The NEC Violation and Classical Stability

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based on

I.Ya. Aref'eva, N.V. Bulatov, L.V. Joukovskaya, S.Yu. V.,
arXiv:0903.5264.

To specify different types of cosmic fluids one usually uses a phenomenological relation between the pressure density p and the energy density ϱ, corresponding to each component of fluid

$$p = w\varrho,$$

where w is the state parameter:

$$w(t) = \frac{p}{\varrho} = -1 - \frac{2}{3}\frac{\dot{H}}{H^2} = -1 + \frac{2E_k}{\varrho}. \quad (1)$$

Contemporary experiments, including WMAP, give strong support that at present time the dark energy state parameter is close to -1: $w_{DE} = -1 \pm 0.2.$

We consider the case $w_{DE} < -1$. Null energy condition (NEC) is violated and there are problems of instability. A possible way to evade the instability problem for models with $w_{DE} < -1$ is to yield a phantom model as an effective one, arising from a more fundamental theory, for example, the string field theory **(I.Ya. Aref'eva, astro-ph/0410443, 2004)**. The concerned models are string field theory approximations.

1 Exactly solvable DE Dominate Model

(I.Ya. Aref'eva, A.S. Koshelev, and S.Yu. Vernov, Theor. Math. Phys. **148** (2006) 895–909, astro-ph/0412619).

This is a model of Einstein gravity interacting with a single *phantom* scalar field in the spatially flat Friedmann Universe:

$$S=\int d^4x\sqrt{-g}\left(\frac{M_p^2}{2M_s^2}R+\frac{1}{g_o^2}\left(\frac{+1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)\right)\right),$$

$M_p = 1/(8\pi G_N)$ is the Planck mass, M_s is a string mass and g_o is a dimensionless open string coupling constant.

Coordinates (t, x_i) and field ϕ are dimensionless.

$$ds^2 = -\,dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2).$$

$$m_p^2 = \frac{g_o^2 M_p^2}{M_s^2}.$$

$$
\begin{aligned}
3H^2 &= \frac{1}{m_p^2}\, \varrho_{DE}, \\
3H^2 + 2\dot{H} &= \, -\frac{1}{m_p^2}\, p_{DE},
\end{aligned} \tag{2}
$$

where

$$H \equiv \frac{\dot{a}(t)}{a(t)},$$

$\phi = \phi(t)$, therefore,

$$
\begin{aligned}
\varrho_{DE} &= -\frac{1}{2}\dot{\phi}^2 + V(\phi), \\
p_{DE} &= -\frac{1}{2}\dot{\phi}^2 - V(\phi),
\end{aligned}
$$

$$
\begin{cases}
\dot{H} = \dfrac{1}{2m_p^2}\dot{\phi}^2, \\
3H^2 = \dfrac{1}{m_p^2}\left(-\dfrac{1}{2}\dot{\phi}^2 + V(\phi)\right).
\end{cases} \tag{3}
$$

The equation of motion for the field ϕ is

$$
\ddot{\phi} + 3H\dot{\phi} - V'_{\phi} = 0, \tag{4}
$$

Equation (4) is a consequence system (3).

System (3) is not integrable. We can find only special solutions.

It is easy to check that for

$$
V(\phi) = \frac{\omega^2}{2A^2}\left(A^2 - \phi^2\right)^2 + \frac{\omega^2\phi^2}{12A^2m_p^2}\left(3\,A^2 - \phi^2\right)^2.
$$

there exists the solution:

$$
\phi(t) = A\tanh(\omega t).
$$

Is the obtained solution stable?

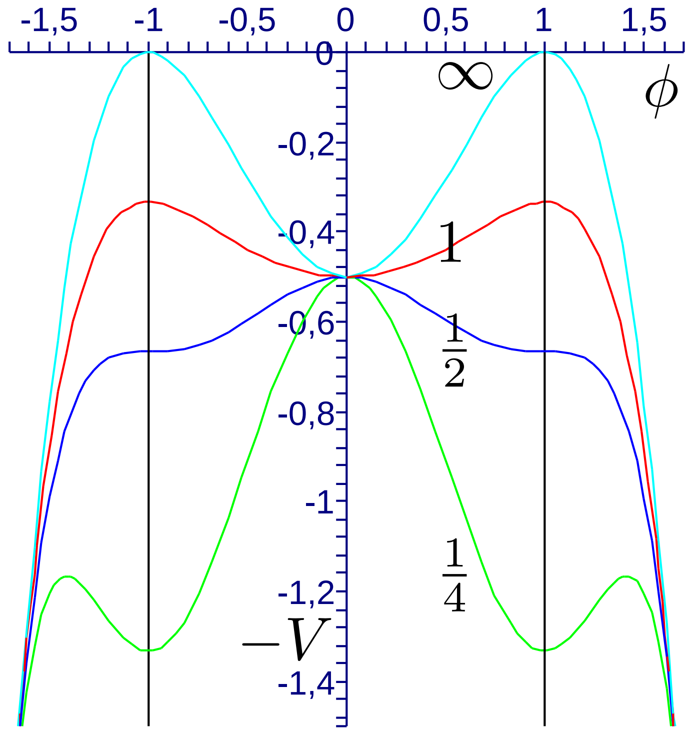

Figure 1: Potential $-V(\phi)$ for various values of m_p^2 $(A = \omega = 1)$.

A few known facts about stability

Let us remind a few facts about stability of solutions for

$$\dot{y}_k = F_k(y), \qquad k = 1, 2, \ldots, N. \tag{5}$$

By definition a solution $y_0(t)$ is attractive if

$$\|\tilde{y}(t) - y_0(t)\| \to 0 \quad \text{at} \quad t \to \infty \tag{6}$$

for all solutions $\tilde{y}(t)$ that start close enough to $y_0(t)$.

If all solutions of the dynamical system that start out near a fixed (equilibrium) point y_f,

$$F_k(y_f) = 0, \tag{7}$$

stay near y_f forever, then y_f is *a Lyapunov stable point.*

If all solutions that start out near the equilibrium point y_f converge to y_f, then the fixed point y_f is *an asymptotically stable* one.

A solution $y_0(t)$ of (5), which tends to the fixed point y_f, is attractive if and only if the point y_f is asymptotically stable.

The Lyapunov theorem states that to prove the stability of fixed point y_f of nonlinear system (5) it is sufficient to prove the stability of this fixed point for the corresponding linearized system

$$\dot{x} = Ax, \quad A_{ik} = \frac{\partial F_i(y)}{\partial y_k}|_{y=y_f}. \tag{8}$$

The stability of the linear system means that real parts of all roots λ_k of the characteristic equations

$$\det(A_{ik} - \lambda I)| = 0 \tag{9}$$

are negative.

If there exist such λ_k that its real part is positive, then the fixed point is unstable.

The gravitational one-field model

Let us consider the gravitational model with a scalar field ϕ and an arbitrary potential $V(\phi)$, described by action:

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}-\frac{C}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi-V(\phi)-\Lambda\right). \qquad (10)$$

In the FRW metric equations are

$$2\dot{H}+3H^2=-8\pi G_N\left(\frac{C\dot{\phi}^2}{2}-V(\phi)-\Lambda\right), \qquad (11)$$

$$3H^2=8\pi G_N\left(\frac{C\dot{\phi}^2}{2}+V(\phi)+\Lambda\right). \qquad (12)$$

The equation of motion for $\phi(t)$ is the following

$$\ddot{\phi}(t)+3H(t)\dot{\phi}(t)+\frac{1}{C}V'_\phi(\phi)=0, \qquad (13)$$

where $V'_\phi\equiv\frac{dV}{d\phi}$. This equation is in fact a consequence of system (11)–(12).

System (11)–(13) can be considered as the following system of the first order equations:

$$\begin{aligned}
\dot{H}(t) = & -\frac{3}{2}H^2 - 4\pi G_N\left(\frac{C\psi^2}{2} - V(\phi) - \Lambda\right), \\
\dot{\phi}(t) = & \psi(t), \\
\dot{\psi}(t) = & -3H(t)\psi(t) - \frac{1}{C}V'_{\phi}(\phi).
\end{aligned} \qquad (14)$$

Equation (12) connects the following integral of motion of system (14) to the cosmological constant:

$$I_1 = \frac{3}{8\pi G_N}H^2 - \left(\frac{C}{2}\psi^2 + V(\phi)\right) = \Lambda. \qquad (15)$$

We are interested in the stability of kink-type solutions.

The Hubble parameter $H(t)$ tends to a finite level at $t \to +\infty$.

In this case $\phi(t)$ tends to a finite level as well.

There exists a fixed point $y_f \equiv (H_f, \phi_f, \psi_f)$, which corresponds to $t = +\infty$.

It is easy to see that

$$\psi_f = 0, \qquad V'_\phi(\phi_f) = 0, \qquad H_f^2 = \frac{8}{3}\pi G_N(\Lambda + V(\phi_f)). \tag{16}$$

To analyse the stability of y_f we present solutions in the following form:

$$H = H_f + \varepsilon h(t) + \mathcal{O}(\varepsilon^2) \tag{17a}$$

$$\phi = \phi_f + \varepsilon\varphi(t) + \mathcal{O}(\varepsilon^2) \tag{17b}$$

$$\psi = \varepsilon\chi(t) + \mathcal{O}(\varepsilon^2), \tag{17c}$$

where ε is a small parameter.

To first order in ε we obtain the following system of the equations:

$$\dot{h}(t) = -3H_f h(t) \tag{18a}$$

$$\dot{\varphi}(t) = \chi(t) \tag{18b}$$

$$\dot{\chi}(t) = -3H_f\chi(t) - \frac{1}{C}V''_\phi(\phi_f)\varphi. \tag{18c}$$

The equation (18a) has the solution

$$h(t) = b_0 e^{-3H_f t}, \tag{19}$$

where b_0 is a constant.

From (18b-18c) we obtain the following solutions:

- at $V''_\phi\left(\phi_f\right) \neq 0$ and $V''_\phi\left(\phi_f\right) \neq \frac{9C}{4} H_f^2$:

$$\varphi(t) = D_1 e^{-\frac{3}{2}\left(H_f + \sqrt{H_f^2 - \frac{4}{9C} V''_\phi(\phi_f)}\right)t} + D_2 e^{-\frac{3}{2}\left(H_f - \sqrt{H_f^2 - \frac{4}{9C} V''_\phi(\phi_f)}\right)t}, \qquad (20)$$

- at $V''_\phi\left(\phi_f\right) = \frac{9C}{4} H_f^2$

$$\varphi(t) = e^{-3H_f t/2}(D_1 + D_2 t), \qquad (21)$$

- at $V''_\phi\left(\phi_f\right) = 0$,

$$\varphi(t) = \tilde{D}_1 - \frac{1}{3H_f} D_2 e^{-3H_f t}, \qquad (22)$$

where $\tilde{D}_1$, D_1 and D_2 are arbitrary constants.

Using the Lyapunov theorem we state that fixed point y_f is asymptotically stable and, therefore, the exact kink-type solution $y_0(t)$ is stable if:

$$\frac{V''_\varphi\left(\phi_f\right)}{C} > 0 \quad \text{and} \quad H_f > 0. \qquad (23)$$

At $V''_\phi(\phi_f) = 0$ we need an additional analysis of stability, because the Lyapunov theorem does not state the correspondence of the behavior of solutions to the initial nonlinear system and the obtained linear system.

At $H_f = 0$ we obtain either saddle or center point.

At $H_f < 0$ the fixed point y_f is unstable, because $h(t)$ tends to infinity.

In our model exact solutions in are stable in the Bianchi I metric at $m_p^2 < 1/2$ and unstable at $m_p^2 > 1/2$. The case of $m_p^2 = 1/2$ needs to more detail analysis, the first corrections are bounded.

Stability in the Bianchi I metric

Let us consider the model with one scalar field (10) in the Bianchi I metric

$$ds^2 = -dt^2 + a_1^2(t)dx_1^2 + a_2^2(t)dx_2^2 + a_3^2(t)dx_3^2. \quad (24)$$

we obtain:

$$\dot{\phi} = \psi, \quad (25)$$

$$\dot{\psi} = -(H_1 + H_2 + H_3)\psi - \frac{1}{C}V'_{\phi}(\phi), \quad (26)$$

$$\dot{H}_1 = -4\pi G_N p - H_1^2 - \frac{1}{2}(H_2H_3 - H_1H_3 - H_1H_2), \quad (27)$$

$$\dot{H}_2 = -4\pi G_N p - H_2^2 - \frac{1}{2}(H_1H_3 - H_2H_3 - H_1H_2), \quad (28)$$

$$\dot{H}_3 = -4\pi G_N p - H_3^2 - \frac{1}{2}(H_1H_2 - H_1H_3 - H_2H_3), \quad (29)$$

where

$$p = C\frac{\dot{\phi}^2}{2} - V(\phi) - \Lambda. \quad (30)$$

$$H_1 = \frac{\dot a_1}{a_1}, \quad H_2 = \frac{\dot a_2}{a_2}, \quad H_3 = \frac{\dot a_3}{a_3} \quad \text{and dot denotes time derivative.} \tag{31}$$

Let the isotropic solution $y_0(t)$ tends to a fixed point $y_f \equiv (H_f, H_f, H_f, \phi_f, \psi_f)$ at $t \to +\infty$. For the fixed point we obtain:

$$\psi_f = 0, \qquad V'_\phi(\phi_f) = 0, \qquad H_f^2 = \frac{8}{3}\pi G_N(\Lambda + V(\phi_f)). \tag{32}$$

To analyse the stability of y_f we present solutions as series in ε:

$$H_i = H_f + \varepsilon h_i(t) + \mathcal{O}(\varepsilon^2), \quad \phi = \phi_f + \varepsilon\varphi(t) + \mathcal{O}(\varepsilon^2), \quad \psi = \varepsilon\chi(t) + \mathcal{O}(\varepsilon^2).$$

To first order in ε we obtain the following system of the equations:

$$\begin{aligned} \dot h_i(t) &= \, - 3H_f h_i(t), \qquad i = 1, 2, 3 \\ \dot\varphi(t) &= \chi(t), \\ \dot\chi(t) &= \, - 3H_f\chi(t) - \frac{1}{C}V''_\phi(\phi_f)\varphi. \end{aligned} \tag{33}$$

The solutions of this system coincide with solutions of system (18). Namely, $\varphi(t)$ and $\chi(t)$ are the same, and equations for $h_i(t)$ and $h(t)$ are the same.

Thus, the isotropic solution which tends to a fixed point is stable in the Bianchi I metric if and only if it is stable in the FRW metric.

The first order corrections to isotropic solutions in the FRW and Bianchi I metrics

We have found that in the case of one-field models the equations for the first corrections near a fixed point in the Bianchi I metric can be automatically solved if the corresponding equations in the FRM metric are solved.

Let us generalize this result and consider consider the cosmological model, which is described by action

$$S=\int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N}-\sum_{k=1}^{N}\frac{C_k}{2}g^{\mu\nu}\partial_\mu\phi_k\partial_\nu\phi_k-V(\phi_1,\dots,\phi_N)-\Lambda\right). \tag{34}$$

In the Bianchi I metric the Einstein equations have the following form:

$$H_1H_2+H_1H_3+H_2H_3=8\pi G_N\varrho, \tag{35}$$

$$\dot{H}_2+H_2^2+\dot{H}_3+H_3^2+H_2H_3=\ -8\pi G_Np, \tag{36}$$

$$\dot{H}_1+H_1^2+\dot{H}_2+H_2^2+H_1H_2=\ -8\pi G_Np, \tag{37}$$

$$\dot{H}_1+H_1^2+\dot{H}_3+H_3^2+H_1H_3=\ -8\pi G_Np, \tag{38}$$

where

$$\varrho = \sum_{k=1}^{N} C_k \frac{\dot{\phi}_k^2}{2} + V(\phi_1, \dots, \phi_N) + \Lambda, \tag{39}$$

$$p = \sum_{k=1}^{N} C_k \frac{\dot{\phi}_k^2}{2} - V(\phi_1, \dots, \phi_N) - \Lambda, \tag{40}$$

It is convenient to write equations (36)–(38) as follows

$$\dot{H}_i + \dot{H}_j = K_{ij}, \quad \text{where} \quad i, j = 1, 2, 3, i \neq j, \tag{41}$$

$$K_{ij} = -H_i^2 - H_j^2 - H_i H_j - 8\pi G_N p. \tag{42}$$

From action (34) we obtain the following equations for the fields ϕ_k:

$$\dot{\phi}_k \; = \; \psi_k, \tag{43}$$

$$\dot{\psi}_k \; = \; -(H_1 + H_2 + H_3)\psi_k - \frac{1}{C_k} V'_{\phi_k}, \tag{44}$$

$$\dot{H}_i \; = \; K_i(\phi_k, \psi_k, H_i), \tag{45}$$

where

$$K_1 = \frac{1}{2}(K_{13}+K_{12}-K_{23}), \quad K_2 = \frac{1}{2}(K_{23}+K_{12}-K_{13}), \quad K_3 = \frac{1}{2}(K_{23}+K_{13}-K_{12}).$$

To study the stability of this solution we present solutions, whose initial conditions are close to the isotropic one, in the following form

$$H_i(t) = H_0(t) + \varepsilon h_i(t) + \mathcal{O}(\varepsilon^2), \quad i = 1, 2, 3 \tag{46}$$

$$\phi_k(t) = \phi_{0k}(t) + \varepsilon \varphi_k(t) + \mathcal{O}(\varepsilon^2), \tag{47}$$

$$\psi_k(t) = \psi_{0k}(t) + \varepsilon \chi_k(t) + \mathcal{O}(\varepsilon^2). \tag{48}$$

and obtain

$$\dot{\varphi}_k = \chi_k, \tag{49}$$

$$\dot{\chi}_k = -(h_1+h_2+h_3)\psi_{0k} - 3H_0\chi_k - \frac{1}{C_k}\sum_{m=1}^{N} V''_{\phi_k\phi_m}(\phi_0)\,\varphi_m, \tag{50}$$

$$\dot{h}_1+\dot{h}_2 = -3H_0(h_1+h_2) + 8\pi G_N \sum_{k=1}^{N}\left(V'_{\phi_k}(\phi_0)\,\varphi_k - C_k\dot{\phi}_{0k}\chi_k\right), \tag{51}$$

$$\dot{h}_1+\dot{h}_3 = -3H_0(h_1+h_3) + 8\pi G_N \sum_{k=1}^{N}\left(V'_{\phi_k}(\phi_0)\,\varphi_k - C_k\dot{\phi}_{0k}\chi_k\right), \tag{52}$$

$$\dot{h}_2+\dot{h}_3 = -3H_0(h_2+h_3) + 8\pi G_N \sum_{k=1}^{N}\left(V'_{\phi_k}(\phi_0)\,\varphi_k - C_k\dot{\phi}_{0k}\chi_k\right). \tag{53}$$

From equations (51)–(53) we obtain

$$\dot{h}_1(t) - \dot{h}_2(t) + 3H_0(t)(h_1(t)-h_2(t))=0, \tag{54}$$

$$\dot{h}_1(t) - \dot{h}_3(t) + 3H_0(t)(h_1(t)-h_3(t))=0, \tag{55}$$

$$H_0(h_1+h_2+h_3) = 4\pi G_N \sum_{k=1}^{N} \Big(C_k \dot{\phi}_{0k} \dot{\varphi}_k + V'_{\phi_k}(\phi_0)\,\varphi_k\Big)\,, \qquad (56)$$

Theorem

Let $H_0(t)$ be a smooth function, bounded at all finite values of time and $\int\limits_0^\infty H_0(\tau)d\tau$ be bounded from below, in other words this integral is equal to a finite number or plus infinity. Functions $h_1(t)$, $h_2(t)$, $h_3(t)$ and $\varphi_k(t)$, which are solutions of (49)–(53), are bounded if and only if isotropic solutions, namely solutions, which satisfy the condition $h_1(t) = h_2(t) = h_3(t)$, are bounded.

Proof. It is trivial that if full set of solutions includes only boundary functions, then any subset, which satisfies an additional condition, includes only boundary functions. Let us prove that the boundedness of isotropic solutions is not only a necessary condition but also a sufficient one.

From equations (54) and (55) we obtain:

$$\begin{aligned} h_1(t) - h_2(t) &= (h_1(0) - h_2(0))e^{-3\int\limits_0^t H_0(\tau)d\tau}, \\ h_1(t) - h_3(t) &= (h_1(0) - h_3(0))e^{-3\int\limits_0^t H_0(\tau)d\tau}. \end{aligned} \tag{57}$$

So we obtain that if the integral $\int\limits_0^t H_0(\tau)d\tau$ is uniformly bounded from below, then anisotropy is bounded at all t. Note, that in the most of cosmological models $H_0(t) > 0$ for all $t > 0$ and the anisotropy tends to zero at $t \to \infty$.

Using (57), one can express $h_2(t)$ and $h_3(t)$ via $h_1(t)$ and reduce system (51)–(53) to one equation.

Let us introduce a new function

$$h_0(t) \equiv h_1(t) - \frac{C_0}{3}e^{-3\int\limits_0^t H_0(\tau)d\tau}. \tag{58}$$

It is easy to check that

$$3h_0(t) = h_1(t) + h_2(t) + h_3(t). \tag{59}$$

In terms of h_0 and φ_k we get the system of equations which coincide with the corresponding Friedmann equations, for which $h_1(t) = h_2(t) = h_3(t) = h_0(t)$.

Therefore, the functions $\varphi_k(t)$ in the Bianchi I and FRW metrics are the same. Functions $h_1(t)$, $h_2(t)$ and $h_3(t)$ differ from the correction for the Hubble parameter $h_0(t)$ on a finite value. Thus the theorem is proven.

Note that Theorem 1 connects the stability properties the the FRM and Bianchi I metrics not only for solutions, which tend to a fixed point, but also for solutions, which tends to infinity at $t \to \infty$.

Model with massless phantom field

Let us consider the stability of solutions of local models, which correspond to the nonlocal model with quadratic potential (I.Ya. Aref'eva, L.V. Joukovskaya, and S.Yu. V., J. Phys. A: Math. Theor. 41 (2008) 304003, arXiv:0711.1364). Let us consider the one-field model with zero potential $V(\phi) = 0$.

The Friedmann equations are:

$$3H^2 = \frac{C}{2m_p^2}\dot{\phi}^2 + \frac{\Lambda}{m_p^2}, \tag{60}$$

$$\dot{H} = -\frac{C}{2m_p^2}\dot{\phi}^2. \tag{61}$$

At $\Lambda > 0$ and $C < 0$ there is the following real solution:

$$\begin{aligned} H_0(t) &= \sqrt{\frac{\Lambda}{3m_p^2}}\tanh\left(\sqrt{\frac{3\Lambda}{m_p^2}}(t-t_0)\right), \\ \phi_0(t) &= \pm\sqrt{-\frac{2m_p^2}{3C}}\arctan\left(\sinh\left(\sqrt{\frac{3\Lambda}{m_p^2}}(t-t_0)\right)\right) + C_1, \end{aligned} \tag{62}$$

where t_0 and C_1 are arbitrary real constant.

Let us consider the stability of the solution (H_0, ϕ_0). Substituting H_0 and ϕ_0 in (46)–(47), to first order in ε we obtain

$$\varphi(t) = \frac{2m_p^2\sqrt{2}e^{2\frac{\sqrt{3m_p^2\Lambda}}{m_p^2}(t-t_0)}}{\sqrt{-C\Lambda}\left(e^{2\frac{\sqrt{3m_p^2\Lambda}}{m_p^2}(t-t_0)}+1\right)}C_3 + C_2, \tag{63}$$

$$h(t) = \frac{2C_3}{\cosh\left(2\frac{\sqrt{3m_p^2\Lambda}}{m_p^2}(t-t_0)\right)+1},$$

where C_2 and C_3 are arbitrary real constants. It is obvious, that functions $h(t)$ and $\varphi(t)$ are bounded. In the Bianchi I metric we have

$$h_i(t) = h(t) + \tilde{C}_i\sqrt{1 - \tanh^2\left(\frac{\sqrt{3m_p^2\Lambda}}{m_p^2}(t-t_0)\right)}. \tag{64}$$

Thus, we have obtained, that kink-type solution (62) in the Bianchi I metric have the bounded first corrections.

Model with quadratic potential and the cosmological constant

Let us consider the model of the scalar field with the quadratic potential and the cosmological constant. In this case the Friedmann equations are

$$H^2 = \frac{8\pi G_N}{3}\left(\frac{C}{2}\dot{\phi}^2 + \frac{B}{2}\phi^2 + \Lambda\right), \tag{65}$$

$$\dot{H} = -4\pi G_N C\dot{\phi}^2, \tag{66}$$

where C and B are arbitrary nonzero real numbers.

System (65)–(66) has the following particular solution

$$H_0(t) = k_1 t, \qquad \phi_0(t) = k_2 t, \tag{67}$$

where

$$k_1 = -\frac{B}{3C}, \qquad k_2^2 = \frac{B}{12\pi G_N A^2}. \tag{68}$$

From (68) it follows, that the function ϕ is real if and only if $B > 0$. The above-mentioned solutions exist only if

$$\Lambda = -\frac{B}{24\pi G_N A}. \tag{69}$$

To analyse stability of these exact solutions we substitute

$$H(t) = k_1 t + \varepsilon h(t) \tag{70}$$

and

$$\phi(t) = k_2 t + \varepsilon \varphi(t). \tag{71}$$

in (65) and (66).

To first order in ε we obtain the following system of equations

$$2\sqrt{3\pi B G_N}(A\dot{\varphi}(t) + Bt\varphi(t)) = 3Bt\, h(t), \tag{72}$$

$$\dot{h}(t) = -8\pi G_N C k_2 \dot{\varphi}(t). \tag{73}$$

Solutions of (72)–(73) are

$$\varphi(t) = \tilde{C}_1 e^{\frac{B}{2C}t^2} + \tilde{C}_2 \tag{74}$$

$$h(t) = \tilde{C}_1 e^{\frac{B}{2C}t^2} + \tilde{C}_2. \tag{75}$$

Therefore, the functions $h(t)$ and $\varphi(t)$ are bounded at $C/B < 0$. Real solutions exist only if $B > 0$, hence, $C < 0$. We come to conclusion that solution (67) can be stable (the first corrections are bounded), only if $C < 0$, in other words, $\phi(t)$ is a phantom scalar field.

Conclusions

- We have analysed the stability of isotropic solutions for the models with the NEC violation in the Bianchi I metric.
- For one-field model we used the Lyapunov theorem and found sufficient conditions for stability of kink-type solutions.
- We found the explicit form of the connection between $h_1(t)$, $h_2(t)$ and $h_3(t)$, which define metric perturbations in the Bianchi I metric, and h_0, which defines perturbations in the FRW metric.
- The first corrections for the fields in both metric are the same.
- In particular we state that for $H_0 \geqslant 0$ the boundedness of h_0 is a sufficient and necessary condition for the boundedness of $h_1(t)$, $h_2(t)$, and $h_3(t)$. This result is valid for both N-field and k-essence models as well as for models with the CDM.
- The exact solutions, found in string inspired phantom models (I. Aref'eva, A. Koshelev, S.V., 2004; I. Aref'eva, L. Joukovskaya, 2005), are stable.