + \sigma_2((1-x)\rho, z)] - \frac{1}{8}\sigma_2(x\rho, z)\,.$$

$$\sigma_2(\rho) = C_T C_F \alpha_s^2 \int d\vec{q} \frac{[1 - \exp(i\vec{q}\,\vec{\rho})]}{(\vec{q}^{\,2} + m_D^2)^2} \ .$$

$\sigma_2(\rho) = C_2(\rho)\rho^2$, $C_2(\rho)$ is smooth for $\rho \ll 1/m_D$

$$C_2(\rho) \approx \frac{C_F C_T \alpha_s \pi^2}{2} \log\left(\frac{1}{\rho m_D}\right) \ .$$

The Hamiltonian takes the oscillator form if one neglects $\rho$-dependence of $C_2(\rho)$ and replaces it by $C_2(\rho_{eff})$. The oscillator frequency reads

$$\Omega(z) = \frac{(1-i)}{\sqrt{2}} \left(\frac{n(z)C_3(x,z)}{E_q x(1-x)}\right)^{1/2} \ ,$$

$C_3(x,z) = \frac{1}{8}\left\{9[1+(1-x)^2] - x^2\right\} C_2$ . The OA is used in BDMPS calculations (for massless partons), $C_2 = \hat{q} C_F / 2nC_A$ [Baier, Dokshitzer, Mueller, Peigne, Schiff (1997)].

The oscillator approximation is clearly not good in the BH regime when $\rho_{eff} \sim 1/\epsilon$. One could expect that the OA should be applicable when $\rho_{eff} \ll 1/\epsilon$. In reality this is not the case when $L \lesssim L_f$.

Analytically the induced gluon spectrum for a quark produced at $z = 0$ reads

$$\frac{dP}{dx} = 2\mathrm{Re}\int_0^\infty dz_1 \int_{z_1}^\infty dz_2\, g(z_1, z_2, x)\left[K(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1) - K_v(\vec{\rho}_2, z_2|\vec{\rho}_1, z_1)\right]\Big|_{\vec{\rho}_1=\vec{\rho}_2=0},$$

$$g(z_1, z_2, x) = \frac{\alpha_s P_{Gq}(x)}{2\mu^2(x)}\frac{\partial}{\partial\vec{\rho}_2}\cdot\frac{\partial}{\partial\vec{\rho}_1}, \qquad P_{Gq}(x) = \frac{C_F[1+(1-x)^2]}{x}.$$

For $L = \infty$ describing interaction with QGP in HTL technique one can obtain from the LCPI formulation the AMY [Arnold, Moore, Yaffe (2002)] form in momentum space [Aurenche, BGZ (2007)].

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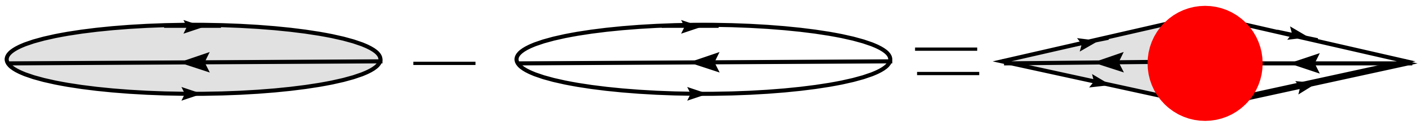
$dP/dx$ can also be written in terms of the $q\bar{q}g$ light-cone wave function



# Effective Bethe-Heitler cross section

$$\frac{dP}{dx} = \int_0^L dz\, n(z) \frac{d\sigma_{eff}^{BH}(x,z)}{dx} \,,$$

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = \mathrm{Re} \int d\vec{\rho}\, \psi^*(\vec{\rho},x)\sigma_3(\rho,x,z)\psi_m(\vec{\rho},x,z) \,,$$

where $\psi(\vec{\rho},x)$ is the LCWF for the $q \to qg$ transition in vacuum, and $\psi_m(\vec{\rho},x,z)$ is the in-medium finite-size modified LCWF for $q \to qg$ transition in medium at the longitudinal coordinate $z$. In the low density limit and $z \to \infty$ $\psi_m(\vec{\rho},x,z) = \psi(\vec{\rho},x)$, and the effective cross section equals the Bethe-Heitler cross section for a quark incident on an isolated color center, $d\sigma^{BH}(x)/dx$. At $z \to 0$

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} \Big/ \frac{d\sigma^{BH}(x)}{dx} \propto z \,.$$

This is a direct consequence of the Schrödinger diffusion relation $\rho^2 \sim z/\mu$ for the transverse size of the $qg$ Fock component of the quark produced at $z = 0$. This effect is responsible for the $L^2$-dependence of $\Delta E_q$ at $E_q \to \infty$. In QED the situations is the same.

# Qualitative pattern of induced gluon emission

The Schrödinger diffusion relation $\rho^2 \sim l/\mu$ relates the typical transverse and longitudinal scales. In an infinite medium

$$\rho_{eff}^{\infty} \sim \min(\rho_{BH}, \rho_{LPM}) \quad l_{eff}^{\infty} \sim (\rho_{eff}^{\infty})^2 \mu$$

$\rho_{BH} = 1/\epsilon$, $l_{BH} = 2Ex(1-x)/\epsilon^2$ (BH regime, weak LPM effect)
$\rho_{LPM} = [Ex(1-x)nC_3]^{-1/4}$, $l_{LPM} = \sqrt{Ex(1-x)/nC_3}$ (strong LPM effect).

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For a finite-size matter two situations are possible.

- Infinite medium regime: $L \gtrsim l_f^{\infty}$, and $\rho_{eff} \sim \rho_{eff}^{\infty}$. The spectrum can roughly be calculated using the effective BH cross section for an infinite medium. There can exist a region with strong LPM effect (number of rescatterings $N \gg 1$) with $dP/dx \propto x^{-3/2}$. But at $x \to 0, 1$ we always have the BH regime ($N = 1$ dominates).
- Diffusion regime: $L \lesssim l_f^{\infty}$ $\rho_{eff} \sim \rho_d(L)$ ($\rho_d(L) = \sqrt{L/2\mu}$ is the diffusion radius). The effective BH cross section is chiefly controlled by the finite-size effects. The $N = 1$ term dominates, the LPM suppression is small.

# Infinite medium regime in the OA

$$\frac{dP}{dx\,dL} = n\frac{d\sigma_{OA}^{BH}}{dx}S(\eta)\,, \qquad \frac{d\sigma_{OA}^{BH}}{dx} = \frac{4\alpha_s C_3(x)(4-4x+2x^2)}{9\pi x[m_q^2 x^2 + m_g^2(1-x)]}\,,$$

$$\eta = L_f|\Omega| = \frac{[4nC_3(x)E_q x(1-x)]^{1/2}}{m_q^2 x^2 + m_g^2(1-x)}\,, \quad \eta \ll 1 \text{ BH regime}, \quad \eta \gg 1 \text{ strong LPM effect.}$$

$$S(\eta) = \frac{3}{\eta\sqrt{2}}\int_0^\infty dy\left(\frac{1}{y^2} - \frac{1}{\mathrm{sh}^2 y}\right)\exp\left(-\frac{y}{\eta\sqrt{2}}\right)\left[\cos\left(\frac{y}{\eta\sqrt{2}}\right) + \sin\left(\frac{y}{\eta\sqrt{2}}\right)\right].$$

At $\eta \gg 1$ $S(\eta) \approx \frac{3}{\eta\sqrt{2}}\left(1 - \frac{\pi}{\eta 2\sqrt{2}}\right)$. The leading term gives mass independent spectrum

$$\frac{dP}{dx\,dL} \approx \frac{\alpha_S(4-4x+2x^2)}{3\pi}\sqrt{\frac{2nC_3(x)}{E_q x^3(1-x)}}\,, \quad \text{[BGZ (1996)].}$$

The factor $\left(1 - \frac{\pi}{\eta 2\sqrt{2}}\right)$ gives the heavy-to-light $K$-factor (strong LPM regime)

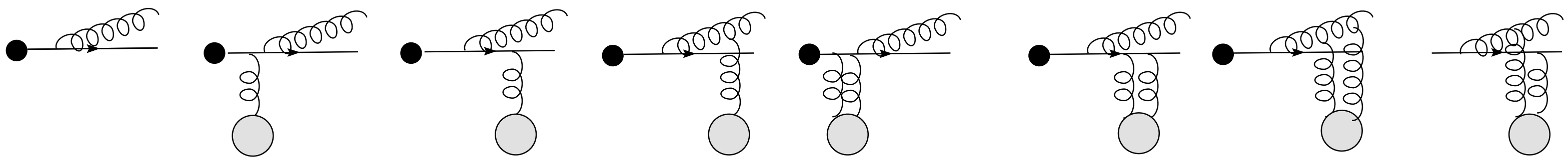
$$K \approx 1 - \frac{\pi}{2\sqrt{2}}\frac{(M_Q^2 - m_q^2)x^{3/2}}{\sqrt{2E(1-x)nC_3(x)}} \qquad \left(K_{DK} = \left[1 + \frac{M_Q^2 x^{3/2}}{\sqrt{18EnC_2/4}}\right]^{-2} \text{ Dokshitzer, Kharzeev (2001)}\right.$$

For massless partons the OA for finite medium gives [BDMPS (1997)]

$$\frac{dP_{BDMPS}}{dx} = \frac{\alpha_s P_{Gq}(x)}{\pi} \log|\cos \Omega L|$$

$$\Rightarrow \quad \frac{dP_{BDMPS}}{dx} \approx \frac{\alpha_s P_{Gq}(x)}{16\pi} \frac{L^4 C_3^2 n^2}{[x(1-x)E]^2}$$ $\Rightarrow$ It is $N=2$ rescattering! [BGZ, (2001)]

The mass effects gives nonzero $N=1$ term, but for massless partons it vanishes.
The OA is equivalent to the collinear expansion in the higher twist method [Wang, Guo (2001); Zhang, Wang (2003)]. The nonzero $N=1$ for massless partons obtained in papers by Wang, Guo and Zhang is a consequence of trivial mistakes [Aurenche, BGZ, Zaraket (2008)].
The $N=1$ can also be calculated using the momentum representation.



Each diagram can be calculated as $\langle bc|M|a\rangle \propto \int dz d\vec{\rho} \phi_b^*(z,\vec{\rho})\phi_c^*(z,\vec{\rho})\phi_a(z,\vec{\rho})$ with the

plane wave functions with sharp $\vec{p}_T$ change at the rescattering point

# $N = 1$ term in momentum representation

$$\frac{d\sigma_{eff}^{BH}(x,z)}{dx} = \frac{\alpha_s^3 P_{Gq}(x)}{\pi^2 C_F}\left[F(1,z) + F(1-x,z) - F(x,z)/9\right],$$

$$F(y,z) = \int \frac{d\vec{k}}{(\vec{k}^{\,2}+m_D^2)^2} H(y\vec{k},\vec{p})\cdot\left[1-\cos\left((\vec{p}^{\,2}+\epsilon^2)\rho_d^2(z)\right)\right], \quad \rho_d^2(z) = \frac{z}{2Ex(1-x)},$$

$$H(\vec{k},\vec{p}) = \frac{\vec{p}^{\,2}}{(\vec{p}^{\,2}+\epsilon^2)^2} - \frac{(\vec{p}-\vec{k})\vec{p}}{(\vec{p}^{\,2}+\epsilon^2)((\vec{p}-\vec{k})^2+\epsilon^2)}, \quad \langle H(\vec{k},\vec{p})\rangle_{\epsilon=0} = \frac{\theta(k-p)}{p^2}$$

$F = F_0 + \delta F$, $F_0 = F(\epsilon = 0)$, $\delta F$ is mass correction.
The momentum integration gives for $\epsilon = 0$ $F_0(y,z) = \pi^3 y^2 \rho_d^2(z)/2$. There is no any $\log$ terms! LLA fails since for $\epsilon = 0$ $\nabla_k^2 H(\vec{k},\vec{p}) = 0$. Integration over the position of the rescattering gives $dP/dx \propto L^2$

$$\left.\frac{dP_{N=1}}{dx}\right|_{\epsilon=0} = \frac{\pi n L^2 \alpha_s^3 P_{Gq}(x)[1+(1-x)^2 - x^2/9]}{8C_F E x(1-x)}$$

The Debye mass does not appear in the spectrum! In massless limit the mass scale is given by $1/\rho_d(z)$. The photon spectrum has the same $L^2$-dpendence! [BGZ (2001,2004)].

# Mass correction

$$\delta F \approx \frac{\pi^2\epsilon^2\rho_d^4 y^2}{2}\left\{2\log^2\left(\frac{1}{\epsilon^2\rho_d^2}\right) + \log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\log\left(\frac{\epsilon^2}{y^4 m_D^4\rho_d^2}\right) - 3\log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\right.$$

$$\left. - \frac{y^2 m_D^2}{\epsilon^2}\log\left(\frac{1}{\epsilon^2\rho_d^2}\right)\right\} \approx \frac{3\pi^2\epsilon^2\rho_d^4 y^2}{2}\log^2\left(\frac{1}{\epsilon^2\rho_d^2}\right) \quad \text{for} \quad \log\left(\frac{1}{\epsilon^2\rho_d^2}\right) \gg 1\,.$$

$\Rightarrow$ The mass correction to $\delta(dP/dx)$ is $\propto L^3$ and positive

$$\delta\frac{dP_{N=1}}{dx} = \frac{\alpha_s^3 P_{Gq}(x)[1+(1-x)^2 - x^2/9]Ln\epsilon^2\rho_d^4(L)}{2C_F}\log^2\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right)\,.$$

In the OA mass correction also has an anomalous mass dependence. To obtain $N=1$ term in the OA one should make replacement

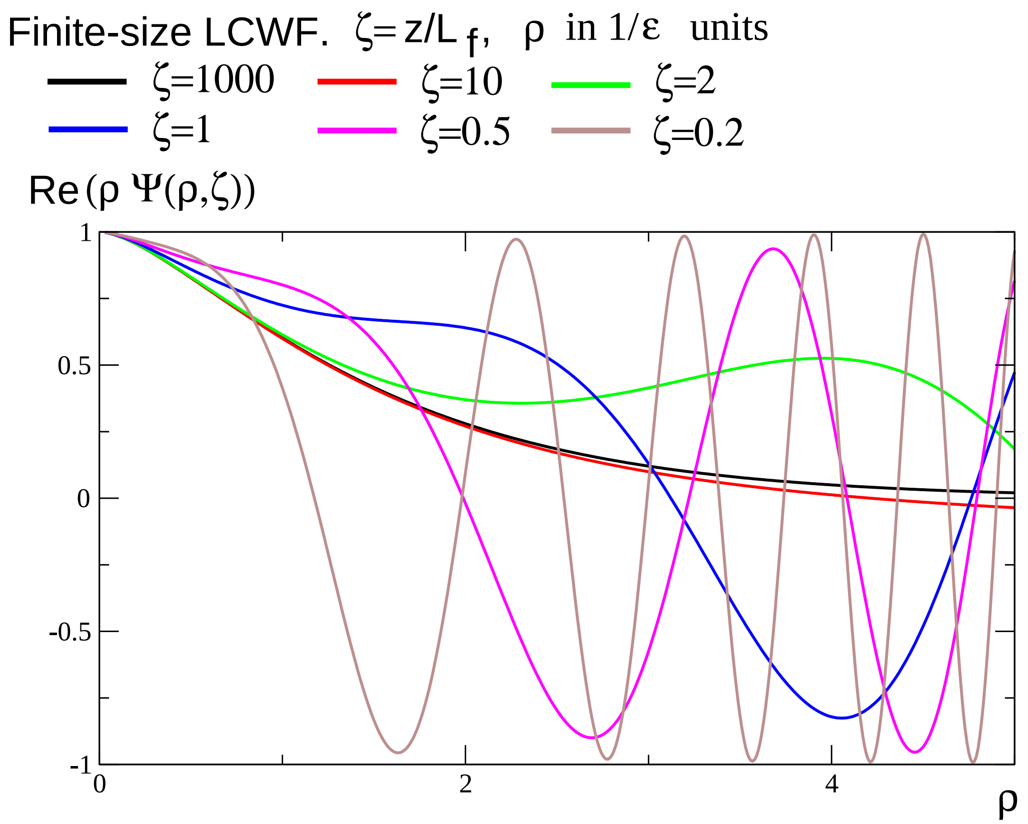
$$\frac{1}{(\vec{k}^2+m_D^2)^2} \Rightarrow \frac{2\hat{q}\delta(\vec{k})}{n\alpha_s^2 C_A C_T}$$

$$\frac{dP_{N=1}^{OA}}{dx} = \frac{4\hat{q}L\alpha_s P_{Gq}(x)[1+(1-x)^2 - x^2/9]\epsilon^2\rho_d^4(L)}{6\pi C_A C_F}\log\left(\frac{1}{\epsilon^2\rho_d^2(L)}\right)$$

# Mass dependence and LCWF in $\rho$ space

$$\frac{d\sigma_{eff}(x,z)}{dx} = \text{Re}\int d\vec{\rho}\,\rho^2\psi^*(\vec{\rho},x)\left(\frac{\sigma_3(\rho,x)}{\rho^2}\right)\psi_m(\vec{\rho},x,z)\,,$$

$\rho|\psi(\vec{\rho},x)| \propto \exp(-\epsilon\rho)$, $\rho|\psi_m(\vec{\rho},x,z=\infty)| \propto \exp(-\epsilon\sqrt{\eta}\rho)$ and the integrand is smooth like that for the BH cross section. At $z \lesssim L_f$ $\psi_m(\rho,x,z)$ oscilates. This can give anomalous mass dependence of the spectrum.



# Numerical calculations

$$\frac{d\sigma^{BH}_{eff}(x,z)}{dx} = \mathrm{Re}\int\limits_0^z dz_1 \int\limits_z^\infty dz_2 \int d\vec{\rho}\, g(x) K_v(z_2,\vec{\rho}_2|z,\vec{\rho})\sigma_3(\rho)K(z,\vec{\rho}|z_1,\vec{\rho}_1)\Big|_{\vec{\rho}_1=\vec{\rho}_2=0}.$$

For the vacuum Green's function $z_2$-integration comes up to infinity. The integral equals the LCWF with the azimuthal quantum number $m = \pm 1$ $\psi(\vec{\rho}, x) \propto K_1(\epsilon\rho)\exp(im\phi)$. It allows one to represent the effective Bethe-Heitler cross section in the form [BGZ (2004)]

$$\frac{d\sigma^{BH}_{eff}(x,z)}{dx} = -\frac{\alpha_s P_{Gq}(x)}{\pi\mu(x)} \mathrm{Im}\int\limits_0^z d\xi\, \frac{\partial}{\partial\rho}\left(\frac{F(\xi,\rho)}{\sqrt{\rho}}\right)\Big|_{\rho=0},$$

$F$ is the solution to the radial Schrödinger equation for $m = 1$

$$i\frac{\partial F(\xi,\rho)}{\partial\xi} = \left[-\frac{1}{2\mu(x)}\left(\frac{\partial}{\partial\rho}\right)^2 - i\frac{n(z-\xi)\sigma_3(\rho)}{2} + \frac{4m^2-1}{8\mu(x)\rho^2} + \frac{1}{L_f}\right] F(\xi,\rho).$$

with the boundary condition $F(\xi=0,\rho) = \sqrt{\rho}\sigma_3(\rho)\epsilon K_1(\epsilon\rho)$. We solve the Schrödinger equation back in time. It allows one to have a smooth boundary condition.

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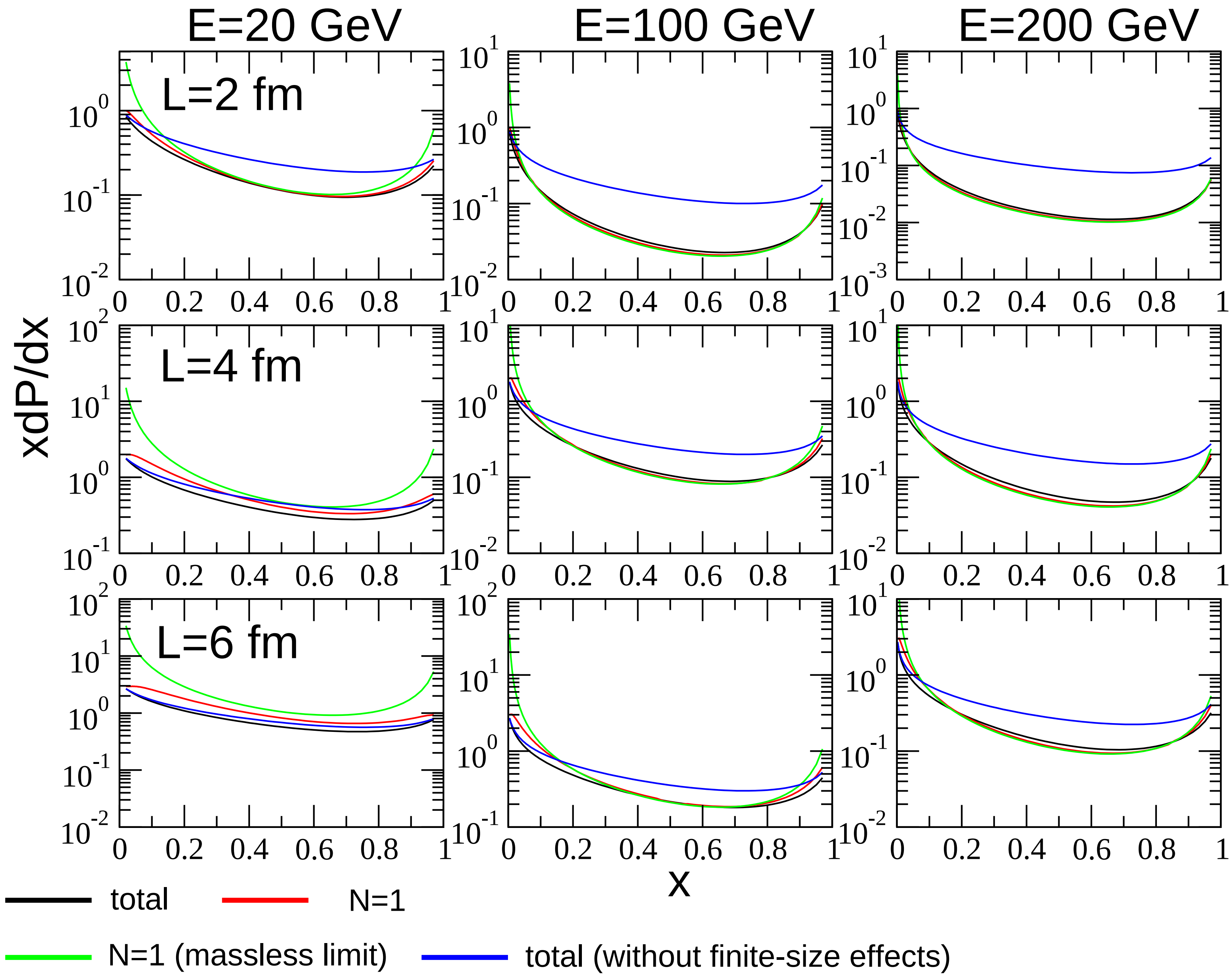
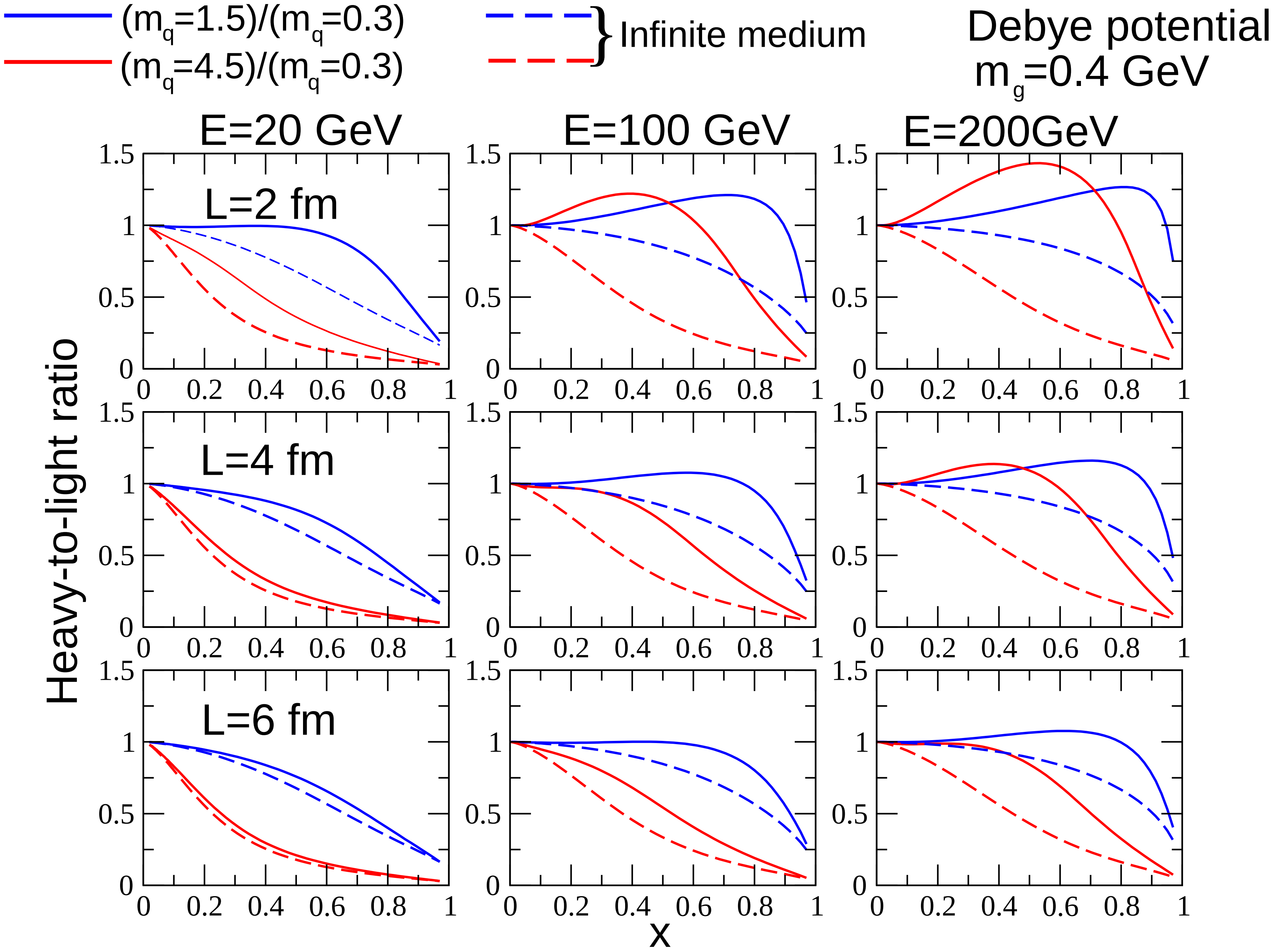
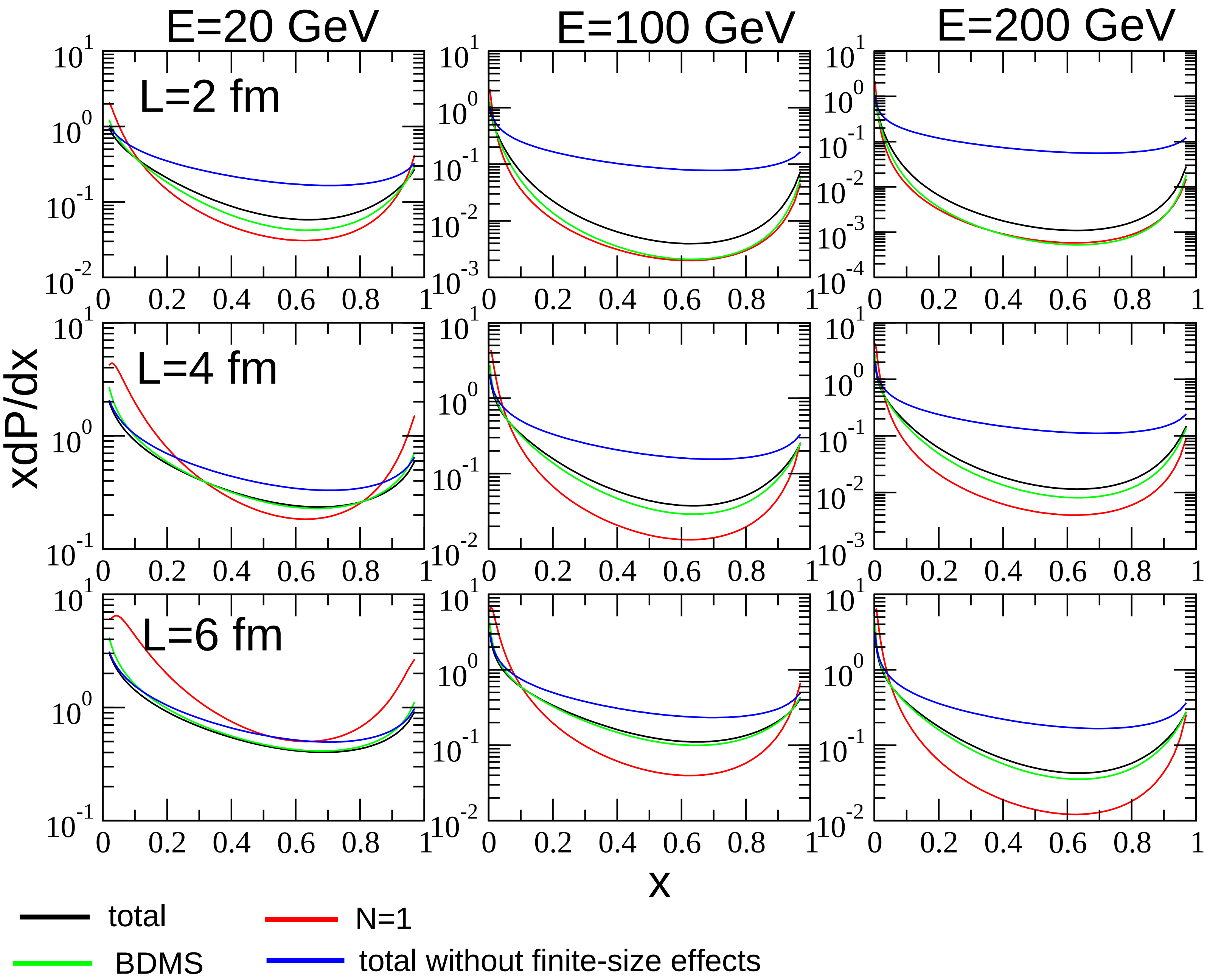
We take $\alpha_s = 0.4$, QGP temperature $T = 250$ MeV, $m_u = 0.3$, $m_c = 1.5$, $m_b = 4.5$, $m_g = 0.4$ GeV, $m_D = \sqrt{2}m_g$. In the OA we use $\hat{q} = 0.3$ GeV$^3$

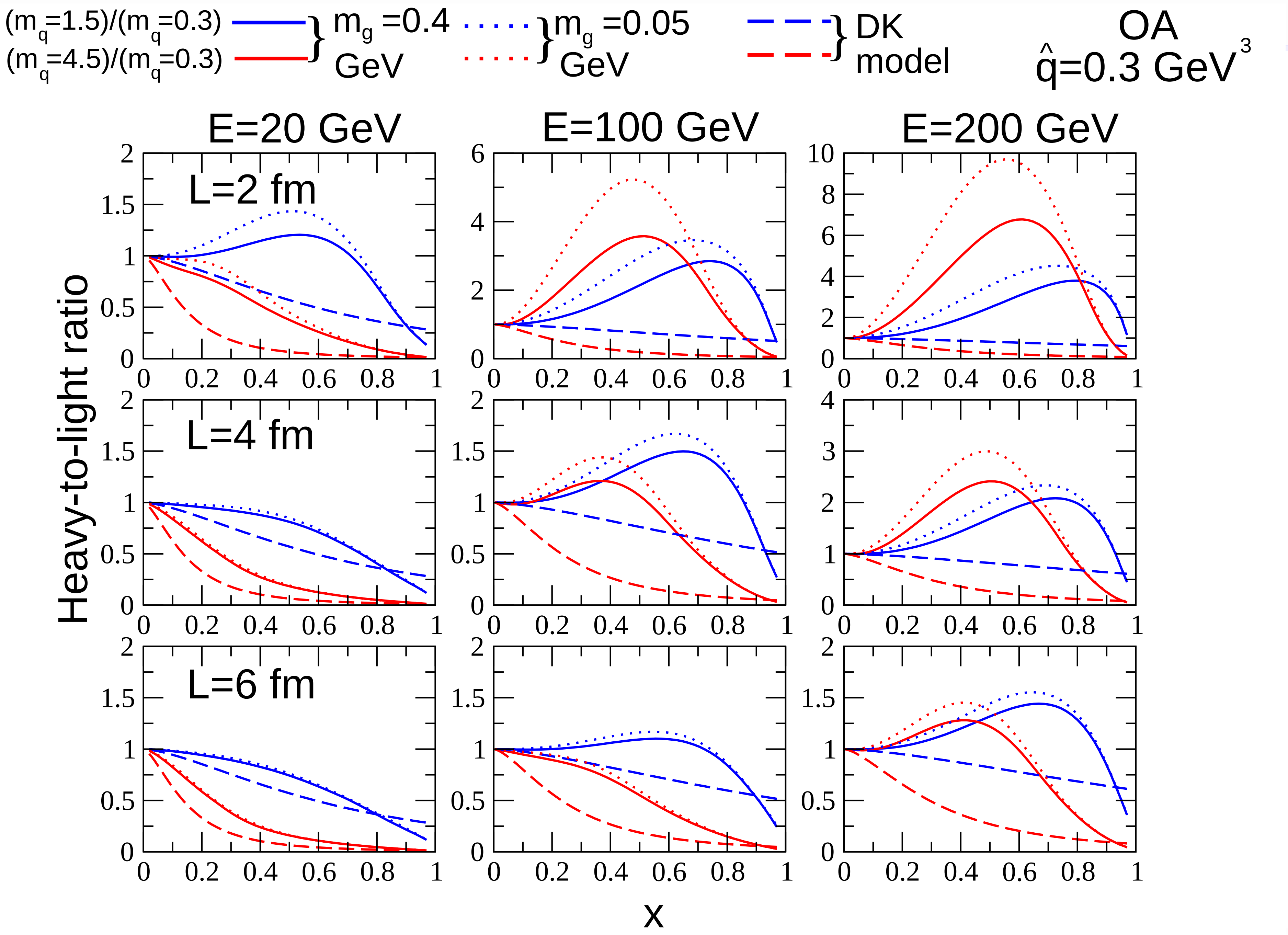
# Gluon spectrum for $m_q$=0.3, $m_g$=0.4 GeV (Debye potential)




(m_q=1.5)/(m_q=0.3)
(m_q=4.5)/(m_q=0.3)
Infinite medium
Debye potential
m_g=0.4 GeV
E=20 GeV
E=100 GeV
E=200GeV
L=2 fm
L=4 fm
L=6 fm
Heavy-to-light ratio
x

# Gluon spectrum for $m_q$=0.3, $m_g$=0.4 GeV (OA $\hat{q}$=0.3 $GeV^3$)




(m_q=1.5)/(m_q=0.3)
(m_q=4.5)/(m_q=0.3)
m_g =0.4 GeV
m_g =0.05 GeV
DK model
OA
q̂=0.3 GeV^3
E=20 GeV
E=100 GeV
E=200 GeV
L=2 fm
L=4 fm
L=6 fm
Heavy-to-light ratio
x

# Conclusions:

- The finite-size effects can lead to an enhancement of the gluon emission from heavy quarks as compared to that from the light quarks.
- The effect is connected with oscillations of the in-medium finite-size modified LCWF. The finite-size effect becomes small only at $L \gtrsim (5-10)L_f$. The anomalous mass dependence is demonstrated explicitely by calculations of the $N=1$ rescattering term in momentum representation.
- For RHIC conditions the gluon emission from the $c$-quark is very similar to that from the light quarks. At the $E \gtrsim 100$ GeV $b$-quark radiates stronger than $c$ and light quarks at $x \lesssim 0.5$.
- The results of our calculations in the OA disagree strongly with Dokshitzer-Kharzeev estimates obtained neglecting the finite-size effects.