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Symmetries of Noncommutative Quantum Field and Gauge Theories

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Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection.

Hermann Weyl

Noncommutative space-time

• Standard space-time = a manifold $\mathcal{M}$;
points $x \in \mathcal{M} \leftrightarrow$ finite number of real coordinates $x^\mu \in R^4$.

• Usual quantum mechanics:

$$\begin{aligned} \left[x_i, x_j\right] &= 0 \ , \ \left[p_i, p_j\right] = 0 \ , \\ \left[x_i, p_j\right] &= i\hbar\delta_{ij} \ . \end{aligned}$$

• This picture of space-time is likely to break down at very short distances $\sim$ Planck length $\lambda_P \approx 1.6 \times 10^{-33}\, cm$.

• A possible approach to description of physics at short distances is

QFT on a NC space-time

• The generalization of commutation relations for the canonical operators of the type

$$x^\mu \to \hat{x}^\mu \quad : \quad [\hat{x}^\mu, \hat{x}^\nu] \neq 0 \ ,$$

was suggested long ago, in particular, by

Snyder (1947); Heisenberg (1954);

Gol'fand (1962)

• The first physical application: particle noncommutativity in the lowest Landau level

Peierls (1933)

– Point particle moving on a plane (x, y) with external magnetic field B perpendicular to the plane

$$L = \frac{1}{2}mv^2 + \frac{e}{c}\vec{v}\cdot\vec{A} - V \quad \text{with} \quad \vec{A} = (0, Bx)$$

– Set m to zero (strong magnetic field)

$$L_0 = \frac{eB}{c}x\dot{y} - V(x, y)$$

which is of the form $p\dot{q} - h(p, q) \Rightarrow \left(\frac{eB}{c}x, y\right)$ form a canonical pair, i.e.

$$\{x, y\}_{PB} = \frac{c}{eB}$$

– Upon quantization

$$[\hat{x}, \hat{y}] = -i\hbar\frac{c}{eB}$$

$\Rightarrow$

Induced noncommutativity of coordinates!

- Practical motivation: the *hope* that QFTs in NC space-time have an improved UV-behaviour.

Snyder (1947)
Grosse, Klimčik and Prešnajder (1996)
Filk (1996)
Chaichian, Demichev and Prešnajder (1998)

- Physical motivations:
  - black hole formation in the process of measurement at small distances ($\sim \lambda_P$) $\Rightarrow$ additional uncertainty relations for **coordinates**

    Doplicher, Fredenhagen and Roberts (1994)

  - open string + D-brane theory in the background with antisymmetric tensor (**NOT induced!**)

    Seiberg and Witten (1999)

- boundary conditions for open string in constant B-field background:

$$\left[g_{mn}(\partial-\bar{\partial})X^n+2\pi\alpha' B_{mn}(\partial+\bar{\partial})X^n|\right]_{z=\bar{z}}=0$$

- corresponding propagator

$$\begin{aligned} < X^m(z,\bar{z})X^n(w,\bar{w}) > \;&=\; -\alpha'(g^{mn}log|z-w|-g^{mn}log|z-\bar{w}| \\ &+\; G^{mn}log|z-\bar{w}|^2+\frac{1}{2\pi\alpha'}\theta^{mn}log(-\frac{z-\bar{w}}{\bar{z}-w}) \end{aligned}$$

- in the limit when both z and w approach the real axis: $z=\bar{z}\rightarrow\tau_1$, $w=\bar{w}\rightarrow\tau_2$, the propagator becomes:

$$< X^m(\tau_1)X^n(\tau_2) >= -\alpha' G^{mn}log(\tau_1-\tau_2)^2+\frac{i}{2}\theta^{mn}sign(\tau_1-\tau_2)$$

implying the commutation relation:

$$[X^m,X^n]=i\theta^{mn},$$
$$\theta^{mn}=-(2\pi\alpha')\left(\frac{1}{g+2\pi\alpha' B}B\frac{1}{g-2\pi\alpha' B}\right)$$

- **Induced noncommutativity?** See gravitational and gauge anomalies

Álvarez-Gaumé and Witten (1984)

Green and Schwartz (1984)

NC space-time and field theory; $\star$-product

Heisenberg-like commutation relations

$$[\hat{X}^\mu, \hat{X}^\nu] \;=\; i\theta^{\mu\nu} \;,$$

$\theta^{\mu\nu}$ - *constant* antisymmetric matrix $\Longrightarrow$ Lorentz invariance violated

$$\textbf{QFT} \;\rightarrow\; \textbf{NC-QFT}: \quad \Phi(x) \;\rightarrow\; \hat{\Phi}(\hat{X}) \;.$$

$$S^{(cl)}[\Phi] \;=\; \int d^4x \left[\frac{1}{2}(\partial^\mu\Phi)(\partial_\mu\Phi) - \frac{1}{2}m^2\Phi^2 - \frac{\lambda}{4!}\Phi^4\right] \;,$$

$$\Downarrow$$

$$S^{(\theta)}[\hat{\Phi}] \;=\; \mathrm{Tr}\left[\frac{1}{2}(\hat{\partial}^\mu\hat{\Phi})(\hat{\partial}_\mu\hat{\Phi}) \;-\frac{1}{2}m^2\hat{\Phi}^2 - \frac{\lambda}{4!}\hat{\Phi}^4\right] \;.$$

Field theory formulation be based on operator (e.g. Weyl) symbols
$\Phi(x)$ = functions on the **commutative** counterpart of the space-time

Weyl-Moyal correspondence

$$\widehat{\Phi}(\hat{X}) \longleftrightarrow \Phi(x)$$

$$\widehat{\Phi}(\hat{X}) = \int e^{i\alpha\hat{X}}\phi(\alpha)d\alpha, \quad \Phi(x) = \int e^{i\alpha x}\phi(\alpha)d\alpha,$$

where α and x are real variables. Then, using the Baker-Campbell-Hausdorff formula:

$$\widehat{\Phi}(\hat{X})\widehat{\Psi}(\hat{X}) = \int e^{i\alpha\hat{X}}\phi(\alpha)e^{i\beta\hat{X}}\psi(\beta)d\alpha d\beta = \int e^{i(\alpha+\beta)\hat{X}-\frac{1}{2}\alpha_\mu\beta_\nu[\hat{X}_\mu,\hat{X}_\nu]}\phi(\alpha)\psi(\beta)$$

Hence the **Moyal $\star$-product** is defined:

$$\widehat{\Phi}(\hat{X})\widehat{\Psi}(\hat{X}) \longleftrightarrow (\Phi \star \Psi)(x),$$

$$(\Phi \star \Psi)(x) \equiv \left[\Phi(x) e^{\frac{i}{2}\theta_{\mu\nu}\frac{\overleftarrow{\partial}}{\partial x_\mu}\frac{\overrightarrow{\partial}}{\partial y_\nu}}\Psi(y)\right]_{x=y}.$$

Thus, all the multiplications (e.g. in the Lagrangian) must be replaced by the $\star$-product

$$S^\theta[\Phi] = \int d^4x \left[\frac{1}{2}(\partial^\mu\Phi)\star(\partial_\mu\Phi) - \frac{1}{2}m^2\Phi\star\Phi - \frac{\lambda}{4!}\Phi\star\Phi\star\Phi\star\Phi\right]$$

Space-time symmetry of NC QFT

• $\theta_{\mu\nu}$ antisymmetric *constant* matrix $\Rightarrow$ **Lorentz invariance violated** (for a dimension of space-time $D > 2$).

• Translational invariance preserved.

• On 4-dimensional space there exists a frame in which the antisymmetric matrix $\theta_{\mu\nu}$ takes the form:

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & \theta' & 0 & 0 \\ -\theta' & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix}.$$

Lorentz group broken to $SO(1,1) \times SO(2)$ subgroup.

Álvarez-Gaumé, Barbón and Zwicky (2001)

• Problem with the representations: both $SO(1,1)$ and $SO(2)$ being Abelian groups, they have only one-dimensional unitary irreducible representations and thus no spinor, vector etc. representations!

Twist deformation of the Poincaré algebra

Chaichian, Kulish, Nishijima and A.T. (2004)

Chaichian, Prešnajder and A.T. (2004)

• Action of NC QFT written with $\star$-product, though it violates Lorentz symmetry, it is invariant under the twisted Poincaré algebra

• Deform the universal enveloping of the Poincaré algebra $\mathcal{U}(\mathcal{P})$ with Abelian twist element $\mathcal{F} \in \mathcal{U}(\mathcal{P}) \otimes \mathcal{U}(\mathcal{P})$

Drinfel'd (1983)

Reshetikhin (1990)

$$\mathcal{F} = exp\left(\frac{i}{2}\theta^{\mu\nu} P_\mu \otimes P_\nu\right)$$

• Commutation relations of Poincaré generators not changed:

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \ , \\
[M_{\mu\nu}, P_\alpha] &= -i(\eta_{\mu\alpha}P_\nu - \eta_{\nu\alpha}P_\mu) \ , \\
[M_{\mu\nu}, M_{\alpha\beta}] &= -i(\eta_{\mu\alpha}M_{\nu\beta} - \eta_{\mu\beta}M_{\nu\alpha} - \eta_{\nu\alpha}M_{\mu\beta} + \eta_{\nu\beta}M_{\mu\alpha})
\end{aligned}$$

Essential physical implication: the representations of the twisted Poincaré algebra are the same as the ones of usual Poincaré algebra

• The twist deforms the action of $\mathcal{U}(\mathcal{P})$ in the tensor product of representations, defined by the coproduct

$$\Delta_0 : \mathcal{U}(\mathcal{P}) \to \mathcal{U}(\mathcal{P}) \otimes \mathcal{U}(\mathcal{P}) \, , \quad \Delta_0(Y) = Y \otimes 1 + 1 \otimes Y \ ,$$

$$\Delta_0(Y) \mapsto \Delta_t(Y) = \mathcal{F} \Delta_0(Y) \mathcal{F}^{-1}$$

Namely the coproduct of the Lorentz algebra generators is changed:

$$\Delta_t(M_{\mu\nu}) = e^{\frac{i}{2}\theta^{\alpha\beta} P_\alpha \otimes P_\beta} \Delta_0(M_{\mu\nu}) e^{-\frac{i}{2}\theta^{\alpha\beta} P_\alpha \otimes P_\beta} \ .$$

• The twist also deforms the multiplication in the algebra of representations of the Poincaré algebra, i.e. algebra of fields $\mathcal{A}_\theta$:

$$m_t(\phi(x) \otimes \psi(x)) = m \circ \mathcal{F}^{-1}(\phi(x) \otimes \psi(x)) =: \phi(x) \star \psi(x)$$

i.e., with the realization on Minkowski space $P_\mu = i\partial_\mu$

$$\begin{aligned} \phi(x) \star \psi(x) &= m \circ e^{-\frac{i}{2}\theta^{\mu\nu} P_\mu \otimes P_\nu}(\phi(x) \otimes \psi(x)) = m \circ e^{\frac{i}{2}\theta^{\mu\nu} \partial_\mu \otimes \partial_\nu}(\phi(x) \otimes \psi(x)) \\ &= \phi(x) e^{\frac{i}{2}\theta^{\mu\nu} \overleftarrow{\partial_\mu} \overrightarrow{\partial_\nu}} \psi(x) \end{aligned}$$

• The twisted Poincaré symmetry exists provided that, in a Lagrangian: (i) we consider $\star$-products among functions instead of the usual one and (ii) we take the proper action of generators specified by the twisted coproduct.

• As a byproduct with major physical implications, the representation content of NC QFT, invariant under the twist-deformed Poincaré algebra, is identical to the one of the corresponding commutative theory with usual Poincaré symmetry $\Rightarrow$ representations (fields) are classified according to their MASS and SPIN.

• New concept of relativistic invariance: while symmetry under usual Lorentz transformations guarantees the relativistic invariance of a theory, in NC QFT the concept of relativistic invariance should be replaced by the requirement of invariance of the theory under twisted Poincaré transformations.

Precursors

-in the context of NC string theory, using $\mathcal{R}$-matrix

Watts (1999)

- mostly in the context of braided field theory, using the dual language of Hopf algebras

Oeckl (2000)

Developments

- differential calculus, twisted diffeomorphisms and NC gravity

Wess (2004)
Aschieri, Blohmann, Dimitrijevic, Meyer, Schupp and Wess (2005)
Aschieri, Dimitrijevic, Meyer and Wess (2005)
Álvarez-Gaumé, Meyer and Vázquez-Mozo (2006)

- supersymmetric twisted Poincaré algebra

Kobayashi and Sasaki (2005)
Zupnik (2005)
Ihl and Saemann (2005)

- global counterpart of the twisted Poincaré algebra

Gonera, Kosinski, Maslanka and Giller (2005)

...

Some known implications...

Twisted Poincaré symmetry and spin-statistics relation

- $\mathcal{R}$-matrix relates the coproduct Δ_t and $\Delta_t^{op} = \tau \circ \Delta_t$, τ - flip operator:

$$\mathcal{R}\Delta_t = \Delta_t^{op}\mathcal{R}, \quad \mathcal{R} = \sum \mathcal{R}_1 \otimes \mathcal{R}_2 \Rightarrow \mathcal{R} = \mathcal{F}_{21}\mathcal{F}^{-1} = exp(-i\theta^{\mu\nu} P_\mu \otimes P_\nu)$$

- Concept of permutation changes

$$P \to \Psi(\mathcal{R}) = P\ \mathcal{R} = P\mathcal{F}^{-2}$$

but $\Psi^{-1} = \Psi \Rightarrow$ "symmetric braiding" $\equiv$ no braiding!

Chari and Pressley (book 1994)

Chaichian and Demichev (book 1996)

- spin-statistics relation all right (as long as the theory can be quantized)

A.T. (2006,2007)

Bu, Kim, Lee, Vac and Yee (2006)

Twisted tensor product of two copies of $\mathcal{A}_\theta$

$$(a_1 \otimes 1)(1 \otimes a_2) = a_1 \otimes a_2, \quad \text{but } (1 \otimes a_2)(a_1 \otimes 1) = (\mathcal{R}_2 a_1) \otimes (\mathcal{R}_1 a_2),$$
$$a_1, a_2 \in \mathcal{A}_\theta$$

$$\Rightarrow \quad x^\mu y^\nu - y^\nu x^\mu := (x^\mu \otimes 1)(1 \otimes y^\nu) - (1 \otimes y^\nu)(x^\mu \otimes 1)$$
$$= (x^\mu \otimes x^\nu) - (\mathcal{R}_2 x^\mu) \otimes (\mathcal{R}_1 y^\nu) = (x^\mu \otimes x^\nu) - (x^\mu \otimes x^\nu) + i\theta^{\mu\nu}$$

$$\Rightarrow \phi(x) \star \phi(y)$$

Oeckl (2000)

• Implications on the axiomatic formulation, Whightman functions defined with $\star$-product etc.

Global counterpart of twisted Poincaré algebra

Oeckl (2000)

Gonera, Kosinski, Maslanka and Giller (2005)

- DUALITY between universal enveloping algebra of the Poincaré algebra $\mathcal{U}(\mathcal{P})$ and the algebra of functions on the Poincaré group, $F(P)$, generated by $\mathbf{\Lambda}^\mu_\nu$ and $\mathbf{a}^\mu$, such that

$$\mathbf{\Lambda}^\mu_\nu \left(e^{i\omega^{\alpha\beta}M_{\alpha\beta}}\right) = \left(\Lambda_{\alpha\beta}(\omega)\right)^\mu_\nu\,, \qquad \mathbf{\Lambda}^\mu_\nu \left(e^{ia^\alpha P_\alpha}\right) = 0$$
$$\mathbf{a}^\mu \left(e^{i\omega^{\alpha\beta}M_{\alpha\beta}}\right) = 0\,, \qquad \mathbf{a}^\mu \left(e^{ia^\alpha P_\alpha}\right) = a^\mu\,,$$

- DUALITY survives the twist, between twisted Poincaré algebra $\mathcal{U}_t(\mathcal{P})$ (twisted coproduct) and $F_\theta(P)$ (twisted multiplication), BUT

$$[\mathbf{a}^\mu, \mathbf{a}^\nu] = i\theta^{\mu\nu} - i\mathbf{\Lambda}^\mu_\alpha \mathbf{\Lambda}^\nu_\beta \theta^{\alpha\beta}\,,$$
$$[\mathbf{\Lambda}^\mu_\nu, \mathbf{a}^\alpha] = [\mathbf{\Lambda}^\mu_\alpha, \mathbf{\Lambda}^\nu_\beta] = 0, \quad \mathbf{\Lambda}^\mu_\alpha, \mathbf{a}^\mu \in F_\theta(P)\,.$$

- The "coordinates" x^μ, generating the algebra of functions with $\star$-product $\mathcal{C}_\theta$, transform by the coaction of the quantum matrix group:

$$\delta : \mathcal{C}_\theta \to F_\theta(P) \otimes \mathcal{C}_\theta$$
$$(x')^\mu = \delta(x^\mu) = \mathbf{\Lambda}^\mu_\alpha \otimes x^\alpha + \mathbf{a}^\mu \otimes 1\,, \quad \text{such that } [x'_\mu, x'_\nu] = i\theta_{\mu\nu}.$$

$$(x')^\mu = \mathbf{\Lambda}^\mu_\alpha \otimes x^\alpha + \mathbf{a}^\mu \otimes 1$$

$$[\mathbf{a}^\mu, \mathbf{a}^\nu] = i\theta^{\mu\nu} - i\mathbf{\Lambda}^\mu_\alpha \mathbf{\Lambda}^\nu_\beta \theta^{\alpha\beta}\,, \quad \mathbf{a}^\mu \in F_\theta(P)$$

- consider Lorentz transformation mixing commutative and noncommutative directions

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \theta \\ 0 & 0 & -\theta & 0 \end{pmatrix} \text{ and } \Lambda_{12} = \mathbf{\Lambda}\left(e^{\omega^{12}(\beta)M_{12}}\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\beta & \sin\beta & 0 \\ 0 & -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

- then

$$\begin{aligned} \left[\mathbf{a}^2\left(e^{\omega^{12}(\beta)M_{12}}\right), \mathbf{a}^3\left(e^{\omega^{12}(\beta)M_{12}}\right)\right] &= [a^2, a^3] = i\,\theta\,(1 - \cos\beta)\,, \\ \left[\mathbf{a}^1\left(e^{\omega^{12}(\beta)M_{12}}\right), \mathbf{a}^3\left(e^{\omega^{12}(\beta)M_{12}}\right)\right] &= [a^1, a^3] = -i\,\theta\sin\beta\,, \end{aligned}$$

By imposing a Lorentz transformation mixing commutative and non-commutative directions we get accompanying noncommuting translations showing up as the *internal mechanism* by which the twisted Poincaré symmetry keeps the commutator $[x_\mu, x_\nu] = i\theta_{\mu\nu}$ invariant.

What is a noncommutative field?

Transformation rules for fields under twisted Poincaré algebra

Chaichian, Kulish, A.T., Zhang and Zhang (2007)

Chaichian, Nishijima, Salminen and A.T. (2008)

- Relativistic classical fields: action of the Poincaré group on them defined by the method of induced representations

$$\Phi = \phi \otimes v\,, \quad \Phi \in \Gamma(V) = (C^{\infty}(\mathbb{R})^{1,3} \otimes_{\mathbb{C}} V)^{L},$$

$L = Spin(1,3)$, $V -$ Lorentz-module

$$\Phi(\Lambda \exp(iPx)) = \rho(\Lambda)\Phi(\exp(iPx)), \quad \phi(x) = \Phi(\exp(iPx))$$

- transformation rule for commutative relativistic classical fields under Poincaré group:

$$(\Lambda \exp(iPa) \cdot \phi)(x) = \rho(\Lambda)\phi(\Lambda^{-1}x + a), \;\; \Lambda \exp(iPa) \in G = Spin(1,3) \ltimes (\mathbb{R})^{1,3}$$

Essential for the construction: Group algebra of L is a Hopf subalgebra of the group algebra of G under the co-multiplication Δ_0 ($\Delta_0(g) = g \times g,\; g \in G$).

Noncommutative classical fields: construction by induced representations fails since enveloping algebra of Lorentz subalgebra is not a Hopf subalgebra of $\mathcal{U}(\mathcal{P})$

- illuminating example (only $\theta_{23} = -\theta_{32} = \theta \neq 0$)

$$\begin{aligned}
\Delta_t(M_{01}) &= \Delta_0(M_{01}) = M_{01} \otimes 1 + 1 \otimes M_{01}, \\
\Delta_t(M_{23}) &= \Delta_0(M_{23}) = M_{23} \otimes 1 + 1 \otimes M_{23}, \\
\Delta_t(M_{02}) &= \Delta_0(M_{02}) + \frac{\theta}{2}(P_0 \otimes P_3 - P_3 \otimes P_0), \\
\Delta_t(M_{03}) &= \Delta_0(M_{03}) - \frac{\theta}{2}(P_0 \otimes P_2 - P_2 \otimes P_0), \\
\Delta_t(M_{12}) &= \Delta_0(M_{12}) + \frac{\theta}{2}(P_1 \otimes P_3 - P_3 \otimes P_1), \\
\Delta_t(M_{13}) &= \Delta_0(M_{13}) - \frac{\theta}{2}(P_1 \otimes P_2 - P_2 \otimes P_1).
\end{aligned}$$

then $M_{02}, M_{03}, M_{12}, M_{13}$ cannot act by twisted coproduct on the field

$$\Phi = \phi \otimes v$$

since $v \in V -$ Lorentz module and does not admit the action of P_μ!

Two ways out:

- take V as $\mathcal{U}(\mathcal{P})$-module with *trivial action* of all the generators P_μ

Chaichian, Kulish, A.T., Zhang and Zhang (2007)

- problems with the finite twisted Poincaré transformations still remain!

- keep V as L-module, but forbid all the transformations requiring the action of the generators P_μ on v

$\Rightarrow$ Only transformations under the stability group of θ-matrix allowed

Chaichian, Nishijima, Salminen and A.T. (2008)

Meaning of the twisted Poincaré symmetry in NC QFT : invariance with respect to the stability group of $\theta_{\mu\nu}$, while the quantum fields still carry representations of the full Lorentz group and the Hilbert space of states has the richness of particle representations of the commutative QFT.

Physical application: A realization of the Cohen-Glashow *Very Special Relativity*

• VSR: symmetry under certain subgroups of Poincaré group, which contain space-time translations and at least a 2-parametric proper subgroup of the Lorentz transformations, isomorphic to that generated by $K_x + J_y$ and $K_y - J_x$, where $\mathbf{J}$ and $\mathbf{K}$ are the generators of rotations and boosts, respectively.

• VSR groups: $T(2)$ $(T_1 = K_x + J_y, T_2 = K_y - J_x)$, $E(2)$, $HOM(2)$, $SIM(2)$
- when supplemented with T, P or CP, they will be enlarged to the full Lorentz group;
- ALL VSR groups have only one-dimensional representations!

• Motivation: the hope that at very high energy scales VSR provides the symmetry of a (most probably nonlocal) "master theory", which gives in the low-energy limit the well-known theories of particle physics.

Cohen and Glashow (2006)

• Problems:
-phenomenological construction is not unique;
-the representation content of the "master theory" is poorer than that of the low-energy limit.

Realization of VSR on the noncommutative space-time

Sheikh-Jabbari and A.T. (2008)

• The only VSR group that admits a realization on NC space-time is $T(2)$

$\Rightarrow$ LIGHT-LIKE NONCOMMUTATIVITY ($\theta^{\mu\nu}\theta_{\mu\nu} = 0$)!

$$\theta^{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \theta & \theta' \\ 0 & -\theta & 0 & 0 \\ 0 & -\theta' & 0 & 0 \end{pmatrix} \quad \text{(in light-cone coordinates) stable under } T(2)$$

• The realization of $E(2)$, $HOM(2)$, $SIM(2)$ on noncommutative space-time necessarily violates translational symmetry.

• Advantages:

- unique realization of VSR as NC QFT on space-time with light-like NCty
- representation content of "master theory" identical to the low-energy limit due to the twisted Poincaré symmetry
- spin-statistics relation, CPT theorem valid
- low-energy limit of string theory
- (perturbative) unitarity all right
- quantization all right (light-cone coordinates).

Is the concept of twist a symmetry principle in constructing NC field theories, i.e. any symmetry that NC field theories may enjoy, be it space-time or internal symmetry, global or local, should be formulated as a twisted symmetry?

Twisted gauge symmetry?

- NC gauge theories - traditional approach

Hayakawa (1999)

The NC QED action:

$$S_{NC\ QED} = \int d^4x \left(-\frac{1}{4} F_{\mu\nu} \star F^{\mu\nu} + \bar{\Psi} \star (\not{D} - m)\Psi + L_{gauge} + L_{ghost} \right)$$

where

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - i(A_\mu \star A_\nu - A_\nu \star A_\mu) \ , \\ D_\mu \Psi &= \partial_\mu \Psi - i A_\mu \star \Psi \ . \end{aligned}$$

NC gauge group elements:

$$\begin{aligned} &U(x) = \exp\star\{i\lambda\} \equiv 1 + i\lambda - \frac{1}{2}\lambda \star \lambda \ + \ .... \ , \\ &U(x) \star U(x)^{-1} = U(x)^{-1} \star U(x) = 1 \ . \end{aligned}$$

Gauge transformations:

$$\begin{aligned} &A_\mu \to A'_\mu(x) = U(x) \star A_\mu \star U^{-1}(x) + iU(x) \star \partial_\mu U(x)^{-1} \ , \\ &\Psi(x) \to \Psi'(x) = U(x) \star \Psi(x) \ . \end{aligned}$$

- Remark: only NC $U(n)$ groups close (not, e.g., $SU(n)$)
- **No-go theorem** - strong restrictions on model building!

Terashima (2000)

Chaichian, Prešnajder, Sheikh-Jabbari and A.T. (2001)

(i) the local NC $u(n)$ algebra only admits the irreducible $n \times n$ matrix-representation. Hence the gauge fields are in the $n \times n$ matrix form, while the matter fields *can only be* in fundamental, adjoint or singlet states;

(ii) for any NC gauge group consisting of several simple-group factors, the matter fields can transform nontrivially under *at most* two group factors.

- Applications:

- NC Standard Model

Chaichian, Prešnajder, Sheikh-Jabbari and A.T. (2001)

Chaichian, Kobakhidze and A.T. (2004)

- NC MSSM

Arai, Saxell and A.T. (2006)

• Attempt to *twist gauge transformations*: extend the Poincaré algebra by semidirect product with the gauge generators and apply the Abelian twist $\mathcal{F} = e^{\left(\frac{i}{2}\theta^{\mu\nu}P_\mu \otimes P_\nu\right)}$ also to the coproduct of the gauge generators

Vassilevich (2006)

Aschieri, Dimitrijevic, Meyer, Schraml and Wess (2006)

- infinitesimal gauge transformation of the individual fields the usual form (without $\star$-product):

$$\delta_\alpha \Phi(x) = \alpha(x)\Phi(x) \ , \quad \alpha(x) = i\alpha^a(x)T_a \ , \quad [T_a, T_b] = if_{abc}T_c$$

- claim

$$\delta_\alpha(\Phi_1(x) \star \Phi_2(x)) = i\alpha^a(x)\left[(\Phi_1(x)T_a^{(1)})\star\Phi_2(x) + \Phi_1(x)\star(T_a^{(2)}\Phi_2(x))\right]$$

- consequences: any gauge algebra would close and any representation is allowed, just as in the commutative case, i.e. contradiction with the no-go theorem!

- Contradiction with the gauge principle:

$$\delta_\alpha(\Phi_1(x)\star\Phi_2(x)) = i\alpha^a(x)\left[(\Phi_1(x)T_a^{(1)})\star\Phi_2(x)+\Phi_1(x)\star(T_a^{(2)}\Phi_2(x))\right].$$

is valid only if one *assumes* that, once $\delta_\alpha\Phi(x)=\alpha(x)\Phi(x)$, then also

$$\delta_\alpha((-i)^n P_{\mu_1}...P_{\mu_n}\Phi(x)) = \delta_\alpha(\partial_{\mu_1}...\partial_{\mu_n}\Phi(x)) = \alpha(x)(\partial_{\mu_1}...\partial_{\mu_n}\Phi(x))$$

which is true only when the "local" parameter α^a is global!

$$\begin{aligned}
\delta_\alpha(\Phi_1\star\Phi_2) &= m_\star\circ\Delta_t(\alpha(x))(\Phi_1(x)\otimes\Phi_2(x)) \\
&= m\circ\mathcal{F}^{-1}\mathcal{F}\Delta_0(\alpha(x))\mathcal{F}^{-1}(\Phi_1(x)\otimes\Phi_2(x)) \\
&= m\circ\Delta_0(\alpha)\mathcal{F}^{-1}(\Phi_1(x)\otimes\Phi_2(x)) \\
&= m\circ\Delta_0(\alpha)e^{\left(\frac{i}{2}\theta^{\mu\nu}\partial_\mu\otimes\partial_\nu\right)}(\Phi_1(x)\otimes\Phi_2(x)) \\
&= m\circ(\delta_\alpha\otimes 1+1\otimes\delta_\alpha)\left[\Phi_1\otimes\Phi_2+\frac{i}{2}\theta^{\mu\nu}(\partial_\mu\Phi_1\otimes\partial_\nu\Phi_2)+\cdots\right.
\end{aligned}$$

Chaichian and A.T. (2006)

However

$$\delta_\alpha(D_{\mu_1}...D_{\mu_n}\Phi(x)) = \alpha(x)(D_{\mu_1}...D_{\mu_n}\Phi(x))$$

• Propose a Non-Abelian twist element of $\mathcal{U}(\mathcal{P} \ltimes \mathcal{G})$:

$$\mathcal{T} = \exp\left(-\frac{i}{2}\theta^{\mu\nu} D_\mu \otimes D_\nu + \mathcal{O}(\theta^2)\right),$$

a power series expansion, such that $\mathcal{T}$ would satisfy the twist conditions:

$$(\mathcal{T} \otimes 1)(\Delta_0 \otimes id)\mathcal{T} = (1 \otimes \mathcal{T})(id \otimes \Delta_0)\mathcal{T}, \quad (\epsilon \otimes id)\mathcal{T} = 1 = (id \otimes \epsilon)\mathcal{T}$$

Chaichian, A.T. and Zet (2006)

- No possible second order terms fulfill the twist condition $\Rightarrow$ a non-Abelian twist element, which would generalize the Abelian twist in a gauge covariant manner cannot exist, i.e. Poincaré symmetry and internal gauge symmetry cannot be unified under a common twist
- situation is reminiscent of the Coleman-Mandula no-go theorem

COULD SUPERSYMMETRY PROVIDE THE SOLUTION?

• Attempts to gauge the twisted Poincaré algebra into a noncommutative theory of gravity

Chaichian, Oksanen, A.T. and Zet (2009)

Some problems to be attacked and clarified

• Dirac quantization condition for magnetic monopole (nonperturbative topological vs. perturbative)

$$e\mu = \frac{n\hbar}{2}c$$

- first attempts in

Chaichian, Ghosh, Langvick and A.T. (2009)

• Looking really at the solutions of NC Gravity, to find out about the singularity of solutions, Schwarzschild, Reissner-Nordström, black holes... and repeat the same arguments for the consistency of emergence of the noncommutativity of space-time based on QM and the NEW way of black hole formation.

• FQHE

QHE description by NC Chern-Simons theory — Susskind (2000)

Field theoretical approach to FQHE. — Hellerman and van Raamsdonk (2001)

Froehlich (1992), (1993), (1995)

• Formulation of noncommutative field theories with finite-range nonlocality with the hope of removing the UV/IR mixing.

• Consistency of the NC QFT with noncommutative time?

- path integral formulation

Fujikawa (2004)

- operator formulation - interaction picture, Tomonaga-Schwinger equation

Chaichian, Nishijima, Salminen and A.T. (2008)

- operator formulation - Heisenberg picture, Yang-Feldman formalism

Meinander, Salminen and A.T. (in preparation)