Interplay between Crossed Andreev reflection and disorder: Zero-bias anomaly and charge imbalance peak

Andrei D. Zaikin

Institute for Nanotechnology
Forschungszentrum Karlsruhe
Karlsruhe Institute for Technology

Tamm Dept. of Theoretical Physics
Lebedev Physics Institute, Moscow

Collaboration: Misha Kalenkov, Dima Golubev

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JETP Lett. 87, 166 (2008)
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arXiv:0409.3455

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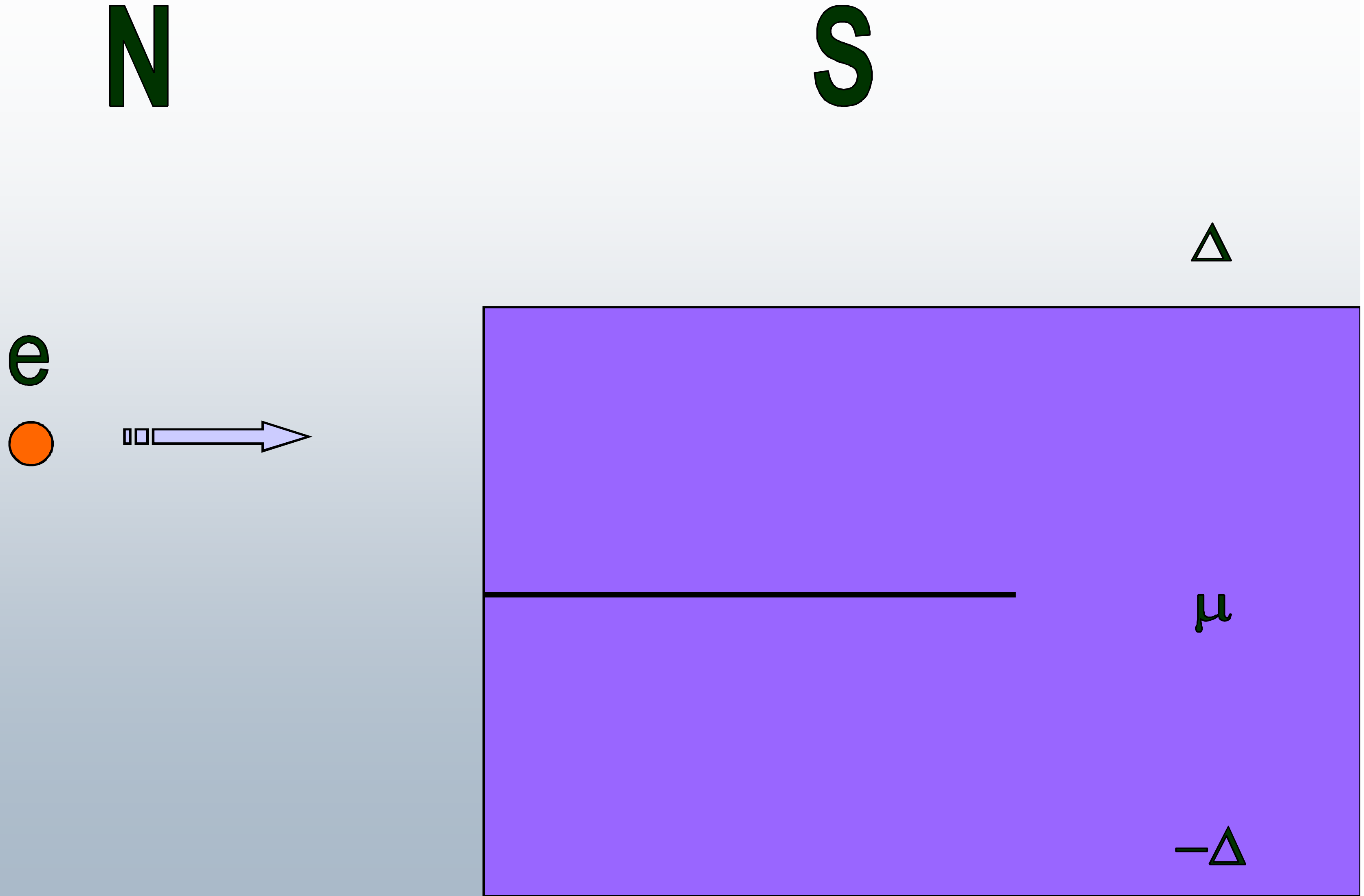
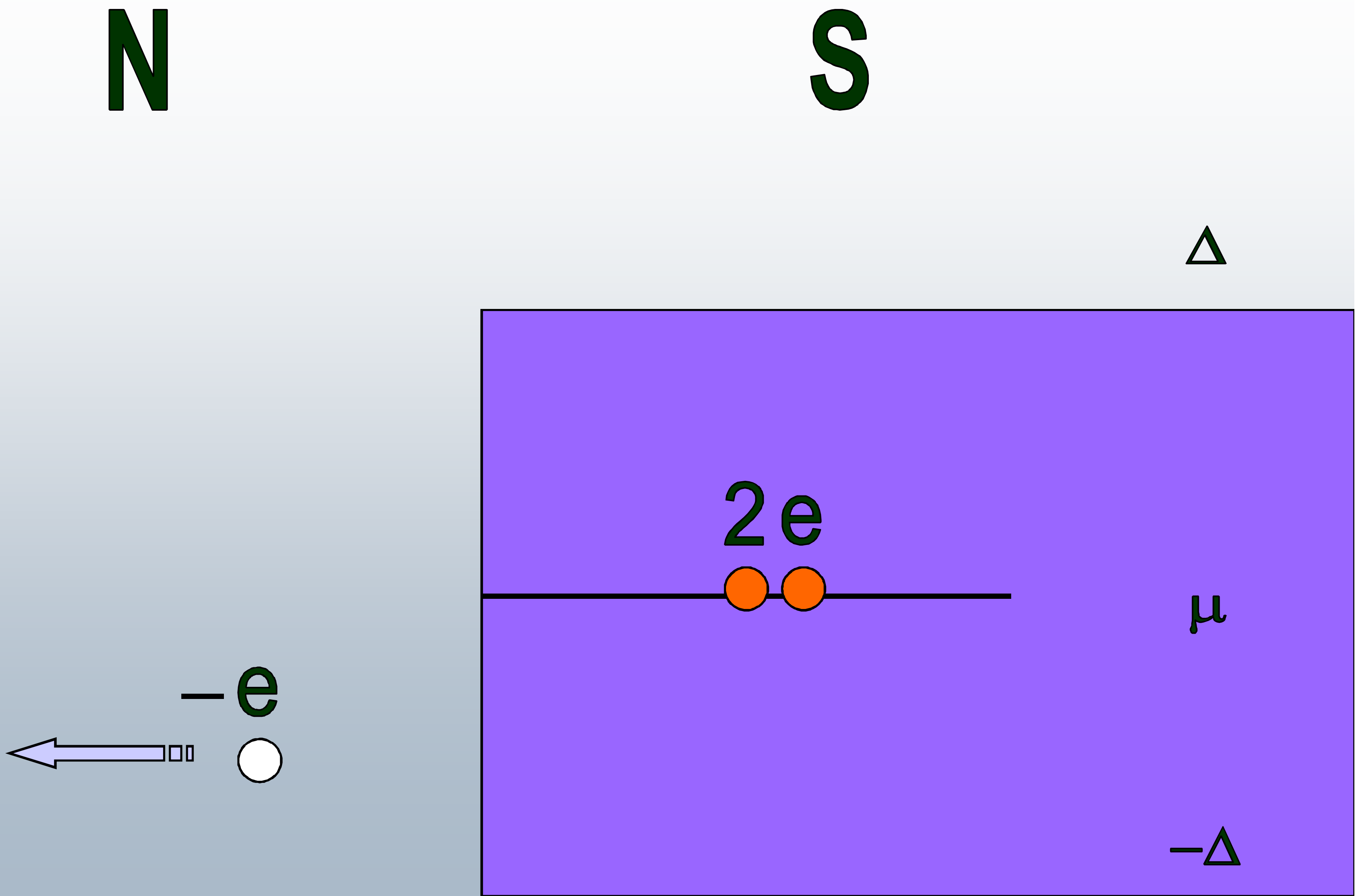
Andreev reflection



Andreev reflection



Blonder, Tinkham, Klapwijk'82:

Non-zero subgap conductance at T=0

$$G_A = dI / dV$$

Blonder, Tinkham, Klapwijk'82:

Non-zero subgap conductance at T=0

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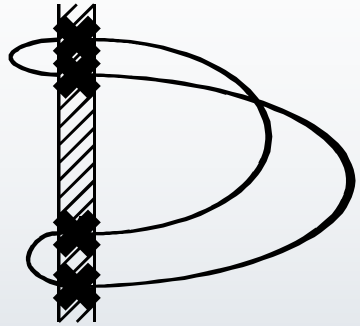
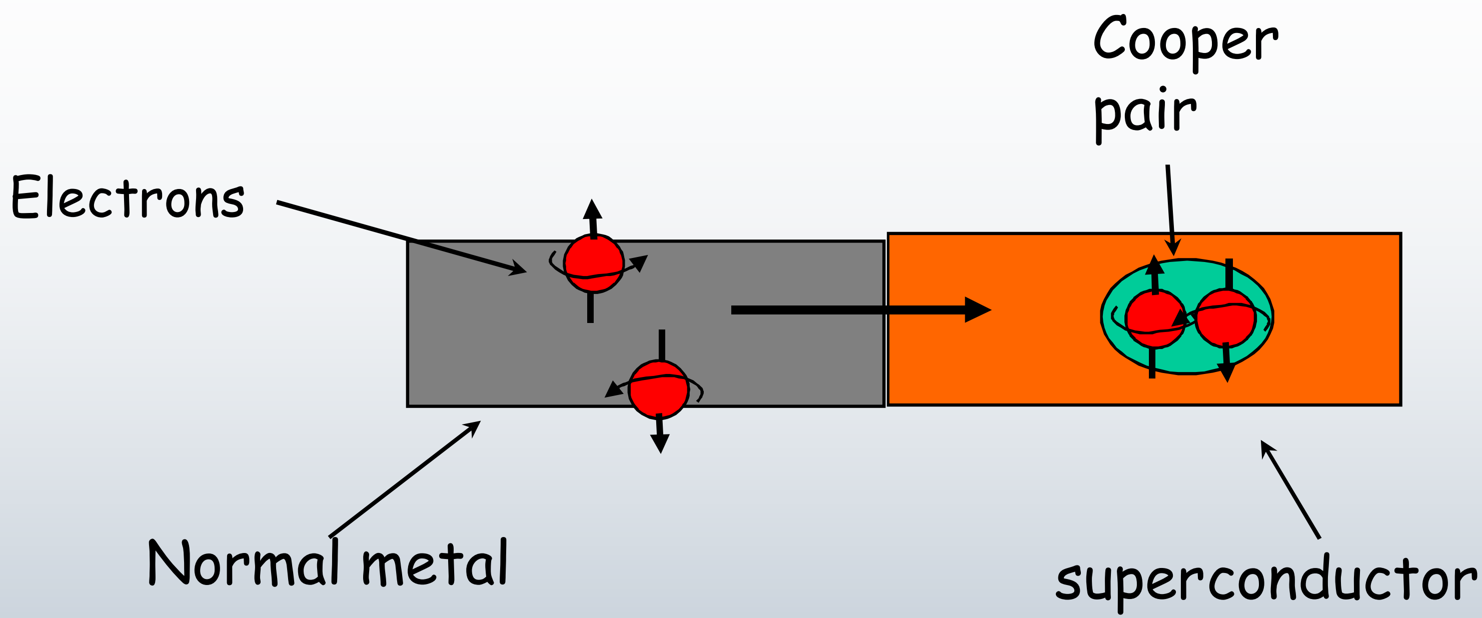
2nd order process:



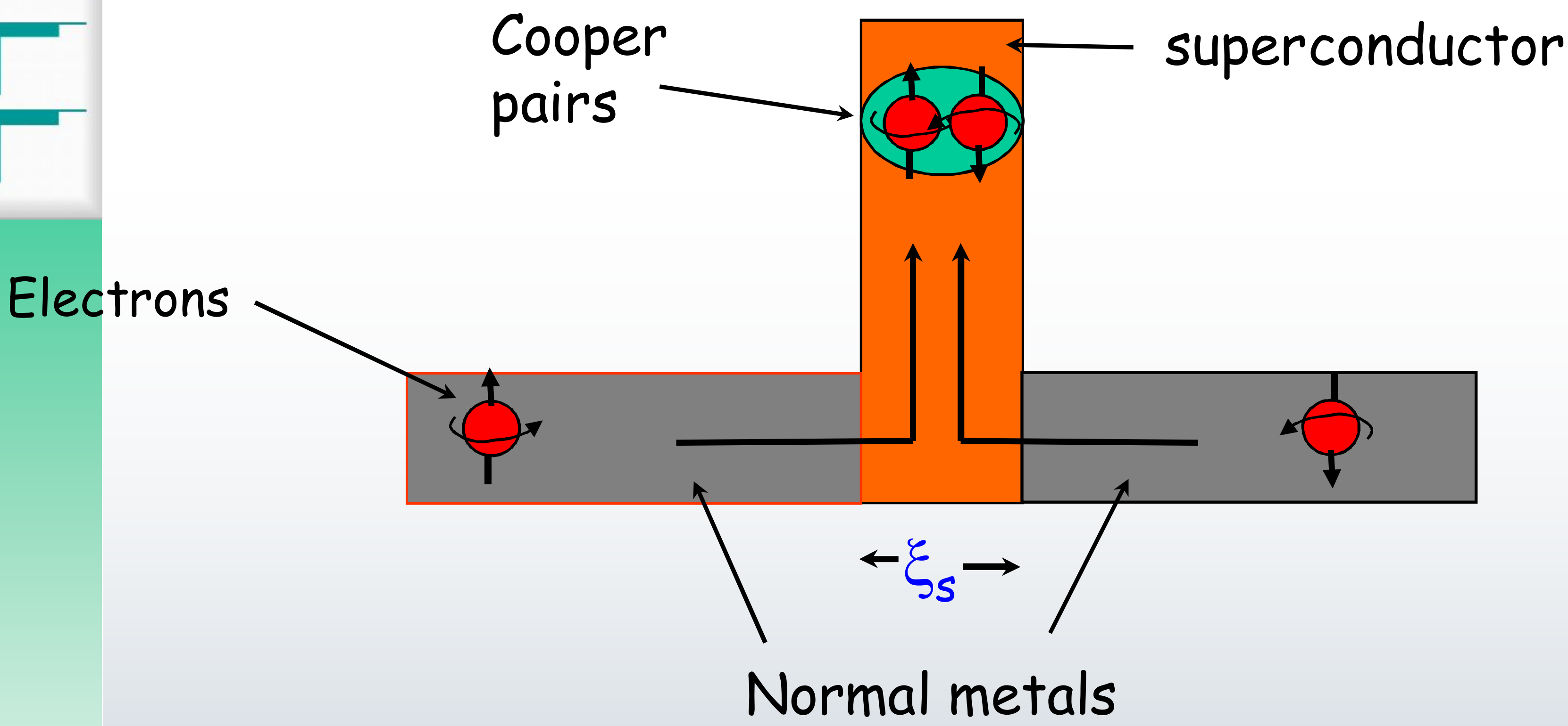
$$G_A \sim R_q / R_t^2 N_{Ch}$$

V- and T-independent!

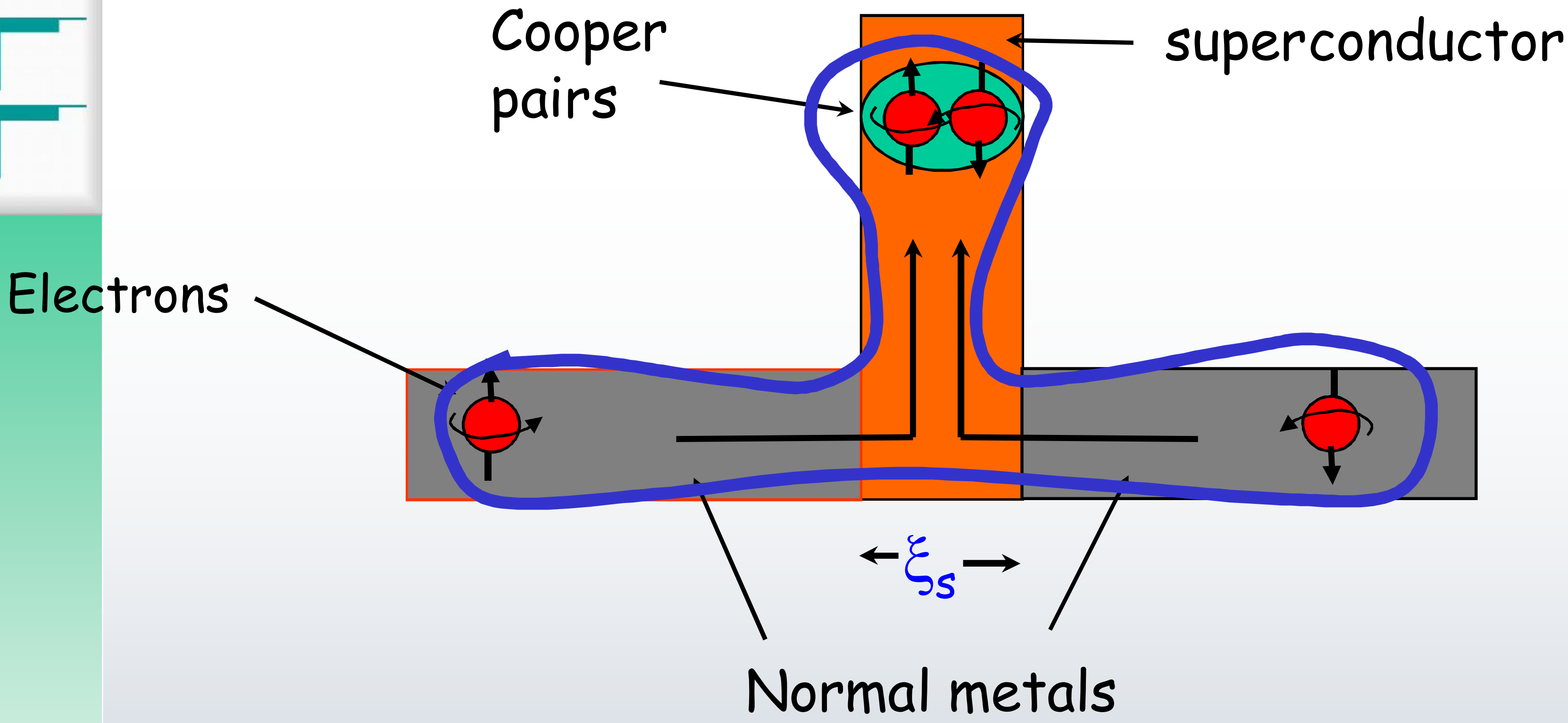
Andreev reflection



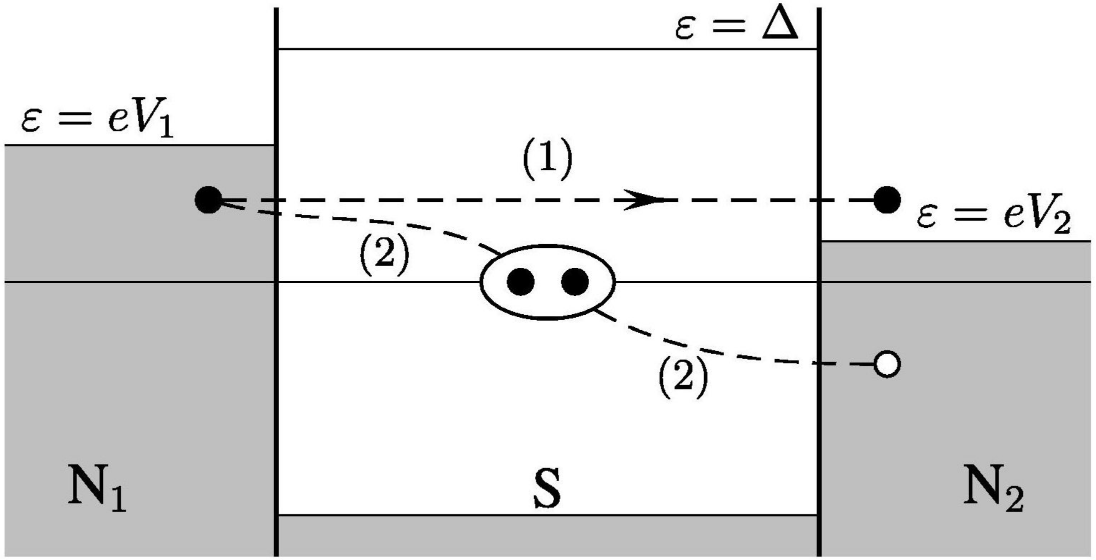
Crossed Andreev Reflection (CAR)



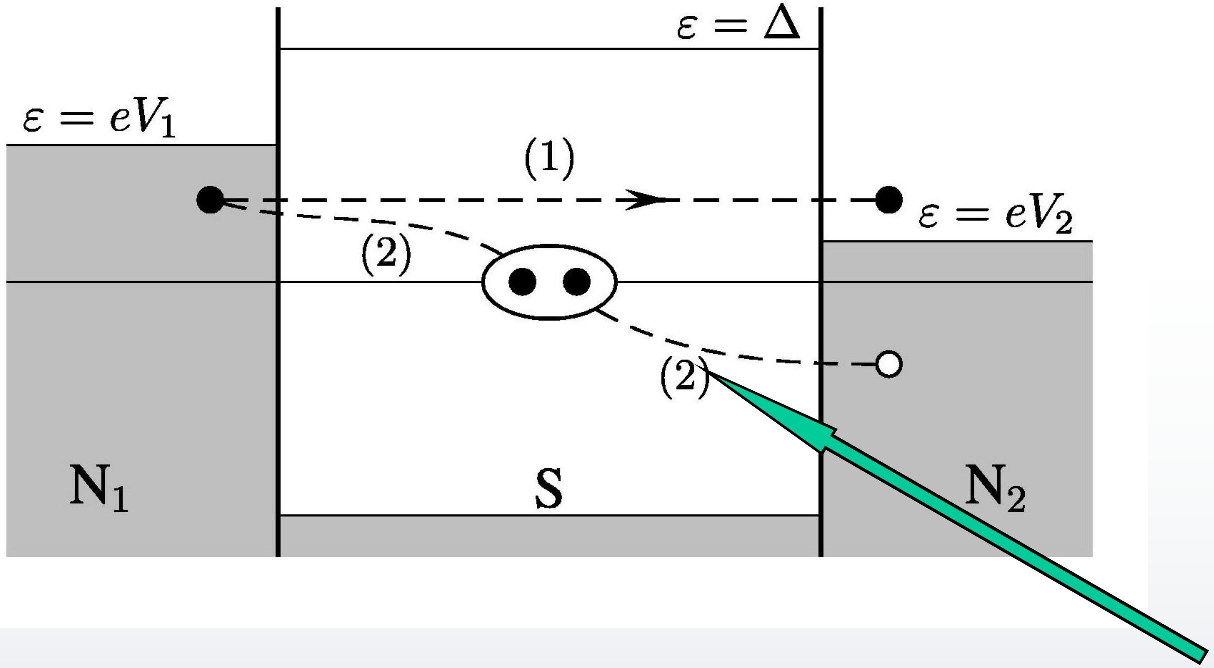
Crossed Andreev Reflection (CAR)



Electrons in two N-metals are "entangled"



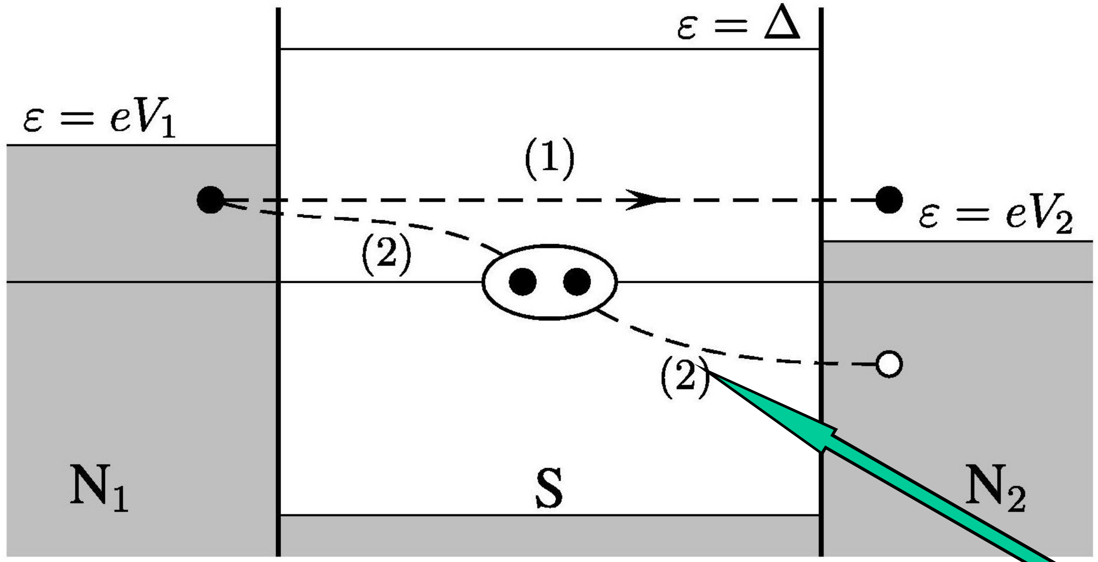
$\varepsilon = \Delta$
$\varepsilon = eV_1$
(1)
$\varepsilon = eV_2$
(2)
(2)
N_1
S
N_2



CAR

Byers, Flatte'95

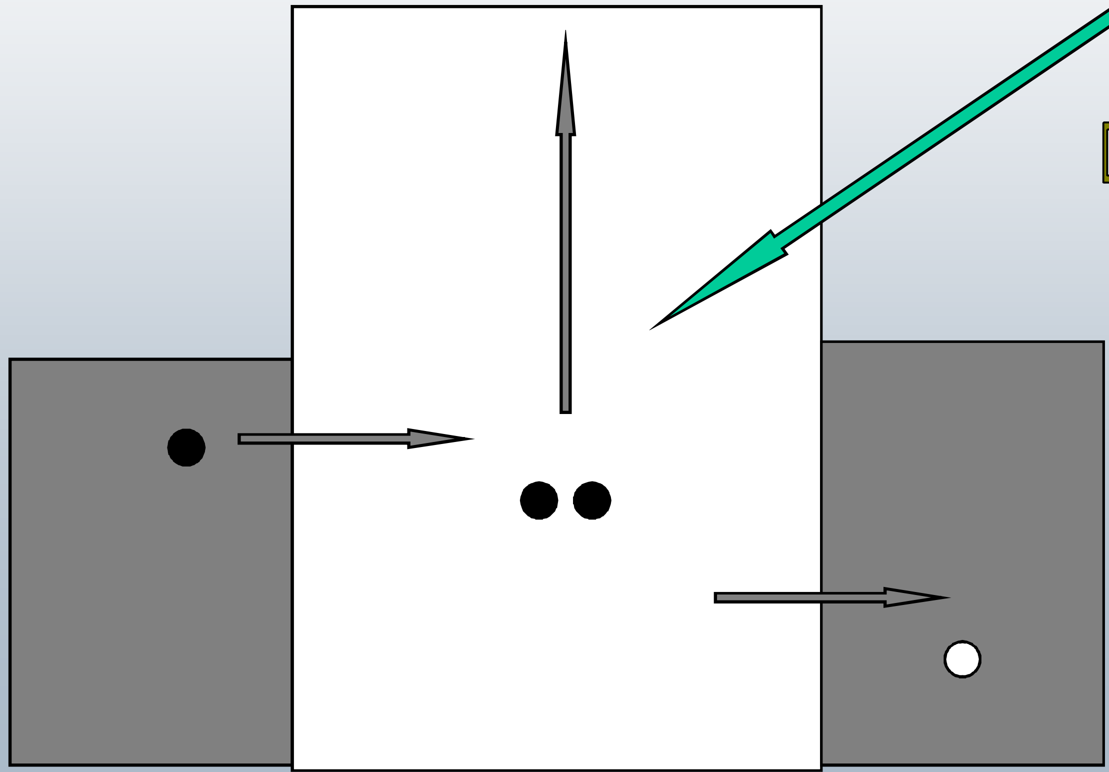
Deutscher, Feinberg'00



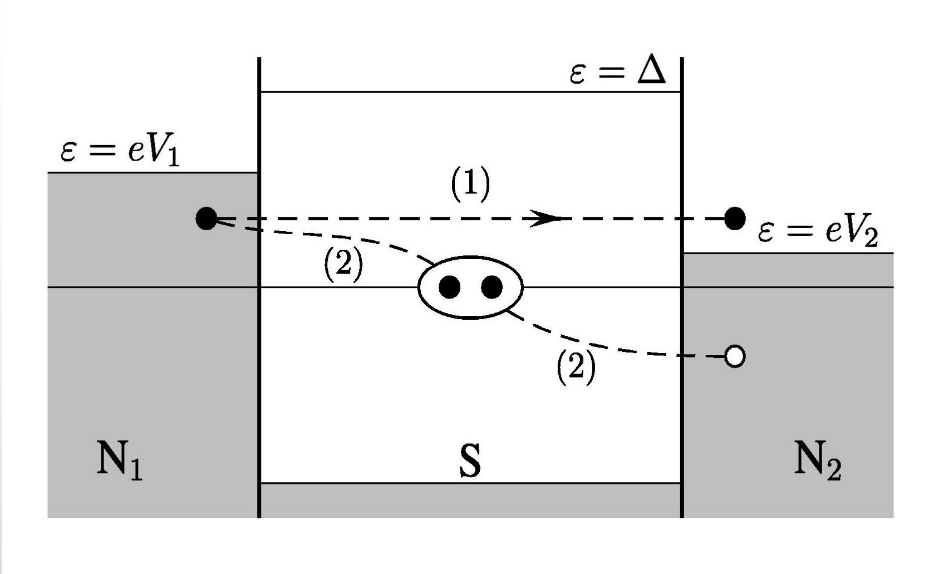
CAR

Byers, Flatte'95

Deutscher, Feinberg'00



T=0, lowest order in transmission:



$$G_{12} = G_{(1)} - G_{(2)} = 0$$

EC CAR

Falci, Feinberg, Hekking'01

VOLUME 93, NUMBER 19 PHYSICAL REVIEW LETTERS week ending 5 NOVEMBER 2004

Evidence for Crossed Andreev Reflection in Superconductor-Ferromagnet Hybrid Structures

D. Beckmann* and H. B. Weber
Institut für Nanotechnologie, Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany

H. v. Löhneysen
Institut für Festkörperphysik, Forschungszentrum Karlsruhe, P.O. Box 3640, D-76021 Karlsruhe, Germany, and Physikalisches Institut, Universität Karlsruhe, D-76128 Karlsruhe, Germany

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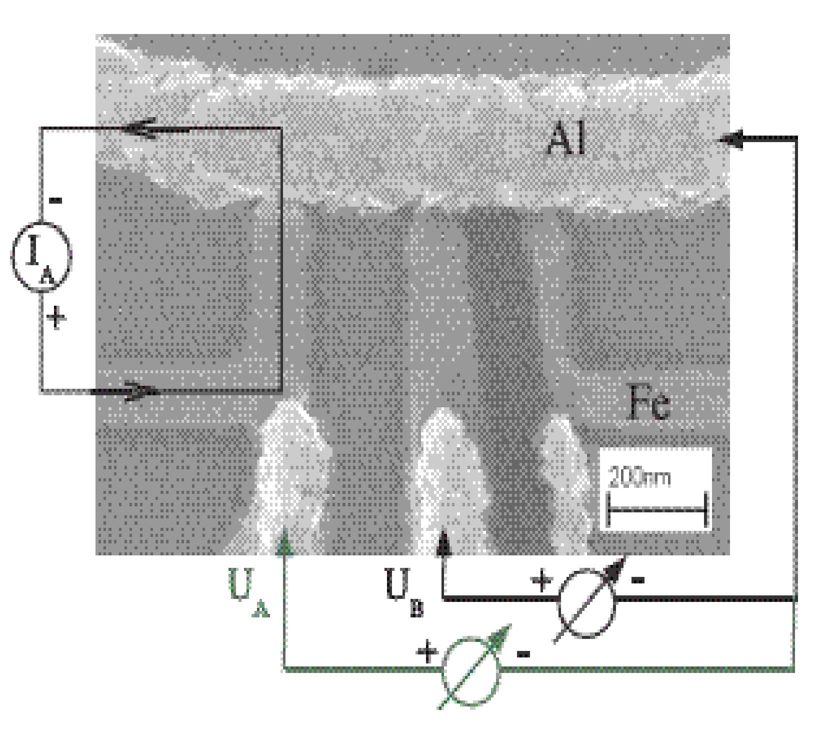
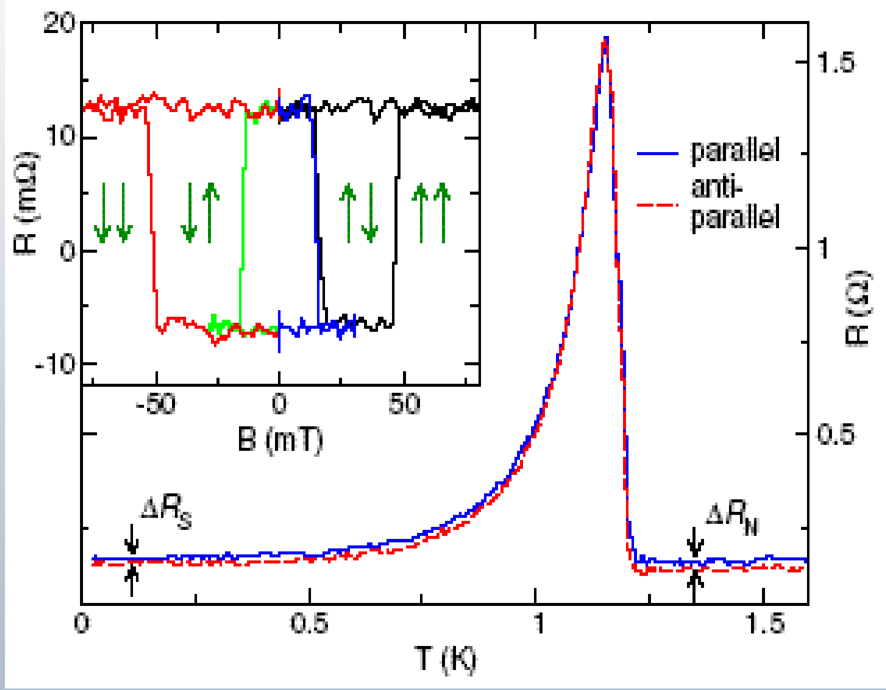
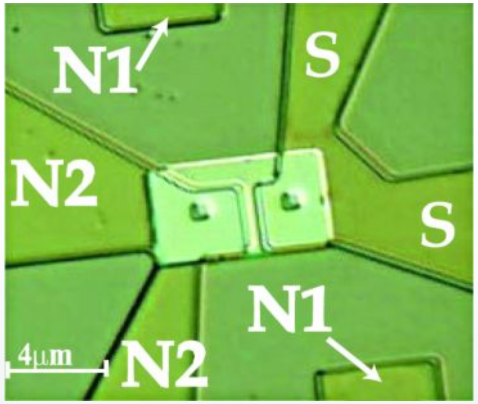
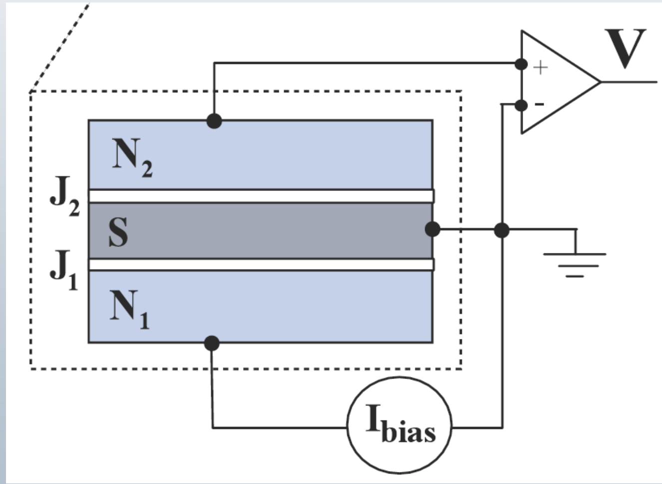
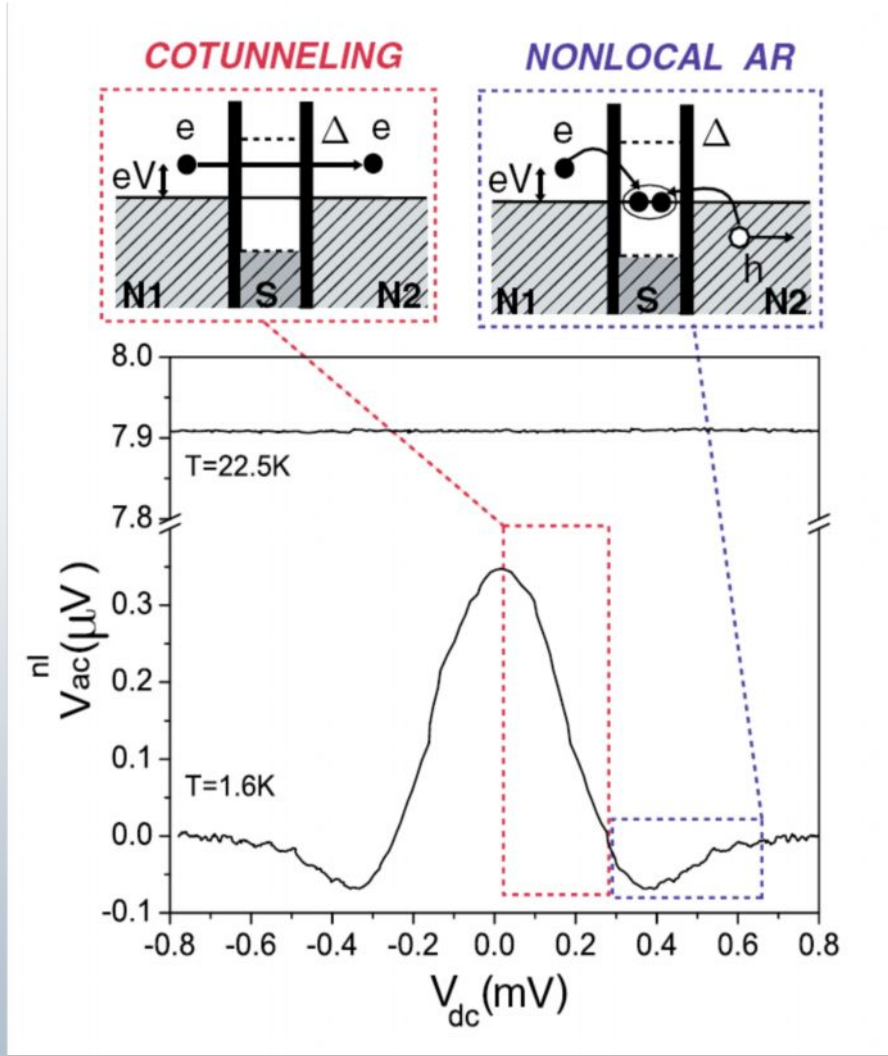




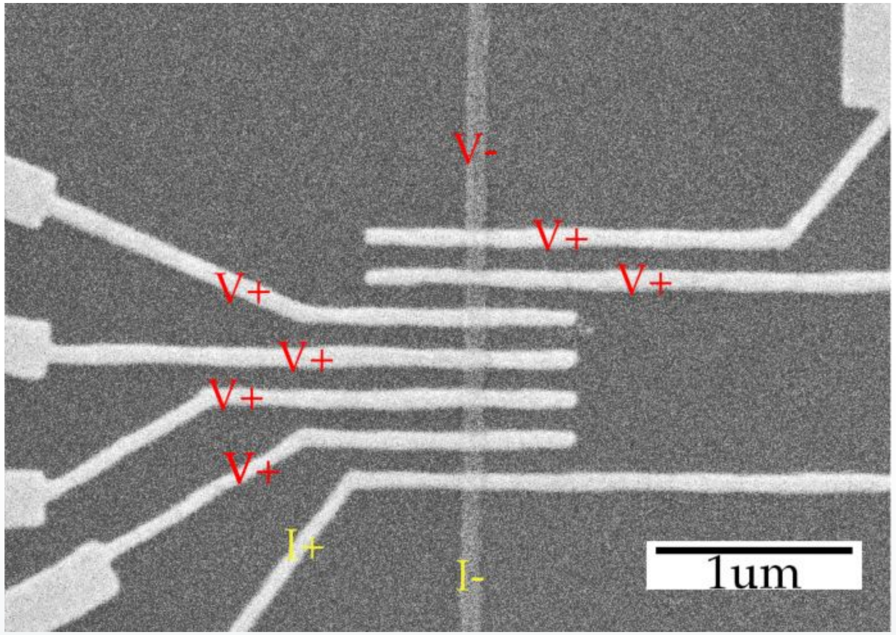
Russo, Kroug, Klapwijk, Morpurgo, PRL'05

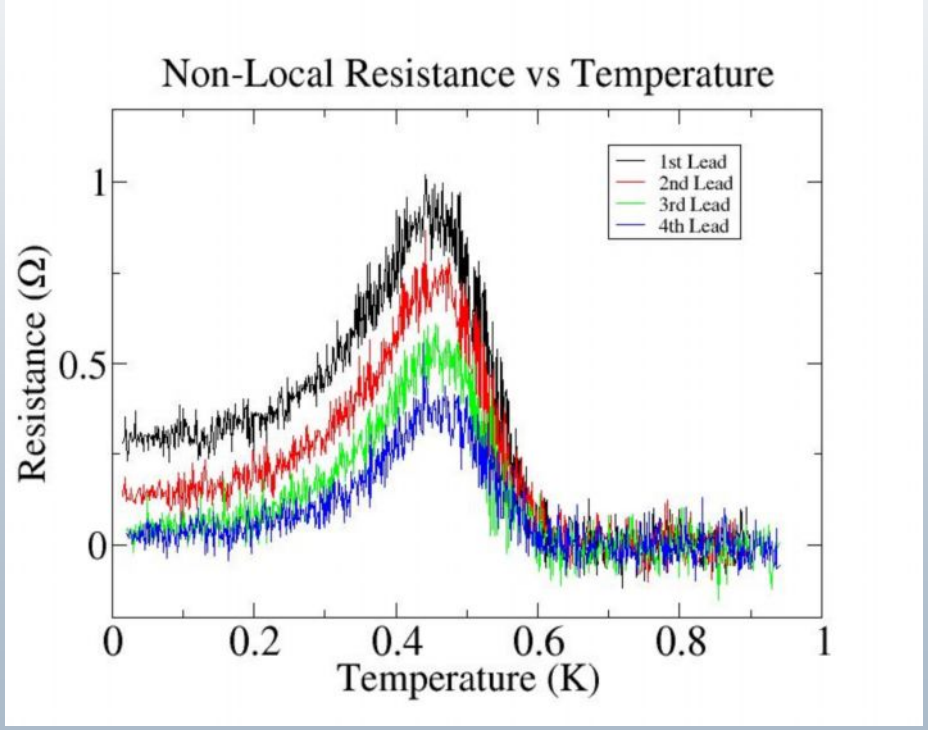


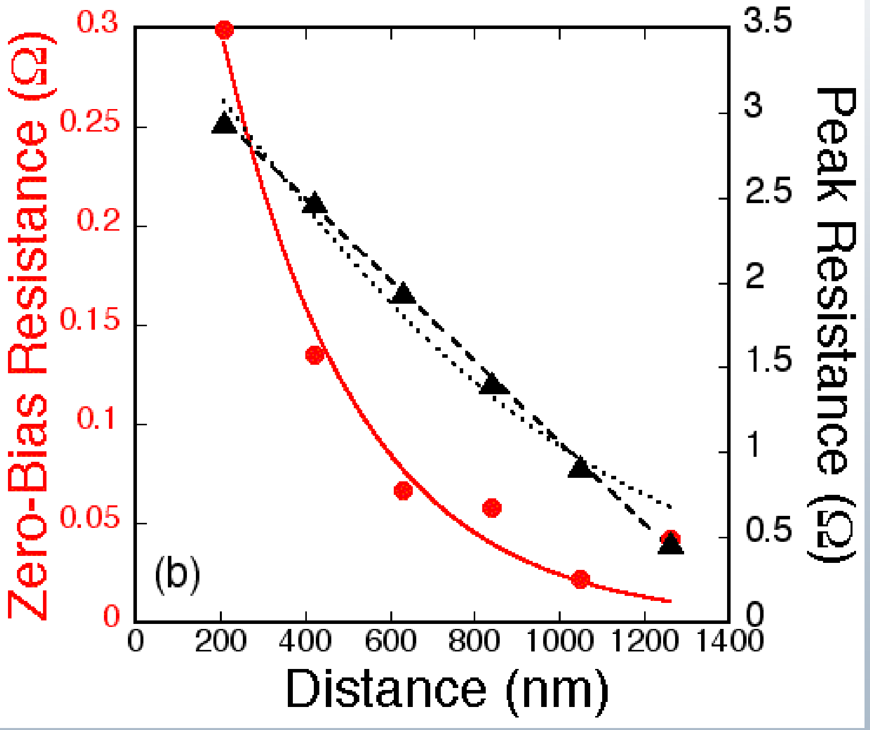




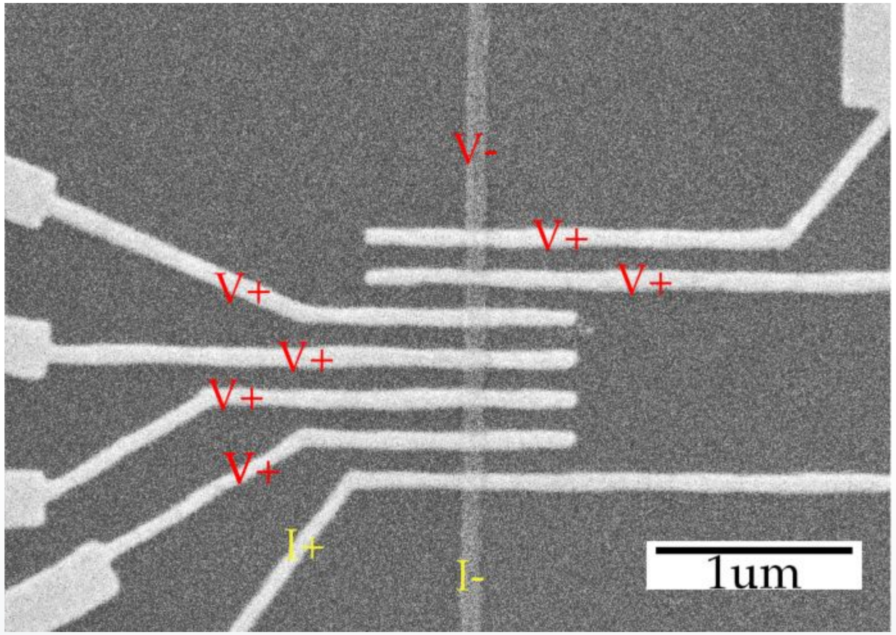
Cadden-Zimansky, Chandrasekhar, PRL'06



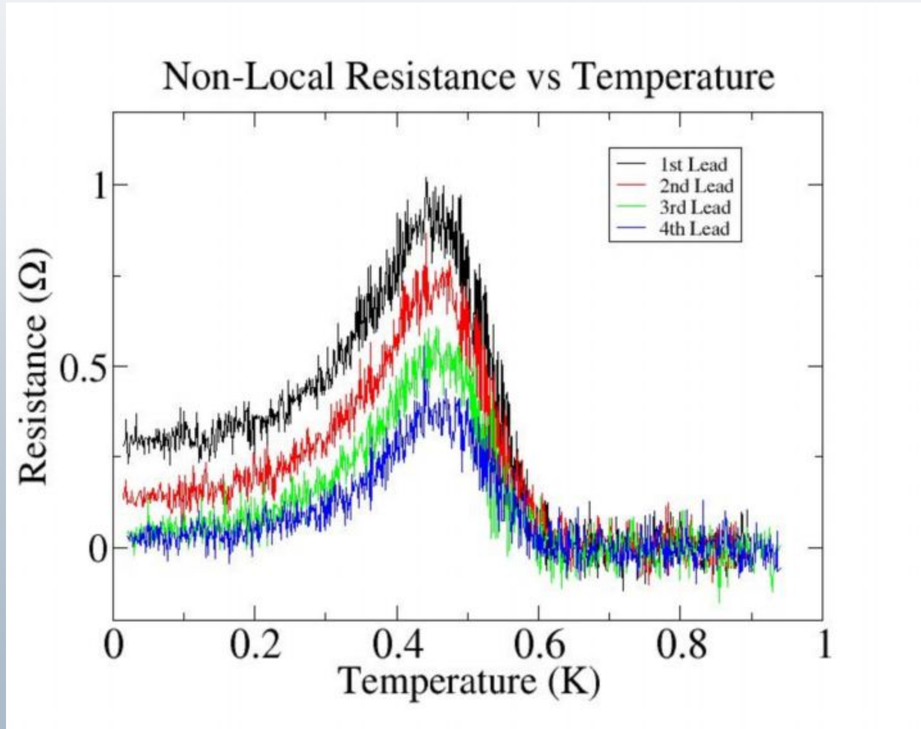


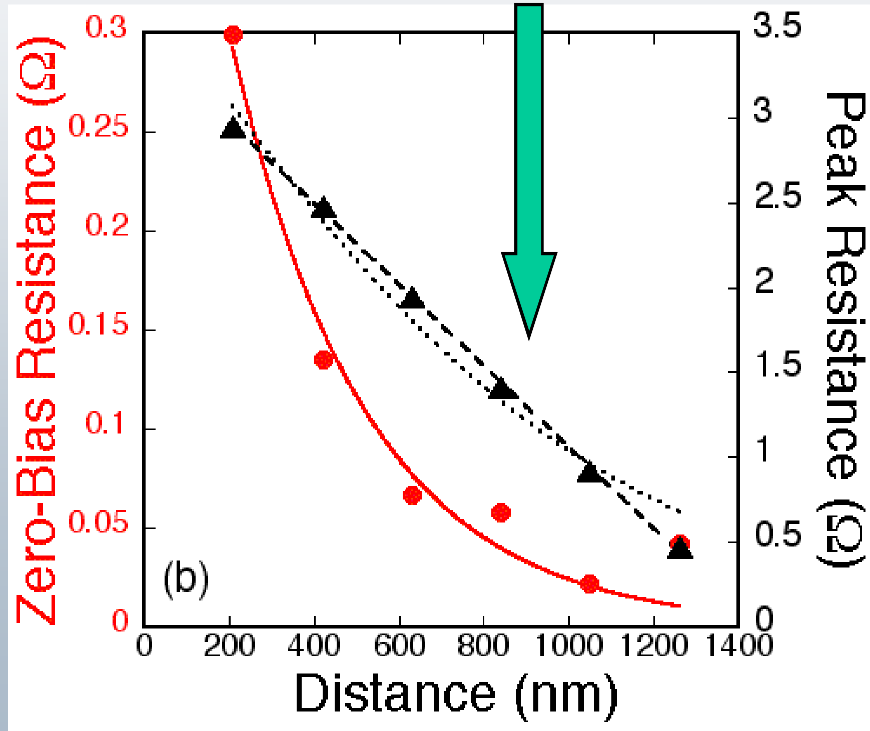


Cadden-Zimansky, Chandrasekhar, PRL'06



Charge imbalance length?





- Higher orders in transmission
- Effect of disorder
- Interactions
- Charge imbalance
- Spin-resolved CAR
- CAR under ac bias
- ...

- Higher orders in transmission
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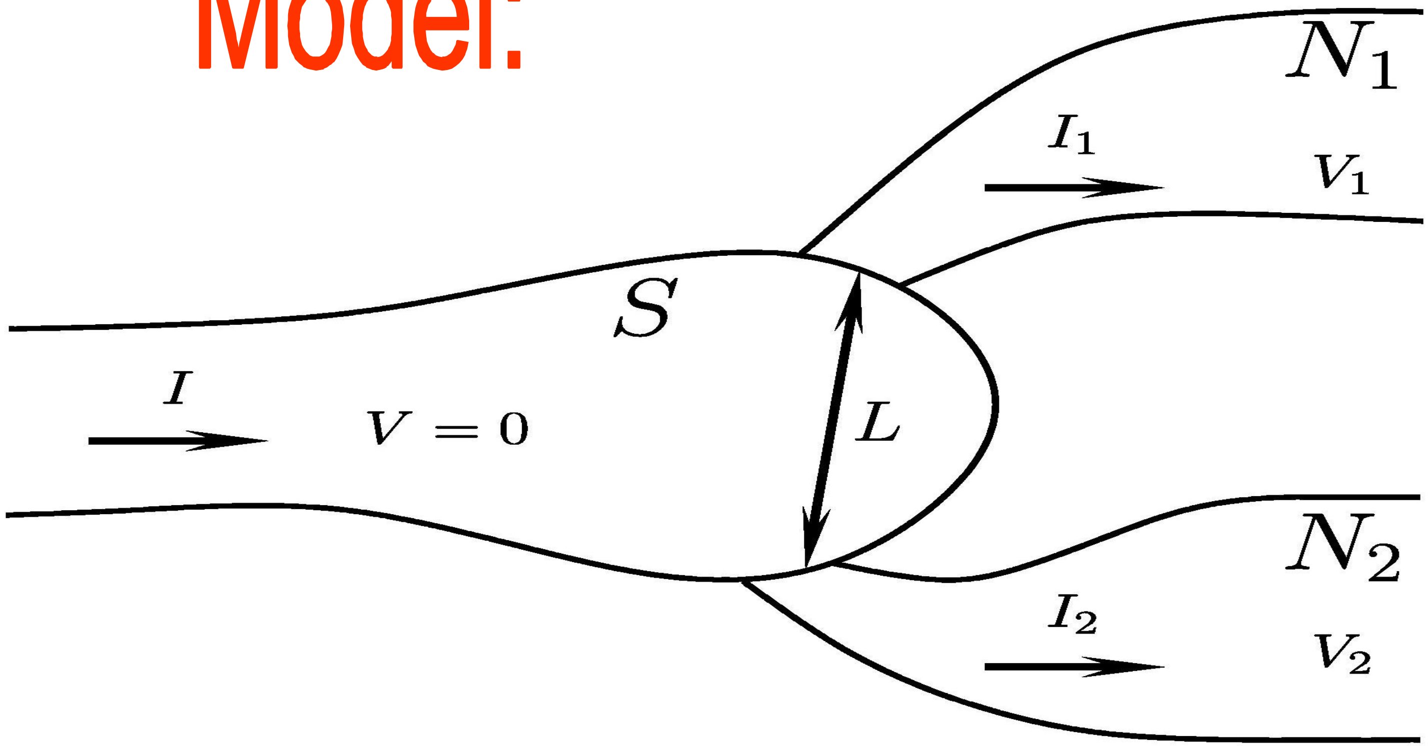
Model:



Ballistic electrodes
High spin-dependent transmissions
NS interfaces are metallic constrictions

Formalism: Eilenberger equations

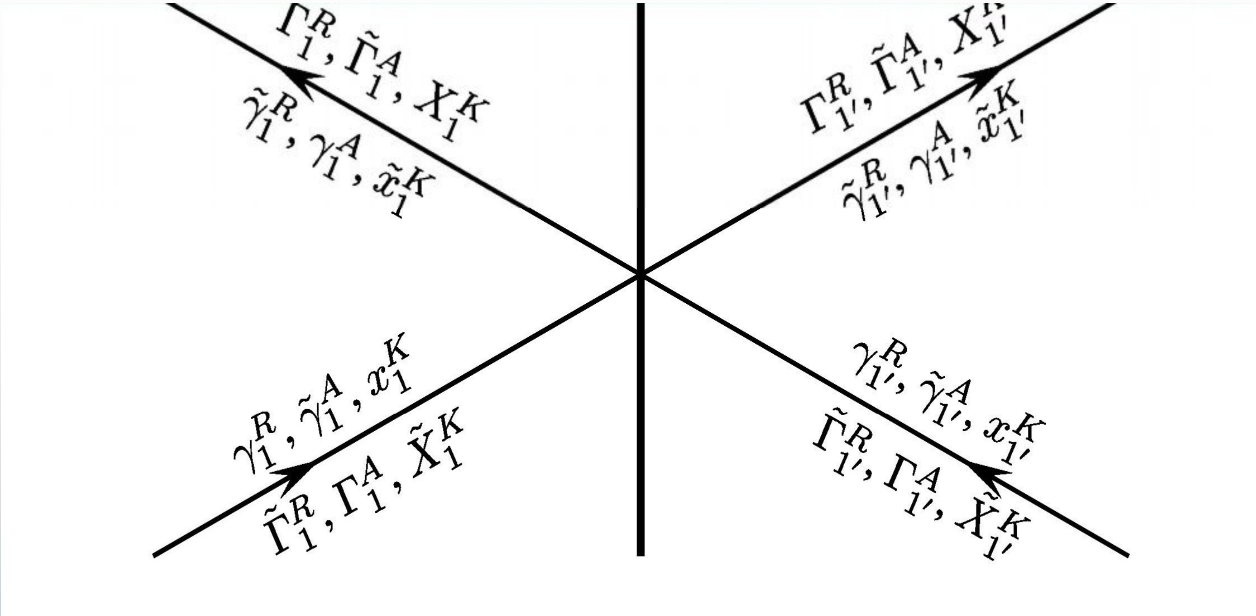
$$[\varepsilon\hat{\tau}_3 + eV(\mathbf{r}, t) - \hat{\Delta}(\mathbf{r}, t), \hat{g}^{R,A,K}(\mathbf{p}_F, \varepsilon, \mathbf{r}, t)] + i\mathbf{v}_F\nabla\hat{g}^{R,A,K}(\mathbf{p}_F, \varepsilon, \mathbf{r}, t) = 0,$$

$$\hat{g}^{R,A,K} = \begin{pmatrix} g^{R,A,K} & f^{R,A,K} \\ \tilde{f}^{R,A,K} & \tilde{g}^{R,A,K} \end{pmatrix}, \quad \hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix},$$

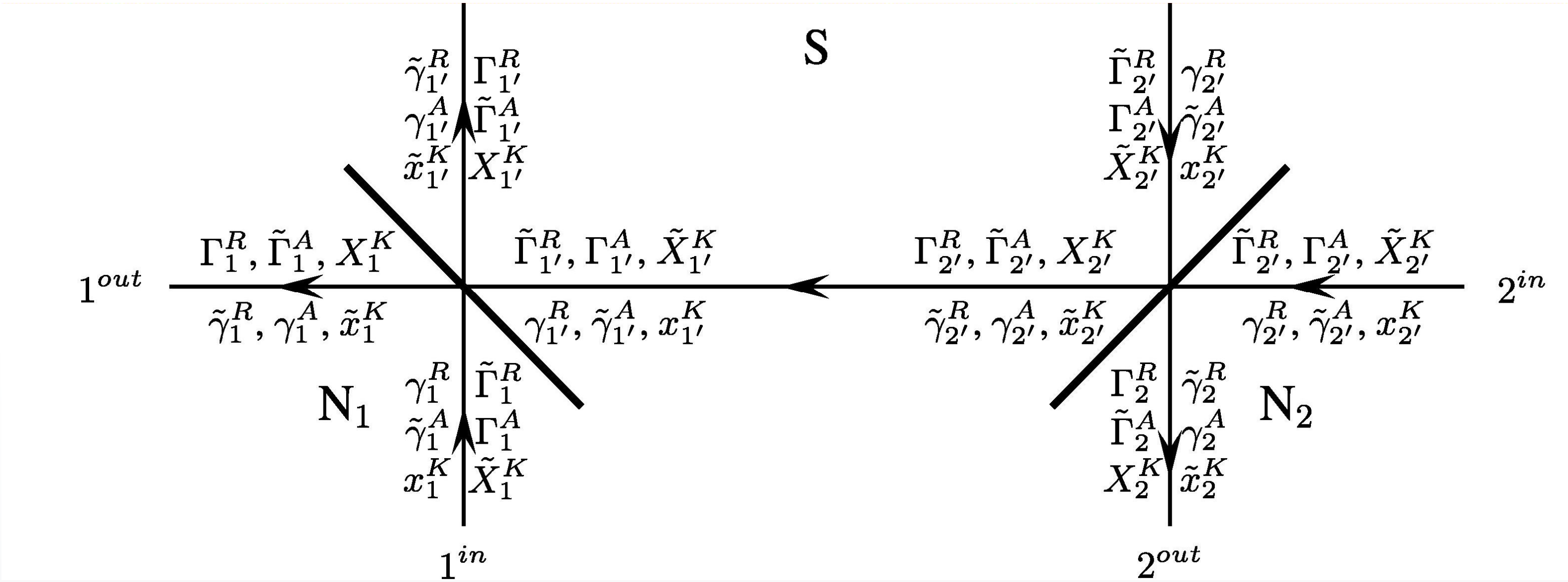
$$\mathbf{j}(\mathbf{r}, t) = -\frac{eN_0}{4}\int d\varepsilon \langle \mathbf{v}_F \mathrm{Sp}[\hat{\tau}_3\hat{g}^K(\mathbf{p}_F, \varepsilon, \mathbf{r}, t)]\rangle,$$

Boundary conditions

Zaitsev'84
Zhao,Löfwander,Sauls'0
4



$$\mathcal{S} = \begin{pmatrix} S_{11} & S_{11'} \\ S_{1'1} & S_{1'1'} \end{pmatrix}, \quad \mathcal{S}\mathcal{S}^+ = 1.$$



$$S_{11} = S_{1'1'} = \underline{S}^T_{11} = \underline{S}^T_{1'1'} = U(\varphi) \times \begin{pmatrix} \sqrt{R_{1\uparrow}} e^{i\theta_1/2} & 0 \\ 0 & \sqrt{R_{1\downarrow}} e^{-i\theta_1/2} \end{pmatrix} U^+(\varphi),$$

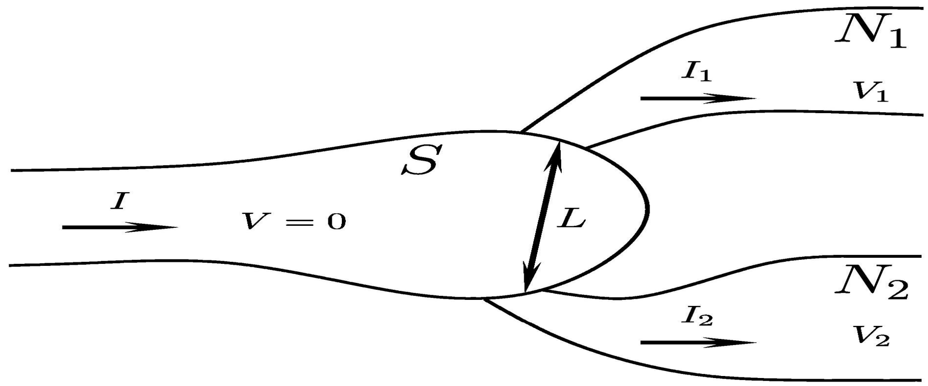
$$S_{11'} = S_{1'1} = \underline{S}^T_{11'} = \underline{S}^T_{1'1} = U(\varphi) i \begin{pmatrix} \sqrt{D_{1\uparrow}} e^{i\theta_1/2} & 0 \\ 0 & \sqrt{D_{1\downarrow}} e^{-i\theta_1/2} \end{pmatrix} U^+(\varphi),$$

$$S_{22} = S_{2'2'} = \underline{S}_{22} = \underline{S}_{2'2'} = \begin{pmatrix} \sqrt{R_{2\uparrow}} e^{i\theta_2/2} & 0 \\ 0 & \sqrt{R_{2\downarrow}} e^{-i\theta_2/2} \end{pmatrix},$$

$$S_{22'} = S_{2'2} = \underline{S}_{22'} = \underline{S}_{2'2} = i \begin{pmatrix} \sqrt{D_{2\uparrow}} e^{i\theta_2/2} & 0 \\ 0 & \sqrt{D_{2\downarrow}} e^{-i\theta_2/2} \end{pmatrix}.$$

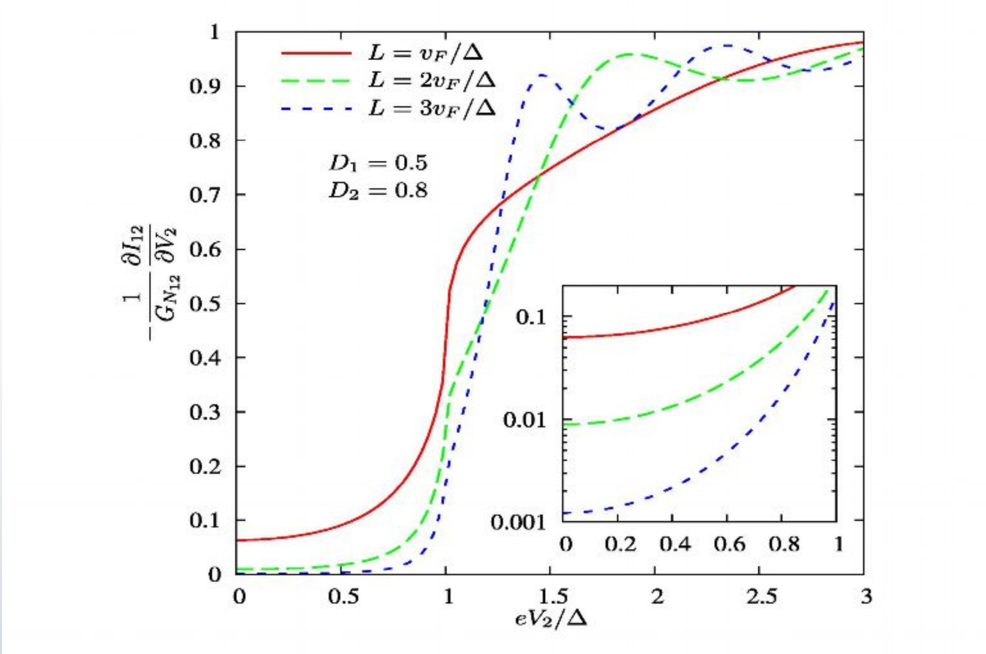
$$U(\varphi) = \exp(-i\varphi\sigma_1/2) = \begin{pmatrix} \cos(\varphi/2) & -i\sin(\varphi/2) \\ -i\sin(\varphi/2) & \cos(\varphi/2) \end{pmatrix}$$

Results

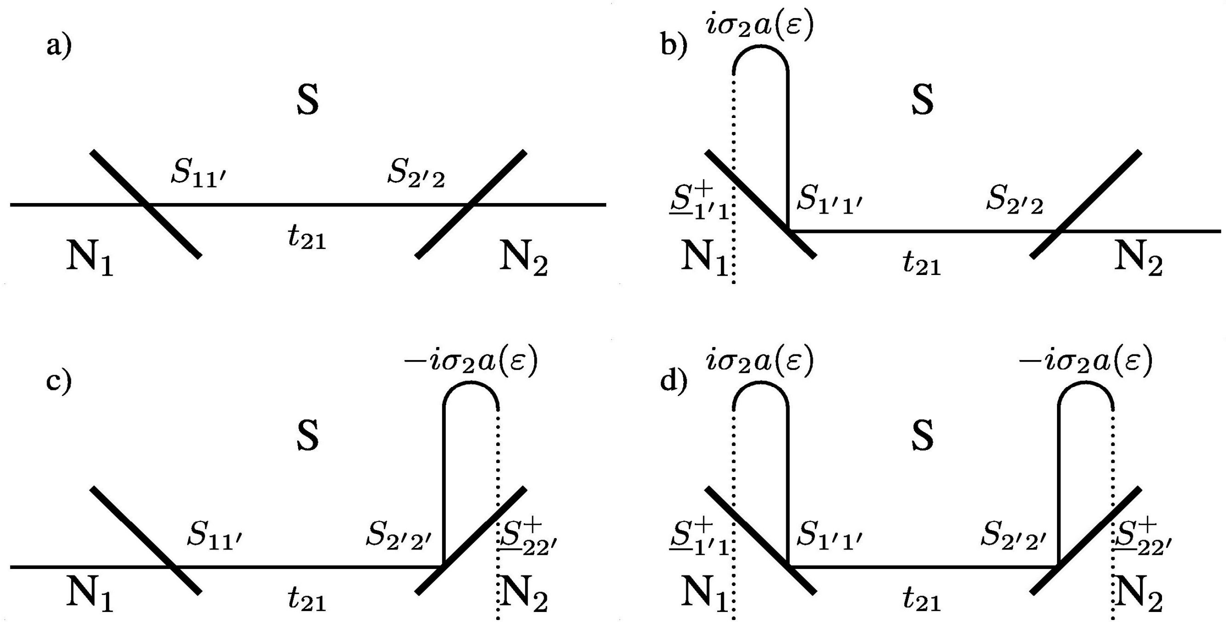


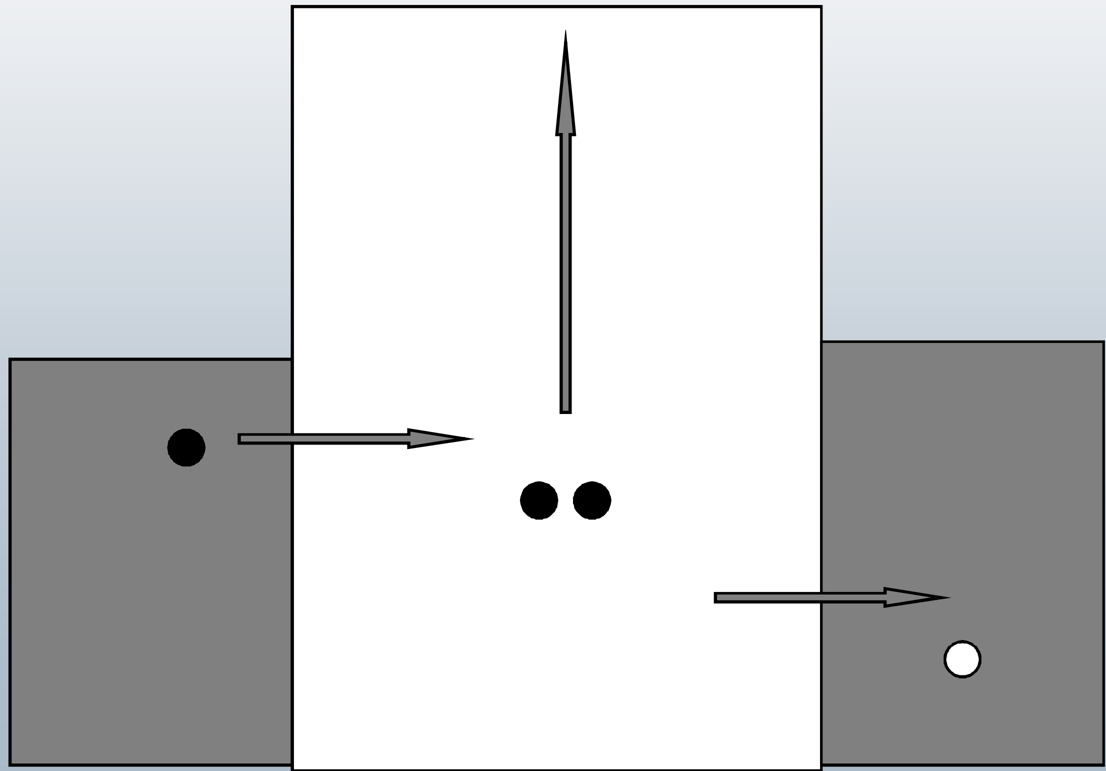
$$I_1 = I_{11}(V_1) + I_{12}(V_2),$$
$$I_2 = I_{21}(V_1) + I_{22}(V_2).$$

Spin-degenerate case:



$$\frac{G_{12}}{G_{N_{12}}} = \frac{\mathcal{D}_1\mathcal{D}_2(1-\tanh^2 L\Delta/v_F)}{[1+\mathcal{R}_1\mathcal{R}_2+(\mathcal{R}_1+\mathcal{R}_2)\tanh L\Delta/v_F]^2}.$$


a)
S
$S_{11'}$
$S_{2'2}$
N_1
t_{21}
N_2
b)
$i\sigma_2 a(\varepsilon)$
S
$\underline{S}^+_{1'1}$
$S_{1'1'}$
$S_{2'2}$
N_1
t_{21}
N_2
c)
$-i\sigma_2 a(\varepsilon)$
S
$S_{11'}$
$S_{2'2'}$
$\underline{S}^+_{22'}$
N_1
t_{21}
N_2
d)
$i\sigma_2 a(\varepsilon)$
$-i\sigma_2 a(\varepsilon)$
S
$\underline{S}^+_{1'1}$
$S_{1'1'}$
$S_{2'2'}$
$\underline{S}^+_{22'}$
N_1
t_{21}
N_2

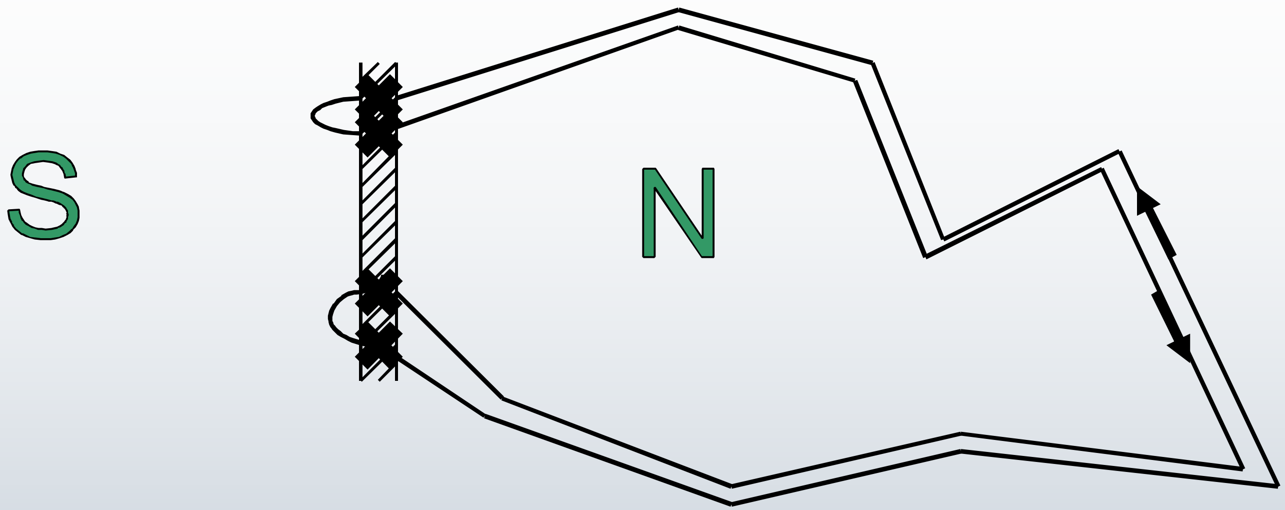


Ballistic NSN structures:

NO CAR

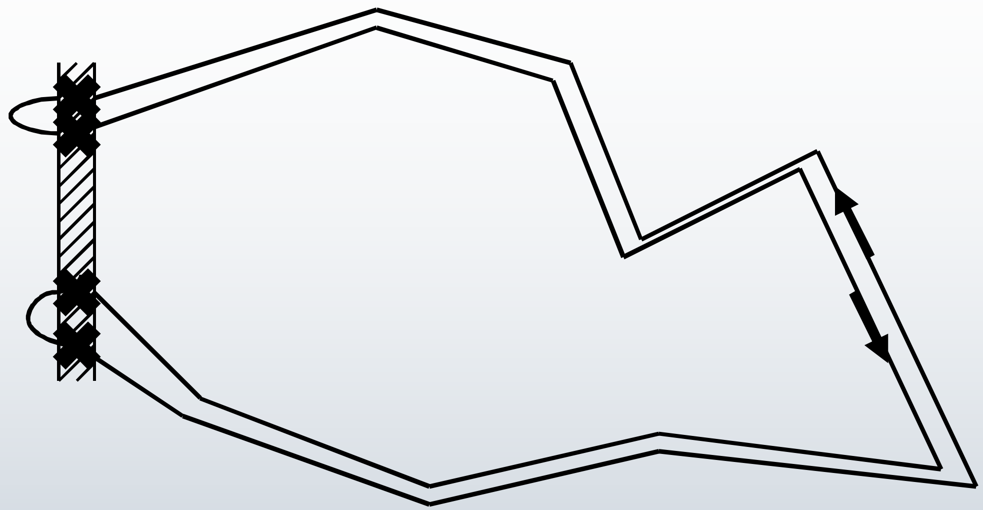
at full transmissions

Effect of disorder



Cooperon: $C(t) = \overline{\mathcal{G}^{\mathcal{R}}\mathcal{G}^{\mathcal{A}}} \sim t^{-d/2}$

Effect of disorder



Andreev conductance: $G_A \sim V^{-1/2}, T^{-1/2}$

Volkov, Zaitsev, Klapwijk'93
Hekking, Nazarov'93,
Beenakker et al.'94
A.D.Z.'94

Experiments

VOLUME 67, NUMBER 21 PHYSICAL REVIEW LETTERS 18 NOVEMBER 1991

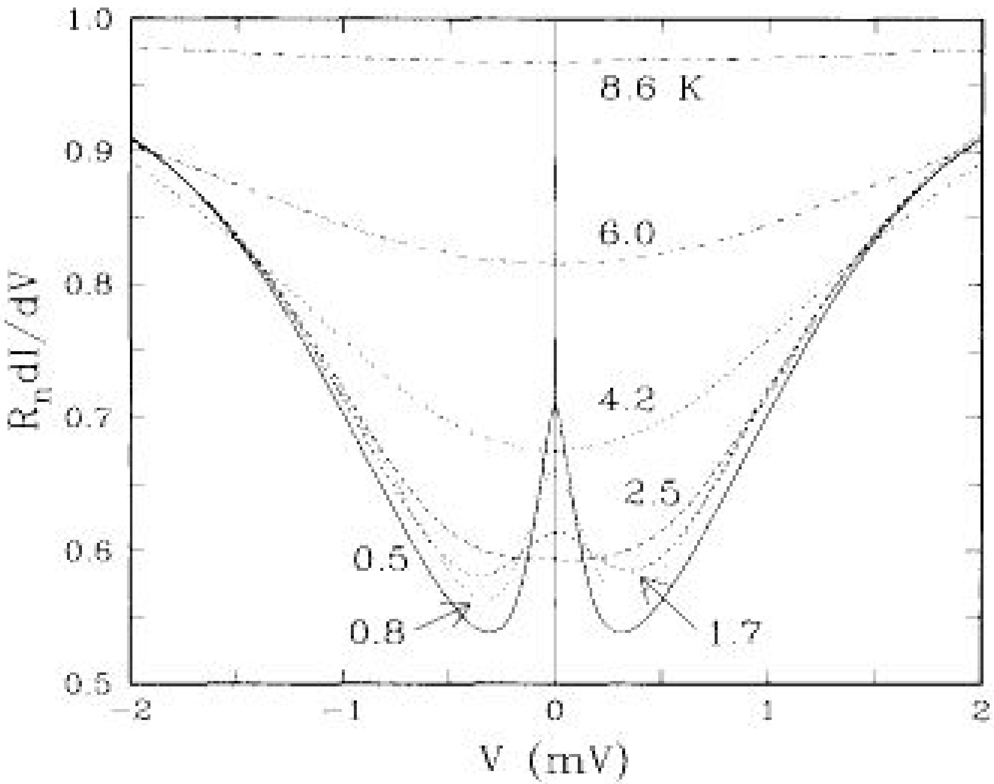
Observation of Pair Currents in Superconductor-Semiconductor Contacts

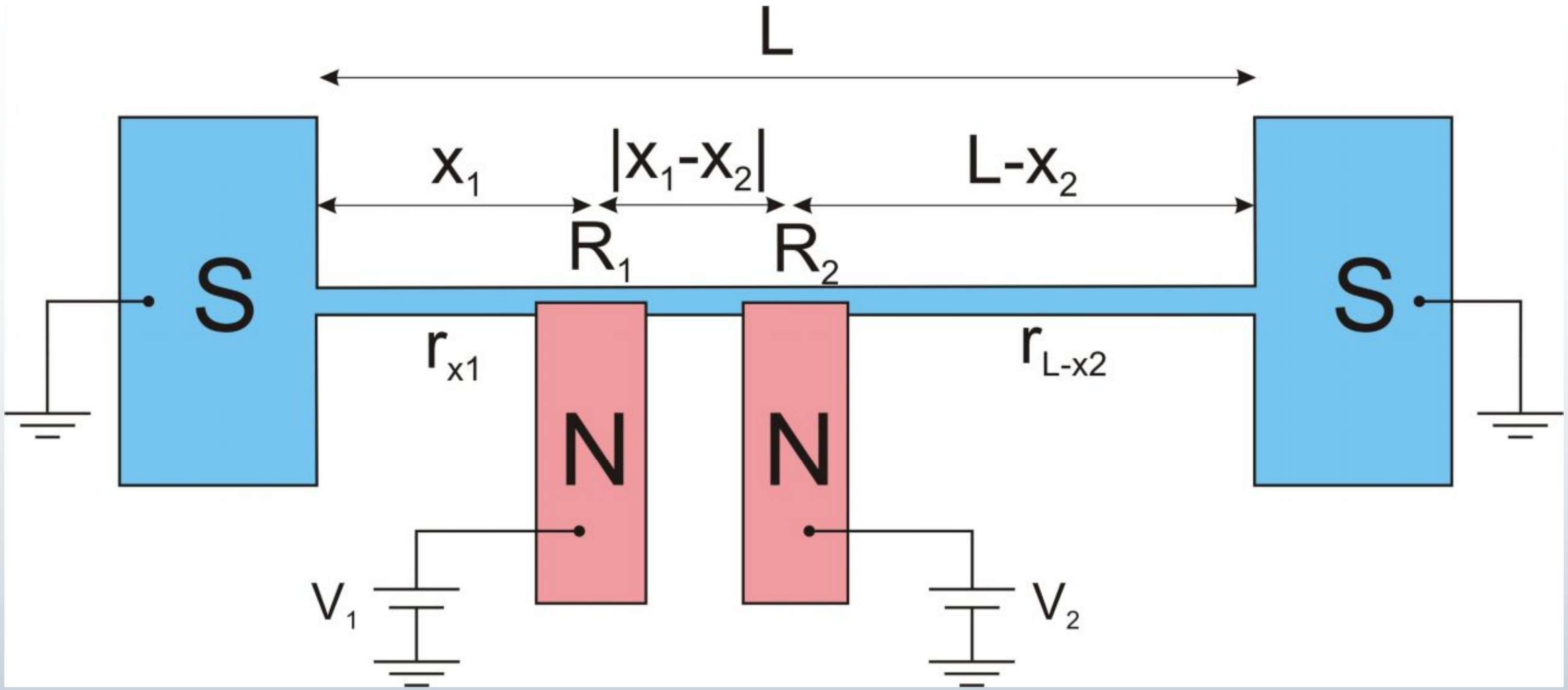
A. Kastalsky,[(1),(a)] A. W. Kleinsasser,[(2)] L. H. Greene,[(1)] R. Bhat,[(1)] F. P. Milliken,[(2)] and J. P. Harbison[(1)]

[(1)]*Bellcore, 331 Newman Springs Road, Red Bank, New Jersey 07701*

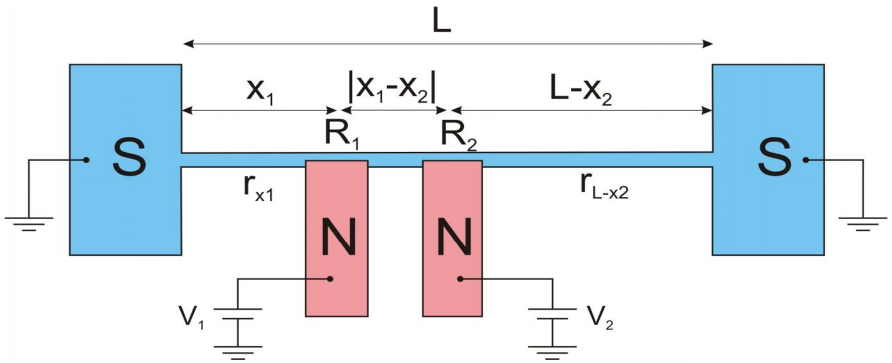
[(2)]*IBM Research Division, T. J. Watson Research Center, P.O. Box 218, Yorktown Heights, New York 10598*

(Received 12 August 1991)





Arbitrary interface transmissions



$$iD\nabla(\check{G}\nabla\check{G}) = \left[\check{\Sigma}, \check{G}\right], \quad \check{G}^2 = 1,$$

Usadel equations

$$\check{G} = \begin{pmatrix} \hat{G}^R & \hat{G}^K \\ 0 & \hat{G}^A \end{pmatrix}, \quad \check{\Sigma} = \check{1}\begin{pmatrix} \varepsilon + eV & \Delta \\ -\Delta^* & -\varepsilon + eV \end{pmatrix}$$

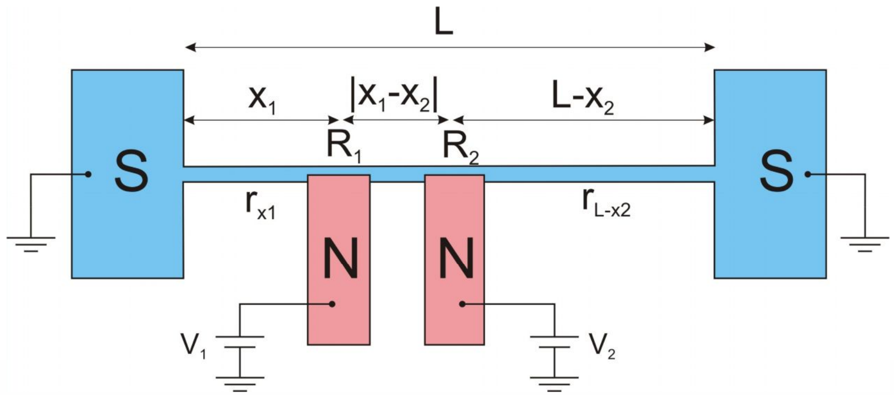
$$\mathcal{A}_1\sigma_1\check{G}_1\partial_x\check{G}_1 = \mathcal{A}_1\sigma_S\check{G}_S\partial_x\check{G}_S$$
$$= \frac{e^2}{\pi}\sum_n \frac{2T_{1,n}[\check{G}_1, \check{G}_S]}{4 + T_{1,n}(\{\check{G}_1, \check{G}_S\} - 2)}$$

Boundary conditions (Nazarov'99)

Current density

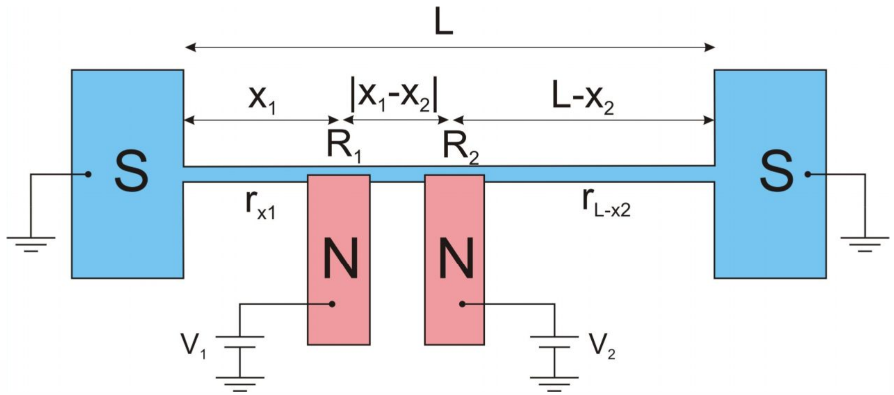
$$\boldsymbol{j} = -\frac{\sigma}{8e}\int \mathrm{tr}[\hat{\tau}_3(\check{G}\nabla\check{G})^K]d\varepsilon.$$

Crucial assumption:



$$\boxed{R_{1,2} \gg r = \max(r_L, r_{N_1}, r_{N_2})}$$

Crucial assumption:

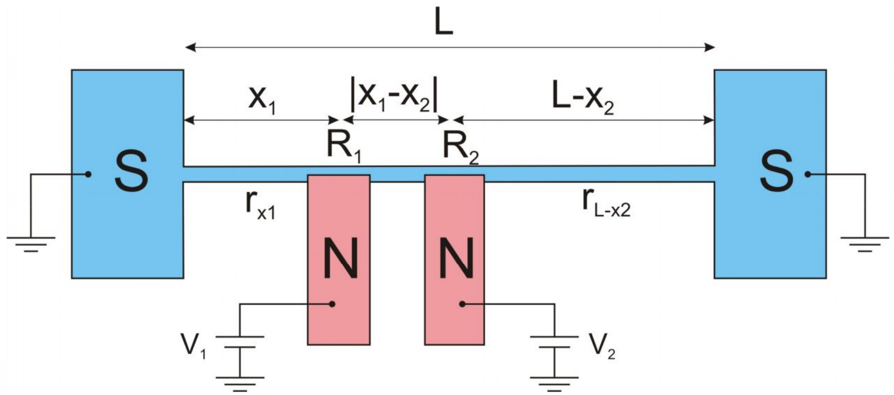


$$\boxed{R_{1,2} \gg r = \max(r_L, r_{N_1}, r_{N_2})}$$

Linearization of Usadel equations

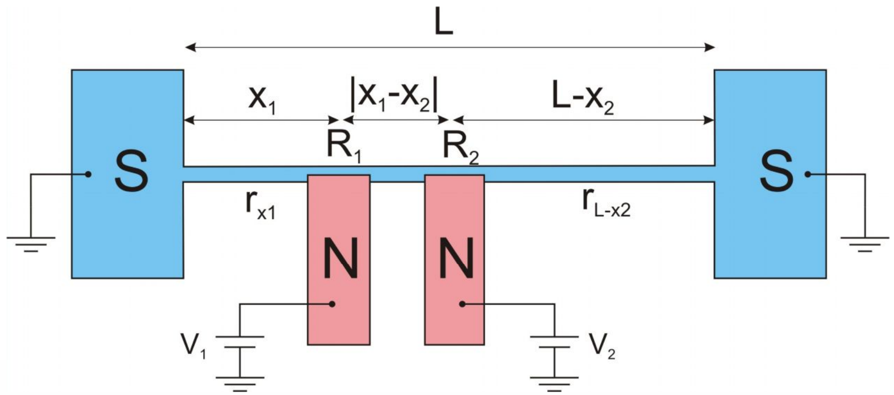
$$\left(-i\omega + \frac{1}{\tau_{Q^*}} - D\nabla^2\right)\mathcal{D}^{\boldsymbol{r}\boldsymbol{r}'}(\omega) = \delta(\boldsymbol{r}-\boldsymbol{r}')$$

$$\tau_{Q^*} \sim \tau_{in} T/\Delta(T)$$



Keldysh function:

$$\hat{G}^K = \hat{G}^R \hat{h} - \hat{h}\hat{G}^A \text{ with } \hat{h} = f_L \hat{1} + f_T \hat{\tau}_3$$

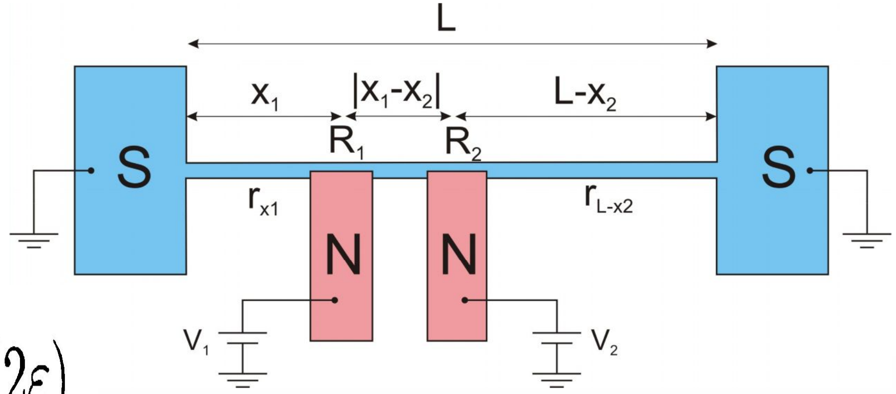


Keldysh function:

$$\hat{G}^K = \hat{G}^R \hat{h} - \hat{h} \hat{G}^A \text{ with } \hat{h} = f_L \hat{1} + f_T \hat{\tau}_3$$

$$I_1 = \frac{1}{2e} \int d\varepsilon g_1(\varepsilon) [f_T^{N_1}(\varepsilon, \boldsymbol{r}_1) - f_T^S(\varepsilon, \boldsymbol{r}_1)].$$

Local conductance:

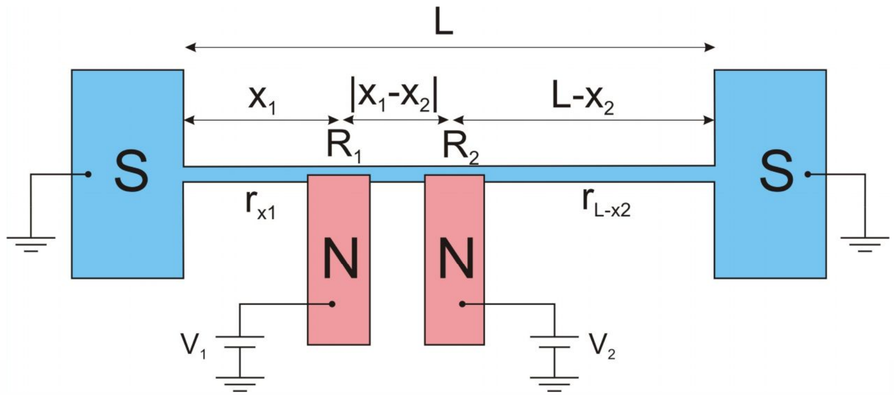


$$g_1(\varepsilon) = g_1^{\mathrm{BTK}}(\varepsilon) + \frac{\theta(\Delta - |\varepsilon|)\Delta^2}{\Omega^2} \frac{\mathrm{Re}\, C_1^{r_1 r_1}(2\varepsilon)}{2e^2 N_1 R_1^2} + \frac{\Delta^2}{\Omega^2} \sum_{j=1,2} \frac{\mathrm{Re}\, C_S^{r_1 r_j}(2W(\varepsilon))}{2e^2 N_S R_1 R_j},$$

$$g_1^{\mathrm{BTK}}(\varepsilon) = \frac{e^2}{\pi} \sum_n \left[\frac{2T_{1,n}^2 \theta(\Delta - |\varepsilon|)\Delta^2}{T_{1,n}^2 \varepsilon^2 + (2 - T_{1,n})^2 \Omega^2} + \frac{2T_{1,n}\theta(|\varepsilon| - \Delta)|\varepsilon|}{T_{1,n}|\varepsilon| + (2 - T_{1,n})|\Omega|} \right]$$

$$\Omega^2 = \Delta^2 - \varepsilon^2$$

Solving kinetic equation...

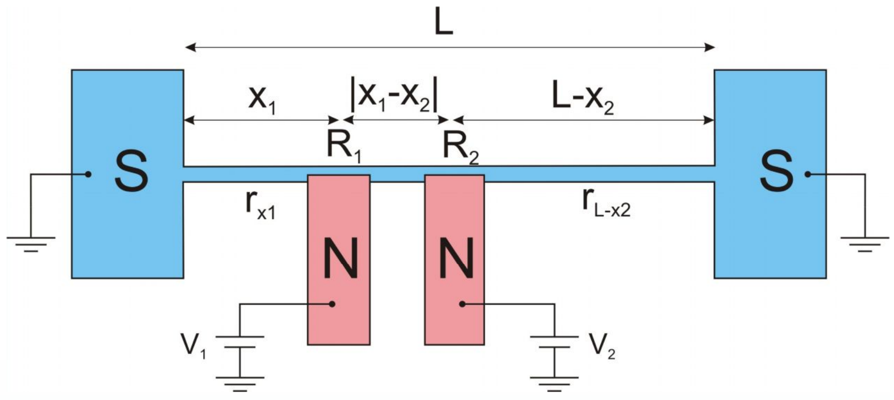


$$r/R_{1,2} \to 0$$

$$f_T^S(\varepsilon) = 0 \text{ and } f_T^{N_j}(\varepsilon, \boldsymbol{r}_j) = h(\varepsilon, V_j) \equiv$$

$$\left(\tanh[(\varepsilon + eV_j)/2T] - \tanh[(\varepsilon - eV_j)/2T]\right)/2$$

Solving kinetic equation...



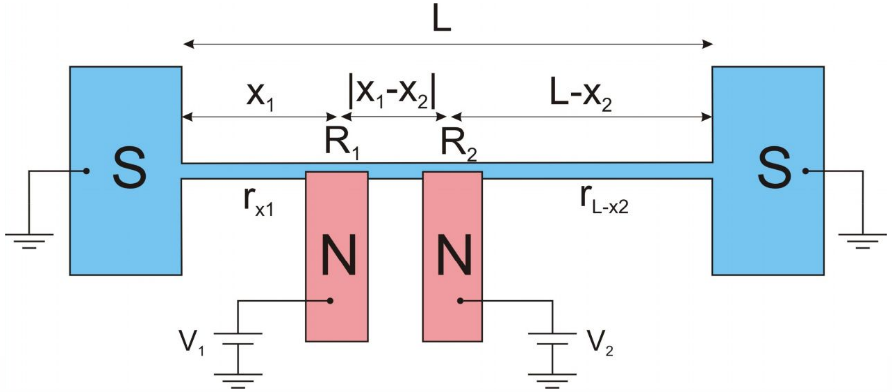
first order in $r/R_{1,2}$

$$\left(2\tilde{\Omega} - D\nabla^2\right) f_T = \sum_{j=1,2} \frac{g_j(\varepsilon) h(\epsilon, V_j)}{2e^2 N_S K(\varepsilon)} \delta(\boldsymbol{r} - \boldsymbol{r}_j),$$

where $K(\varepsilon) = \theta(\Delta - |\varepsilon|)\Delta^2/\Omega^2 - \theta(|\varepsilon| - \Delta)\varepsilon^2/\Omega^2$

$$\tilde{\Omega} = \theta(\Delta - |\varepsilon|)\Omega$$

Non-local current response:



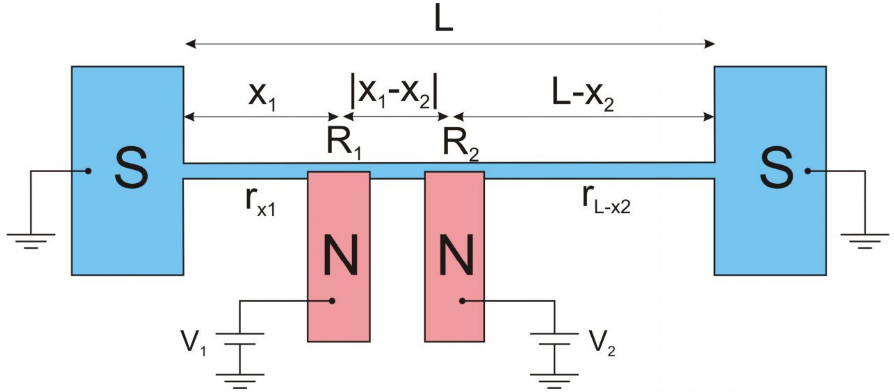
$$I_1(V_1,V_2)=\frac{1}{2e}\int d\varepsilon\left[g_{11}(\varepsilon)h(\varepsilon,V_1)-g_{12}(\varepsilon)h(\varepsilon,V_2)\right]$$

$$g_{11}(\varepsilon)=g_1(\varepsilon)-\frac{\mathcal{D}_S^{r_1r_1}(2i\tilde{\Omega})}{2e^2N_S}\frac{g_1^2(\varepsilon)}{K(\varepsilon)}-\frac{\mathcal{D}_1^{r_1r_1}(0)}{2e^2N_S}g_1^2(\varepsilon),$$

$$\boxed{g_{12}(\varepsilon)=g_{21}(\varepsilon)=\frac{\mathcal{D}_S^{r_1r_2}(2i\tilde{\Omega})}{2e^2N_S}\frac{g_1(\varepsilon)g_2(\varepsilon)}{K(\varepsilon)}}$$

Tunneling limit

$$E_{1,2} \equiv D_{1,2}/L_{1,2}^2 \ll |\varepsilon| < \Delta$$

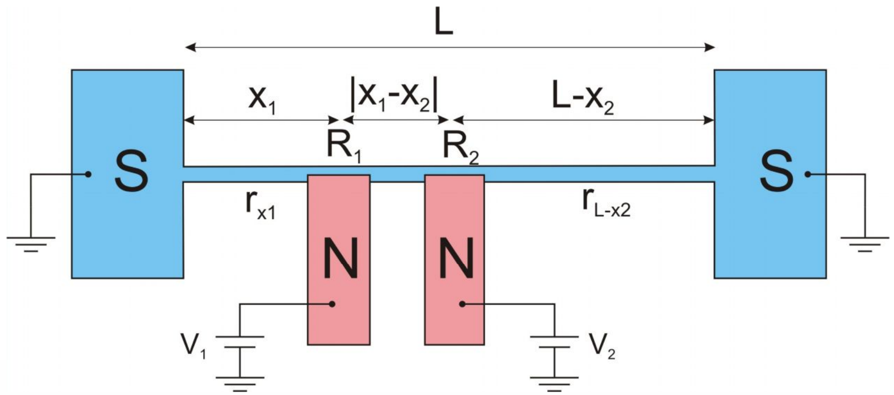


$$g_{11}(\varepsilon) = \frac{\Delta^2}{\Omega^2}\left[\frac{r_{\xi_S}(\varepsilon) + r_{\xi_1}(\varepsilon)}{2R_1^2} + \frac{r_{\xi_S}(\varepsilon)}{2R_1R_2} e^{-\frac{|x_2-x_1|}{\xi_S(\varepsilon)}}\right]$$

$$g_{12}(\varepsilon) = \frac{\Omega^2}{2\Delta^2} r_{\xi_S}(\varepsilon) g_{11}(\varepsilon) g_{22}(\varepsilon) e^{-\frac{|x_2-x_1|}{\xi_S(\varepsilon)}}$$

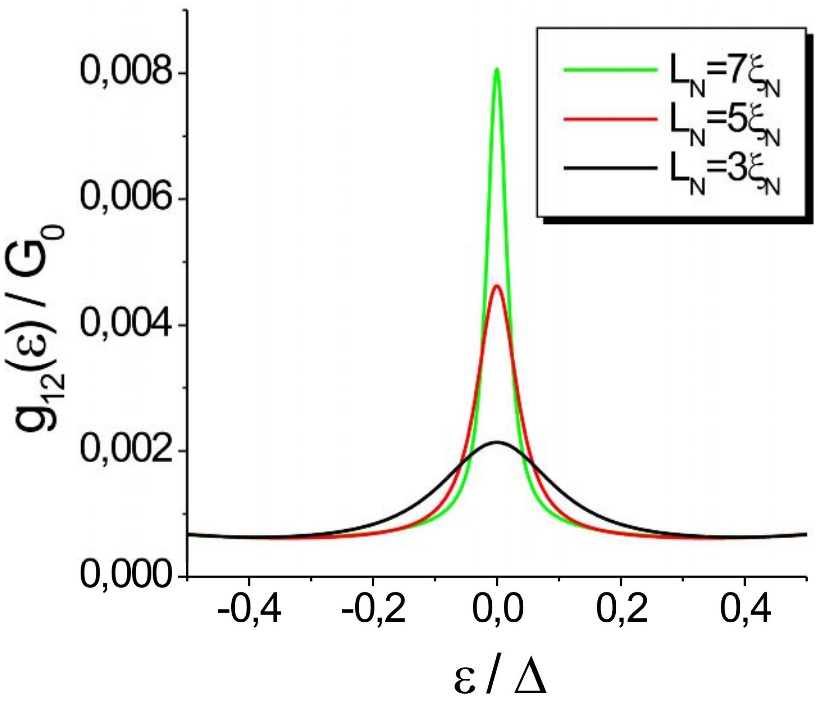
$$\xi_{1,2}(\varepsilon) = \sqrt{D_{1,2}/|\varepsilon|}$$

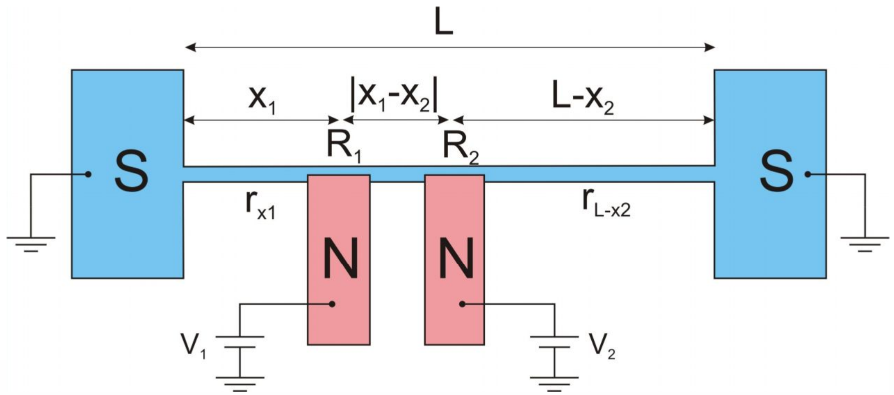
Zero-bias anomaly



$$g_{11}(\varepsilon) \propto 1/\sqrt{\varepsilon}$$

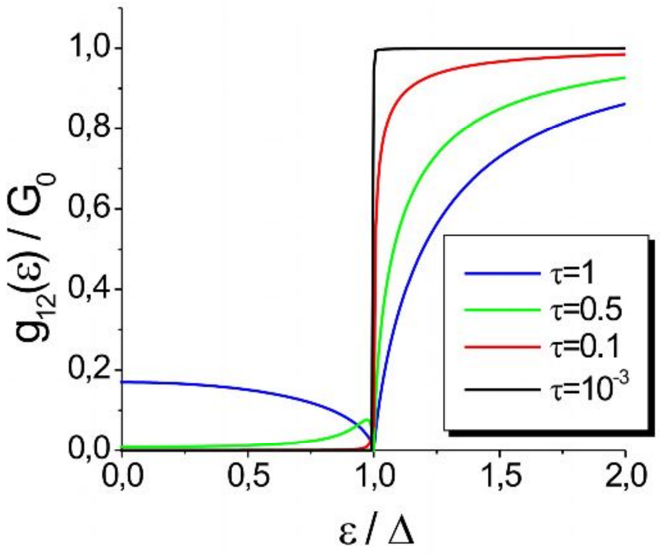
$$\boxed{g_{12}(\varepsilon) \propto 1/\varepsilon}$$

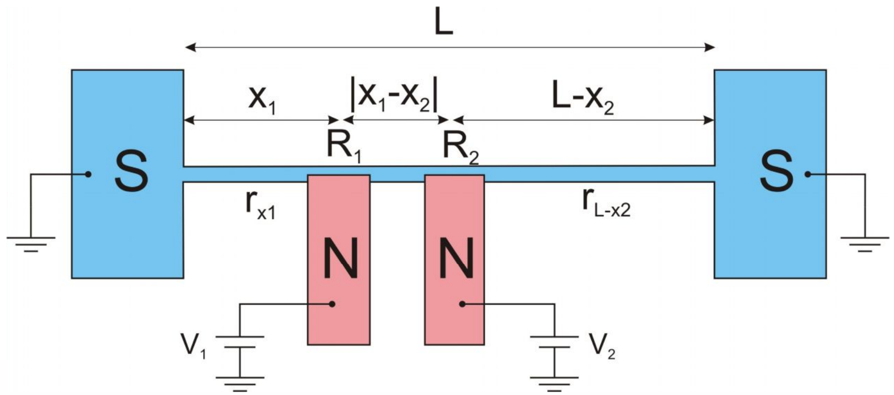




Zero-energy, small transmissions:

$$g_{12}(0) = G_{12}(0,0) = \frac{r_{\xi_S} r_{N_1} r_{N_2}}{2R_1^2 R_2^2} e^{-|x_2 - x_1|/\xi_S}$$





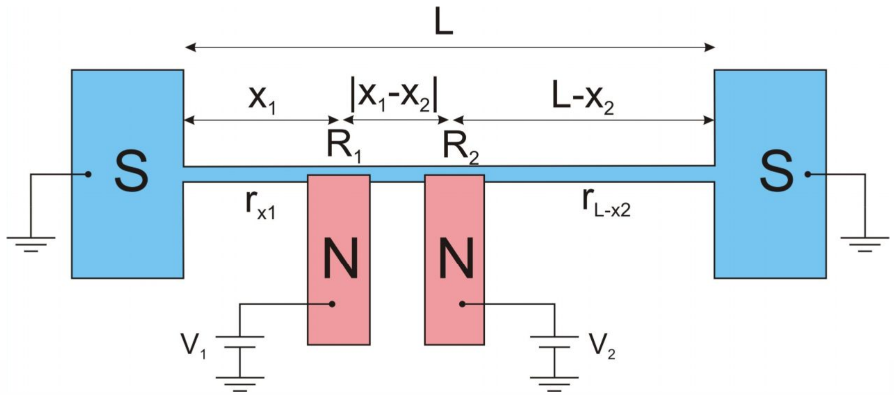
Zero-energy, small transmissions:

$$g_{12}(0) = G_{12}(0,0) = \frac{r_{\xi_S} r_{N_1} r_{N_2}}{2R_1^2 R_2^2} e^{-|x_2-x_1|/\xi_S}$$

Fully open barriers:

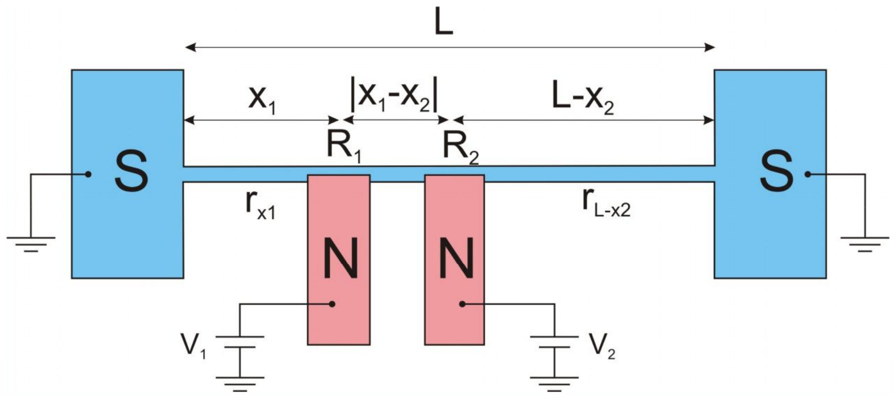
$$g_{12}(\varepsilon) = \frac{\Omega^2}{\Delta^2}\frac{2r_{\xi_S(\varepsilon)}}{R_1 R_2} e^{-|x_2-x_1|/\xi_S(\varepsilon)}, \quad |\varepsilon| < \Delta.$$

Non-local resistance:



$$R_{12}(T) = \frac{G_{12}(0,T)}{G_{11}(0,T)G_{22}(0,T) - G_{12}(0,T)G_{21}(0,T)}$$

Non-local resistance:



$$R_{12}(T) = \frac{G_{12}(0,T)}{G_{11}(0,T)G_{22}(0,T) - G_{12}(0,T)G_{21}(0,T)}$$

$T \ll \Delta$ $\longrightarrow$ $\boxed{R_{12} = \frac{r_{\xi_S}}{2} e^{-|x_2 - x_1|/\xi_S}}$

Charge imbalance resistance peak

$$R_{12}(T) = \frac{G_{12}(T)}{G_{11}(T)G_{22}(T) - G_{12}(T)G_{21}(T)},$$

Quasi-ballistic limit:

$$G_{11}(T) = G_{11}(0) + G_{N_{11}}\sqrt{2\pi}\sqrt{\frac{\Delta}{T}}e^{-\Delta/T},$$

$$G_{12}(T) = \begin{cases} 2G_{N_{12}}e^{-\Delta/T}, & \mathcal{D}_1\mathcal{D}_2 \ll e^{-\Delta/T}, \\ G_{12}(0), & T \ll \dfrac{\Delta}{\ln(1/[\mathcal{D}_1\mathcal{D}_2])}, \end{cases}$$

Charge imbalance resistance peak

$$R_{12}(T) = \frac{2R_{N_{12}} e^{-\Delta/T}}{\left[G_{11}(0)/G_{N_{11}} + \sqrt{2\pi\Delta/T} e^{-\Delta/T}\right]} \times$$

$$\times \frac{1}{\left[G_{22}(0)/G_{N_{22}} + \sqrt{2\pi\Delta/T} e^{-\Delta/T}\right]},$$

local Andreev

quasiparticles
(charge imbalance)

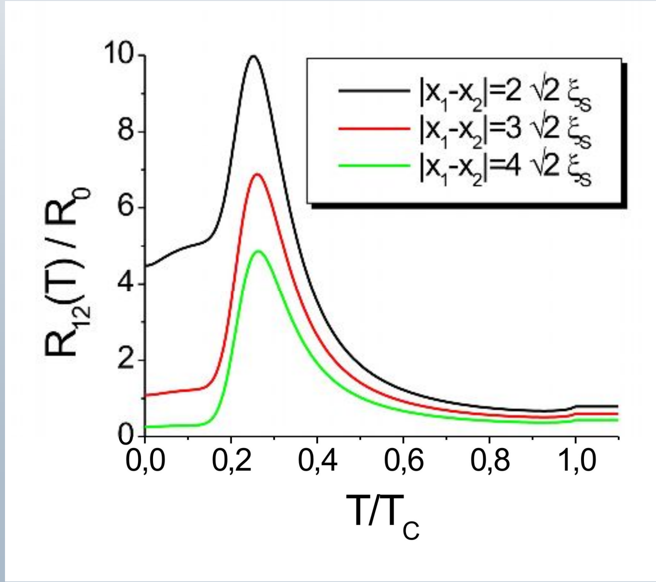
$$R_{N_{12}} = G_{N_{12}}/(G_{N_{11}} G_{N_{22}})$$

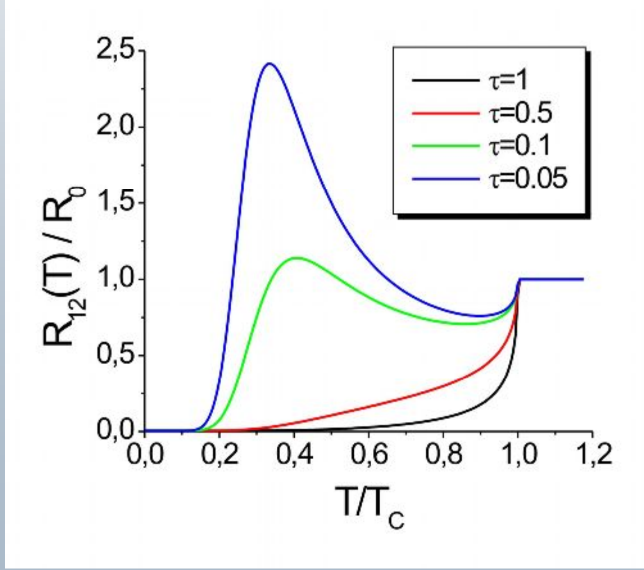
Diffusive limit:

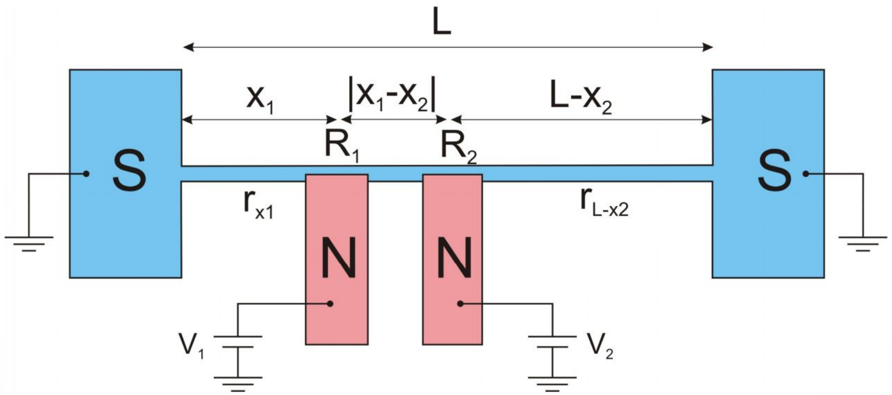
$$T^* \simeq 2\Delta / \ln(R_1 R_2 / r_{\xi_S}^2)$$

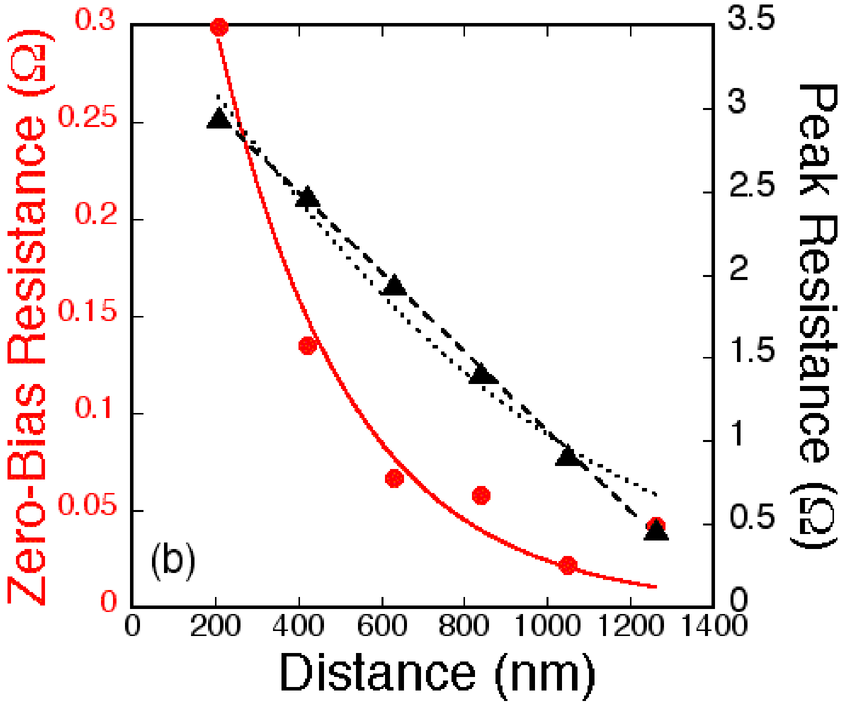
$$R_{12}(T^*) \approx \frac{\alpha r_{x_1}\sqrt{\frac{8T^*}{\pi\Delta}}\left(1-\frac{|x_2-x_1|}{\lambda}\right)}{\left(\sqrt{\frac{r_{\xi_S}+r_{\xi_1}(T^*)}{R_1}}+\sqrt{\frac{r_{\xi_S}+r_{\xi_2}(T^*)}{R_2}}\right)^2},$$

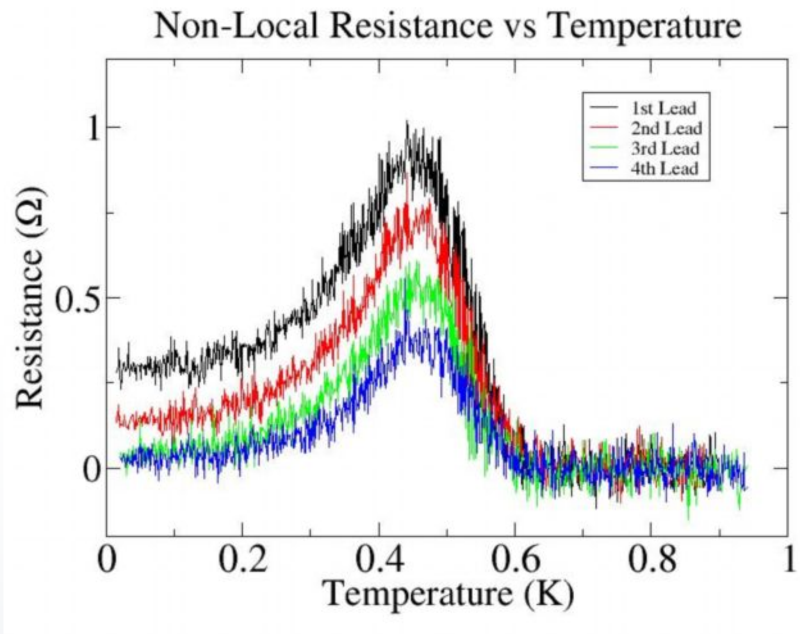
where $\lambda = \alpha L$ and $\alpha = 1 - r_{x_1}/r_L$











$$R_{12} = \frac{r_{\xi_S}}{2}\, e^{-|x_2 - x_1|/\xi_S}$$

$$R_{12}(T^*) \approx \frac{\alpha r_{x_1} \sqrt{\frac{8T^*}{\pi\Delta}} \left(1 - \frac{|x_2 - x_1|}{\lambda}\right)}{\left(\sqrt{\frac{r_{\xi_S} + r_{\xi_1}(T^*)}{R_1}} + \sqrt{\frac{r_{\xi_S} + r_{\xi_2}(T^*)}{R_2}}\right)^2},$$

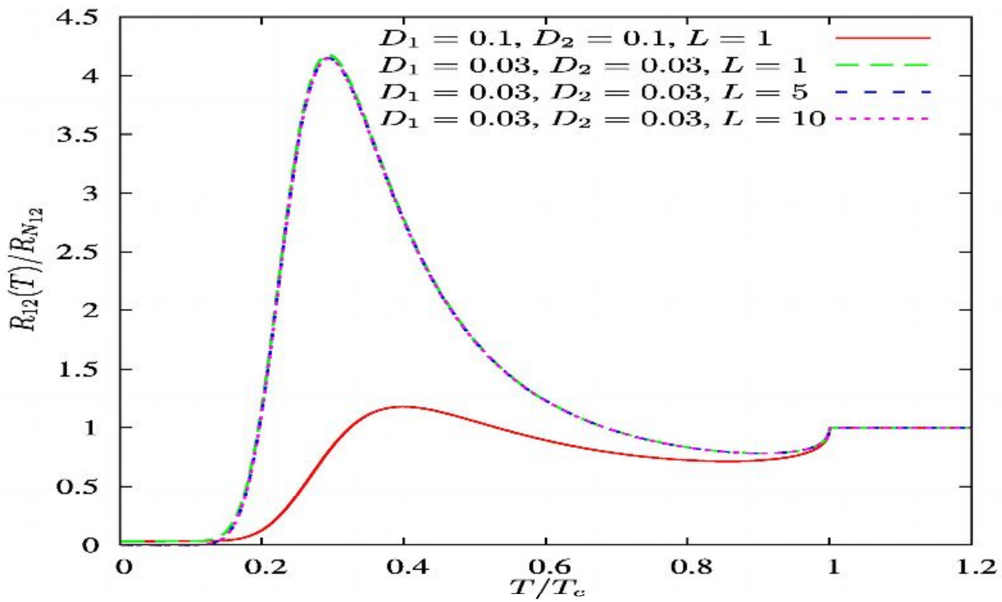
Summary and outlook

- Full quasiclassical theory of non-local transport in both ballistic and diffusive 3-terminal NSN structures with arbitrary interface transmissions
- Ballistic limit: Non-exponential length dependence at high transmissions, no CAR at full transmissions
- Diffusive structures: disorder-induced ZBA, universality of cross-resistance
- Quantitative explanation for the charge imbalance peak
- Next step: effect of e-e interactions

The End

Peak resistance:

$$\frac{R_{12}(T^*)}{R_{N_{12}}} = \frac{\sqrt{\frac{2T^*}{\pi\Delta}}}{\left[\sqrt{\frac{G_{11}(0)}{G_{N_{11}}}} + \sqrt{\frac{G_{22}(0)}{G_{N_{22}}}}\right]^2}.$$



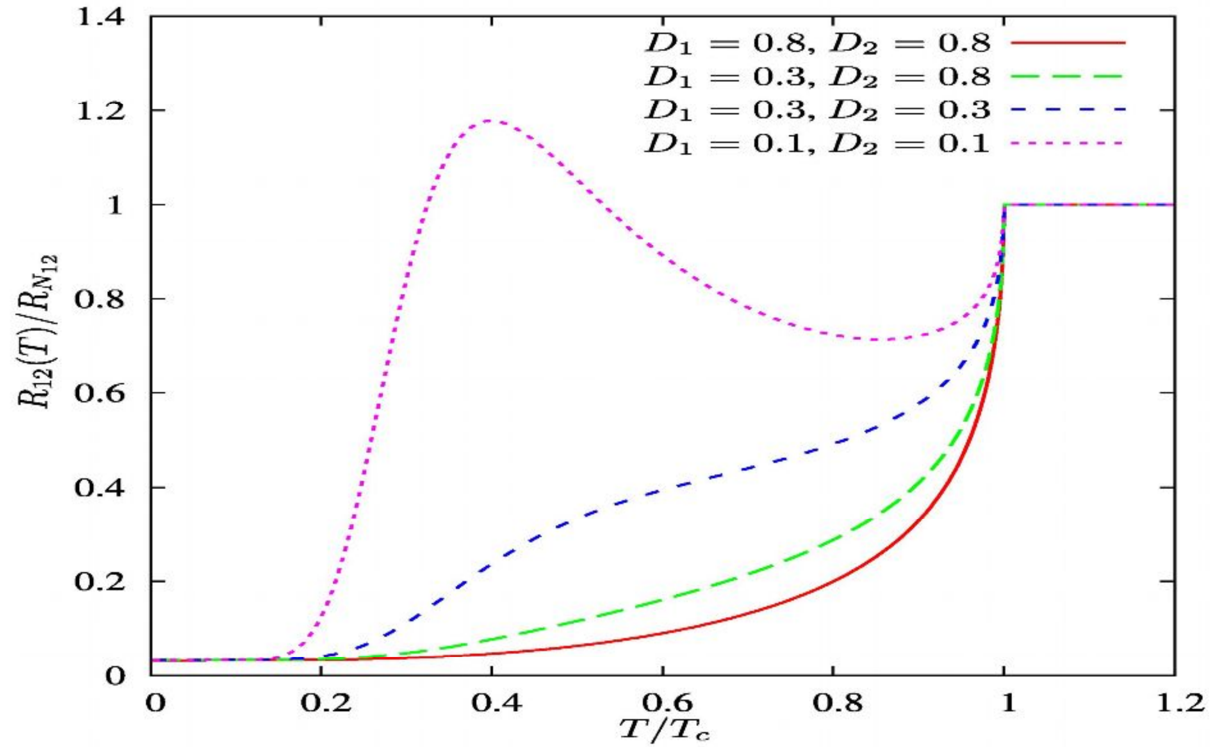
Peak temperature:

$$T^* \simeq \frac{\Delta}{\ln \sqrt{\dfrac{G_{N_{11}} G_{N_{22}}}{G_{11}(0) G_{22}(0)}}},$$

$$D_{1,2} = D$$

$$T^* \simeq \Delta / \ln(1/D)$$

Charge imbalance resistance peak



Cross-current

$$I_{12}(V_2) = -\frac{G_0}{4e}\int d\varepsilon \left[\tanh\frac{\varepsilon + eV_2}{2T} - \tanh\frac{\varepsilon}{2T}\right]\frac{1-\tanh^2 iL\Omega/v_F}{W(z_1,z_2,\varepsilon,\varphi)}\times$$
$$\times\Big\{\left[D_{1\downarrow}D_{2\downarrow} - |a|^2 D_{1\uparrow}D_{2\downarrow}(R_{1\downarrow}+R_{2\uparrow}) + |a|^4 D_{1\downarrow}R_{1\uparrow}D_{2\downarrow}R_{2\uparrow}\right]|K(z_1,z_2,\varepsilon)|^2\cos^2(\varphi/2)+$$
$$+\left[D_{1\uparrow}D_{2\uparrow} - |a|^2 D_{1\downarrow}D_{2\uparrow}(R_{1\uparrow}+R_{2\downarrow}) + |a|^4 D_{1\uparrow}R_{1\downarrow}D_{2\uparrow}R_{2\downarrow}\right]|K(z_1^*,z_2^*,\varepsilon)|^2\cos^2(\varphi/2)+$$
$$+\left[D_{1\uparrow}D_{2\downarrow} - |a|^2 D_{1\downarrow}D_{2\downarrow}(R_{1\uparrow}+R_{2\uparrow}) + |a|^4 D_{1\uparrow}R_{1\downarrow}D_{2\downarrow}R_{2\uparrow}\right]|K(z_1^*,z_2,\varepsilon)|^2\sin^2(\varphi/2)+$$
$$+\left[D_{1\downarrow}D_{2\uparrow} - |a|^2 D_{1\uparrow}D_{2\uparrow}(R_{1\downarrow}+R_{2\downarrow}) + |a|^4 D_{1\downarrow}R_{1\uparrow}D_{2\uparrow}R_{2\downarrow}\right]|K(z_1,z_2^*,\varepsilon)|^2\sin^2(\varphi/2)\Big\},$$

where we define

$$K(z_1,z_2,\varepsilon) = (1-a^2 z_1 z_2) - \left[\varepsilon(1+a^2 z_1 z_2) + \Delta a(z_1+z_2)\right]Q,$$
$$W(z_1,z_2,\varepsilon,\varphi) = |K(z_1,z_2,\varepsilon)K(z_1^*,z_2^*,\varepsilon)\cos^2(\varphi/2) + K(z_1^*,z_2,\varepsilon)K(z_1,z_2^*,\varepsilon)\sin^2(\varphi/2)|^2$$

and $z_i = \sqrt{R_{i\uparrow}R_{i\downarrow}}\exp(i\theta_i)$ ($i=1,2$).

Polarized interfaces

Linear conductance at T=0, zero spin mixing

$$G_{12} = G_0 \frac{1 - \tanh^2 L\Delta/v_F}{|K(z_1, z_2, 0)|^2} \{ D_{1\uparrow} D_{1\downarrow} D_{2\uparrow} D_{2\downarrow} + \\ + (D_{1\uparrow} - D_{1\downarrow})(D_{2\uparrow} - D_{2\downarrow}) \cos\varphi \}.$$

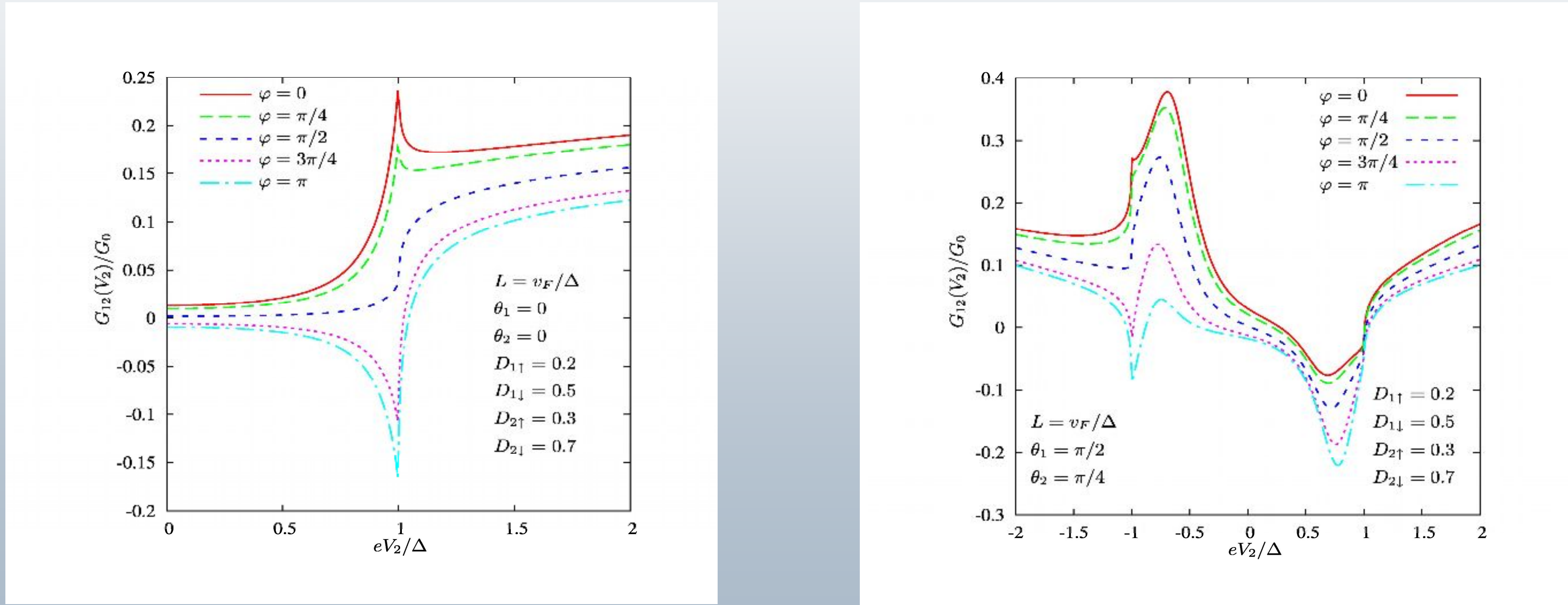
Vanishes identically for

half-metal/superconductor/N-metal structures

Polarized interfaces

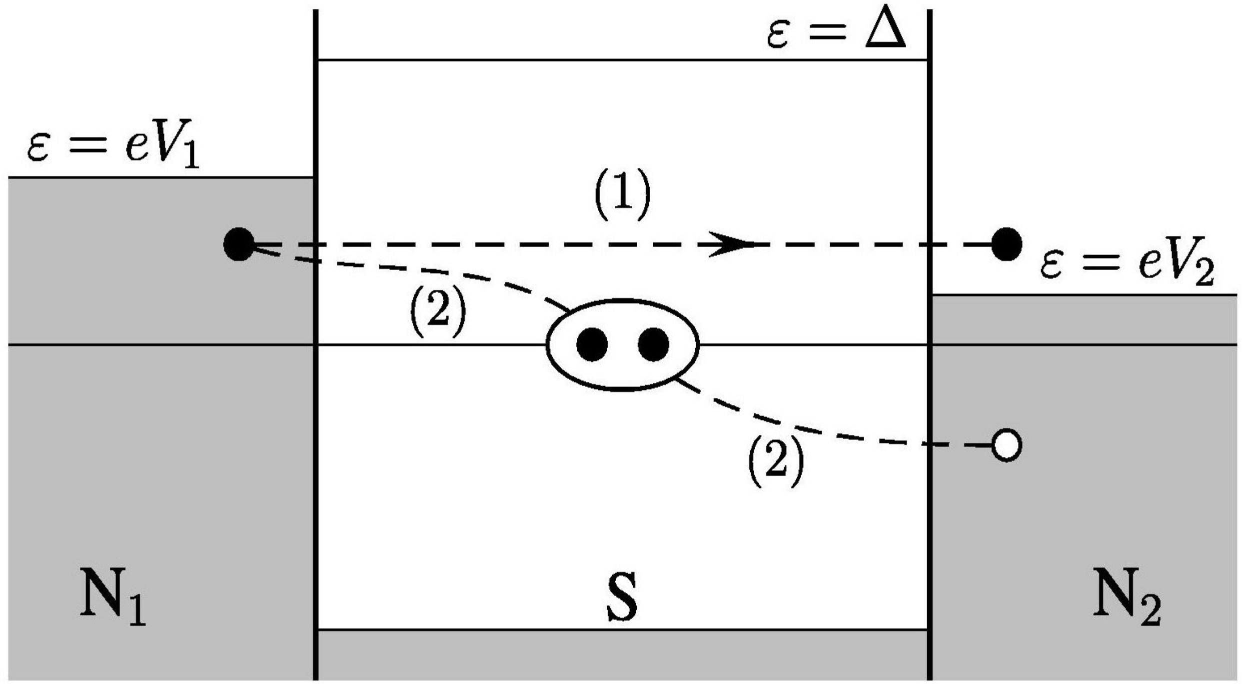
Linear conductance at T=0, zero spin mixing

$$G_{12} = G_0 \frac{1-\tanh^2 L\Delta/v_F}{|K(z_1,z_2,0)|^2}\{D_{1\uparrow}D_{1\downarrow}D_{2\uparrow}D_{2\downarrow} + \\ + (D_{1\uparrow}-D_{1\downarrow})(D_{2\uparrow}-D_{2\downarrow})\cos\varphi\}.$$



Spin-degenerate case:

$$\frac{\delta G_{11}}{G_{N_{12}}} = \frac{2(1+\mathcal{R}_2)(1-\tanh^2 L\Delta/v_F)}{[1+\mathcal{R}_1\mathcal{R}_2+(\mathcal{R}_1+\mathcal{R}_2)\tanh L\Delta/v_F]^2}$$
$$+\frac{\mathcal{D}_1\left[(1+\mathcal{R}_2\tanh L\Delta/v_F)^2+3(\mathcal{R}_2+\tanh L\Delta/v_F)^2\right]}{\mathcal{D}_2[1+\mathcal{R}_1\mathcal{R}_2+(\mathcal{R}_1+\mathcal{R}_2)\tanh L\Delta/v_F]^2}$$



Non-local correction to BTK

$$I_{11}(V_1) = \frac{G_0}{2e}\int d\varepsilon (h_0(\varepsilon+eV_1)-h_0(\varepsilon))\frac{1}{W(z_1,z_2,\varepsilon,\varphi)}\Big\{2W(z_1,z_2,\varepsilon,\varphi)-$$

$$-R_{1\uparrow}\left|\cos^2(\varphi/2)K(z_1/R_{1\uparrow},z_2,\varepsilon)K(z_1^*,z_2^*,\varepsilon)+\sin^2(\varphi/2)K(z_1/R_{1\uparrow},z_2^*,\varepsilon)K(z_1^*,z_2,\varepsilon)\right|^2-$$

$$-R_{1\downarrow}\left|\cos^2(\varphi/2)K(z_1^*/R_{1\downarrow},z_2^*,\varepsilon)K(z_1,z_2,\varepsilon)+\sin^2(\varphi/2)K(z_1^*/R_{1\downarrow},z_2,\varepsilon)K(z_1,z_2^*,\varepsilon)\right|^2\Big\}+$$

$$+\frac{G_0}{4e}\int d\varepsilon (h_0(\varepsilon+eV_1)-h_0(\varepsilon))\frac{D_{1\uparrow}D_{1\downarrow}}{W(z_1,z_2,\varepsilon,\varphi)}\Big\{$$

$$|a|^2\left|\cos^2(\varphi/2)K(0,z_2,\varepsilon)K(z_1^*,z_2^*,\varepsilon)+\sin^2(\varphi/2)K(0,z_2^*,\varepsilon)K(z_1^*,z_2,\varepsilon)\right|^2+$$

$$+|a|^2\left|\cos^2(\varphi/2)K(0,z_2^*,\varepsilon)K(z_1,z_2,\varepsilon)+\sin^2(\varphi/2)K(0,z_2,\varepsilon)K(z_1,z_2^*,\varepsilon)\right|^2+$$

$$+\frac{1}{|a|^2}\left|\cos^2(\varphi/2)K'(z_2^*,\varepsilon)K(z_1,z_2,\varepsilon)+\sin^2(\varphi/2)K'(z_2,\varepsilon)K(z_1,z_2^*,\varepsilon)\right|^2+$$

$$+\frac{1}{|a|^2}\left|\cos^2(\varphi/2)K'(z_2,\varepsilon)K(z_1^*,z_2^*,\varepsilon)+\sin^2(\varphi/2)K'(z_2^*,\varepsilon)K(z_1^*,z_2,\varepsilon)\right|^2\Big\}+$$

$$+\frac{G_0}{e}R_{2\uparrow}R_{2\downarrow}\sin^2(\theta_2/2)\sin^2(\varphi/2)\cos^2(\varphi/2)\int d\varepsilon (h_0(\varepsilon+eV_1)-h_0(\varepsilon))\frac{(1-\tanh^2 iL\Omega/v_F)^2}{W(z_1,z_2,\varepsilon,\varphi)}\times$$

$$\times\left[|a|^2(D_{1\uparrow}^2+D_{1\downarrow}^2)-2|a|^4D_{1\uparrow}D_{1\downarrow}(R_{1\uparrow}+R_{1\downarrow})+|a|^6(D_{1\uparrow}^2R_{1\downarrow}^2+D_{1\downarrow}^2R_{1\uparrow}^2)\right],$$

$$G_{11} = G^{BTK} + \delta G_{11}$$

$$D_1 << 1, \quad D_2 \sim 1$$

$$\delta G_{11} \sim D_1 >> G^{BTK} \sim D_1^2$$