Superconducting non-Abelian strings in Weinberg-Salam theory – electroweak thunderbolts

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- Constructing new vortex solutions in the electroweak theory and studying their stability

M.S.V. Phys.Lett. B648, 249 (2007)
J.Garaud and M.S.V. Nucl.Phys. B799, 430 (2008)
J.Garaud and M.S.V. hep-ph/0905.xxxx

superconductivity – constant currents due to non-zero values of scalar fields (condensate)

Witten's U(1)×U(1) model '85

$$\begin{aligned}\mathcal{L}_W = & - \frac{1}{4}(F^{(1)}_{\mu\nu})^2 + |D_\mu\phi_1|^2 - \frac{\lambda_1}{4}\left(|\phi_1|^2 - \eta_1^2\right)^2 \\ & - \frac{1}{4}(F^{(2)}_{\mu\nu})^2 + |D_\mu\phi_2|^2 - \frac{\lambda_2}{4}\left(|\phi_2|^2 - \eta_2^2\right)^2 \\ & - \gamma|\phi_1|^2|\phi_2|^2,\end{aligned}$$

Vacuum: $|\phi_1| = \eta_1,\ \phi_2 = 0 \Rightarrow A^{(2)}_\mu$ is massless

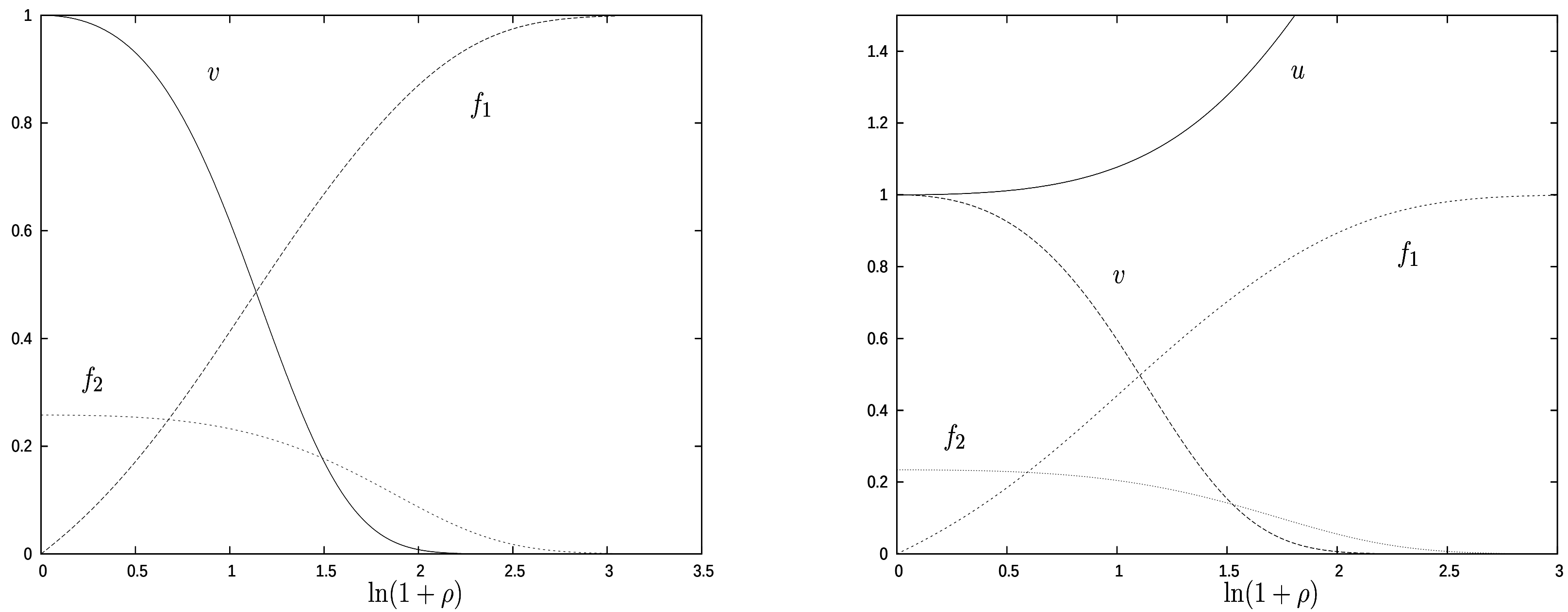
- $A^{(2)}_\mu = \phi_2 = 0 \Rightarrow$ ANO vortex made of $A^{(1)}_\mu, \phi_1$
- Unstable, relaxes to dressed vortex with $\phi_2 \neq 0$
- Phase excitation of $\phi_2 \neq 0 \Rightarrow$ superconducting string

$$J_\mu = \partial^\nu F^{(2)}_{\nu\mu} \neq 0$$

Witten's superconducting strings

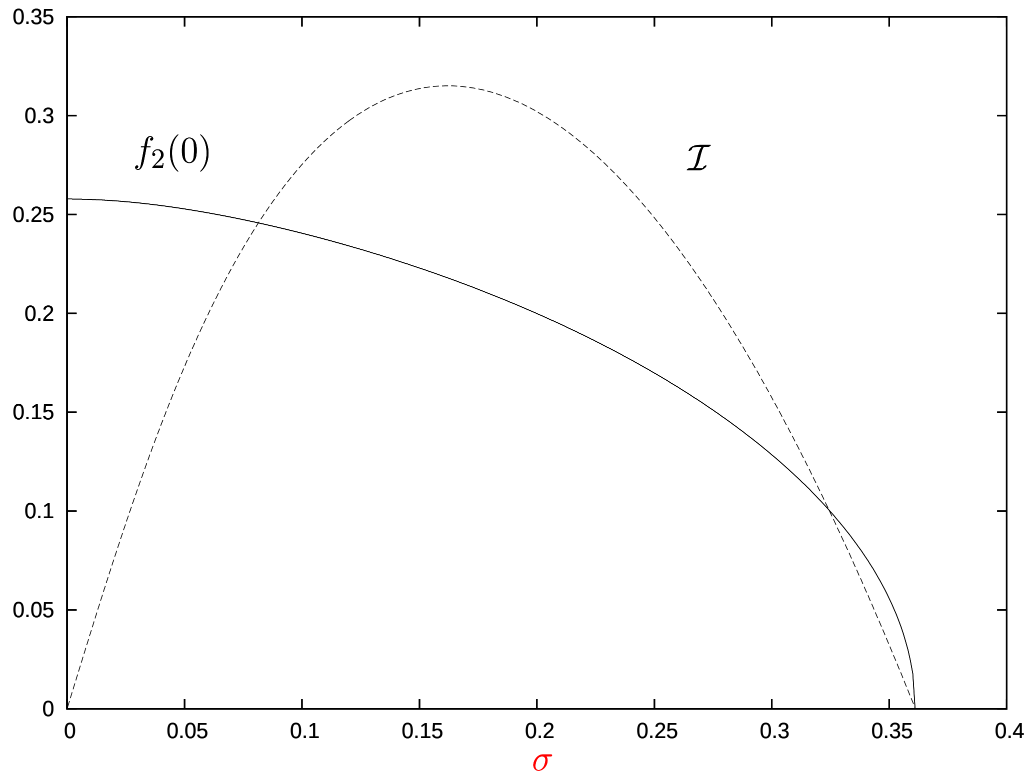
$$A^{(1)} = (n - v(\rho))\, d\varphi, \quad \phi_1 = f_1(\rho) e^{in\varphi},$$

$$A^{(2)} = (\sigma_0 dt + \sigma_3 dz)\, (1 - u(\rho)), \quad \phi_2 = f_2(\rho) e^{i\sigma_0 t + i\sigma_3 z},$$



twist $\sigma^2 = \sigma_3^2 - \sigma_0^2 > 0$ magneitc; $\sigma^2 < 0$ electric; $\sigma^2 = 0$ chiral

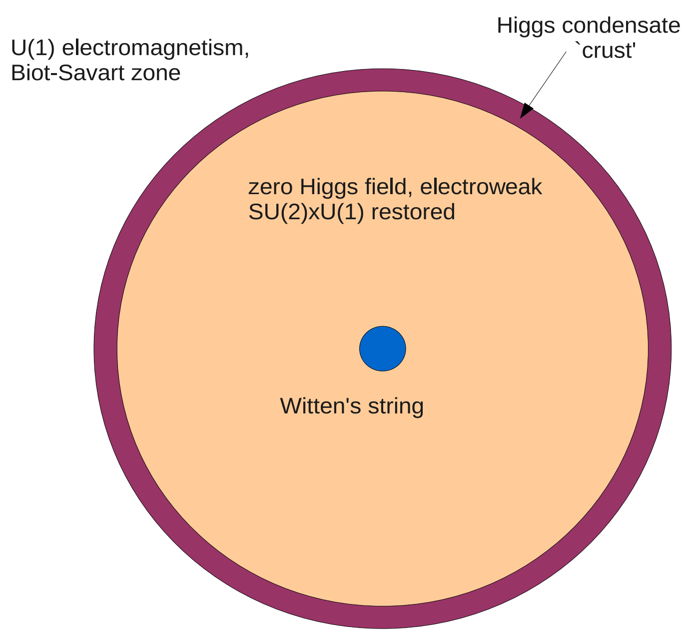
Current quenching



Witten's model $\in$ GUT theories $\Rightarrow \mathcal{I}_{\max} \sim 10^{20}$ Amperes
$\Rightarrow$ cosmological applications

Electroweak vacuum polarization

Higgs vacuum for $B < m_w^2/e$
electroweak condensate for $m_w^2/e < B < m_h^2/e$
symmetry restoration for $B > m_h^2/e$



Ambjorn and Olesen '89

What about electroweak theory ?

- It contains two complex scalars that could be the vortex field and condensate field
- U(1)$\times$U(1) is contained in SU(2)$\times$U(1)
- Z strings = embedded ANO vortices **/Vachaspati '93/**
- unstable but non-topological $\Rightarrow$ relax to zero
- no dressed strings **/Achucarro et al '94/**
- $\Rightarrow$ no superconducting strings ???

loophole: one can have superconducting strings without dressed strings

Weinberg-Salam theory

$$\mathcal{L} = -\frac{1}{4g^2}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} - \frac{1}{4g'^2}\,B_{\mu\nu}B^{\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2,$$

$$\mathrm{W}^a_{\mu\nu} = \partial_\mu \mathrm{W}^a_\nu - \partial_\nu \mathrm{W}^a_\mu + \epsilon_{abc}\mathrm{W}^b_\mu \mathrm{W}^c_\nu\,, \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\,,$$

$$\Phi = \begin{pmatrix} \phi^1 \\ \phi^2 \end{pmatrix}, \qquad D_\mu\Phi = \left(\partial_\mu - \frac{i}{2}\,A_\mu - \frac{i}{2}\,\tau^a \mathrm{W}^a_\mu\right)\Phi.$$

$$g = \cos\theta_{\mathsf{W}}, \qquad g' = \sin\theta_{\mathsf{W}}, \quad m_{\mathsf{Z}} = 1/\sqrt{2},$$

$$m_{\mathsf{W}} = m_{\mathsf{Z}}\cos\theta_{\mathsf{W}}, \quad \beta = \left(\frac{m_{\mathsf{h}}}{m_{\mathsf{Z}}}\right)^2 \qquad \boxed{1.5 \le \beta \le 3.5}$$

Field equations

$$\partial_\mu B^{\mu\nu} = g'^2\,\Re(i\Phi^\dagger D^\nu\Phi)\,,$$
$$\partial_\mu \mathrm{W}_a^{\mu\nu} + \epsilon_{abc}\mathrm{W}_\sigma^b\mathrm{W}^{c\sigma\nu} = g^2\,\Re(i\Phi^\dagger\tau^a D^\nu\Phi)\,,$$
$$D_\mu D^\mu\Phi = \frac{\beta}{4}(\Phi^\dagger\Phi-1)\Phi.$$

$n^a = \Phi^\dagger\tau^a\Phi/(\Phi^\dagger\Phi) \Rightarrow$ electromagnetic, Z fields **/Nambu '77/**

$$F_{\mu\nu} = \frac{g}{g'}\,B_{\mu\nu} - \frac{g'}{g}\,n^a\mathrm{W}^a_{\mu\nu}\,, \qquad Z_{\mu\nu} = B_{\mu\nu} + n^a\mathrm{W}^a_{\mu\nu}\,,$$

$\Rightarrow$ electromagnetic current density

$$J_\mu = \partial^\nu F_{\nu\mu}$$

Vortex symmetries

symmetry generators

$$K_{(t)} = \frac{\partial}{\partial t}, \qquad K_{(z)} = \frac{\partial}{\partial z}, \qquad K_{(\varphi)} = \frac{\partial}{\partial \varphi}$$

$\Rightarrow$ energy, momentum, angular momentum

$$\int T^0_\mu K^\mu_{(t)} d^2x, \qquad \int T^0_\mu K^\mu_{(z)} d^2x, \qquad \int T^0_\mu K^\mu_{(\varphi)} d^2x,$$

electric charge and current $(\alpha = 0, 3)$

$$\mathcal{I}^\alpha = \int J^\alpha d^2x$$

Field ansatz

Symmetries commute $\Rightarrow\ \exists$ a gauge where the fields depend only on ρ. With $\sigma_\alpha = (\sigma_0, \sigma_3)$

$$\mathcal{W} = u(\rho)\,\sigma_\alpha dx^\alpha - v(\rho)\,d\varphi + \tau^1\,[u_1(\rho)\,\sigma_\alpha dx^\alpha - v_1(\rho)\,d\varphi]$$

$$+\,\tau^3\,[u_3(\rho)\,\sigma_\alpha dx^\alpha - v_3(\rho)\,d\varphi], \qquad \Phi = \begin{pmatrix} f_1(\rho) \\ f_2(\rho) \end{pmatrix}$$

- $\mathcal{W}_\rho = 0$ – gauge condition
- $\mathcal{W} = \mathcal{W}^*, \qquad \Phi = \Phi^*$
- Boosts along $z = x^3$ axis
- Residual global symmetry $(f_1 + if_2) \to e^{\frac{i}{2}\Gamma}(f_1 + if_2)$, $(u_1 + iu_3) \to e^{-i\Gamma}(u_1 + iu_3)$, $(v_1 + iv_3) \to e^{-i\Gamma}(v_1 + iv_3)$

U(1) + Higgs equations

$$\frac{1}{\rho}(\rho u')' = \frac{g'^{\,2}}{2}\left\{ (u+u_3)f_1^2 + 2\,u_1 f_1 f_2 + (u-u_3)f_2^2 \right\},$$

$$\rho\left(\frac{v'}{\rho}\right)' = \frac{g'^{\,2}}{2}\left\{ (v+v_3)f_1^2 + 2\,v_1 f_1 f_2 + (v-v_3)f_2^2 \right\},$$

$$\frac{1}{\rho}(\rho f_1')' = \left\{ \frac{\sigma^2}{4}\left[(u+u_3)^2 + u_1^2\right] + \frac{1}{4\rho^2}\left[(v+v_3)^2 + v_1^2\right] + \frac{\beta}{4}(f_1^2 + f_2^2 - 1)\right\} f_1$$
$$+ \left(\frac{\sigma^2}{2}\,uu_1 + \frac{1}{2\rho^2}\,vv_1\right) f_2,$$

$$\frac{1}{\rho}(\rho f_2')' = \left\{ \frac{\sigma^2}{4}\left[(u-u_3)^2 + u_1^2\right] + \frac{1}{4\rho^2}\left[(v-v_3)^2 + v_1^2\right] + \frac{\beta}{4}(f_1^2 + f_2^2 - 1)\right\} f_2$$
$$+ \left(\frac{\sigma^2}{2}\,uu_1 + \frac{1}{2\rho^2}\,vv_1\right) f_1.$$

$$\sigma^2 = \sigma_3^2 - \sigma_0^2$$

SU(2) equations

$$
\begin{aligned}
\frac{1}{\rho}(\rho u_1')' &= -\frac{1}{\rho^2}\,(v_1 u_3 - v_3 u_1)\,v_3 + \frac{g^2}{2}\left[u_1(f_1^2+f_2^2)+2uf_1f_2\right], \\
\frac{1}{\rho}(\rho u_3')' &= +\frac{1}{\rho^2}\,(v_1 u_3 - v_3 u_1)\,v_1 + \frac{g^2}{2}\left[(u_3+u)f_1^2+(u_3-u)f_2^2\right], \\
\rho\left(\frac{v_1'}{\rho}\right)' &= +\sigma^2\,(v_1 u_3 - v_3 u_1)\,u_3 + \frac{g^2}{2}\left[v_1(f_1^2+f_2^2)+2vf_1f_2\right], \\
\rho\left(\frac{v_3'}{\rho}\right)' &= -\sigma^2\,(v_1 u_3 - v_3 u_1)\,u_1 + \frac{g^2}{2}\left[(v_3+v)f_1^2+(v_3-v)f_2^2\right].
\end{aligned}
$$

Boundary conditions

- At the symmetry axis, $\rho = 0$, the fields are regular, energy density is finite.
- At infinity, $\rho \to \infty$, one has the Biot-Savart field of an infinitely long electric wire:

$$A_\mu = \frac{Q}{gg'}\,\sigma_\alpha dx^\alpha \ln\frac{\rho}{\rho_0} + C\,d\varphi$$

$$\Rightarrow Z_\mu = 0, \quad \mathbf{W}^\pm_\mu = 0, \quad \Phi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

The current of the wire

$$\mathcal{I}_\alpha = -\frac{2\pi Q}{gg'}\,\sigma_\alpha$$

Local solutions at the origin

$$u = a_1 + \ldots, \quad u_1 = a_2\rho^{\nu} + \ldots, \quad u_3 = 1 + \ldots,$$

$$v_1 = O(\rho^{\nu+2}), \quad v_3 = \nu + a_3\,\rho^2 + \ldots, \quad v = 2n - \nu + a_4\,\rho^2 + \ldots,$$

$$f_1 = a_5\,\rho^n + \ldots, \quad f_2 = q\rho^{|n-\nu|} + \ldots,$$

$n, \nu \in \mathbb{Z}$. Regular gauge

$$\mathcal{W} = \left\{u(\rho) + 1 + \tau^1_\psi\, u_1(\rho) + \tau^3[u_3(\rho) - 1]\right\}\,\sigma_\alpha dx^\alpha$$
$$+ \left\{2n - \nu - v(\rho) - \tau^1_\psi\, v_1(\rho) + \tau^3\,[\nu - v_3(\rho)]\right\}\,d\varphi,$$

$$\Phi = \left[\begin{array}{c} e^{in\varphi} f_1(\rho) \\ e^{i(n-\nu)\varphi + i\,\sigma_\alpha x^\alpha} f_2(\rho) \end{array}\right]$$

Infinity = Biot-Savart+corrections

$$u = Q\ln\rho + c_1 + \frac{c_3 g'^2}{\sqrt{\rho}}\,e^{-m_{\mathsf{z}}\rho} + \ldots$$

$$v = c_2 + c_4 g'^2 \sqrt{\rho}\,e^{-m_{\mathsf{z}}\rho} + \ldots$$

$$u_1 + iu_3 = e^{-i\gamma}\left\{\frac{c_7}{\sqrt{\rho}}\,e^{-\int m_\sigma d\rho} + i\left[-Q\ln\rho - c_1 + \frac{c_3 g^2}{\sqrt{\rho}}\,e^{-m_{\mathsf{z}}\rho}\right]\right\} + \ldots$$

$$v_1 + iv_3 = e^{-i\gamma}\left\{c_8\sqrt{\rho}\,e^{-\int m_\sigma d\rho} + i\left[-c_2 + c_4 g^2 \sqrt{\rho}\,e^{-m_{\mathsf{z}}\rho}\right]\right\} + \ldots$$

$$f_1 + if_2 = e^{\frac{i}{2}\gamma}\left\{1 + \frac{c_5}{\sqrt{\rho}}\,e^{-m_{\mathsf{h}}\rho} + i\,\frac{c_6}{\sqrt{\rho}}\,e^{-\int m_\sigma d\rho}\right\} + \ldots$$

$$m_\sigma = \sqrt{m_{\mathsf{w}}^2 + \sigma^2 (Q\ln\rho + c_1)^2} \quad \Rightarrow \sigma^2 \geq 0$$

Global solutions

- numerically propagating the local solutions at small and large ρ and matching them at $\rho \sim 1$ within the multiple shooting method.
- there are 16 matching conditions and 17 parameters to resolve them: $a_1, \ldots, a_5$ and q at the origin, also $c_1, \ldots, c_8$, Q, γ at infinity and also σ^2.
- there is one parameter left to label the global solutions: $q = f_2(0)$.

$q = 0 \Rightarrow$ zero current Z strings

$q = f_2(\rho) = 0 \Rightarrow$ Z strings

$$\mathcal{W}_Z = 2(g'^2 + g^2\tau^3)(n - v_{\mathsf{ANO}}(\rho))\, d\varphi, \quad \Phi_Z = \begin{pmatrix} e^{in\varphi} f_{\mathsf{ANO}}(\rho) \\ 0 \end{pmatrix}.$$

$$\frac{1}{\rho}(\rho f'_{\mathsf{ANO}})' = \left(\frac{v^2_{\mathsf{ANO}}}{\rho^2} + \frac{\beta}{4}(f^2_{\mathsf{ANO}} - 1)\right) f_{\mathsf{ANO}}\,,$$

$$\rho\left(\frac{v'_{\mathsf{ANO}}}{\rho}\right)' = \frac{1}{2} f^2_{\mathsf{ANO}}\, v_{\mathsf{ANO}},$$

$$0 \leftarrow f_{\mathsf{ANO}} \rightarrow 1, \quad n \leftarrow v_{\mathsf{ANO}} \rightarrow 0$$

$n = 1, 2, \ldots$

$q = f_2(0) \ll 1 \Rightarrow$ small currents

small Z string deformations, $(\mathcal{W}, \Phi) = (\mathcal{W}_Z, \Phi_Z) + (\delta\mathcal{W}, \delta\Phi)$,

$$(\delta\mathcal{W}, \delta\Phi) \sim e^{i\sigma_\alpha x^\alpha}\Psi(\rho)$$

$\Rightarrow$ eigenvalue problem

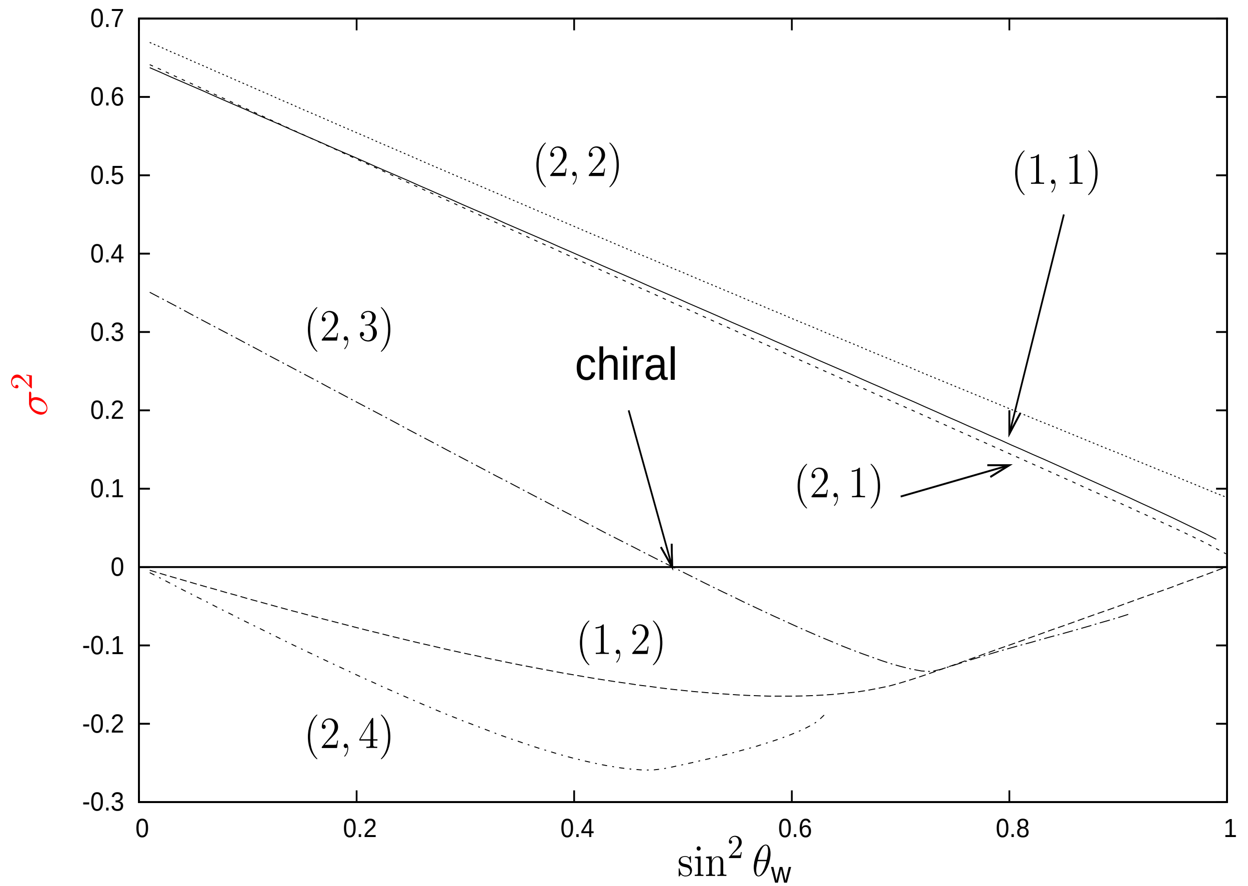
$$\Psi'' = (\sigma^2 + V_Z[\beta, \theta_{\mathrm{W}}, n, \nu, \rho])\Psi,$$

$\Rightarrow 2n$ bound states labeled by $\nu = 1, 2, \ldots 2n$

$$\Psi \sim \exp(-m_\sigma \rho), \qquad m_\sigma^2 = m_{\mathrm{W}}^2 + \sigma^2$$

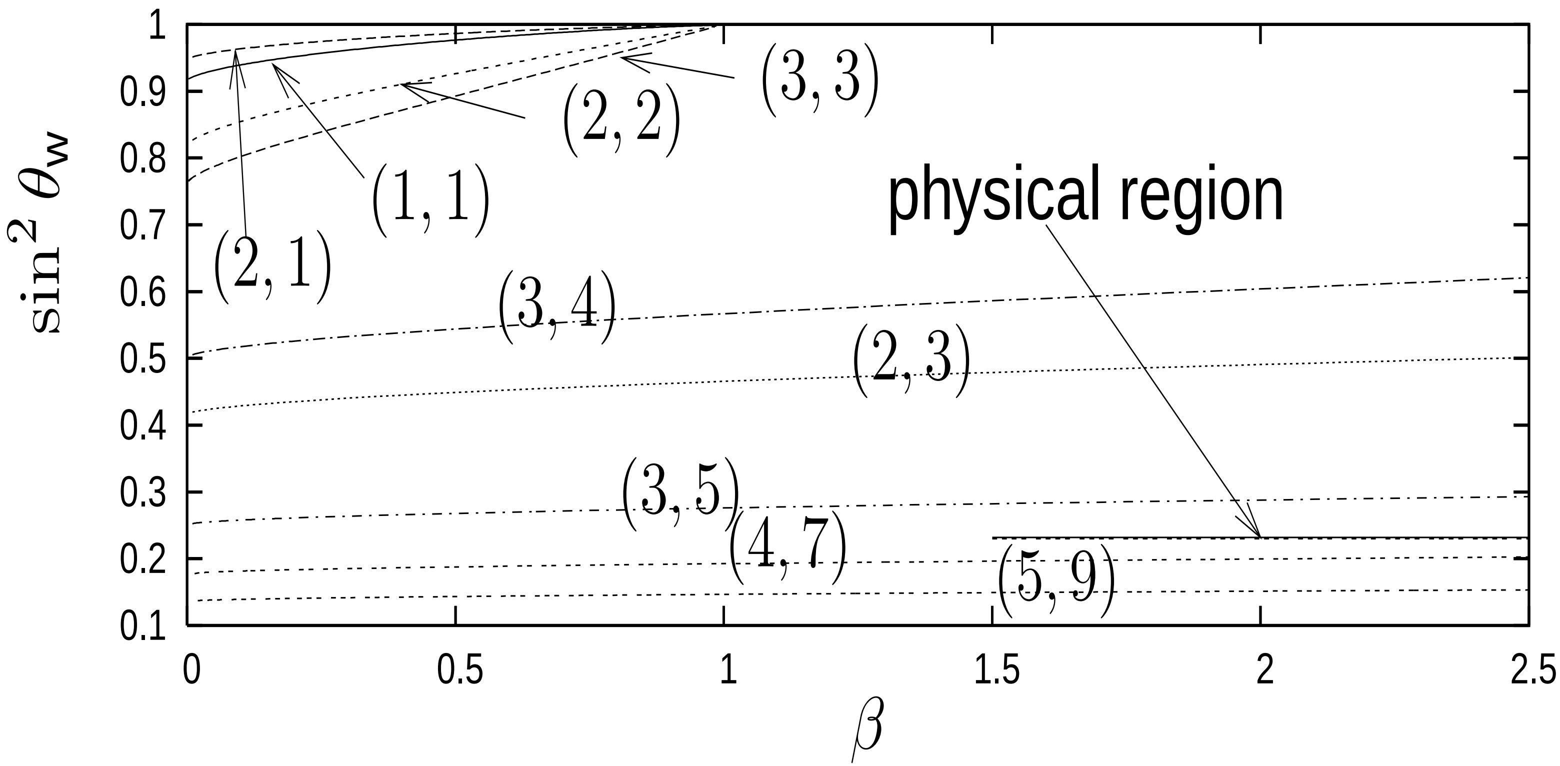
$\Rightarrow$ small deformations of Z strings by a spacelike ($\sigma^2 > 0$), timelke ($-m_{\mathrm{W}}^2 < \sigma^2 < 0$), or isotropic ($\sigma^2 = 0$) current $\mathcal{I}_\alpha \sim \sigma_\alpha$

$\sigma^2(n,\nu)$-eigenvalue $(\beta = 2)$

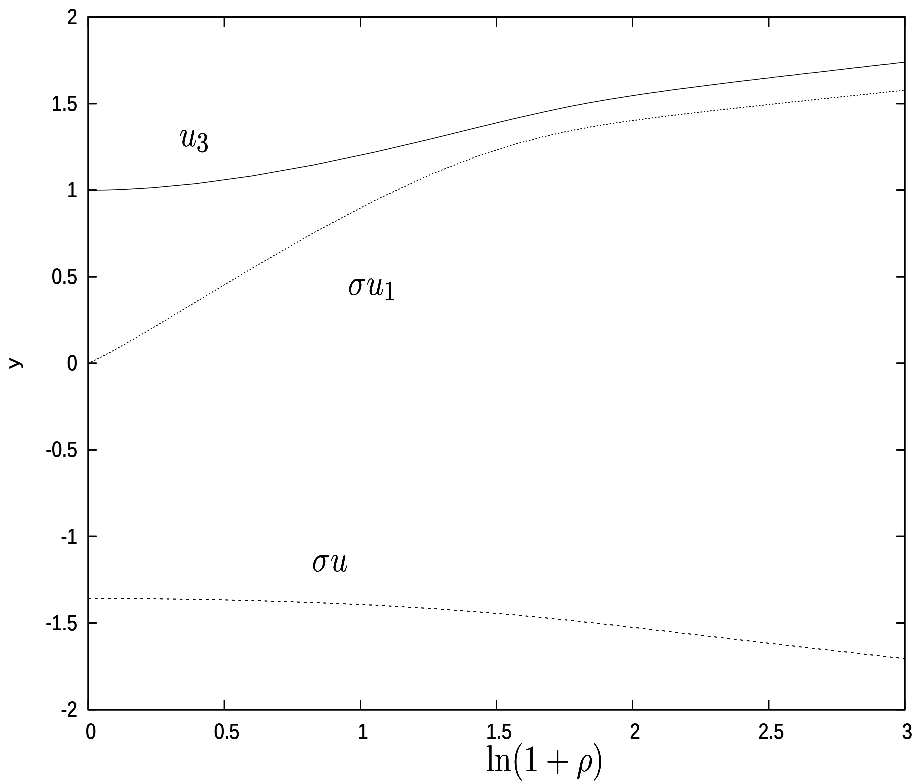


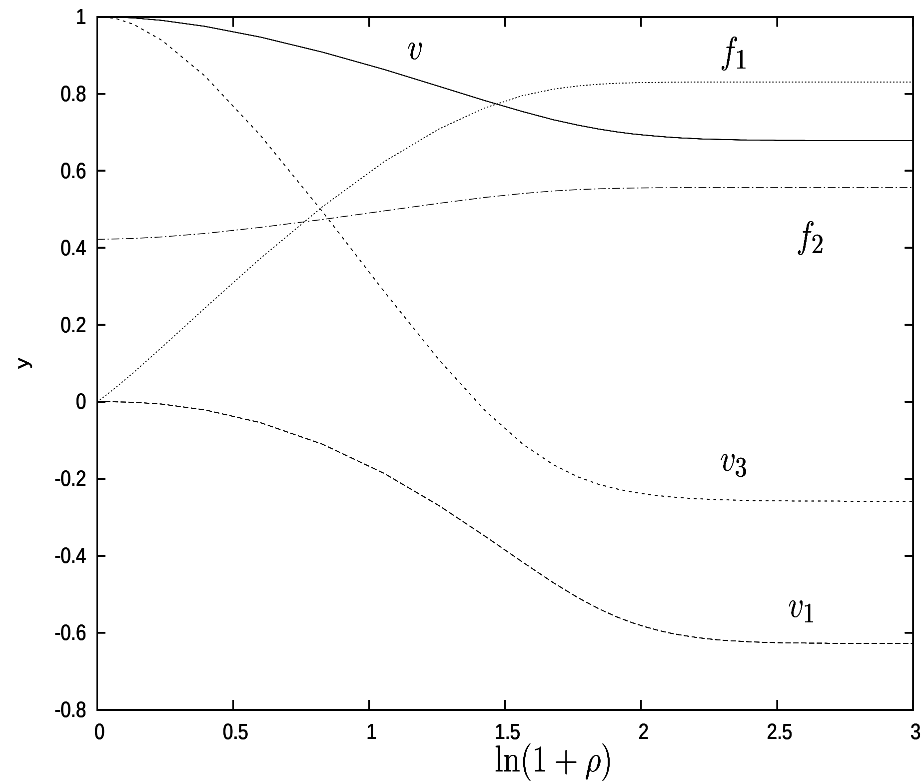
$\sigma^2 = 0$ $\exists$ only for special values of $\beta, \theta_{\mathrm{W}}, n, \nu$

Chiral solutions



Generic $q = f_2(0)$



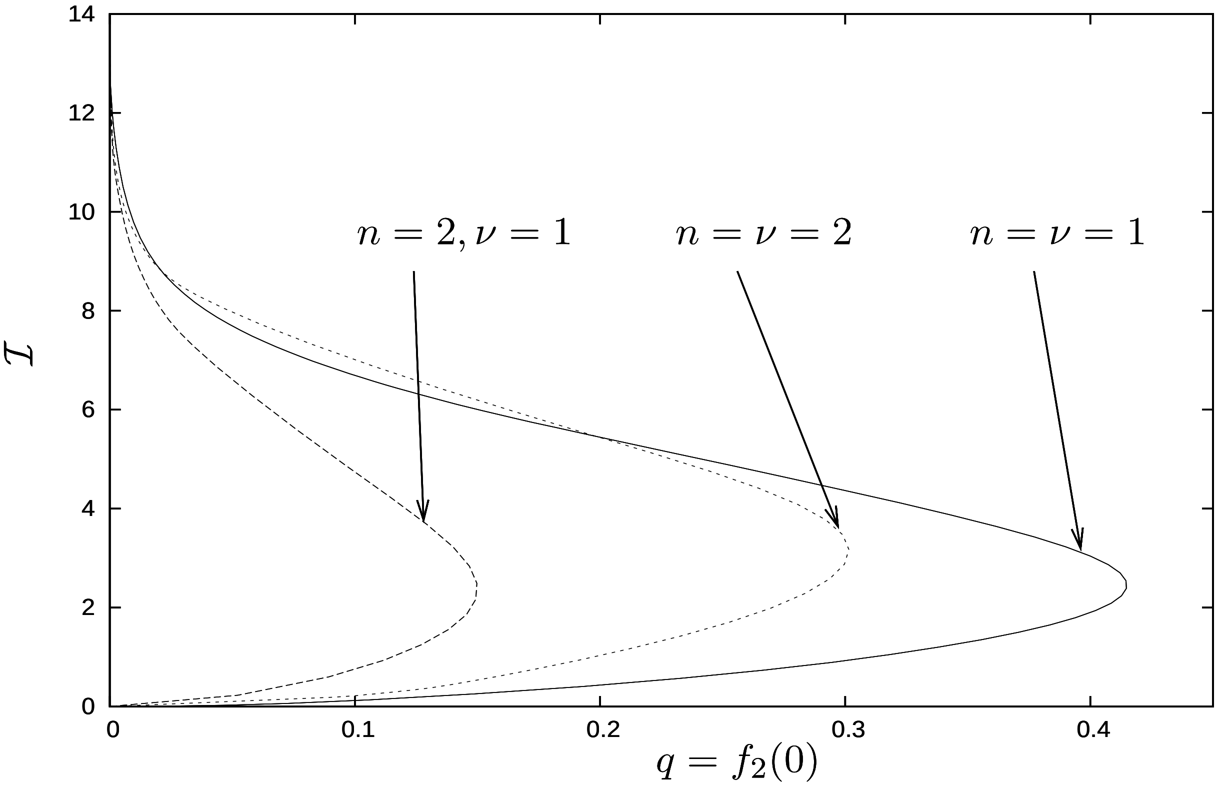


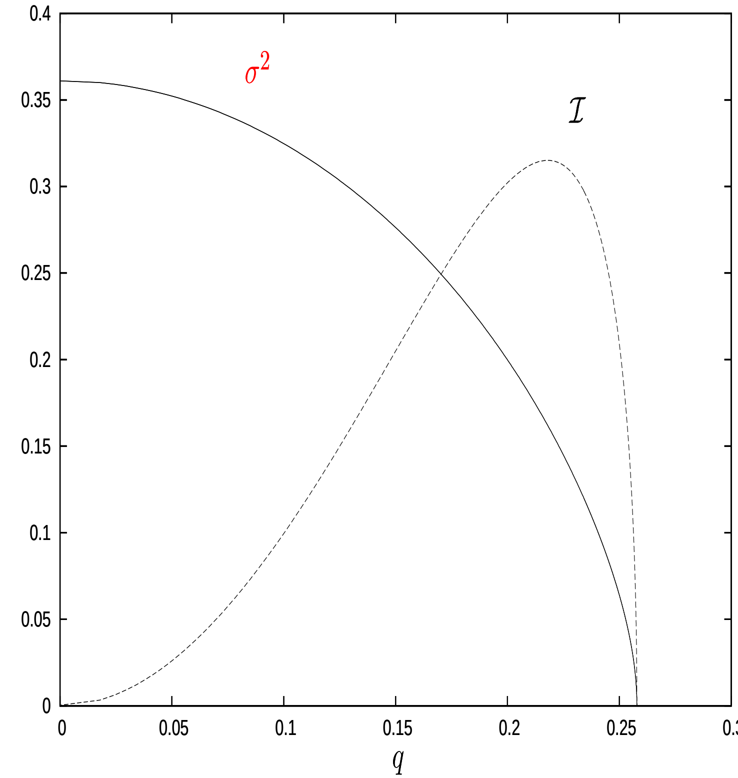
Generic superconducting vortices

are globally regular, with a regular vortex core condaining massive W-condensate that creates a current. The current produces a Biot-Savart field outside the core.

- Exist for any value of $m_{\rm h}$ and for any $\sin^2\theta_{\rm w} \in [0, 1]$
- Comprise a four parameter family labeled by q, n, ν, σ_0. Related to these are the current, momentum, angular momentum, magnetic and Z fluxes of the vortex.
- When current tends to zero, they reduce to Z-strings.

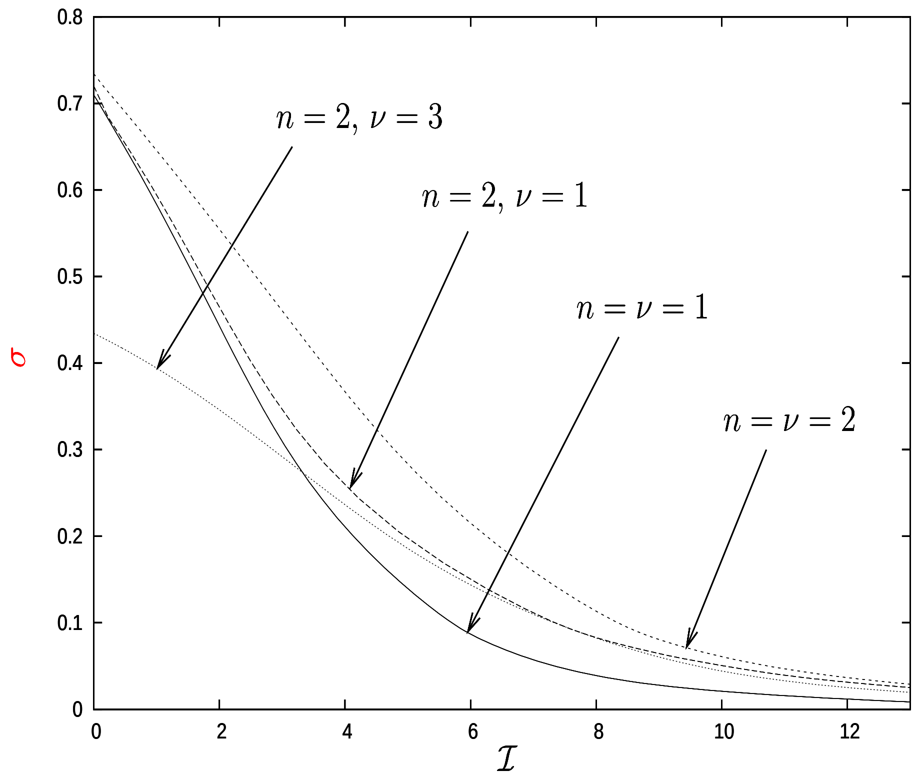
Current $\mathrm{I}/\mathrm{I}_0 = \mathcal{I}$

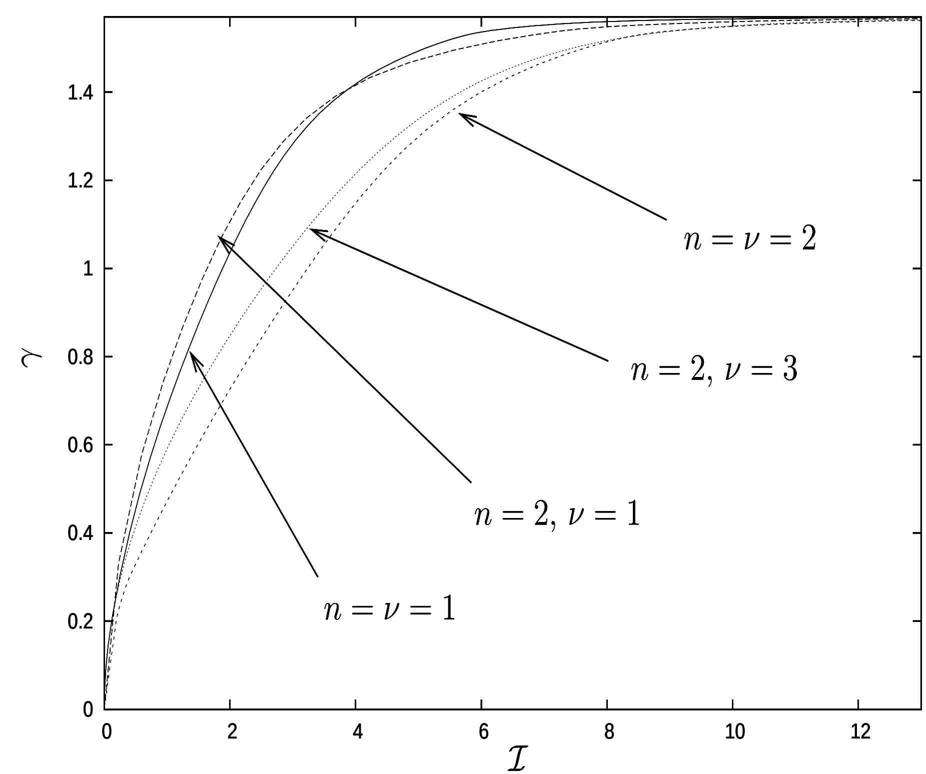




$$\mathbf{I}_0 = \mathbf{c}\mathbf{\Phi}_0 = \mathbf{c} \times 52.68 \times 10^9 \text{ Volts} = 1.75 \times 10^9 \text{ Amperes}.$$

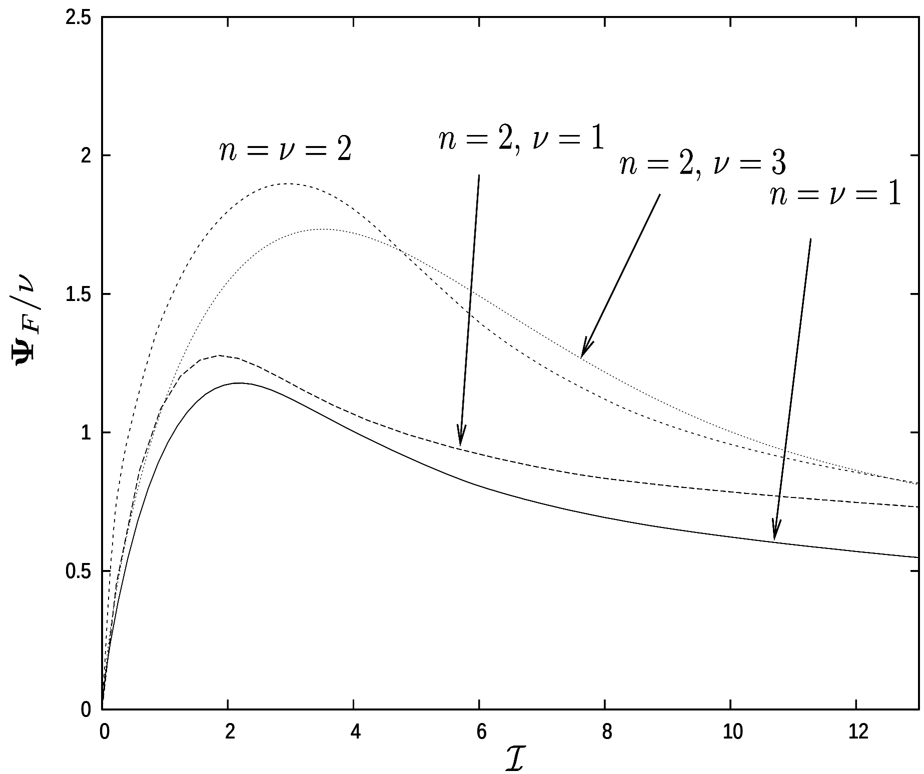
$\sigma^2(\mathcal{I})$, $\gamma(\mathcal{I})$

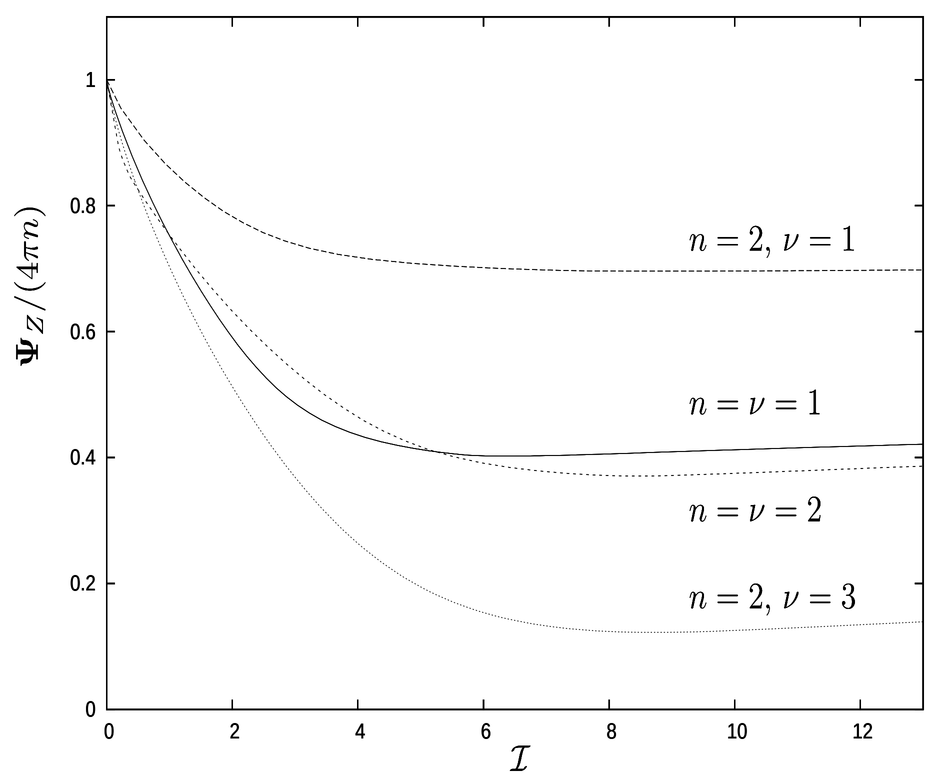




$$\tan\gamma = \frac{f_2(\infty)}{f_1(\infty)}$$

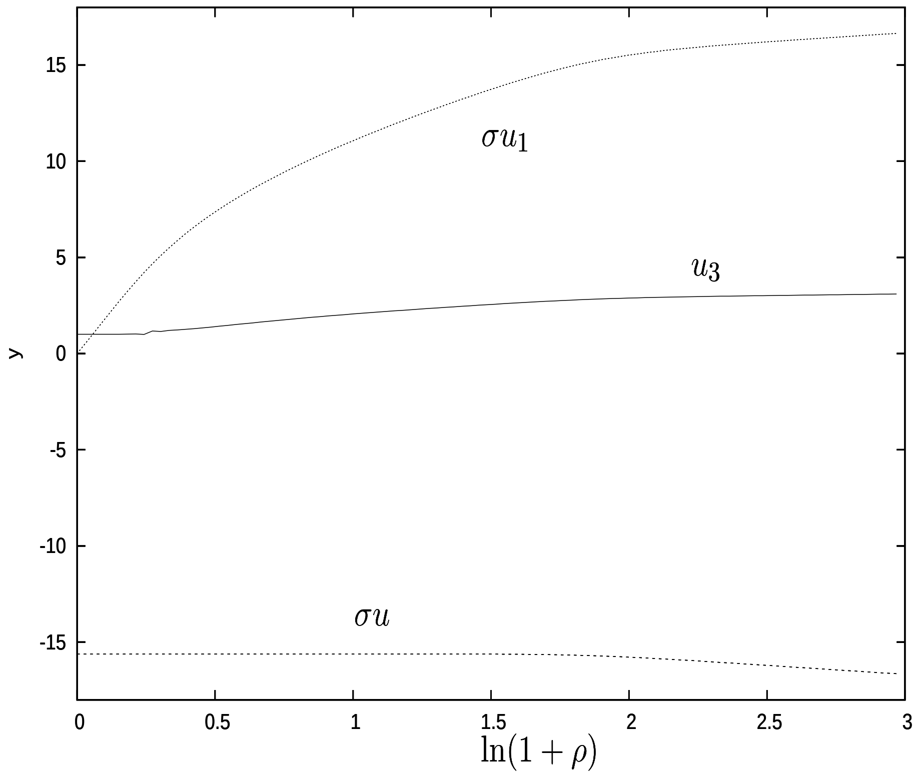
Fluxes

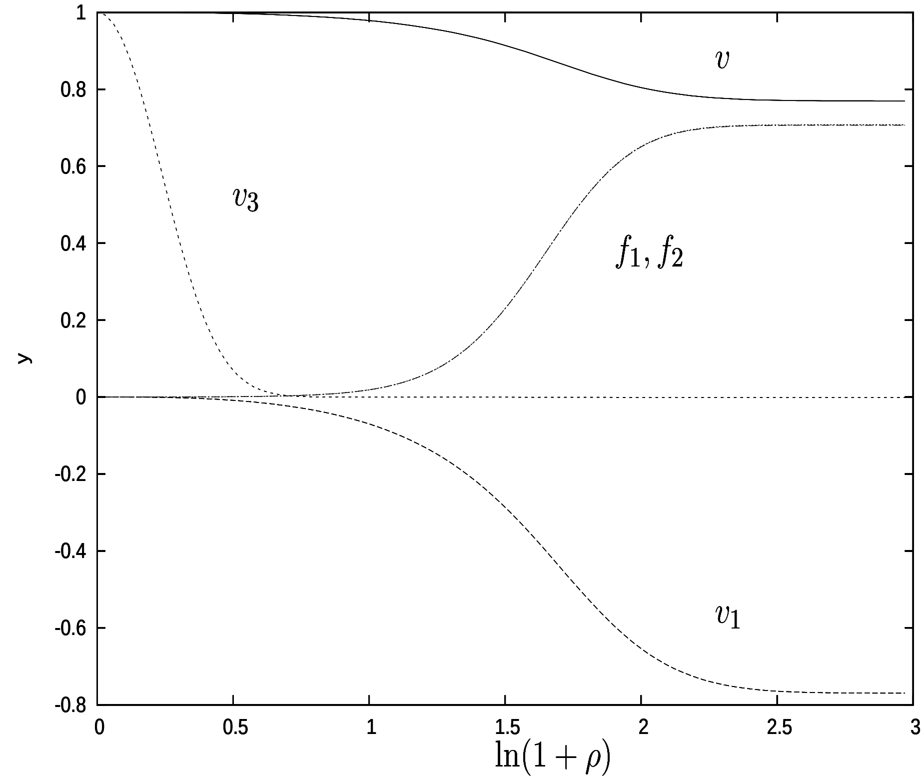




The electromagnetic flux Ψ_F/ν and Z-flux $\Psi_Z/(4\pi n)$ against the current for the vortices with $\beta = 2$, $\sin^2\theta_{\rm w} = 0.23$.

Large current limit $\sigma \to 0$





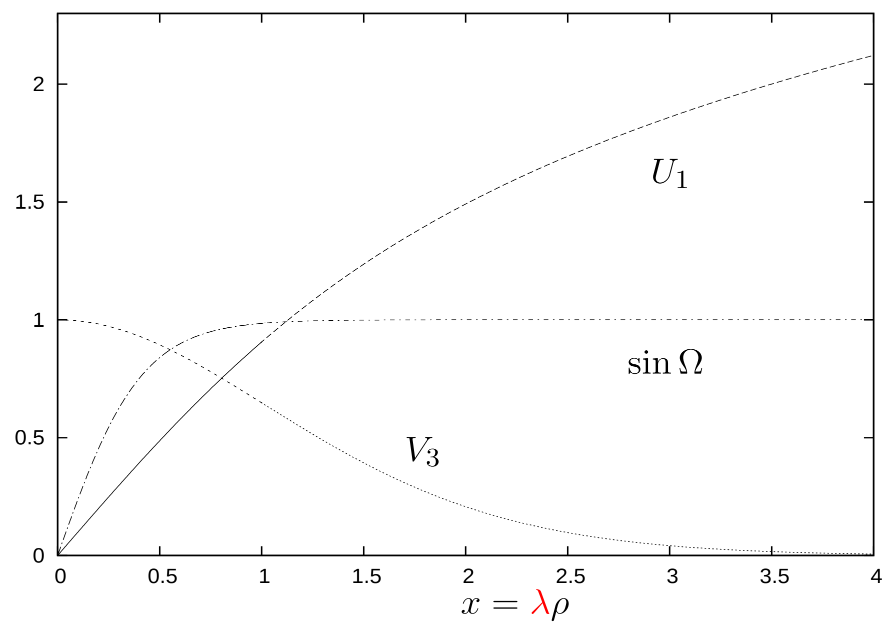
profiles for $\beta = 2$, $g'^2 = 0.23$, $n = \nu = 1$ and $\sigma = 0.008$.

Central condensate region, $\rho < 4/\lambda$

$B_\mu \approx$const., $\Phi \approx 0 \Rightarrow$ pure Yang-Mills $\quad \mathcal{L} = -\frac{1}{4g^2}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu}$

Yang-Mills string: $\quad \tau^a \mathrm{W}^a_\nu dx^\mu = \tau^1 \lambda U_1(\lambda\rho)dz + \tau^3 V_3(\lambda\rho)d\varphi.$

$$x\left(\frac{V_3'}{x}\right)' = U_1^2 V_3, \qquad \frac{1}{x}(xU_1')' = \frac{V_3^2}{x^2}\,U_1 \;\leftarrow\; \text{current density}$$



External region, $\rho > 4/\lambda$

$$A_\mu = \frac{g}{g'}\, B_\mu - \frac{g'}{g}\, \mathrm{W}^1_\mu, \qquad Z_\mu = B_\mu + \mathrm{W}^1_\mu, \ \ \phi_1 \approx \phi_2 \equiv \phi$$

$$\mathcal{L} = -\frac{1}{4}(F_{\mu\nu})^2 - \frac{1}{4}(Z_{\mu\nu})^2 + |(\partial_\mu - \frac{i}{2}\, Z_\mu)\phi|^2 - \frac{\beta}{8}(|\phi|^2 - 1)^2$$

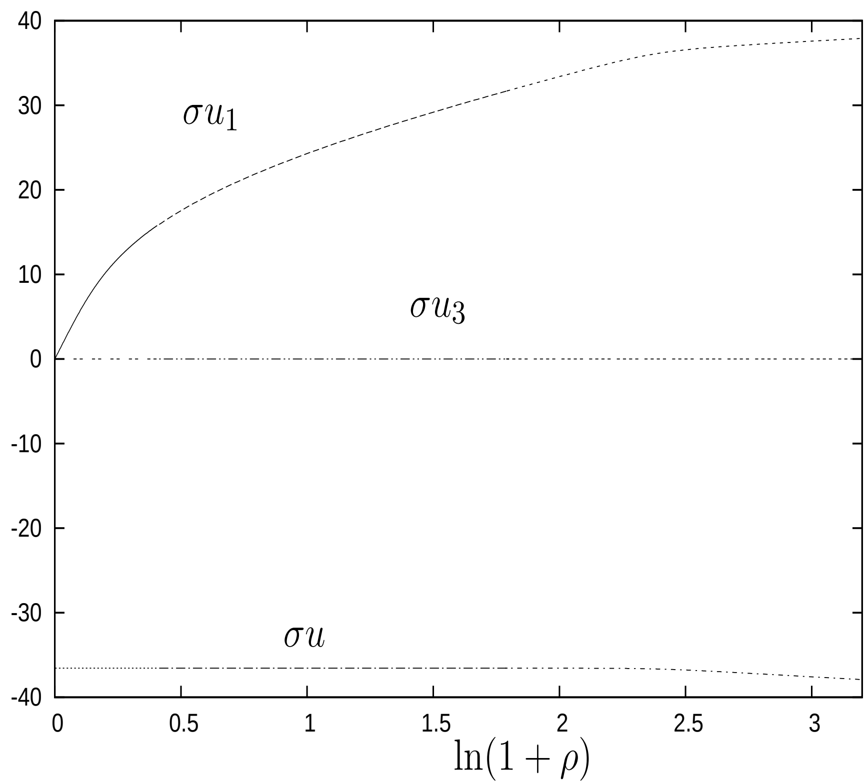
$$A = (a + b\ln\rho)dz + g^2 d\varphi, \quad Z = U(\rho)dz + V(\rho)d\varphi, \quad \phi = f(\rho)$$

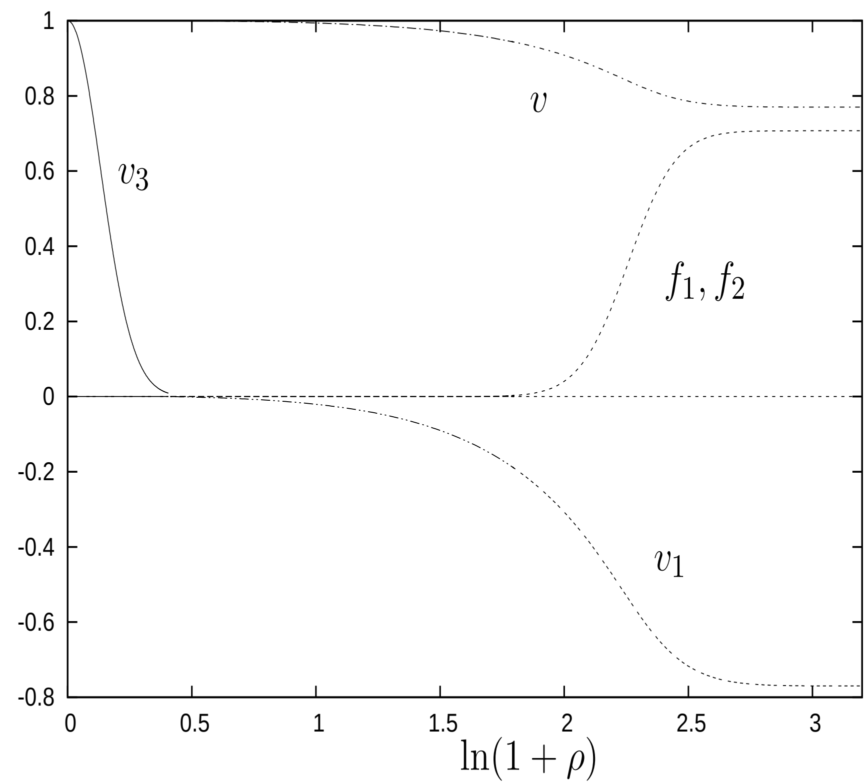
$$\frac{1}{\rho}(\rho f')' = \left(U^2 + \frac{V^2}{\rho^2} + \frac{\beta}{4}(f^2 - 1)\right) f$$

$$\rho\left(\frac{V'}{\rho}\right)' = \frac{1}{2}\, f^2 V$$

$$\frac{1}{\rho}(\rho U')' = \frac{1}{2}\, f^2 U$$

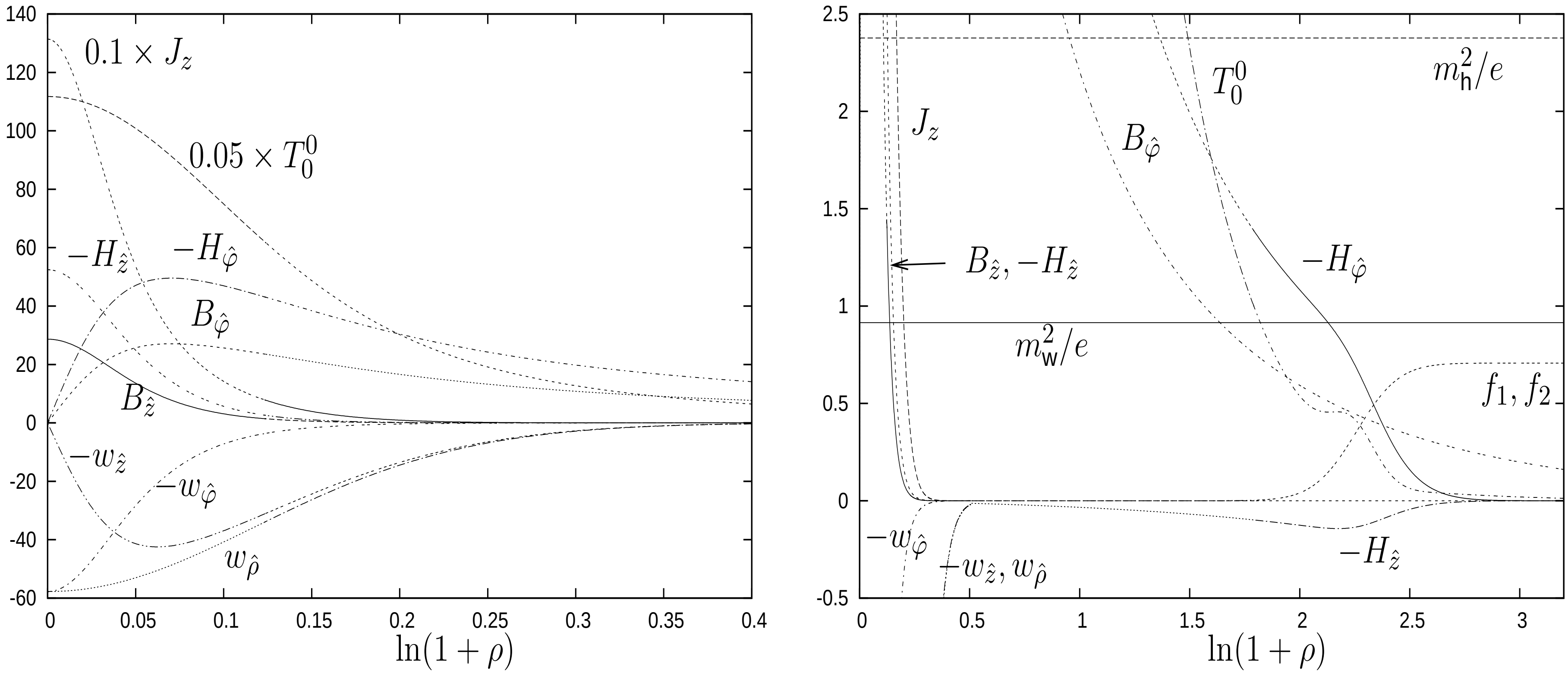
Matching at $\rho = 4/\lambda$





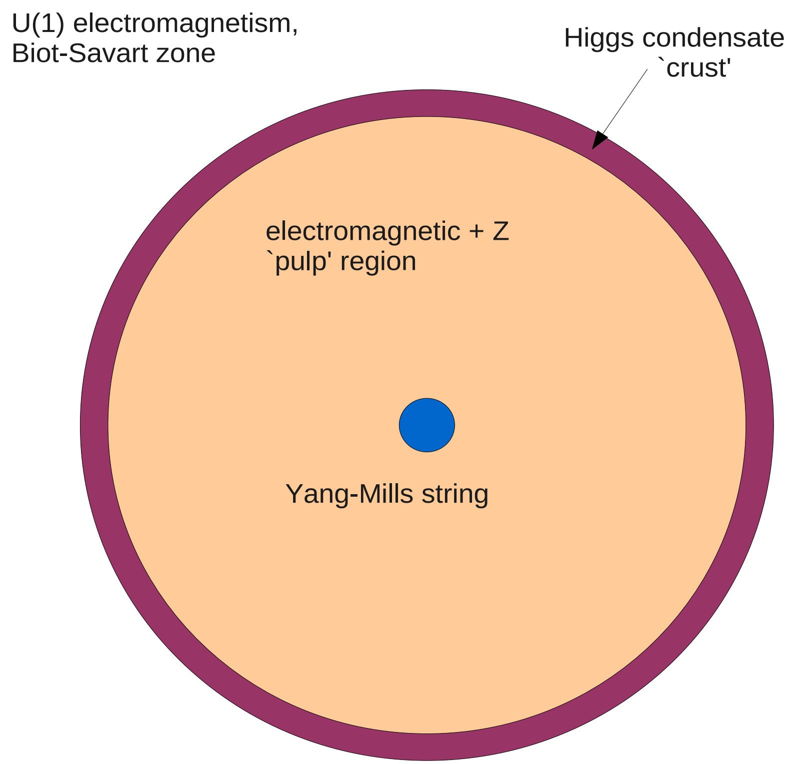
$$\lambda = 0.17\,\frac{g}{g'}\,\mathcal{I} \qquad\qquad \rho_* = 0.28 \times \frac{g}{g'}\frac{\mathcal{I}}{\sqrt{\beta}}$$

Inner structure of large $\mathcal{I}$ vortex



supercritical field in condensate core+‘pulp’ $\Rightarrow$ SU(2)$\times$U(1)
critical field in the ‘crust’ $\Rightarrow$ non-trivial Higgs
undercritical field in Biot-Savart region $\Rightarrow$ Higgs vacuum

Vortex cross section

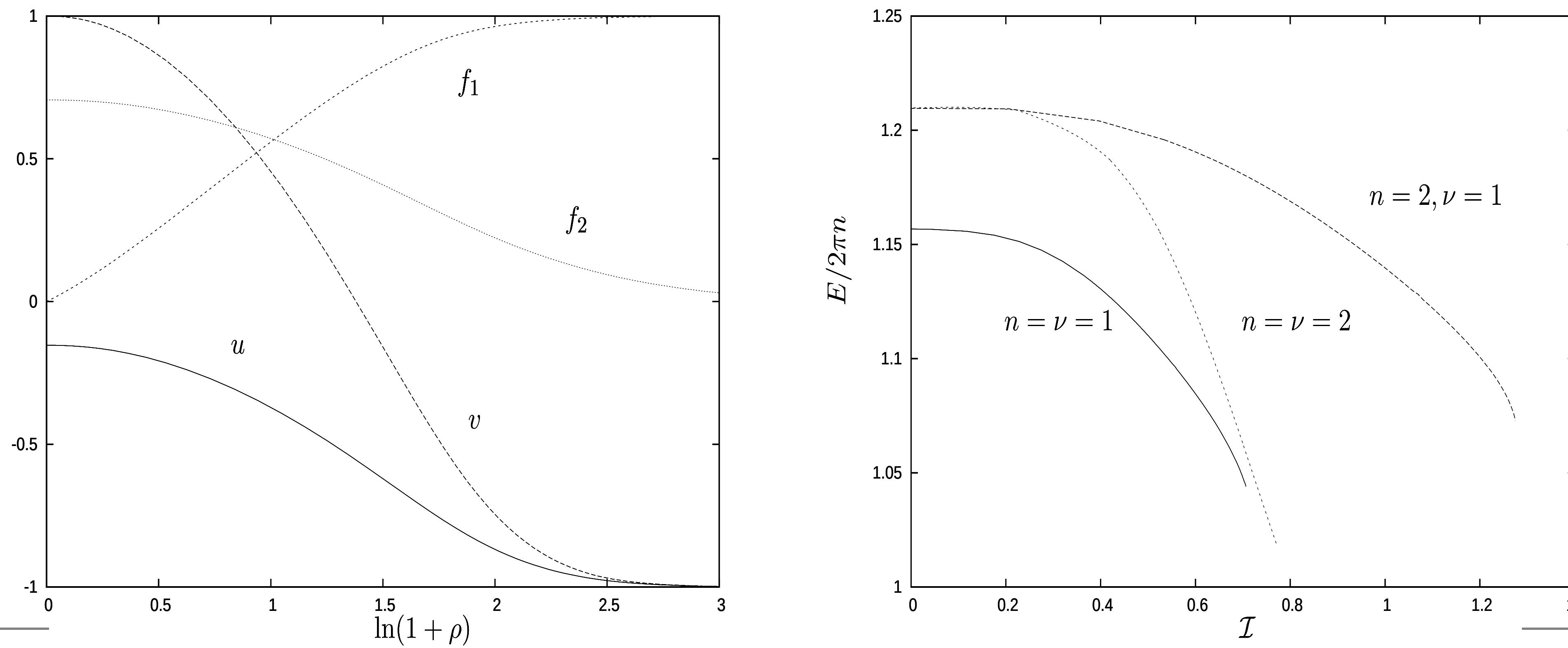


No need of GUT-originating Witten’s string

Semilocal limit $\theta_w = \pi/2$

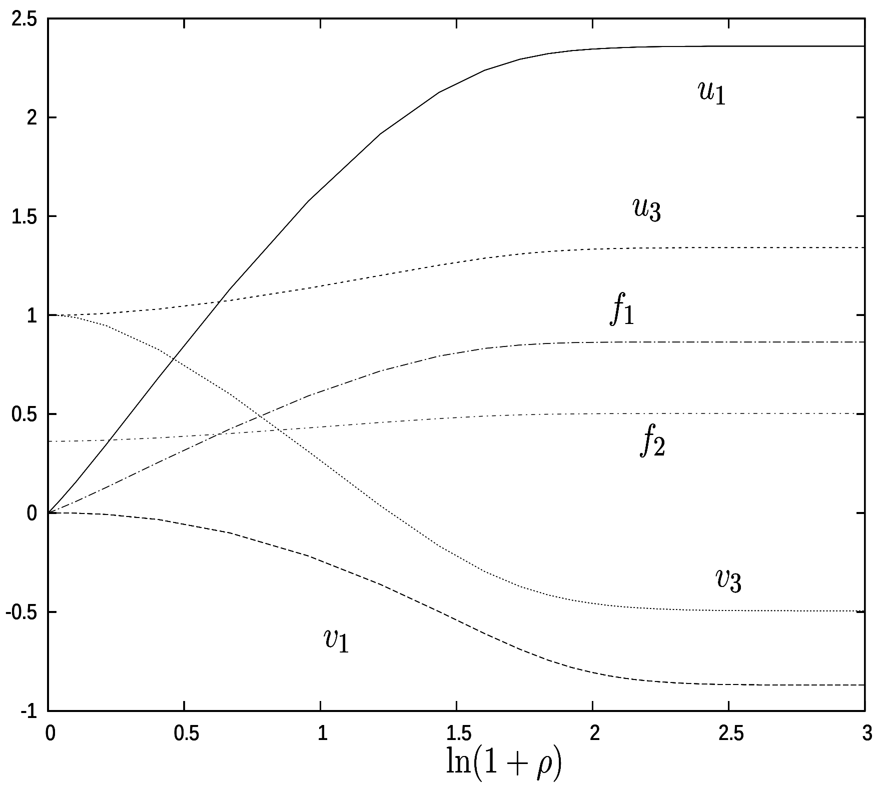
$$\mathcal{L} = -\frac{1}{4}\, B_{\mu\nu} B^{\mu\nu} + (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8}\left(\Phi^\dagger \Phi - 1\right)^2,$$

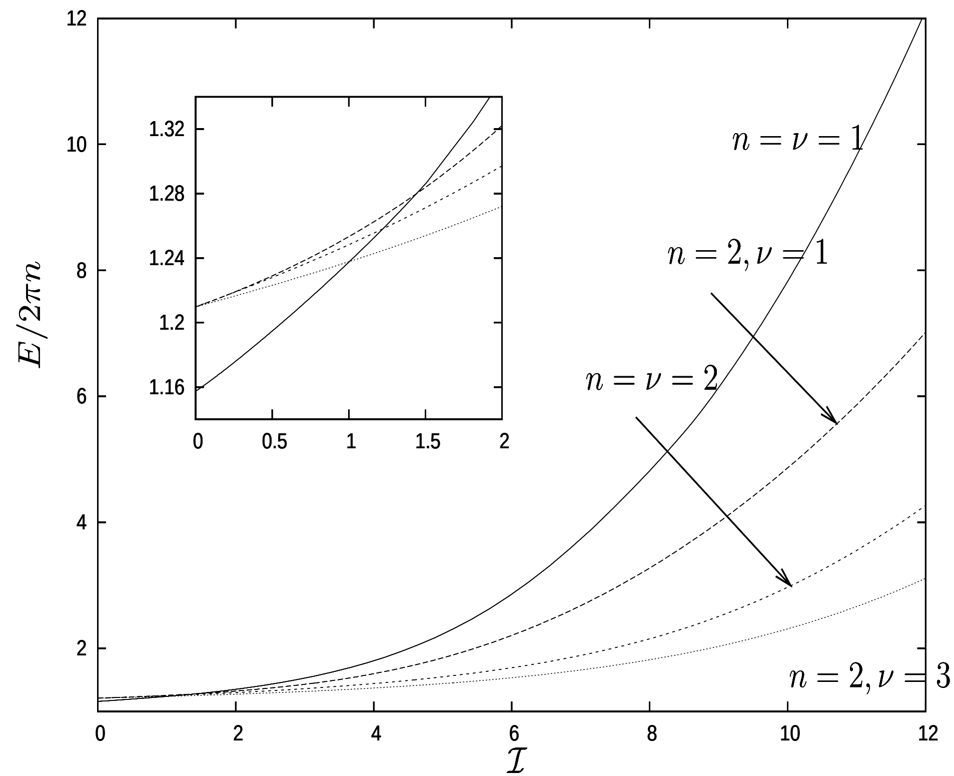
only massive fields $\Rightarrow$ no longrange modes $\Rightarrow$ finite energy, current is global



Isospin limit $\theta_w = 0$

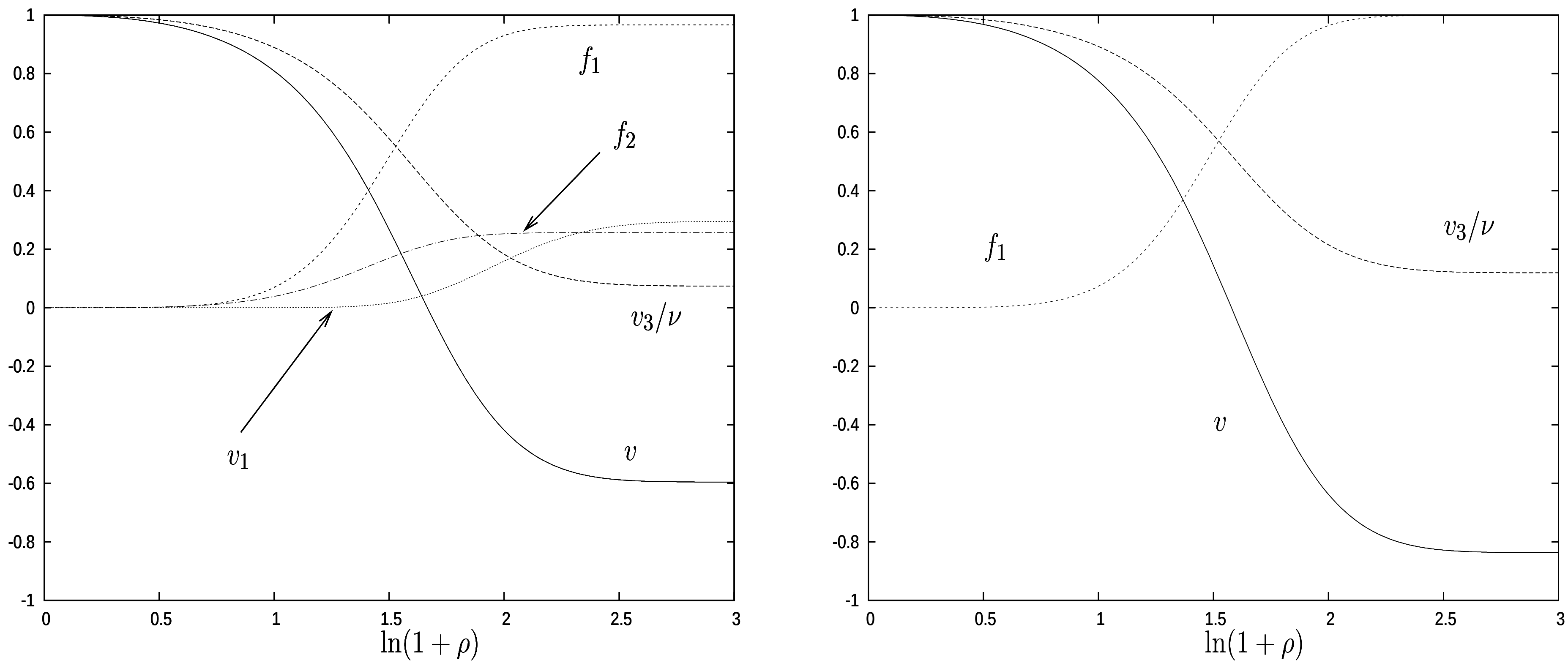
$$\mathcal{L} = -\frac{1}{4}\,\mathrm{W}^a_{\mu\nu}\mathrm{W}^{a\mu\nu} + (D_\mu\Phi)^\dagger D^\mu\Phi - \frac{\beta}{8}\left(\Phi^\dagger\Phi - 1\right)^2$$





Chiral solutions, $\sigma^2 = 0$. Non-generic !

$\Rightarrow \sigma_0 = \pm\sigma_3$ in particular $\sigma_\alpha = 0 \Rightarrow$ finite energy.



chiral versus Z string for $n = 4$, $\nu = 7$, $\beta = 2$, $g'^2 = 0.23$.

Stability in the semilocal limit, $\theta_{\mathrm{w}} = \pi/2$

$$\mathcal{L} = -\frac{1}{4}\, B_{\mu\nu} B^{\mu\nu} + (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\beta}{8}\left(\Phi^\dagger \Phi - 1\right)^2,$$

$$\Phi \to \Phi + \delta\Phi, \qquad B_\mu \to B_\mu + \delta B_\mu, \qquad \delta B_0 = 0$$

$$\delta\Phi_1 = e^{iN\varphi}\delta\tilde{\Phi}_1, \qquad \delta\Phi_2 = e^{i\sigma z}\delta\tilde{\Phi}_2,$$

$$\begin{aligned}
\delta\tilde{\Phi}_a &= \sum_{\omega,\kappa,m} \cos(\omega t + m\varphi + \kappa z)\left(\phi_a^{\omega,\kappa,m}(\rho) + i\psi_a^{\omega,\kappa,m}(\rho)\right) \\
&+ \sin(\omega t + m\varphi + \kappa z)\left(\pi_a^{\omega,\kappa,m}(\rho) + i\nu_a^{\omega,\kappa,m}(\rho)\right), \\
\delta A_\mu &= \sum_{\omega,\kappa,m} \xi_\mu^{\omega,\kappa,m}(\rho)\cos(\omega t + m\varphi + \kappa z) \\
&+ \chi_\mu^{\omega,\kappa,m}(\rho)\sin(\omega t + m\varphi + \kappa z)
\end{aligned}$$

Perturbation equations

Variables separate to give a Schrodinger system

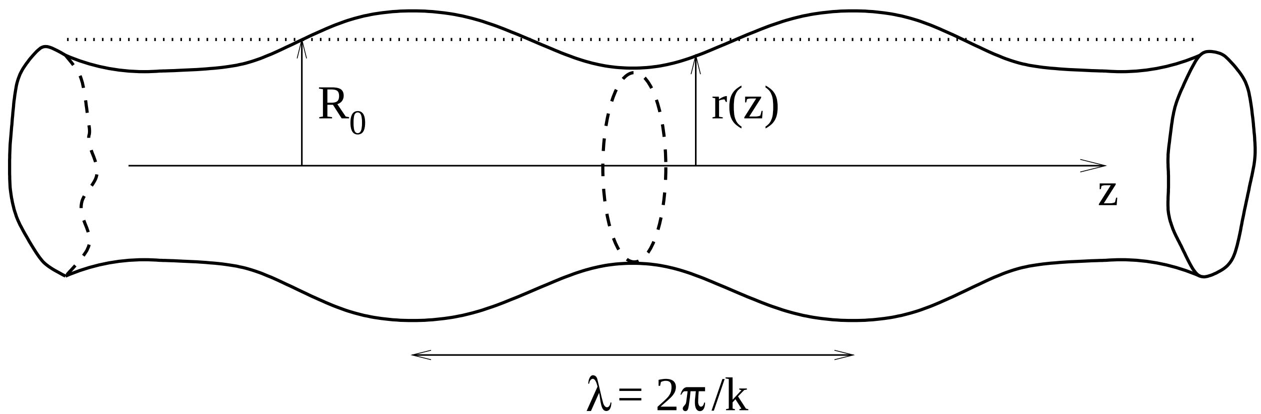
$$-\Psi'' + \mathbf{U}_{m,\kappa}\Psi = \omega^2\Psi\,,$$

$\Psi(\rho)$ is a 6-component vector, $\mathrm{U}_{m,\kappa}$ is a potential matrix determined by the background fields.

solutions with $\omega^2 < 0 \Rightarrow$ unstable modes

$\exists$ only one negative mode

proportional to $\exp\{ikz\}$



where $k < \sigma \Rightarrow$

$$\lambda > \lambda_{\rm min}(\mathcal{I}) = \frac{2\pi}{\sigma}$$

$\Rightarrow$ one can eliminate the instability by imposing periodic boundary conditions with period $L < \lambda_{\rm min}(\mathcal{I})$!
This does not work if $\mathcal{I} = 0$ due to the homogeneous mode

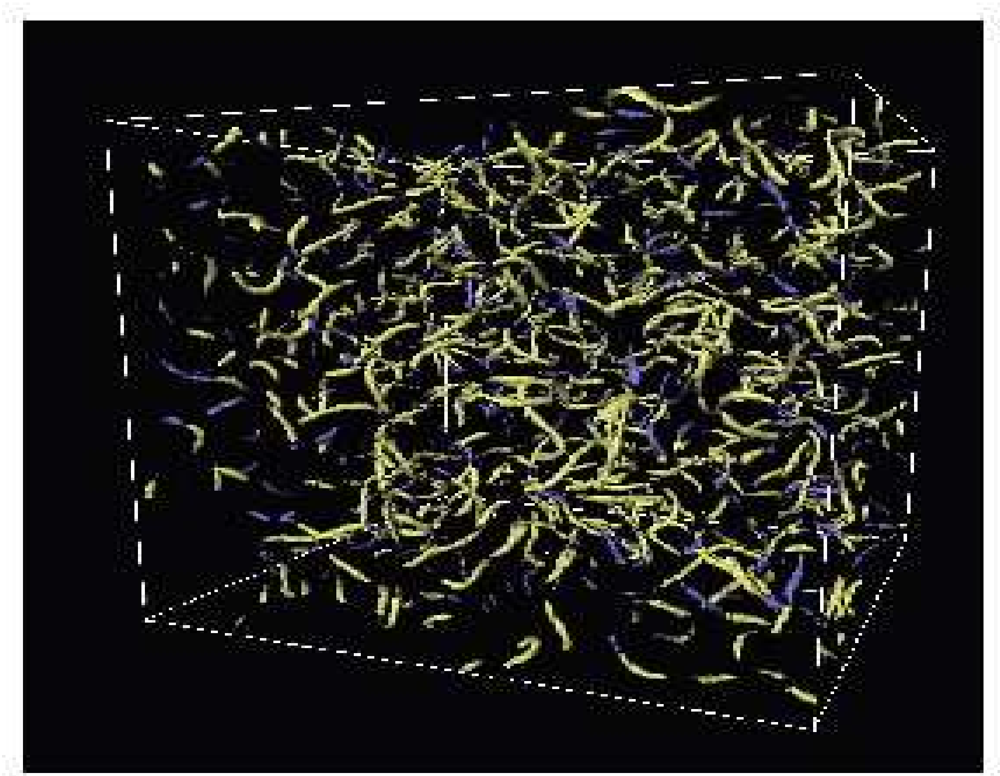
Conclusion of the stability analysis

- Short vortex segments stable – no room to accommodate inhomogeneous unstable modes.
- The length of stable segments increases with current and tends to infinity for $\mathcal{I} \to \infty$.
- Hydrodynamical analogy: Plateau-Rayleigh instability of a water jet: if the jet is long enough, ripples appear.
- It seems that the same conclusions apply for any θ_{W}.

Perhaps small vortex loops are stable ?

Electroweak thunderbolts

Z strings are non-topological – can exist as finite segments. Current carrying vortices can perhaps also exist as finite segments joining electrically polarized regions of space – ‘thunderbolts between clouds’.



Summary

- New type of solutions describing vortices carrying a constant electric current is constructed in the electroweak sector of Standard Model.
- The vortex current can typically attain billions of Amperes, and there seem to be no upper bound for it.
- For large currents the electroweak gauge symmetry is completely restored inside the vortex by a very strong magnetic field.
- Short vortex segments whose length increases with current are stable. Could perhaps transfer charge between different regions of space (? thunderbolts?)
- Loops made of stable segments could perhaps be stable (?? stable electroweak solitons ??)