Conformal Higher Spin Gauge Theory and Unfolded Dynamics

To the memory of

Efim Samoilovich Fradkin

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Efim Samoilovich Fradkin was fully devoted to science
and had fantastically broad scope of interests:
Hamiltonian Quantization, SUSY, SUGRA, String Theory,
Higher Spin theory,...

Many unpublished or improperly published important results:
E.S.Fradkin, Proceed. 10 Winter Karpacz School, 1973, Hamiltonian formalism...
E.S.Fradkin, M.V., Light-cone gravity (1974), N=2 SUGRA (1976)

Fradkin's scientific career started from his work on massive fermionic HS fields in 1950

because Ginzburg and Tamm worked in the field

Fradkin, Tseytlin: Symmetric conformal HS gauge fields (1985)

Fradkin, Vasiliev: HS interactions in AdS_4 (1987)

Fradkin, Linetsky: Conformal HS interactions (1989)

HS Symmetries

$\varphi_{n_1 \dots n_s}$ - **rank s double traceless symmetric tensor** Fronsdal 1978

Gauge transformation:

$$\delta \varphi_{k_1 \dots k_s} = \partial_{(k_1} \varepsilon_{k_2 \dots k_s)} = 0$$

$\varepsilon_{k_1 \dots k_{s-1}}(x)$- **symmetric traceless tensor gauge parameter**

Study of HS interactions is the search of symmetries beyond ad hoc geometric pictures.

Metric tensor (spin two) a member of an infinite family of gauge fields.

Formalism of differential forms

Unfolded Dynamics

First-order form of differential equations

$$\dot{q}^i(t) = \varphi^i(q(t)) \qquad \text{initial values:} \quad q^i(t_0)$$

DOF = # of dynamical variables

Field theory: infinite number of DOF = spaces of functions

Maxwell $q \sim \overrightarrow{A}(x)$, $p \sim \overrightarrow{E}(x)$.

Covariant extension $t \to x^n$?

Unfolded dynamics: multidimensional generalization (1988)

$$\frac{\partial}{\partial t} \to d\,, \qquad q^i(t) \to W^\alpha(x) = dx^{n_1} \wedge \ldots \wedge dx^{n_p} W^\alpha_{n_1 \ldots n_p}(x)$$

a set of differential forms

Unfolded equations

$$dW^\alpha(x) = G^\alpha(W(x))\,, \qquad d = dx^n \partial_n$$

$G^\alpha(W)$: **function of "supercoordinates"** W^α

$$G^\alpha(W) = \sum_{n=1}^{\infty} f^\alpha{}_{\beta_1\ldots\beta_n} W^{\beta_1} \wedge \ldots \wedge W^{\beta_n}$$

Covariant first-order differential equations

$d > 1$: **Nontrivial compatibility conditions:** $G^\beta(W) \wedge \frac{\partial G^\alpha(W)}{\partial W^\beta} = 0$ **equivalent to the generalized Jacobi identities**

$$\sum_{n=0}^{m} (n+1) f^\gamma{}_{[\beta_1\ldots\beta_{m-n}} f^\alpha{}_{\gamma\beta_{m-n+1}\ldots\beta_m\}} = 0$$

Any solution to generalized Jacobi identities: FDA **(Sullivan (1968))**

FDA is universal if the generalized Jacobi identity holds for W interpreted as supercoordinates. HS FDAs are universal.

Every universal FDA = some L_∞ algebra

Equivalent form of compatibility condition

$$Q^2 = 0\,, \qquad Q = G^\alpha(W)\frac{\partial}{\partial W^\alpha}$$

Q-manifolds

Hamiltonian-like form of the unfolded equations

$$dF(W(x)) = Q(F(W(x))\,, \qquad \forall F(W)\,.$$

Invariant functionals: Q cohomology

$$S = \int L(W(x))\,, \qquad QL = 0 \qquad \textbf{(2005)}$$

The unfolded equation is invariant under the gauge transformation

$$\delta W^\alpha = d\varepsilon^\alpha + \varepsilon^\beta \frac{\partial G^\alpha(W)}{\partial W^\beta}$$

Properties

- General applicability
- Manifest (HS) gauge invariance
- Invariance under diffeomorphisms

  Exterior algebra formalism
- Interactions: nonlinear deformation of $G^{\alpha}(W)$
- Degrees of freedom are in 0-forms $C^{i}(x_0)$ at any $x = x_0$ (as $q(t_0)$) instead of phase coordinates in the Hamiltonian approach
- Natural realization of infinite symmetries with higher derivatives
- Lie algebra cohomology interpretation

Unfolding as a covariant twistor transform

Twistor transform

$$\begin{array}{ccc} & C(Y|x) & \\ \eta \swarrow & & \searrow \nu \\ M(x) & & T(Y)\,. \end{array}$$

$W^{\alpha}(Y|x)$ **are functions on the "correspondence space"** C.

Space-time M : **coordinates** x. **Twistor space** T : **coordinates** Y.

Unfolded equations: Penrose transform mapping functions on T **to solutions of field equations in** M.

Independence of ambient space-time: Geometry is encoded by $G^{\alpha}(W)$

Physical dimension and metric emerge from unfolded equations 2002

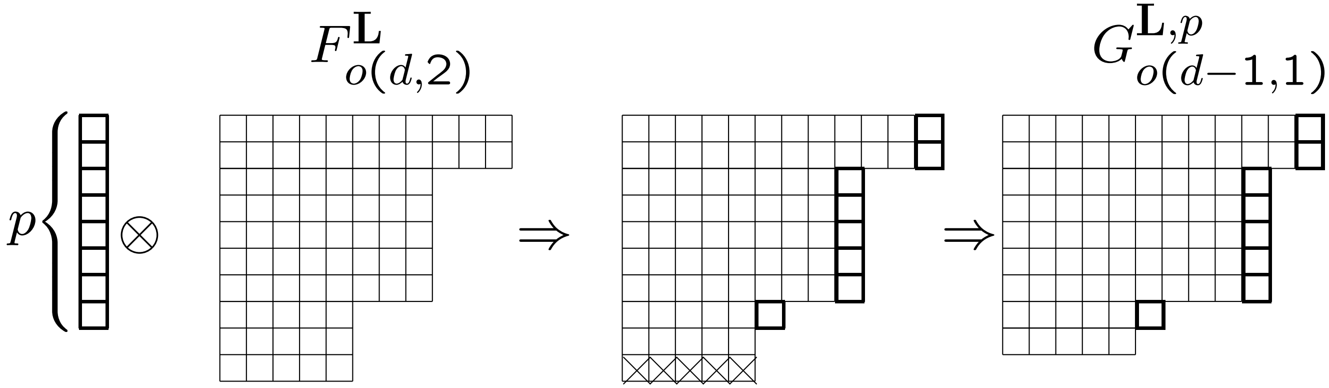
Physical space-times of different dimensions can coexist in an ambient space-time of higher (possibly infinite) dimension.

Branes are not localized while HS symmetries are unbroken

Mixed symmetry conformal fields

σ_- **cohomology determines content of all mixed symmetry conformal fields in any dimension**

p-**form gauge field in** $o(d,2)$-**module** $F^{\mathbf{L}}_{o(d,2)}$: **Lorentz tensor field in the** $o(d-1,1)$-**module** $G^{\mathbf{L},p}_{o(d-1,1)}$



(1)

Dynamical field: $G^{\mathbf{L},p}_{o(d-1,1)}$

Gauge symmetry parameter: $G^{\mathbf{L},p-1}_{o(d-1,1)}$, $L_p - L_{p+1} + 1$ **derivatives**

Ground gauge invariant Weyl tensor: $G^{\mathbf{L},p-1}_{o(d-1,1)}$, $L_{p+1} - L_p + 1$ **derivatives.**

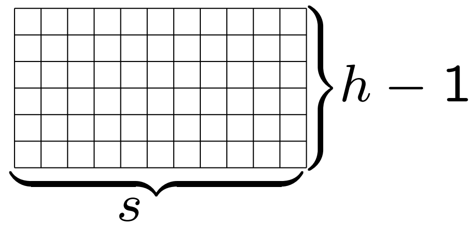
Order of field equations is $d + 2L_{p+1} - 2p$

Example of Block

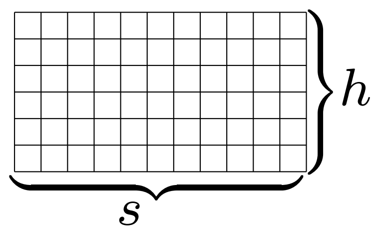
$$\mathbf{L} = (\underbrace{s-1,\ldots,s-1}_{h},0,\ldots 0\ldots 0)$$

$p = h - 1$ **Fradkin-Tseytlin case:** $h = 1, p = 2.$

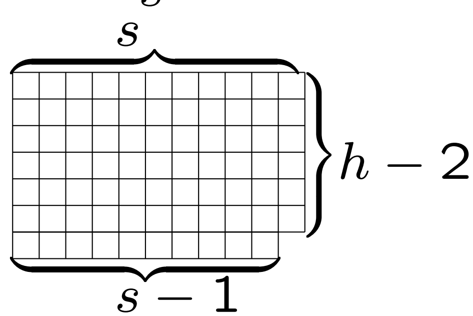
physical fields: ϕ^{dyn} :



Weyl tensor: C :



gauge parameter ε^{dif} :



Physical content via supersymmetric mechanics

Supersymmetric Hamiltonian

$$\mathcal{H} = \frac{1}{4}\Big(T^{L\,ab}T^{L}_{ab} - T^{AB}T_{AB}\Big) - \frac{1}{2}(\Delta + p)(\Delta + p - d)\,,$$

$$T^{AB}T_{AB} = -2\sum_{i=1}^{h} L_i(L_i + d + 2 - 2i)$$

Dynamical fields, Weyl tensors and gauge symmetry parameters: supersymmetric vacua of $\mathcal{H}$

Conclusions

Realization of old Fradkin's idea that Conformal HS gauge theory should help to understand unitary HS gauge theory