Overview of Electromagnetic Properties of Neutrinos

IV Sakharov Conference
FIAN, 18/05/09

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Carlo Giunti, A.S. :

- *"Neutrino electromagnetic properties"*
  arXiv:0812.3646, to appear in Phys.Atom.Nucl. 73 (2009)

A.S. :

- *"Neutrino magnetic moment: a window to new physics"*
  arXiv:0812.4716,
  Nucl.Phys.B (Proc.Supl.) 188, 220 (2009)

…Why

Electromagnetic properties of ν

provide a kind of ***window / bridge*** *to*

NEW Physics ?

… Up to now, in spite of reasonable efforts,

- ***NO** any unambiguous experimental confirmation in favour of nonvanishing ν **em** properties ,*
- *available experimental data in the field do not rule out possibility that ν have **"ZERO" em** properties.*
- *… However, in course of recent development of knowledge on ν mixing and oscillations,*

Recent studies (exp. & theor.) of
flavour conversion of
solar, atmospheric, reactor and accelerator
neutrinos have conclusively established that

! neutrinos have non-zero mass !

and they mix among themselves
that provides the first evidence of new physics
beyond the standard model

Neutrino mass

$$m_\nu \neq 0 \;!$$

⇒

Neutrino mass

$m_\nu \neq 0$!

⇒

Neutrino magnetic moment ν ⊙

$\mu_\nu \neq 0$? (!)

⊛ Lee, Shrock 1977

Shrock, Fujikawa 1980

...Massive neutrino electromagnetic properties...

Theory (Standard Model with ν_R)

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e} \sim 3\cdot 10^{-19}\mu_B\left(\frac{m_{\nu_e}}{1eV}\right), \quad \mu_B = \frac{e}{2m_e}$$

Lee
Shrock, 1977; Fujikawa
Shrock, 1980

In the Standard Model : $m_\nu = 0$,
there is no ν_R $\Longrightarrow$
ν magnetic moment $\mu_\nu = 0$.

Thus, $\mu_\nu \neq 0$ ← beyond the SM. !

… puzzling

ν electromagnetic properties

something that is tiny or probably even do not exist at all…

ν exhibits unexpected properties (puzzles)

W. Pauli, 1930

Pauli **himself wrote to** **Baade**:
"Today I did something a physicist should never do. I predicted something which will never be observed experimentally...".

- neutron **now we know that it is** ***neutrino*** ***E.Fermi, 1933***
- neutral **now we know that** $\boxed{q_\nu \neq 0}$ **in plasma and beyond SM (?)**
- and probably $m_\nu \neq 0$? !
- massless particle **now we know that** $m_\nu \neq 0$
- ν **very important player** **(astrophysics, cosmology etc. . .)**

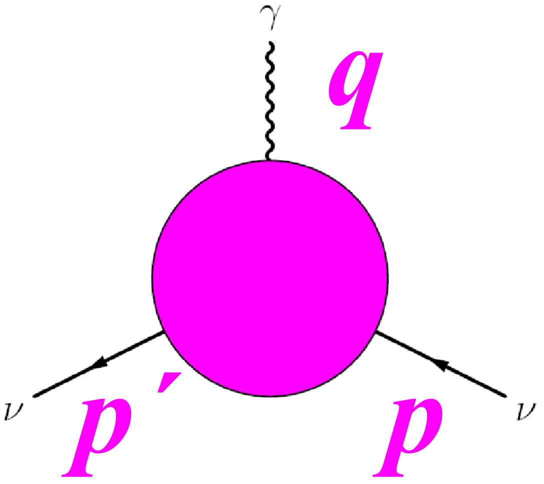
Outline

- ν electromagnetic properties - theory
- ν magnetic moment - experiment
- constraints on ν electromagnetic properties

● Outline ●

0. Introduction
1. ν magnetic moment in experiments
2. New experimental result on μ_ν
3. ν electromagnetic properties - theory
   - 3.1 ν vertex function
   - 3.2 μ_ν (arbitrary masses)
   - 3.3 relationship between m_ν and μ_ν
   - 3.4 ν vertex function in case of flavour mixing
   - 3.5 ν dipole moments in case of mixing
   - 3.6 μ_ν in left-right symmetry models
   - 3.7 astrophysical bounds on μ_ν
   - 3.8 ν millicharge (Red Gaints cooling etc)
   - 3.9 ν charge radius and anapole moment
   - 3.10 ν electromagnetic properties in matter and e.m.f.
4. Effects of ν electromagnetic properties
   - 3.11 ν radiative decay, *Ch* radiation and *Spin Light of* ν in matter
   - 3.12 ν radiative $2\times\gamma$ - decay
   - 3.13 ν spin-flavour oscillations
5. Direct-Indirect influence of e.m.f. on ν
6. Conclusion

ν electromagnetic vertex function



$$< \psi(p')|J_{\mu}^{EM}|\psi(p) >= \bar{u}(p')\Lambda_{\mu}(q,l)u(p)$$

Matrix element of electromagnetic current is a Lorentz vector

$\Lambda_{\mu}(q,l)$ *should be constructed using*

matrices $\hat{1},\quad \gamma_5,\quad \gamma_\mu,\quad \gamma_5\gamma_\mu,\quad \sigma_{\mu\nu},$

tensors $g_{\mu\nu},\quad \epsilon_{\mu\nu\sigma\gamma}$

vectors q_μ *and* l_μ

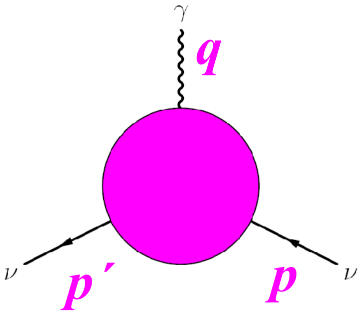
$$q_\mu = p'_\mu - p_\mu,\ l_\mu = p'_\mu + p_\mu$$

Vertex function $\Lambda_\mu(q,l)$ → **there are three sets of operators:**

- $\hat{\mathbf{1}} q_\mu, \quad \hat{\mathbf{1}} l_\mu, \quad \gamma_5 q_\mu, \quad \gamma_5 l_\mu$

  $\not{q} q_\mu, \quad \not{l} q_\mu, \quad \gamma_5 q_\mu, \quad \gamma_5 \not{q} q_\mu, \quad \gamma_5 \not{l} q_\mu, \quad \sigma_{\alpha\beta} q^\alpha l^\beta q_\mu, \quad \{q_\mu \leftrightarrow l_\mu\}$
- $\gamma_\mu, \quad \gamma_5 \gamma_\mu, \quad \sigma_{\mu\nu} q^\nu, \quad \sigma_{\mu\nu} l^\nu.$
- $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} q^\nu, \quad \epsilon_{\mu\nu\sigma\gamma} \sigma^{\alpha\beta} l^\nu, \quad \epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} q_\beta q^\sigma l^\gamma,$

  $\epsilon_{\mu\nu\sigma\gamma} \sigma^{\nu\beta} l_\beta q^\sigma l^\gamma, \quad \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \hat{\mathbf{1}}, \quad \epsilon_{\mu\nu\sigma\gamma} \gamma^\nu q^\sigma l^\gamma \gamma_5$



vertex function **(using Gordon-like identities)**

$$\boxed{\begin{aligned}\Lambda_\mu(q,l) = {} & f_1(q^2) q_\mu + f_2(q^2) q_\mu \gamma_5 + f_3(q^2)\gamma_\mu + \\ & f_4(q^2)\gamma_\mu\gamma_5 + f_5(q^2)\sigma_{\mu\nu} q^\nu + f_6(q^2)\epsilon_{\mu\nu\rho\gamma}\sigma^{\rho\gamma} q^\nu,\end{aligned}}$$

the only dependence on q^2 **remains because** $p^2 = p'^2 = m^2, \; l^2 = 4m^2 - q^2$

Gordon-like identities

$$\begin{aligned}
\bar{u}(\mathbf{p}_1)\gamma^\mu u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[l^\mu + i\sigma^{\mu\nu}q_\nu]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)\gamma^\mu\gamma_5 u(\mathbf{p}_2) &= \frac{1}{2m}\bar{u}(\mathbf{p}_1)[\gamma_5 q^\mu + i\gamma_5\sigma^{\mu\nu}l_\nu]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}l_\nu u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)q^\nu u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}q_\nu u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)[2m\gamma^\mu l^\mu]u(\mathbf{p}_2)
\end{aligned}$$

$$\begin{aligned}
\bar{u}(\mathbf{p}_1)i\sigma^{\mu\nu}\gamma_5 q_\nu u(\mathbf{p}_2) &= -\bar{u}(\mathbf{p}_1)l^\mu\gamma_5 u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_5\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{-i[q^\alpha \not{l} - l^\alpha \not{q}] + i(q^2 - 4m^2)\gamma^\alpha + \\
&\quad 2im(l^\alpha + q^\alpha)\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\alpha\mu\nu\beta}\gamma_\beta q_\mu l_\nu]u(\mathbf{p}_2) &= \bar{u}(\mathbf{p}_1)\{i[q^\alpha \not{l} - l^\alpha \not{q}]\gamma_5 + iq^2\gamma_5\gamma^\alpha - \\
&\quad 2im(l^\alpha + q^\alpha)\gamma_5\}u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\gamma_\nu\gamma_5]u(\mathbf{p}_2) &= \frac{i}{2m}\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}q^\rho]u(\mathbf{p}_2) \\
\bar{u}(\mathbf{p}_1)[\epsilon^{\mu\nu\alpha\beta}q_\alpha l_\beta\sigma_{\nu\rho}l^\rho]u(\mathbf{p}_2) &= 0
\end{aligned}$$

Requirement of current conservation (electromagnetic gauge invariance)

$$\boxed{\partial_\mu j^\mu = 0}$$

$$f_1(q^2)q^2 + f_2(q^2)q^2\gamma_5 + 2mf_4(q^2)\gamma_5 = 0,$$

$$f_1(q^2) = 0, \qquad f_2(q^2)q^2 + 2mf_4(q^2) = 0$$

$$\Lambda_\mu(q) \;=\; f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu + f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

charge

dipole electric and magnetic

anapole

Form Factors

Matrix element of electromagnetic current between neutrino states

$$\boxed{\langle\nu(p')|J_\mu^{EM}|\nu(p)\rangle = \bar{u}(p')\Lambda_\mu(q)u(p)}$$

where vertex function generally contains **4** *form factors*

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5+f_A(q^2)(q^2\gamma_\mu-q_\mu\not{q})\gamma_5$$

1. electric

dipole

2. magnetic

3. electric

4. anapole

- **Hermiticity** and **discrete symmetries** of EM current J_μ^{EM} put constraints on form factors

Dirac ν

1) *CP invariance + hermiticity* ⟹ $f_E = 0$,

2) *at zero momentum transfer only electric charge* $f_Q(0)$ *and magnetic moment* $f_M(0)$ *contribute to* $H_{int} \sim J_\mu^{EM} A^\mu$,

3) *hermiticity* itself ⟹ *three form factors are real:* $Imf_Q = Imf_M = Imf_A = 0$.

Majoran ν

1) *from CPT invariance (regardless CP or ~~CP~~) .*

$$f_Q = f_M = f_E = 0$$

…as early as 1939, W.Pauli…

EM properties ⟹ a way to distinguish **Dirac** and **Majorana** ν

Effective Lagrangian for the spin component of ν vertex

$$\boxed{L = \frac{1}{2}\bar{\nu}_j \sigma_{\eta\xi}(\beta_{ij} + \varepsilon_{ij}\gamma_5)\nu_i F^{\eta\xi} + \text{h.c.}\,,}$$

e.m. field tensor

magnetic and **electric** moments
which couple together mass eigenstates
$(\nu_i)_L$ and $(\nu_j)_R$ ⇨ change of the helicity states

- $\nu_i = \nu_j$ ➡ diagonal moments
- $\nu_i \neq \nu_j$ ➡ transitional moments
- $\varepsilon_{ii} = \beta_{ii} = 0$ for Majorana ν

E.M. properties
↳ a way to distinguish Dirac and Majorana ν

In general case matrix element of J_μ^{EM} can be considered between ν ν
different initial $\psi_i(p)$ and final $\psi_j(p')$ states of different masses $p^2 = m_i^2, \ p'^2 = m_j^2$:

$$\boxed{< \psi_j(p')|J_\mu^{EM}|\psi_i(p) >= \bar{u}_j(p')\Lambda_\mu(q)u_i(p)}$$

and

$$\Lambda_\mu(q) = \Big(f_Q(q^2)_{ij} + f_A(q^2)_{ij}\gamma_5\Big)(q^2\gamma_\mu - q_\mu \not{q}) + f_M(q^2)_{ij} i\sigma_{\mu\nu}q^\nu + f_E(q^2)_{ij}\sigma_{\mu\nu}q^\nu\gamma_5$$

! *form factors are matrices in* ν *mass eigenstates space.*

Dirac ν *(off-diagonal case $i \neq j$)* **Majorana** ν

1) ***hermiticity*** itself ***does not apply restrictions on form factors,***

2) ***CP invariance + hermiticity***

⇓

$$f_Q(q^2), \ f_M(q^2), \ f_E(q^2), \ f_A(q^2)$$

are relatively real (no relative phases) .

1) ***CP invariance + hermiticity***

$$\boxed{\mu_{ij}^M = 2\mu_{ij}^D \ \ and \ \ \epsilon_{ij}^M = 0}$$

or

$$\boxed{\mu_{ij}^M = 0 \ \ and \ \ \epsilon_{ij}^M = 2\epsilon_{ij}^D}$$

…two remarks ….

1

Difference between electromagnetic vertex function of massive and massless ν

Dirac Form factor

- *m=0* **:**

$$\boxed{\bar{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\bar{u}(p')\gamma_\mu(1+\gamma_5)u(p)}$$

electric charge $f_Q(q^2)$ and anapole moment $f_A(q^2)$ ***FF*** *are related to* ***DF*** *(and to each other):*

$$\boxed{f_Q(q^2) = f_D(q^2), \quad f_A(q^2) = f_D(q^2)/q^2}$$

- *In case $m \neq 0$ there is no such simple relation (because term $q_\mu \not{q}\gamma_5$ in anapole* ***FF*** *cannot be neglected).*

2 ν form factors in gauge models

$$\boxed{<\psi_j(p')|J_\mu^{EM}|\psi_i(p)>=\bar{u}_j(p')\Lambda_\mu(q)u_i(p)}$$

- *Form Factors at zero momentum transfer ($q^2 = 0$) are elements of scattering matrix* ⇒ *in any consistent theoretical model FF in matrix element* ⇒ (*gauge independent and finite.*)

  Then

  FF at $q^2 = 0$ determine static properties of ν *that can be probed (measured) in direct interaction with external em fields .*

This is the case for
$f_Q(q^2),\ f_M(q^2),\ f_E(q^2)$
in minimally extended SM
($f_A(q^2)$ is an exceptional case)

- *In non-Abelian gauge models,*

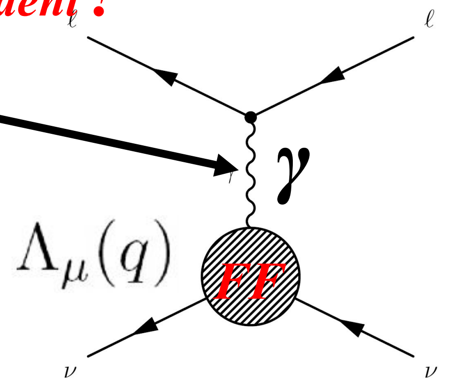
  FF at $q^2 \neq 0$ can be not invariant under gauge transformation

  because (in general) off-shell photon propagator *is gauge dependent !*

  … One-photon approximation is not enough to get physical quantity…

  … FF in matrix element cannot be directly measured in experiment with em field …

  … FF can contribute to higher order processes accessible for experimental observation.



Dipole magnetic $\boxed{f_M(q^2)}$ **and electric** $\boxed{f_E(q^2)}$

are most well studied and theoretically understood among form factors

…because even in the limit $q^2 \to 0$ they may have nonvanishing values

$\boxed{\mu_\nu = f_M(0)}$ ⇐ ν magnetic moment

$\boxed{\epsilon_\nu = f_E(0)}$ ⇐ ν electric moment ???

ν magnetic moment ？

1

ν magnetic moment in experiments

Samuel Ting
(wrote on the wall at Department of Theoretical Physics of Moscow State University) :

“Physics is an experimental science”

Studies of ν-e scattering - most sensitive method of experimental investigation of μ_ν

Cross-section:

- $$\frac{d\sigma}{dT}(\nu+e\to\nu+e)=\left(\frac{d\sigma}{dT}\right)_{\rm SM}+\left(\frac{d\sigma}{dT}\right)_{\mu_\nu},$$

where the Standard Model contribution

- $$\left(\frac{d\sigma}{dT}\right)_{\rm SM}=\frac{G_{\rm F}^2m_e}{2\pi}\left[(g_V+g_A)^2+(g_V-g_A)^2\left(1-\frac{T}{E_\nu}\right)^2+(g_A^2-g_V^2)\frac{m_eT}{E_\nu^2}\right],$$

T is the electron recoil energy and

$$g_V=\begin{cases}2\sin^2\theta_W+\dfrac{1}{2} & \text{for}\quad \nu_e\,,\\ 2\sin^2\theta_W-\dfrac{1}{2} & \text{for}\quad \nu_\mu,\nu_\tau\,,\end{cases}\qquad g_A=\begin{cases}\dfrac{1}{2} & \text{for}\quad \nu_e\,,\\ -\dfrac{1}{2} & \text{for}\quad \nu_\mu,\nu_\tau\end{cases}$$

for anti-neutrinos $g_A\to -g_A$,

to incorporate **charge radius**: $g_V\to g_V+\boxed{\tfrac{2}{3}M_W^2\langle r^2\rangle\sin^2\theta_W}$.

$$\boxed{\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\text{SM}} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}}$$

ν*-*γ coupling

- $$\boxed{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} = \frac{\pi\alpha_{em}^2}{m_e^2}\left[\frac{1-T/E_\nu}{T}\right]\mu_\nu^2}$$

with change of helicity,
contrary to SM

T is the electron recoil energy: $0 \leq T \leq \dfrac{2E_\nu^2}{2E_\nu + m_e}$

If neutrino has electric dipole moment,
or electric or magnetic transition moments,
these quantities would also contribute to scattering cross section

$$\mu_\nu^2 = \sum_{j=\,\nu_e,\,\nu_\mu,\,\nu_\tau} \left| \mu_{ij} - \epsilon_{ij} \right|^2 ,$$ *i refers to initial neutrino* ***flavour***

Possibility of ***distractive interference*** between **magnetic** and **electric** transition moments of **Dirac** neutrino (**Majorana** neutrino has only magnetic or electric transition moment, but not both if CP is conserved)

Effective ν_e magnetic moment measured in ν-*e* scattering experiments ? $\boxed{\mu_e^2}$

Two steps:

1) consider ν_e as superposition of mass eigenstates (*i=1,2,3*) at some distance *L*, and then sum up magnetic moment contributions to ν-*e* scattering amplitude (of each of mass components) induced by their magnetic moments

$$\boxed{A_j \sim \sum_i U_{ei} e^{-iE_i L} \mu_{ji}}$$

2) amplitudes combine incoherently in total cross section

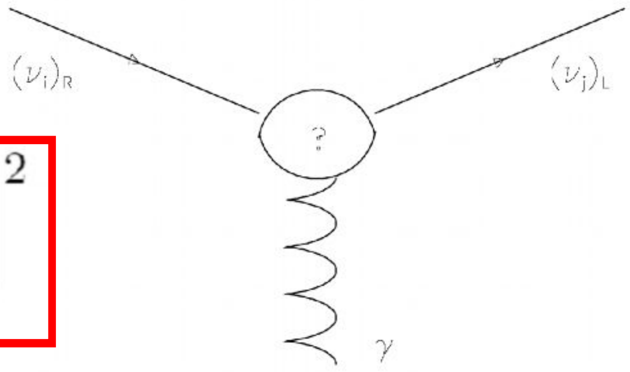
$$\boxed{\sigma \sim \mu_e^2 = \sum_j \left| \sum_i U_{ei} e^{-iE_i L} \mu_{ji} \right|^2}$$

J.Beacom, P.Vogel, 1999

NB! Summation over ***j=1,2,3*** is outside the square because of **incoherence** of different final mass states contributions to cross section.

ν magnetic moment in experiments

(for neutrino produced as ν_l with energy E_ν
and after traveling a distance L)



$$\boxed{\mu_\nu^2(\nu_l, L, E_\nu) = \sum_j \left| \sum_i U_{li} e^{-iE_iL} \mu_{ji} \right|^2}$$

where neutrino mixing matrix $\mu_{ij} \equiv |\beta_{ij} - \varepsilon_{ij}|$

Observable μ_ν is an effective parameter that depends on neutrino flavour composition at the detector.

H.Wong,
H.-B.Li, 2005

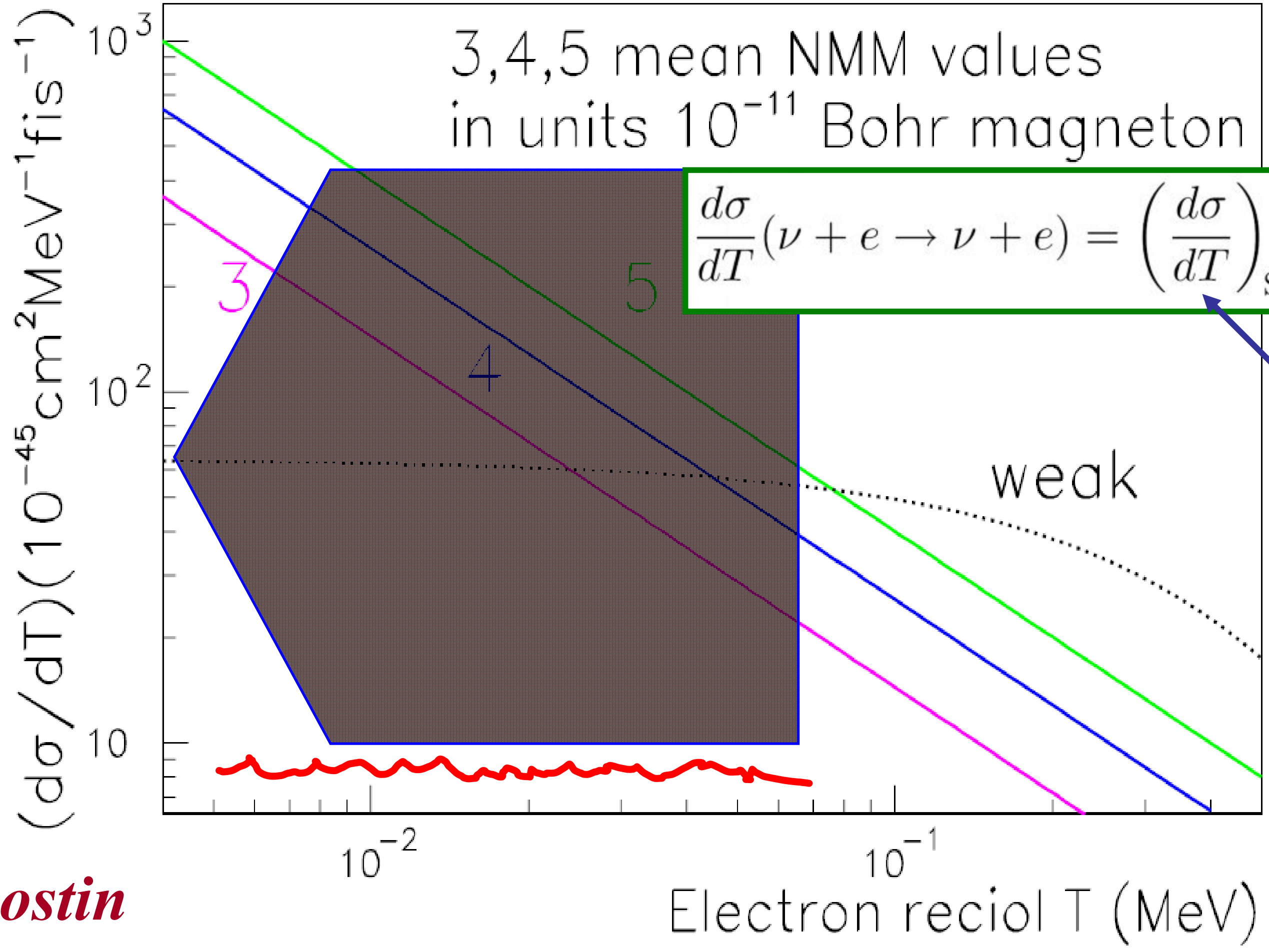
Implications of μ_ν limits from different experiments (reactor , solar ^{8}B and ^{7}Be) are different.

Magnetic moment contribution is dominated at low electron recoil energies

and $\boxed{\left(\frac{d\sigma}{dT}\right)_{\mu_\nu} > \left(\frac{d\sigma}{dT}\right)_{SM}}$ when $\boxed{\frac{T}{m_e} < \frac{\pi^2 \alpha_{em}}{G_F^2 m_e^4}\mu_\nu^2}$

… the **lower** the smallest measurable electron recoil energy is,

the **smaller** values of μ_ν^2 can be probed in scattering experiments **…**



$$\frac{d\sigma}{dT}(\nu + e \to \nu + e) = \left(\frac{d\sigma}{dT}\right)_{\rm SM} + \left(\frac{d\sigma}{dT}\right)_{\mu_\nu}$$

…dynamical zeros…

Bernabeu, Papavassiliou, Passera, PLB 2005

from A.Starostin

First and future ν-*e* scattering experiments

- $\boxed{\mu_\nu \leq 2 \div 4 \times 10^{-10} \mu_B}$

Savannah River (1976), *first observation of ν - e*

Vogel, Engel, 1989

Kurchatov, Krasnoyarsk (1992), Rovno (1993) reactors

- $\boxed{\mu_\nu \leq \ 1.1 \times 10^{-10} \ \mu_B}$

SuperKamiokande (2004)

- $\boxed{\mu_\nu \leq \ few \times 10^{-11} \mu_B}$

…in the future…

Beta-beams

McLaughlin, Volpe, 2004

MUNU experiment at Bugey reactor (2005)

$$\boxed{\mu_\nu \leq 9 \times 10^{-11} \mu_B}$$

TEXONO **collaboration** at Kuo-Sheng power plant (2006)

$$\boxed{\mu_\nu \leq 7 \times 10^{-11} \mu_B}$$

GEMMA **(2007)**

$$\boxed{\mu_\nu \leq \ 5.8 \times 10^{-11} \mu_B}$$

GEMMA I 2005 - 2007

BOREXINO **(2008)**

$$\boxed{\mu_\nu \leq \ 5.4 \times 10^{-11} \mu_B}$$

$$\boxed{\mu_\nu \leq \ 8.5 \times 10^{-11} \mu_B \quad (\nu_\tau, \ \nu_\mu)}$$

Montanino,
Picariello,
Pulido, PRD 2008

2

New Result of
Neutrino Magnetic Moment Measurement
in GEMMA Experiment (2008)

A.Starostin et al, in: “Particle Physics on the Eve of LHC”,
ed. by A.Studenikin, World Scientific (Singapore), p.112, 2009,
www.icas.ru (13th Lomonosov Conference)

A.Beda et al, Phys.Atom.Nucl. 70 (2007) 1873

“The New Result of the Neutrino magnetic Moment measurement in the GEMMA Experiment”

A.Starostin et al, in: “Particle Physics on the Eve of LHC”, ed. by A.Studenikin, World Scientific (Singapore), 2009, www.icas.ru (13th Lomonosov Conference)

GEMMA I (2008)

Status :
“on” (operation of reactor) 9426 hours
“off” (reactor shutdown) 2965 hours

!

$$\boxed{\mu_\nu \leq \; 3.1 \times 10^{-11} \mu_B}$$

and

$$\boxed{\mu_\nu \leq \; 4.9 \times 10^{-11} \mu_B}$$

…obtained with **more conservative data analysis** method

3

… a bit of ν electromagnetic properties theory

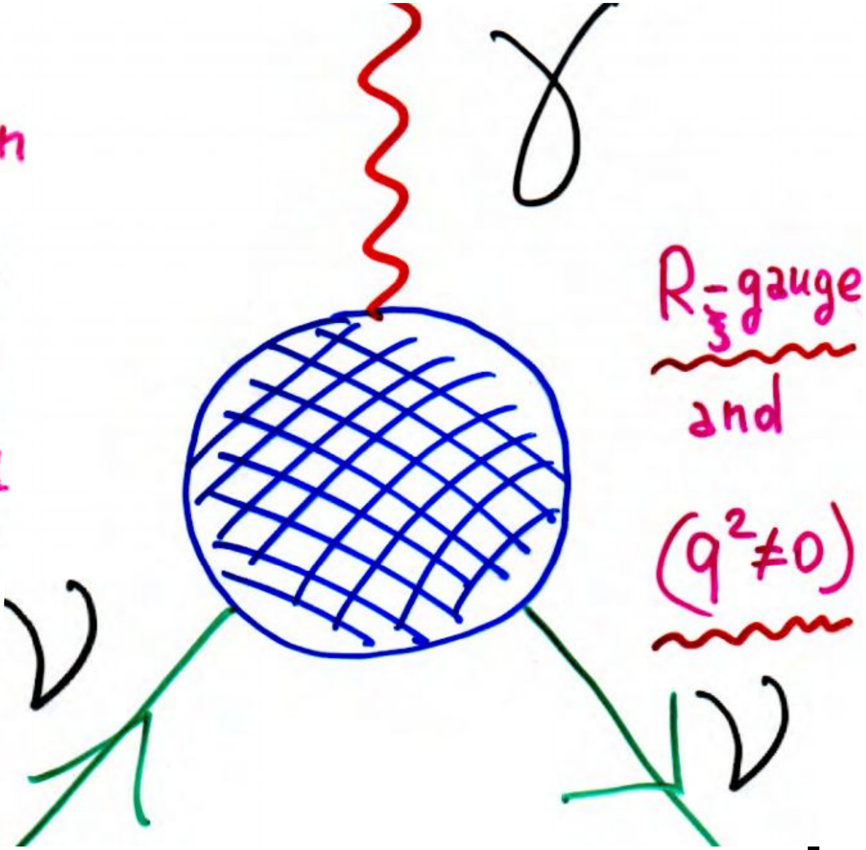
3.1 ν vertex function

The most general study of the massive neutrino vertex function (including electric and magnetic form factors) in arbitrary R_ξ gauge in the context of the SM + SU(2)-singlet ν_R accounting for masses of particles in polarization loops

M. Dvornikov, A. Studenikin

⊛ Phys. Rev. D 63, 073001, 2004,
"Electric charge and magnetic moment of massive neutrino";
JETP 126 (2004), N8, 1

⊛ "Electromagnetic form factors of a massiv neutrino."



R_ξ-gauge

and

$(q^2 \neq 0)$

$$\Delta_\mu(q) = \overset{\text{charge}}{\underbrace{f_Q(q^2)}}\gamma_\mu + \overset{\text{magnetic moment}}{\underbrace{f_M(q^2)}}\, i\sigma_{\mu\nu} q^\nu -$$

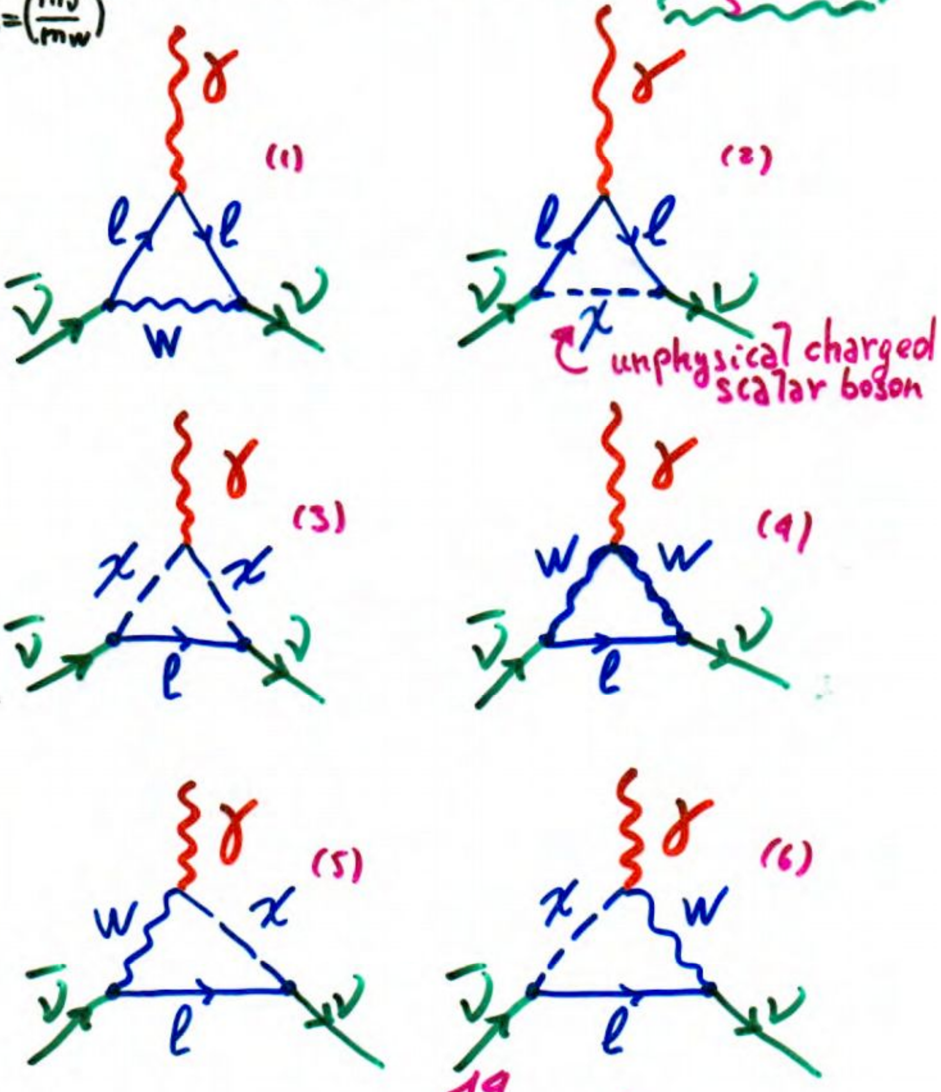
$$- \underbrace{f_E(q^2)}_{\text{electric moment}}\, i\sigma_{\mu\nu} q^\nu \gamma_5 - \underbrace{f_A(q^2)}_{\text{anapole moment}}(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

$a = \left(\frac{m_\ell}{m_W}\right)^2,$

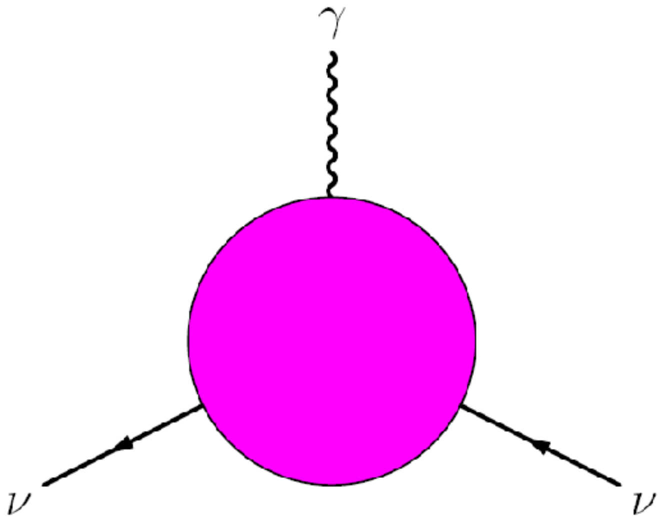
$B = \left(\frac{m_\nu}{m_W}\right)^2$

Proper vertices

R_ξ-gauge



$$\Lambda_\mu(q) = \sum_{i=1}^{14} \Lambda_\mu^i(q)$$



$$\Lambda_\mu(q)$$

Contributions of proper vertices diagrams

(dimensional-regularization scheme)

- $$\Lambda_\mu^{(1)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\left[g^{\kappa\lambda}-(1-\alpha)\frac{k^\kappa k^\lambda}{k^2-\alpha M_W^2}\right]\times\frac{\gamma_\kappa^L(\not p\,'-\not k+m_\ell)\gamma_\mu(\not p-\not k+m_\ell)\gamma_\lambda^L}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-M_W^2]},$$

- $$\Lambda_\mu^{(2)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}\frac{(m_\nu P_L-m_\ell P_R)(\not p\,'-\not k+m_\ell)\gamma_\mu(\not p-\not k+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-m_\ell^2][(p-k)^2-m_\ell^2][k^2-\alpha M_W^2]},$$

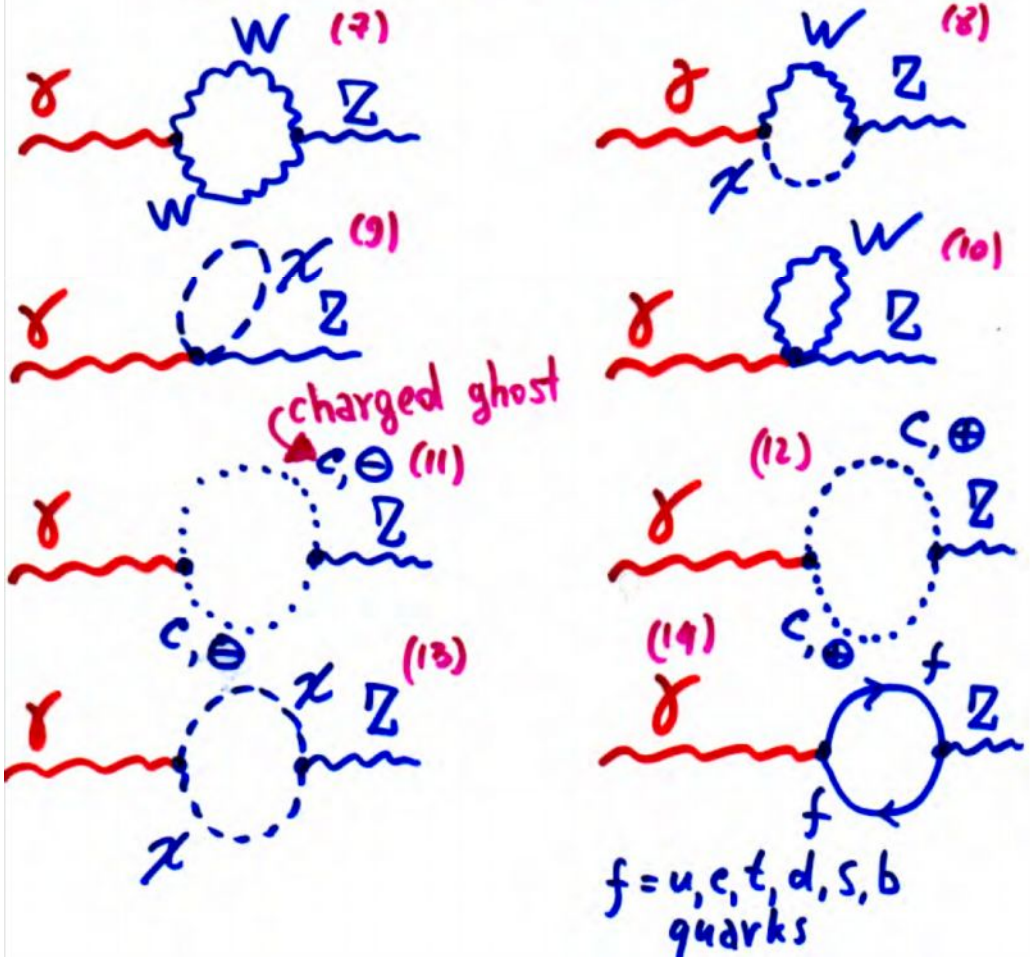
- $$\Lambda_\mu^{(3)}=i\frac{eg^2}{2M_W^2}\int\frac{d^Nk}{(2\pi)^N}(2k-p-p')_\mu\frac{(m_\nu P_L-m_\ell P_R)(\not k+m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-\alpha M_W^2][k^2-m_\ell^2]},$$

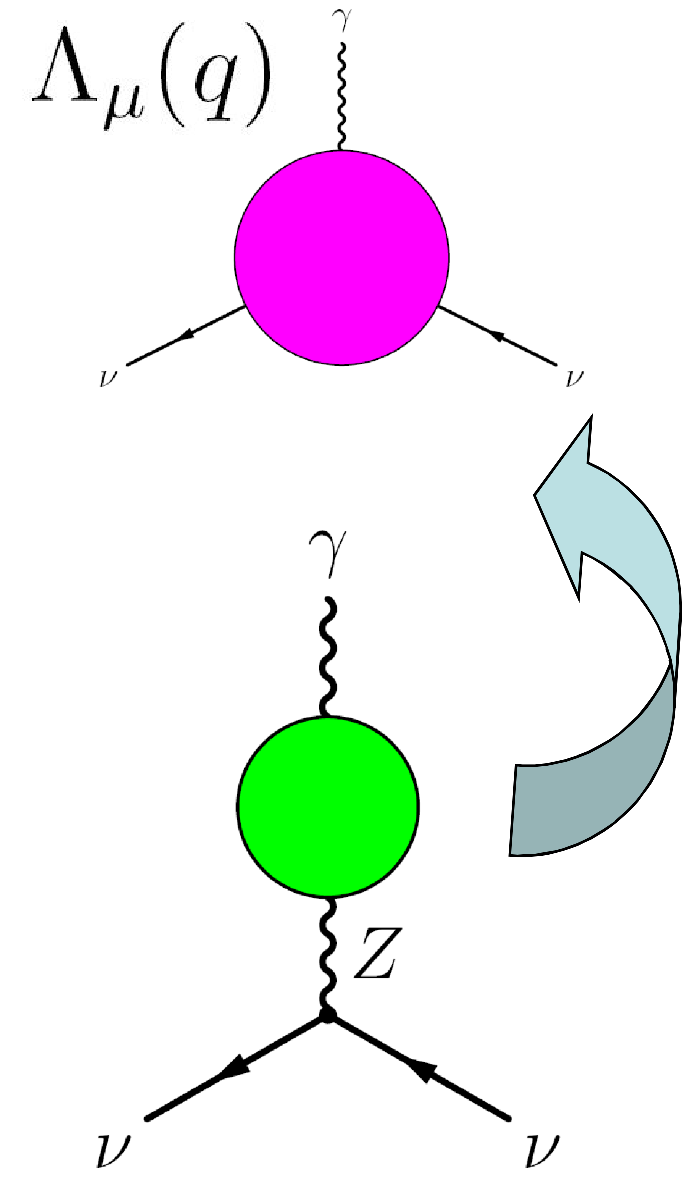
- $$\Lambda_\mu^{(4)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}\gamma_\kappa^L(\not k+m_\ell)\gamma_\lambda^L\left[\delta_\beta^\kappa-(1-\alpha)\frac{(p'-k)^\kappa(p'-k)_\beta}{(p'-k)^2-\alpha M_W^2}\right]\left[\delta_\gamma^\lambda-(1-\alpha)\frac{(p-k)^\lambda(p-k)_\gamma}{(p-k)^2-\alpha M_W^2}\right]$$
$$\times\frac{\delta_\mu^\beta(2p'-p-k)^\gamma+g^{\beta\gamma}(2k-p-p')_\mu+\delta_\mu^\gamma(2p-p'-k)^\beta}{[(p'-k)^2-M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]},$$

- $$\Lambda_\mu^{(5)+(6)}=i\frac{eg^2}{2}\int\frac{d^Nk}{(2\pi)^N}$$
$$\times\left\{\frac{\gamma_\beta^L(\not k-m_\ell)(m_\ell P_L-m_\nu P_R)}{[(p'-k)^2-M_W^2][(p-k)^2-\alpha M_W^2][k^2m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p'-k)^\beta(p'-k)_\mu}{(p'-k)^2-\alpha M_W^2}\right]\right.$$
$$\left.-\frac{(m_\nu P_L-m_\ell P_R)(\not k-m_\ell)\gamma_\beta^L}{[(p'-k)^2-\alpha M_W^2][(p-k)^2-M_W^2][k^2-m_\ell^2]}\left[\delta_\mu^\beta-(1-\alpha)\frac{(p-k)^\beta(p-k)_\mu}{(p-k)^2-\alpha M_W^2}\right]\right\}$$

$$\Lambda^{j}_{\mu}(q) = \frac{g}{2\cos\theta_W}\,\Pi^{(j)}_{\mu\nu}(q)\,\frac{1}{q^2 - M_Z^2} \times \left\{ g^{\nu\alpha} - (1-\alpha_Z)\frac{q^\nu q^\alpha}{q^2 - \alpha_Z M_Z^2} \right\} \gamma^{L}_{\alpha}, \; j=7,\dots,14$$

γ-Z self-energy diagrams





γ − Z self-energy diagrams

Matrix element of electromagnetic current between massive and zero-mass neutrino states differ radically

- **For massless** ν

$$f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

$$\boxed{\overline{u}(p')\Lambda_\mu(q)u(p) = f_D(q^2)\overline{u}(p')\gamma_\mu(1+\gamma_5)u(p)}$$

$\boxed{f_Q(q^2)=f_D(q^2)}$ *electric* **form factor** *anapole* $\boxed{f_A(q^2)=f_D(q^2)/q^2}$

- **For massive** ν

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu + f_M(q^2)i\sigma_{\mu\nu}q^\nu - f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5$$
$$+f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

one cannot disregard

- **Calculations of massive** ν **vertex function (calculation the complete set of Feynman diagrams)**

Dvornikov, Studenikin, 2004

→ **additional term** $\boxed{\Lambda_\mu(q) \sim f_5(q^2)\gamma_\mu\gamma_5}$

- **Direct calculation of these contributions**

$$\boxed{f_5(q^2)=f_5^{(\gamma-Z)}(q^2)+f_5^{(\text{prop. vert.})}(q^2)=0}$$

Direct calculations of complete set of one-loop contributions to ν vertex function in minimally extended Standard Model

for a massive Dirac neutrino:

M.Dvornikov,
A.Studenikin,
PRD, 2004

… in case CP conservation

- $\Lambda_\mu(q) \Rightarrow f_Q(q^2),\ f_M(q^2),\ \cancel{f_E(q^2)},\ f_A(q^2)$
- Electric charge $f_Q(0) = \mathbf{0}$ and is gauge-independent
- Magnetic moment $f_M(0)$ is finite and gauge-independent
- Gauge and qxq dependence … ⇒

ν magnetic moment

(for arbitrary neutrino
mass, heavy neutrino…)

- LEP data →

only 3 light νs coupled to Z°,

for any additional neutrino

$$\boxed{m_{\nu} \geq 45 \;\; Gev}$$

3.2

Calculation of ν magnetic moment (massive ν, arbitrary R_ξ - gauge)

Dvornikov,
Studenikin, PRD 2004

$$\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\underline{\sigma_{\mu\nu}q^\nu}-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5+f_A(q^2)(q^2\gamma_\mu-q_\mu \not{q})\gamma_5$$

magnetic moment

$$\mu(a,b,\alpha)=f_M(q^2=0)$$

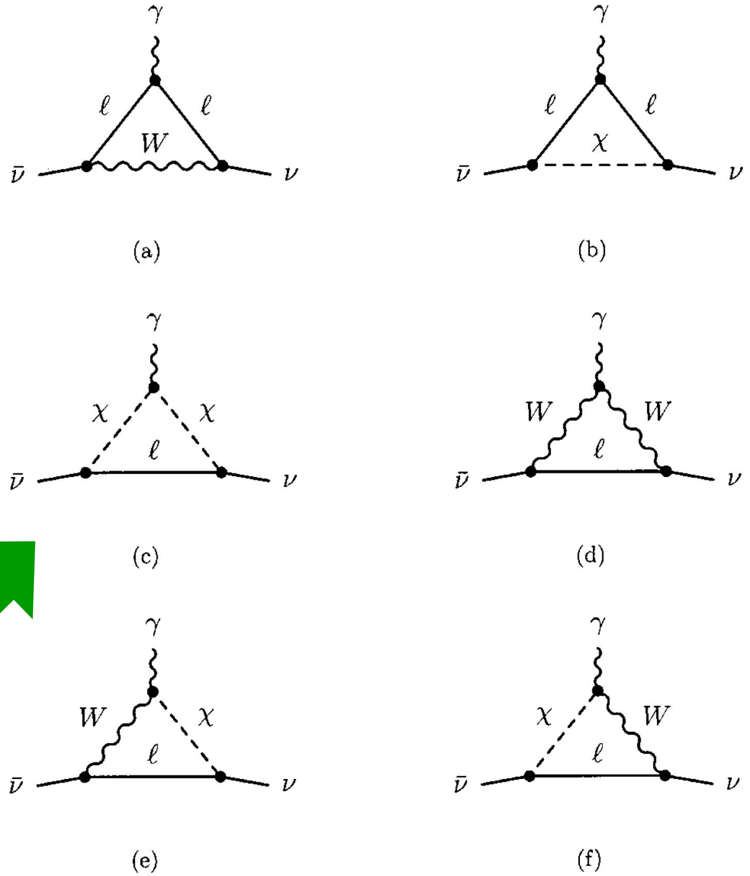
! two mass parameters $a=\left(\frac{m_\ell}{M_W}\right)^2$ $b=\left(\frac{m_\nu}{M_W}\right)^2$

and gauge-fixing parameter $\alpha=\frac{1}{\xi}$

$\xi=0$ - unitary gauge, $\xi=1$ - ‘t Hooft-Feynman gauge

$$\mu(a,b,\alpha)=\sum_{i=1}^{6}\mu^{(i)}(a,b,\alpha)$$

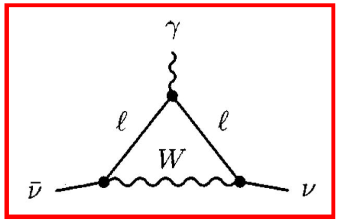
Proper vertices



… after loop integrals calculations (e.g., for diagrams (a) and (d) contributing in unitary gauge)

$$\mu^{(1)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\int_0^1 dz\, z(1-z^2)\frac{1}{D}-\frac{1}{2}\int_0^1 dz(1-z)^3(a-bz)\left[\frac{1}{D_\alpha}-\frac{1}{D}\right]\right.$$

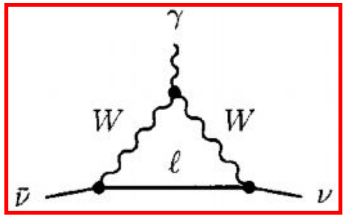
$$\left.-\frac{1}{2}\int_0^1 dz(1-z)(1-3z)[\ln D_\alpha-\ln D]\right\},$$



$$\mu^{(4)}(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\left\{\frac{1}{2}\int_0^1 dz\, z^2(1+2z)\frac{1}{D}\right.$$

$$+\frac{b}{2}\int_0^1 dz\int_0^z dy(1-z)^2[z(1-z)-2y]\left[\frac{1}{D_\alpha+y(1-\alpha)}-\frac{1}{D}\right]$$

$$\left.+\frac{1}{2}\int_0^1 dz\int_0^z dz(-2+9z-4z^2-6y)\{\ln[D_\alpha+y(1-\alpha)]-\ln D\}\right\},$$



where $D_\alpha=a+(\alpha-a)z-bz(1-z)$ **and** $D=D_{\alpha=1}$

Dvornikov, Studenikin,
PRD 2004, JETP 2004

… within exact calculations it is possible to expand over mass parameter $b=\left(\frac{m_\nu}{M_W}\right)^2$

$$\mu(a,b,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\sum_{i=1}^{6}\{\overline{\mu}_0^{(i)}(a,\alpha)+b\,\overline{\mu}_1^{(i)}(a,\alpha)+\mathcal{O}(b^2)\}$$

Cabral-Rosetti,
Bernabéu,
Vidal, Zepeda,
EPJ 2000

$$\mu_0(a,\alpha)=\frac{eG_F}{4\pi^2\sqrt{2}}m_\nu\frac{3}{4(1-a)^3}(2-7a+6a^2-2a^2\ln a-a^3)+\mathcal{O}(a^2)$$

$a=\left(\frac{m_\ell}{M_W}\right)^2$

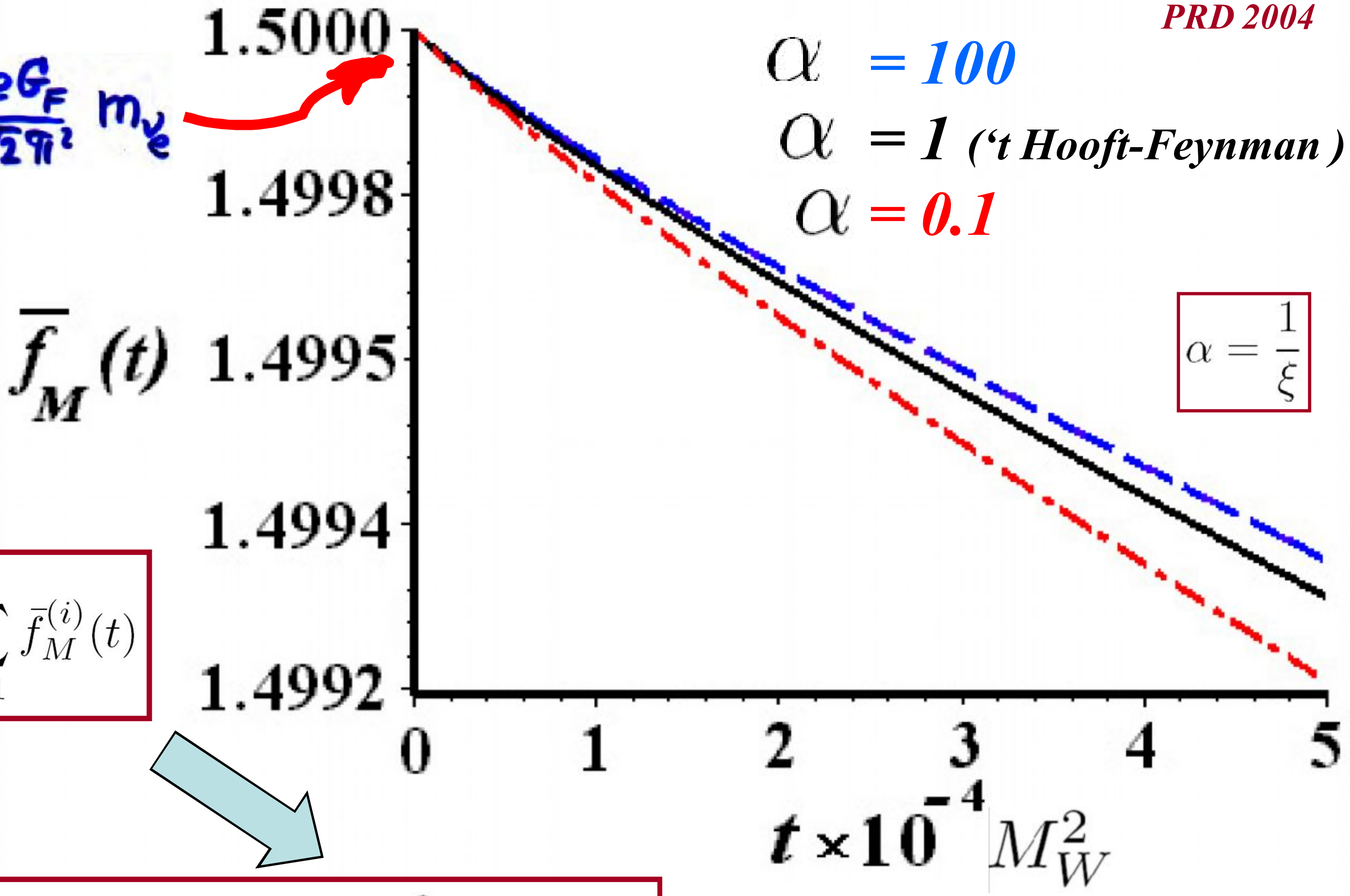
$$\overline{\mu}_1(a,\alpha)=\sum_{i=1}^{6}\overline{\mu}_1^{(i)}(a,\alpha)=\frac{1}{12(1-a)^5}(5-26a+6a\ln a-36a^2-60a^2\ln a+58a^3-18a^3\ln a-a^4)$$

… μ_ν **gauge independent and finite value…**

Gauge and $q \times q$ dependence …

Dvornikov, Studenikin, PRD 2004

● $\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$



$$\alpha = \frac{1}{\xi}$$

$$\bar{f}_M(t) = \sum_{i=1}^{6} \bar{f}_M^{(i)}(t)$$

$$f_M(q^2) = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \sum_{i=1}^{6} \bar{f}_M^{(i)}(q^2)$$

- $m_\nu \ll m_e \ll M_W$ **light** ν

$$\boxed{\mu_\nu = \frac{eG_F}{4\pi^2\sqrt{2}} m_\nu \frac{3}{4(1-a)^3}\left(2-7a+6a^2-2a^2\ln a-a^3\right)}, \quad a=\left(\frac{m_e}{M_W}\right)^2$$

$$\mu_e = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

Gabral-Rosetti, Bernabeu, Vidal, Zepeda, Eur.Phys.J C 12 (2000) 633

Dvornikov, Studenikin, Phys.Rev.D 69 (2004) 073001; JETP 99 (2004) 254

- $m_e \ll m_\nu \ll M_W$ **intermediate** ν

$$\boxed{\mu_\nu = \frac{3eG_F}{8\pi^2\sqrt{2}} m_\nu \left\{1+\frac{5}{18}b\right\}}, \quad b=\left(\frac{m_\nu}{M_W}\right)^2$$

- $m_e \ll M_W \ll m_\nu$ **heavy** ν

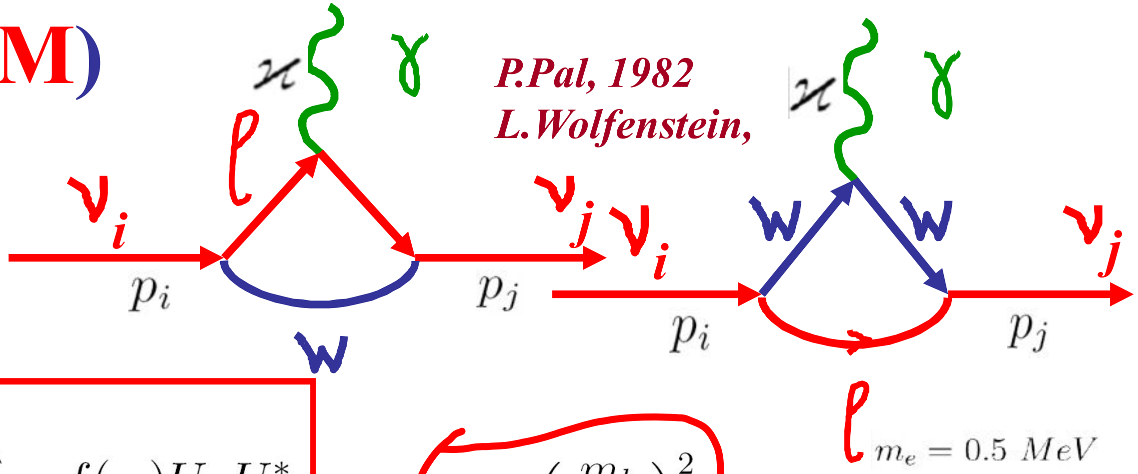
$$\boxed{\mu_\nu = \frac{eG_F}{8\pi^2\sqrt{2}} m_\nu}$$

$$\sim 10^{-19}\mu_B\left(\frac{m_{\nu_e}}{1eV}\right)$$

3.5

Neutrino (SM) dipole moments

(+ transition moments)



P.Pal, 1982
L.Wolfenstein,

- **Dirac neutrino**

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{eG_F m_i}{8\sqrt{2}\pi^2}\left(1 \pm \frac{m_j}{m_i}\right)\sum_{l=\,e,\,\mu,\,\tau} f(r_l)U_{lj}U^*_{li}$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

$m_e = 0.5\ MeV$
$m_\mu = 105.7\ MeV$
$m_\tau = 1.78\ GeV$
$m_W = 80.2\ GeV$

- $m_i, m_j \ll m_l, m_W$

$$\Rightarrow \quad f(r_l) \approx \frac{3}{2}\left(1 - \frac{1}{2}r_l\right), \quad r_l \ll 1$$

transition moments vanish because unitarity of ***U*** implies that its rows or columns represent orthogonal vectors

- **transition moments are suppressed, Glashow-Iliopoulos-Maiani cancellation,**
- **for diagonal moments there is no GIM cancelation**

- **Majorana neutrino**

only for $i \neq j$

$$\mu^M_{ij} = 2\mu^D_{ij} \quad and \quad \epsilon^M_{ij} = 0$$

or

$$\mu^M_{ij} = 0 \quad and \quad \epsilon^M_{ij} = 2\epsilon^D_{ij}$$

… depending on relative ***CP*** phase of ν_i and ν_j

The first nonzero contribution from neutrino transition moments

$$f_{r_l} \to \cancel{-\frac{3}{2}} + \frac{3}{4}\left(\frac{m_l}{m_W}\right)^2 \ll 1$$

GIM cancellation

$$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = \frac{3eG_F m_i}{32\sqrt{2}\pi^2}\left(1\pm\frac{m_j}{m_i}\right)\left(\frac{m_\tau}{m_W}\right)^2 \sum_{l=\,e,\,\mu,\,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

$$\mu_B = \frac{e}{2m_e}$$

- $$\left.\begin{matrix}\mu_{ij}\\ \epsilon_{ij}\end{matrix}\right\} = 4\times 10^{-23}\mu_B\left(\frac{m_i \pm m_j}{1\ eV}\right) \sum_{l=\,e,\,\mu,\,\tau}\left(\frac{m_l}{m_\tau}\right)^2 U_{lj}U^*_{li}$$

… neutrino radiative decay is very slow

- **Dirac ν diagonal *(i=j)* magnetic moment**

$\epsilon^D_{ii} = 0$ **for CP-invariant interactions**

$\mu^M_{ii} = \epsilon^M_{ii} = 0$

$$\mu_{ii} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2}\left(1 - \frac{1}{2}\sum_{l=\,e,\mu,\tau} r_l \mid U_{li}\mid^2\right) \approx 3.2\times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

$$r_l = \left(\frac{m_l}{m_W}\right)^2$$

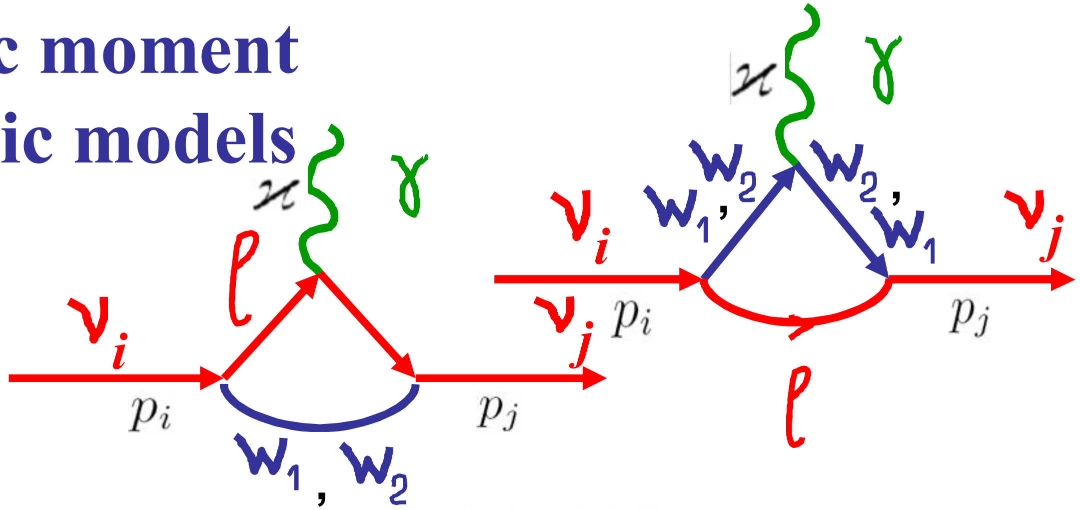
- *no GIM cancellation*

Lee, Shrock, Fujikawa, 1977

- μ^D_{ii} **- to leading order - independent on U_{li} and $m_{\ l=e,\ \mu,\ \tau}$**
- $$\mu_e^2 = \sum_{i=1,2,3} \mid U_{ie}\mid^2 \mu_{ii}^2$$ **…possibility to measure fundamental μ^D_{ii}**
- $\mu^D_{ii} = 0$ **for massless ν (in the absence of right-handed charged currents)**

3.6 Neutrino magnetic moment in left-right symmetric models

$SU_L(2) \times SU_R(2) \times U(1)$



Gauge bosons mass states

$$W_1 = W_L \cos\xi - W_R \sin\xi$$
$$W_2 = W_L \sin\xi + W_R \cos\xi$$

with mixing angle ξ **of gauge bosons** $W_{L,R}$ **with pure** $(V \pm A)$ **couplings**

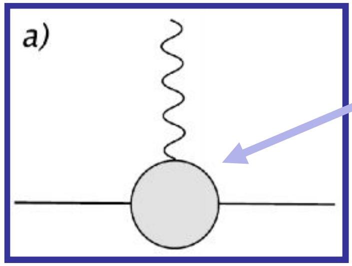
Kim, 1976; Marciano, Sanda, 1977;
Beg, Marciano, Ruderman, 1978

$$\boxed{\mu_{\nu_l} = \frac{eG_F}{2\sqrt{2}\pi^2}\left[m_l\left(1 - \frac{m^2_{W_1}}{m^2_{W_2}}\right)\sin 2\xi + \frac{3}{4}m_{\nu_l}\left(1 + \frac{m^2_{W_1}}{m^2_{W_2}}\right)\right]}$$

3.3 Naïve relationship between the size of m_ν and μ_ν

If μ_ν is generated by physics beyond the SM at energy scale Λ ,

P.Vogel e.a., 2006

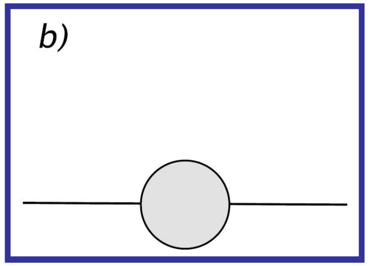


then

$$\boxed{\mu_\nu \sim \frac{eG}{\Lambda},}$$

…combination of constants and loop factors…

contribution to m_ν given by



, then

$$\boxed{m_\nu \sim G\Lambda}$$

$$m_\nu \sim \frac{\Lambda^2}{2m_e}\frac{\mu_\nu}{\mu_B} \sim \frac{\mu_\nu}{10^{-18}\mu_B}[\Lambda(\mathrm{TeV})]^2 \ \mathrm{eV}$$

Voloshin, 1988; Barr, Freire, Zee, 1990

from quadratic divergence appearing in renormalization of dimension four neutrino mass operator

Large magnetic moment

$\mu_\nu = \mu_\nu(m_\nu, m_{B^+}, m_{e^-})$

$\Uparrow$

- In the L-R symmetric models
  $(SU(2)_L \times SU(2)_R \times U(1))$

Kim, 1976
Beg, Marciano, Ruderman, 1978

- *Voloshin, 1988*

"On compatibility of small m_ν with large μ_ν of neutrino", Sov.J.Nucl.Phys. 48 (1988) 512

… there may be $SU(2)_\nu$ symmetry that forbids m_ν but not μ_ν

- ***Bar, Freire, Zee, 1990***
- supersymmetry
- extra dimensions
- model-independent constraint μ_ν

considerable enhancement of to experimentally relevant range

$\boxed{\mu_\nu^D \leq 10^{-15} \mu_B}$

$\boxed{\mu_\nu^M \leq 10^{-14} \mu_B}$

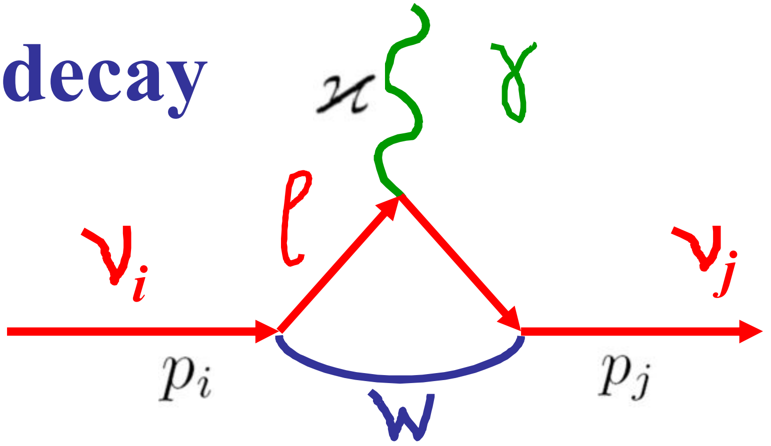
for BSM* ($\Lambda \sim 1$ TeV) *without fine tuning and under the assumption that $\boxed{\delta m_\nu \leq 1 \text{ eV}}$

Bell, Cirigliano, Ramsey-Musolf, Vogel, Wise, 2005

3.7 Neutrino radiative decay

$\nu_i \longrightarrow \nu_j + \gamma$

$m_i > m_j$



$$\boxed{L_{int} = \frac{1}{2}\bar{\psi}_i \sigma_{\alpha\beta}(\sigma_{ij} + \epsilon_{ij}\gamma_5)\psi_j F^{\alpha\beta} + h.c.}$$

Matrix element squired :

$$| M |^2 = 8\mu_{eff}^2 (\varkappa \cdot p_i)(\varkappa \cdot p_j)$$

Radiative decay rate

$$\boxed{\mu_{eff}^2 = | \mu_{ij} |^2 + | \epsilon_{ij} |^2}$$

$$\boxed{\Gamma_{\nu_i \to \nu_j + \gamma} = \frac{\mu_{eff}^2}{8\pi}\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3} \approx 5\left(\frac{\mu_{eff}}{\mu_B}\right)^2\left(\frac{m_i^2 - m_j^2}{m_i^2}\right)^3\left(\frac{m_i}{1\ eV}\right)^3 s^{-1}$$

- Radiative decay has been constrained from absence of decay photons:

  1) reactor $\bar{\nu}_e$ and solar ν_e fluxes,

  2) SN 1987A ν burst (all flavours),

  3) spectral distortion of CMBR

Raffelt 1999

Kolb, Turner 1990;

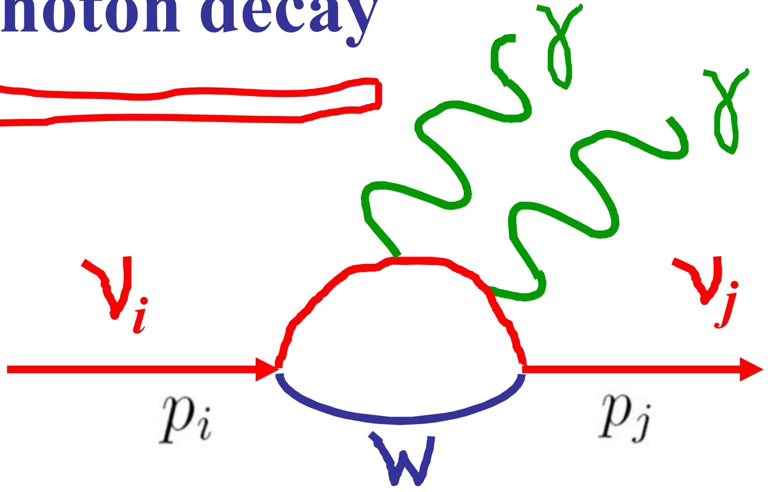
Ressell, Turner 1990

3.8

Neutrino radiative two-photon decay

$\nu_i \longrightarrow \nu_j + \gamma + \gamma$

$m_i > m_j$



fine structure constant

$\ll 1$

$$\boxed{\Gamma_{\nu_i\to\nu_j+\gamma+\gamma} \sim \frac{\alpha_{QED}}{4\pi}\Gamma_{\nu_i\to\nu_j+\gamma}}$$

✓ *… there is no GIM cancellation…*

$$\boxed{f(r_l) \approx \frac{3}{2}\left(\cancel{1} - \frac{1}{2}\left(\frac{m_l}{m_W}\right)^2\right) \rightarrow (m_i/m_l)^2}$$

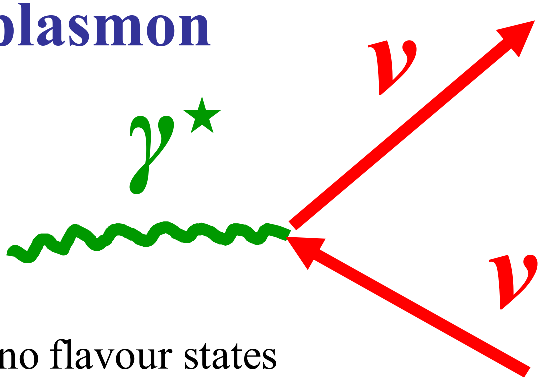
Nieves, 1983; Ghosh, 1984

… can be of interest for certain range of ν masses…

3.9 The tightest astrophysical bound on μ_ν

G.Raffelt, PRL 1990

comes from cooling of red giant stars by plasmon decay $\gamma^\star \longrightarrow \nu\nu$



$$\boxed{L_{int} = \frac{1}{2}\sum_{a,b}\left(\mu_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\psi_b + \epsilon_{a,b}\bar{\psi}_a\sigma_{\mu\nu}\gamma_5\psi_b\right)}$$

neutrino flavour states

Matrix element

$$\epsilon_\alpha k^\alpha = 0$$

$$|M|^2 = M_{\alpha\beta}p^\alpha p^\beta, \quad M_{\alpha\beta} = 4\mu^2(2k_\alpha k_\beta - 2k^2\epsilon^*_\alpha\epsilon_\beta - k^2 g_{\alpha,\beta}),$$

Decay rate

$$\boxed{\Gamma_{\gamma\to\nu\bar{\nu}} = \frac{\mu^2}{24\pi}\frac{(\omega^2-k^2)^2}{\omega}}$$

= 0 in vacuum $\omega = k$

In the classical limit $\gamma^\star$ - like a massive particle with $\omega^2 - k^2 = \omega_{pl}^2$

Energy-loss rate per unit volume

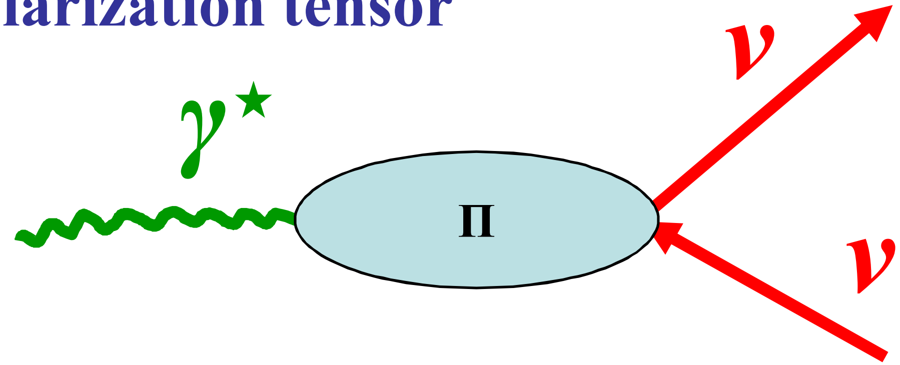
$$\boxed{Q_\mu = g\int\frac{d^3k}{(2\pi)^3}\omega f_{BE}\Gamma_{\gamma\to\nu\bar{\nu}}}$$

$$\mu^2 \to \sum_{a,b}\left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2\right)$$

distribution function of plasmons

$$\boxed{Q_\mu = g \int \frac{d^3k}{(2\pi)^3} \omega f_{BE} \Gamma_{\gamma \to \nu\bar{\nu}}}$$

Magnetic moment plasmon decay enhances the Standard Model photo-neutrino cooling by photon polarization tensor



more fast cooling of the star.

In order not to delay helium ignition (≤5% in *Q*)

$$\boxed{\mu^2 \leq 3 \times 10^{-12} \mu_B}$$

$$\mu^2 \to \sum_{a,b} \left(|\mu_{a,b}|^2 + |\epsilon_{a,b}|^2 \right)$$

G.Raffelt,
PRL 1990

Astrophysics bounds on μ_ν

$$\boxed{\mu_\nu(\text{astro}) < 10^{-10}\text{–}10^{-12}\ \mu_{\text{B}}}$$

Mostly derived from consequences of **helicity-state change** in astrophysical medium:

- available degrees of freedom in BBN,
- stellar cooling via plasmon decay,
- cooling of SN1987a.

Red Giant lumin.
! $\mu_\nu \leq 3 \cdot 10^{-12} \mu_B$
G. Raffelt, D. Dearborn,
J. Silk, 1989.

Bounds depend on

- modeling of astrophysical systems,
- on assumptions on the neutrino properties.

Generic assumption:

- absence of other nonstandard interactions except for μ_ν .

A global treatment would be desirable, incorporating **oscillation** and **matter** **effects** as well as the complications due to interference and competitions among various channels

… A remark on electric charge of ν

ν neutrality $Q=0$
is attributed to

gauge invariance
+
anomally cancellation constraints

imposed in SM of
electroweak
interactions

Foot, Joshi, Lew, Volkas, 1990;
Foot, Lew, Volkas, 1993;
Babu, Mohapatra, 1989, 1990

…General proof:

$$SU(2)_L \times U(1)_Y$$

- *In SM :* $\boxed{Q = I_3 + \frac{Y}{2}}$
- *In SM (without ν_R) triangle anomalies cancellation constraints* ⇒ *certain relations among particle hypercharges Y, that is enough to fix all Y so that they, and consequently Q, are quantized*

$\boxed{Q=0}$

- $\boxed{Q=0}$ *is proven also by direct calculation in SM within different gauges and methods*

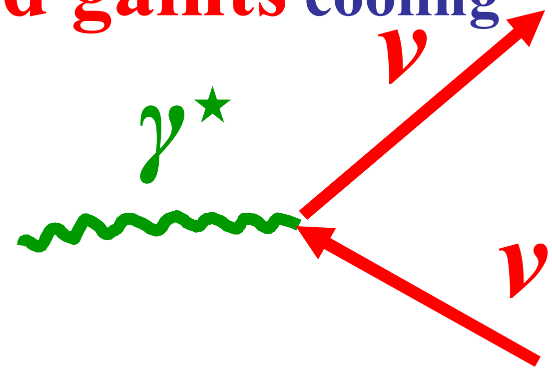
Bardeen, Gastmans, Lautrup, 1972;
Cabral-Rosetti, Bernabeu, Vidal, Zepeda, 2000;
Beg, Marciano, Ruderman, 1978;
Marciano, Sirlin, 1980; Sakakibara, 1981;
- *M.Dvornikov, A.S., 2004 (for extended SM in one-loop calculations)*

- *… However, strict requirements for Q quantization may disappear in extensions of standard $SU(2)_L \times U(1)_Y$ EW model if ν_R with $Y \neq 0$ are included : in the absence of Y quantization electric charges Q gets dequantized* ⇒ **millicharged ν**

Dobroliubov, Ignatiev (1990); Babu, Volkas (1992);
Mohapatra, Nussinov (1992) …

● Constraints on neutrino millicharge from red gaints cooling

$\gamma^{\star} \longrightarrow \nu\nu$



Interaction Lagrangian

$$\boxed{L_{int} = -iq_\nu \bar{\psi}_\nu \gamma^\mu \psi_\nu A^\mu}$$

millicharge

Decay rate

$$\boxed{\Gamma_{q_\nu} = \frac{{q_\nu}^2}{12\pi}\omega_{pl}\left(\frac{\omega_{pl}}{\omega}\right)}$$

- $\boxed{q_\nu \leq 2 \times 10^{-14} e}$ …to avoid helium ignition in low-mass **red gaints**

Halt,Raffelt, Weiss, PRL1994

- $\boxed{q_\nu \leq 3 \times 10^{-17} e}$ … absence of anomalous energy-dependent dispersion of SN1987A ν signal, most model independent

- **… from “charge neutrality” of neutron…** $\boxed{q_\nu \leq 3 \times 10^{-21} e}$

ν charge radius

Even if the electric charge of a neutrino is vanishing, the electric form factor $f_Q(q^2)$ can still contain nontrivial information about neutrino static properties. A neutral particle can be characterized by a superposition of two charge distributions of opposite signs so that the particle's form factor $f_Q(q^2)$ can be non zero for $q^2 \neq 0$. The application of this notion to neutrinos has a long-standing history and is puzzling. In the case of a electrically neutral neutrino, one usually introduces the mean charge radius, which is determined by the second term in the expansion of the neutrino charge form factor $f_Q(q^2)$ in series of powers of q^2,

$$\boxed{f_Q(q^2) = f_Q(0) + q^2 \frac{df_Q(q^2)}{dq^2}_{|q^2=0} + \ \ldots}$$

$$\boxed{\langle r_\nu^2 \rangle = -6 \frac{df_Q(q^2)}{dq^2}|_{q^2=0}}$$

Carlo Giunti, A.S. arXiv:0812.3646

The definition of the neutrino charge radius follows an analogy with the elastic electron scattering off a static spherically symmetric charged distribution of density $\rho(r)$ ($r = |\mathbf{x}|$), for which the differential cross section is determined [79–81] by the point particle cross section $\frac{d\sigma}{d\Omega}|_{point}$,

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{|point} |f(q^2)|^2, \tag{90}$$

where the correspondent form factor $f(q^2)$ in the so-called *Breit frame*, in which $q_0 = 0$, can be expressed as

$$f(q^2) = \int \rho(r) e^{i\mathbf{qx}} d^3x = 4\pi \int dr r^2 \rho(r) \frac{\sin(qr)}{qr}, \tag{91}$$

here $q = |\mathbf{q}|$. Thus, one has

$$\frac{df_Q}{dq^2} = \int \rho(r) \frac{qr\cos(qr) - \sin(qr)}{2q^{3/2}r} d^3x. \tag{92}$$

In the case of small q, we have $\lim_{q^2\to 0} \frac{qr\cos(qr)-\sin(qr)}{2q^{3/2}r} = -\frac{r^2}{6}$ and

$$f(q^2) = 1 - |\mathbf{q}|^2 \frac{\langle r^2 \rangle}{6} + \; ... \; . \tag{93}$$

Therefore, the neutrino charge radius (in fact, it is the charge radius squared) is usually defined by

$$\boxed{\langle r_\nu^2 \rangle = -6 \frac{df_Q(q^2)}{dq^2}|_{q^2=0}.} \tag{94}$$

Since the neutrino charge density is not a positively defined quantity, $\langle r_\nu^2 \rangle$ can be negative.

3.11

ν anapole moment and charge radius

$$\boxed{\Lambda_\mu(q)=f_Q(q^2)\gamma_\mu+f_M(q^2)i\sigma_{\mu\nu}q^\nu-f_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + f_A(q^2)(q^2\gamma_\mu-q_\mu\not{q})\gamma_5}$$

1. electric

dipole

2. magnetic

3. electric

4. anapole

Although it is usually assumed that ν are electrically neutral

(charge quantization implies $Q \sim \frac{1}{3}e$),

ν can dissociates into charged particles so that $f_Q(q^2) \neq 0 \ for \ q^2 \neq 0$:

$$\boxed{f_Q(q^2)=f_Q(0)+q^2\frac{df_Q}{dq^2}(0)+\cdots,}$$

where the massive ν charge radius

$$\boxed{\langle r_\nu^2\rangle = -6\,\frac{df_Q}{dq^2}(0)}$$

For massless ν

anapole moment

$$\boxed{a_\nu = f_A(q^2) = \frac{1}{6}\langle r_\nu^2\rangle}$$

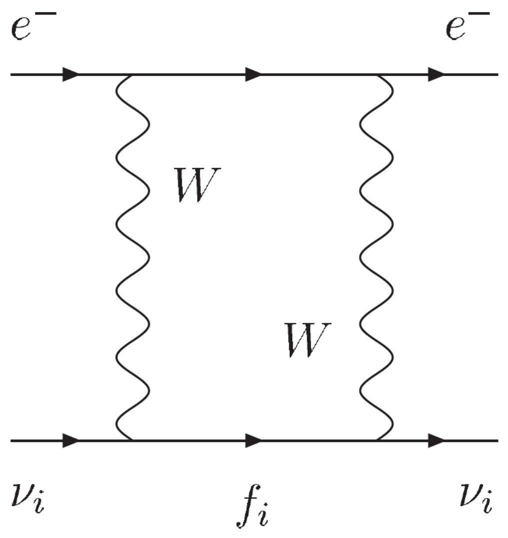
Interpretation of charge radius as an observable is rather delicate issue: $\langle r_\nu^2\rangle$ represents a correction to tree-level electroweak scattering amplitude between ν and charged particles, which receives radiative corrections from several diagrams (including γ exchange) to be considered simultaneously ⇨ calculated CR is infinite and gauge dependent quantity. For massless ν, a_ν and $\langle r_\nu^2\rangle$ can be defined (finite and gauge independent) from scattering cross section. ⇨

Bernabeu, Papavassiliou, Vidal, 2004

For massive ν ? ? ?

To obtain ν charge radius as physical (finite, not divergent) quantity

Bernabeu, Papavassiliou, Vidal, 2004



Contribution of box diagram to

$$\nu_l + l' \rightarrow \nu_l + l'.$$

!

$i = e, \mu, \tau$

$$\boxed{\langle r^2_{\nu_i}\rangle = \frac{G_F}{4\sqrt{2}\pi^2}\left[3 - 2\log\left(\frac{m_i^2}{m_W^2}\right)\right]}$$

$$\boxed{\langle r^2_{\nu_e}\rangle = 4\times 10^{-33}\,\mathrm{cm}^2}$$

…contribution to ν - e scattering experiments through

$$\boxed{g_V \rightarrow \frac{1}{2} + 2\sin^2\theta_W + \frac{2}{3}m_W^2\langle r^2_{\nu_e}\rangle\sin^2\theta_W}$$

… theoretical predictions and present experimental limits are in agreement within one order of magnitude…

ν anapole form factor

Zeldovich, JETP, 1957

Giunti, AS, 2008
Dubovik, Kuznetsov, 1998;
Bukina, Dubovik, Kuznetsov

- **Anapole form factor is the most mysterious one!**

To understand the physical meaning of the anapole form factor, as well as the meaning of other form factors, it is instructive to couple the correspondent term of the current to an external electromagnetic field (given by a potential A_μ), to derive the corresponding Dirac equation of motion for a neutrino field ψ of mass m, and finally to obtain the interaction energy with a static electromagnetic field in the nonrelativistic limit. From

$$\Lambda_\mu(q)_A = f_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5$$

- *In nonrelativistic limit, the correspondent interaction energy*

$$\boxed{H_{int} \propto f_A(0)\big(\boldsymbol{\sigma} \cdot curl\ \mathbf{B} - \dot{\mathbf{E}}\big),}$$

which corresponds to a T-invariant toroidal (anapole) interaction of the neutrino that does not conserve the P and C parities. This interaction defines the axial-vector interaction with an external electromagnetic field. The poloidal currents on a torus can be considered as a geometrical model for the anapole

Direct calculation of γ - Z and proper-vertex diagrams contribution

ν anapole moment is infinite and gauge dependent

- *$m=0$,* *Lucio, Rosado, Zepeda, 1985*
- *$m \neq 0$,* *Dvornikov, Studenikin, 2004*

is not a static quantity,
can’t be measured with external field

Physical definition of anapole moment:

Dubovik,
Kuznetsov, 1998

- through diagrammes contributing to $\boxed{\nu_l \; l' \to \nu_l \; l'}$
- with inclusion of all ν anapole diagrammes
- finite and gauge independent
- does not depend on charged lepton l' .

Giunti, AS, 2008
Dubovik, Kuznetsov, 1998;
Bukina, Dubovik, Kuznetsov

As it was discussed in [63], since the anapole form factor does not correspond to a multipole distribution, the anapole moment has a quite intricate classical analog. A more convenient and transparent characteristic, the toroidal dipole moment, was proposed instead for the description of T-invariant interactions. In this case, the electromagnetic vertex of a neutrino can be rewritten in an alternative multipole (toroidal) parameterization. In some sense this parameterization has a more transparent and clear physical interpretation, because it provides a one-to-one correspondence between the multipole moments and the corresponding form factors.

ν e.m. form factors are affected by matter and B

- ⊛ magnetic moment $\mu_\nu = \mu_\nu(B)$

  Egorov, Studenikin, 1997

  Borisov, Zhukovsky, Kurilin, Ternov, 1985

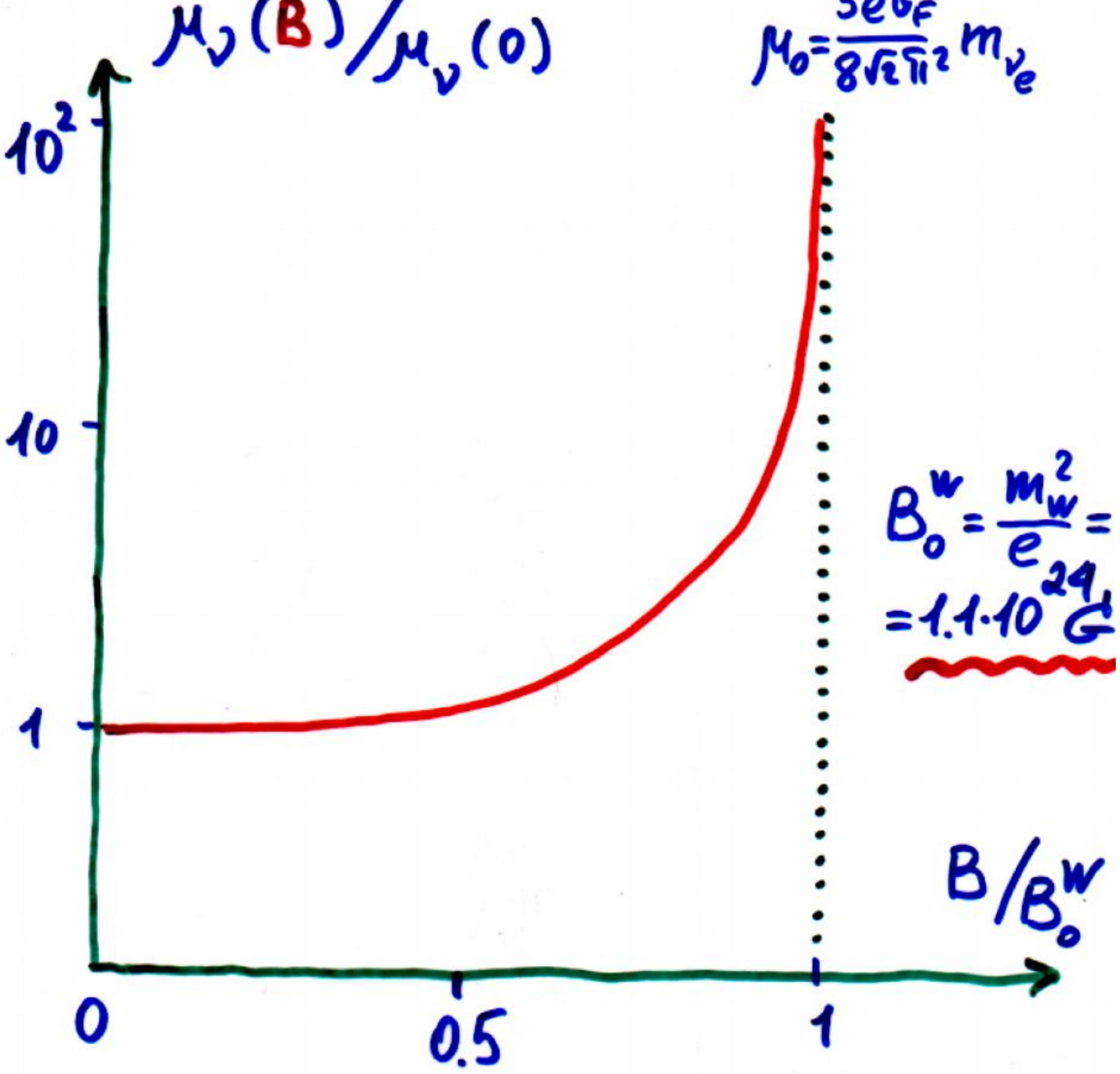
- ⊛ induced electric charge of ν in magnetized matter

  ⊛ Oraevsky, Semikoz, Smorodinsky, 1986

  Bhattacharaya, Ganguly, Konar, 2002

  Nieves, 2003

Neutrino magnetic moment



$$\mu_0 = \frac{3eG_F}{8\sqrt{2}\pi^2} m_{\nu_e}$$

Borisov,
Zhukovsky,
Kurilin,
Ternov, 1985;

$$B_0^W = \frac{m_W^2}{e} = 1.1 \cdot 10^{24} G$$

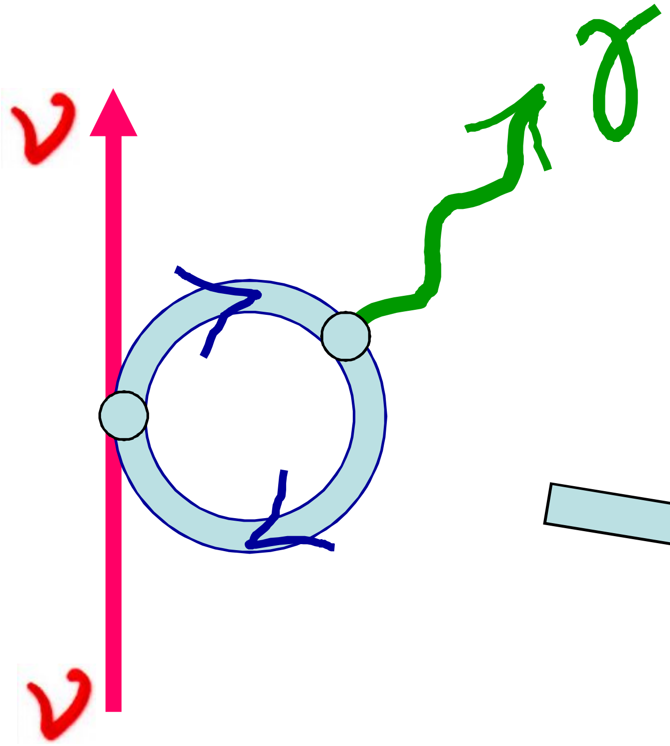
Masood,
Perez Rojas,
Gaitan,
Rodrigues-Romo,
1999

ν "effective electric charge" in magnetized plasma

- νs do not couple with γs in vacuum,

  . . . however, when

- ν in thermal medium (e^- and e^+)



✓ V.Oraevsky, V.Semikoz, Ya.Smorodinsky, JETP Lett. 43 (1986) 709;
J.Nieves, P.Pal, Phys.Rev.D 49 (1994) 1398;
T.Altherr, P.Salati, Nucl.Phys.B421 (1994) 662;
K.Bhattacharya, A.Ganguly, 2002

…different $\nu\gamma$ interactions in astrophysical and cosmological media

4 ν spin and spin-flavour oscillations in $B_\perp$

Consider two different neutrinos: $\nu_{e_L}, \quad \nu_{\mu_R}, \quad m_L \neq m_R$

with magnetic moment interaction

$$\boxed{L \sim \bar{\nu}\sigma_{\lambda\rho}F^{\lambda\rho}\nu\ ' = \bar{\nu}_L\sigma_{\lambda\rho}F^{\lambda\rho}\nu_R\ ' + \ \bar{\nu}_R\sigma_{\lambda\rho}F^{\lambda\rho}\nu_L\ '}\ .$$

Twisting magnetic field $B = |\mathbf{B}_\perp| e^{i\phi(t)}$ ⟸ **for solar** ν **etc …**

ν **evolution equation**

$$\boxed{i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = H\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}}$$

$$\boxed{H = \begin{pmatrix} E_L & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & E_R \end{pmatrix} = \ldots \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \tilde{H}}$$

!

$$\boxed{\tilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{V_{\nu_e}}{2} & \mu_{e\mu}Be^{\boxed{-i\phi}} \\ \mu_{e\mu}Be^{\boxed{+i\phi}} & \frac{\Delta m^2}{4E} - \frac{V_{\nu_e}}{2} \end{pmatrix}}$$

After unitary transformation

$$\nu = U\nu', \quad U = \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$i\left[\frac{i}{2}\begin{pmatrix} -e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}\dot{\phi}\nu' + \begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}\frac{d}{dt}\nu'\right] = H\begin{pmatrix} e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}\nu'$$

$$U' = \begin{pmatrix} -e^{-i\phi} & 0 \\ 0 & e^{i\phi} \end{pmatrix}, \quad U^\dagger = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}$$

conjugated

$$U^\dagger U' = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$i\frac{d}{dt}\nu' = (U^\dagger \tilde{H} U + \frac{\dot{\phi}}{2}U^\dagger U')\nu'$$

$$U^\dagger \tilde{H} U = \tilde{H}_{|\phi=0}$$

$$\tilde{H} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{V_{\nu_e}}{2} & \mu_{e\mu}Be^{-i\phi} \\ \mu_{e\mu}Be^{+i\phi} & \frac{\Delta m^2}{4E} - \frac{V_{\nu_e}}{2} \end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \frac{V_{\nu e}}{2} - \underline{\frac{\dot{\phi}}{2}} & \mu_{e\mu}B \\ \mu_{e\mu}B & \frac{\Delta m^2}{4E} - \frac{V_{\nu e}}{2} + \underline{\frac{\dot{\phi}}{2}} \end{pmatrix}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$

$$i\frac{d}{dt}\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix} = \left(-\frac{\Delta_{LR}}{4E}\sigma_3 + \mu_{e\mu}B\sigma_1\right)\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}$$

$$\sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \sigma_3 = \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$$

For relativistic ν : $\frac{d}{dt} \to \frac{d}{dz}$ **and solution is**

$$\nu(z) = e^{-i\Omega z(\mathbf{n}\boldsymbol{\sigma})}\nu(0)$$

where

$$\mathbf{n} = \frac{\mathbf{k}}{\Omega}, \quad \mathbf{k} = \left(\mu_{e\mu}B, 0, -\frac{\Delta_{LR}}{4E}\right)$$

$$\Omega^2 = (\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2$$

and

$$\Delta_{LR} = \frac{\Delta m^2}{2}\underline{(\cos 2\theta + 1)} - 2E\underline{V_{\nu_e}} + 2E\dot{\phi}$$

… Flavour oscillations ⇔ Spin oscillations…

$$\boxed{P_{\nu_e\nu_\mu} = \sin^2 2\theta \sin^2 \frac{\Delta m^2}{4E} z} \Longleftrightarrow \boxed{P_{\nu_L\nu_R} = \sin^2\beta \, \sin^2\Omega z}$$

$$\boxed{\Omega^2 = (\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$$

$$\boxed{\sin^2 2\theta} \Longleftrightarrow \boxed{\frac{(\mu_{e\mu}B)^2}{(\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2} = \sin^2\beta}$$

$$\boxed{\frac{\Delta m^2}{4E}} \Longleftrightarrow \boxed{\sqrt{(\mu_{e\mu}B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}}$$

Probability of $\nu_{e_L} \Leftrightarrow \nu_{\mu_R}$ oscillations in $\boxed{B = |\mathrm{B}_\perp| e^{i\phi(t)}}$ and matter

- $$\boxed{P_{\nu_L \nu_R} = \sin^2\beta \, \sin^2\Omega z, \quad \sin^2\beta = \frac{(\mu_{e\mu} B)^2}{(\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}}$$

$$\boxed{\Delta_{LR} = \frac{\Delta m^2}{2}(\cos 2\theta + 1) - 2EV_{\nu_e} + 2E\dot{\phi}}$$

$$\boxed{\Omega^2 = (\mu_{e\mu} B)^2 + \left(\frac{\Delta_{LR}}{4E}\right)^2}$$

- **Resonance amplification of oscillations in matter:**

$$\boxed{\Delta_{LR} \rightarrow 0} \Rightarrow \boxed{\sin^2\beta \rightarrow 1}$$

Akhmedov, 1988
Lim, Marciano

In magnetic field

$$\nu_{e_L} \ \nu_{\mu_R}$$

$$i\frac{d}{dz}\nu_{e_L} = -\frac{\Delta_{LR}}{4E}\nu_{e_L} + \mu_{e\mu} B \nu_{\mu_R}$$

$$i\frac{d}{dz}\nu_{\mu_L} = \frac{\Delta_{LR}}{4E}\nu_{\mu_L} + \mu_{e\mu} B \nu_{e_R}$$

Neutrino conversions and oscillations in magnetic field

- ⊛ $\nu_\odot$ problem $\boxed{\nu_L \overset{B}{\leftrightarrow} \nu_R}$ ⇐ …for recent analysis see

  J.Pulido, 2006

  A.Balantekin,
  C.Volpe, 2005

  Cisneros, 1971

  * { Voloshin, Vysotsky, Okun, 1986
  Barbieri, Fiorentini, 1988

  $\odot$ twisting B { Smirnov, 1991
  Akhmedov, Petcov, Smirnov 1993

- ⊛ Supernova $\nu_L \overset{B}{\leftrightarrow} \nu_R$

  Dar, 1987

  Fujikawa, Shrock, 1988

  Voloshin, 1988

✳ *Spin-flavour oscillations in early universe – strong* $B_\perp$
⇒ *population of* ν *wrong-helicity states (r.h.) would accelerate expansion of universe (???)*

Periodicity of the active solar neutrino flux is probably the most important issue to be investigated after LMA has been ascertained as the dominant solution to the $\odot\ \nu$ problem. If confirmed it will imply the existence of a sizable neutrino magnetic moment μ_ν and hence a wealth of new physics.

- **Idea was introduced in 1986 by**

  Voloshin, Vysotsky and Okun

$$Strong\ B_\odot \ \rightarrow\ large\ \mu_\nu B_\odot \ \rightarrow\ large\ conversion$$

- **For recent analysis see**

  J.Pulido, 2006 …see also A.Balantekin and C.Volpe, 2005

- **… Spin-flavour precession resonance and MSW resonance take place very close to each other inside sun…**

SPIN FLAVOUR PRECESSION AND LMA

João M. Pulido

CFTP - Instituto Superior Técnico, Lisbon

12th Lomonosov Conference on Elementary Particle Physics, Moscow, August 2005

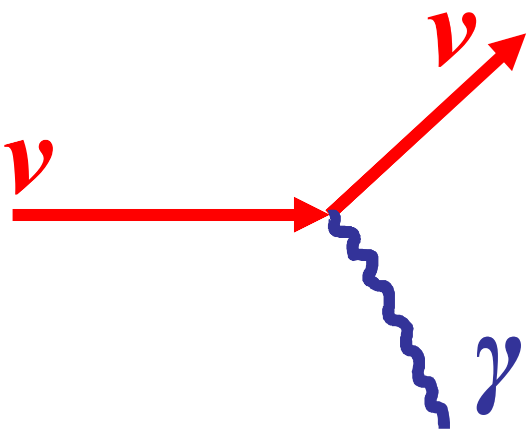
Long term periodicity may have been observed by the Gallium experiments. In fact

Period	1991-97	1998-03
SAGE+Ga/GNO	77.8 ± 5.0	63.3 ± 3.6
Ga/GNO only	77.5 ± 7.7	62.9 ± 6.0
no. of suspots	52	100

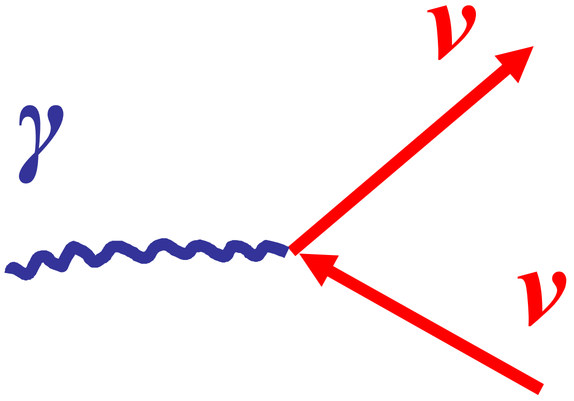
Notice a 2.4σ discrepancy in the combined results over the two periods. This is suggestive of an anticorrelation of Ga event rate with the 11-year solar sunspot cycle.

Conclusion

Neutrino – photon couplings (I)

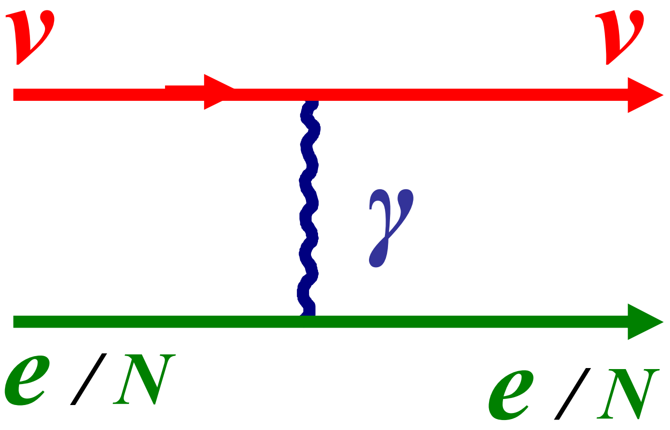


ν **decay, Cherenkov radiation**

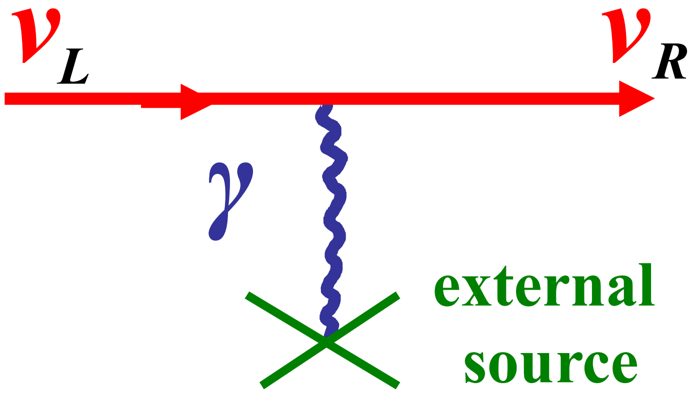


γ **decay in plasma**

!!!



Scattering



Spin precession

New mechanism of electromagnetic radiation

ν spin evolution in presence of general external fields

M.Dvornikov, A.Studenikin,
JHEP 09 (2002) 016

General types non-derivative interaction with external fields

$$\boxed{\begin{gathered}-\mathcal{L} = g_s s(x)\bar{\nu}\nu + g_p \pi(x)\bar{\nu}\gamma^5\nu + g_v V^\mu(x)\bar{\nu}\gamma_\mu\nu + g_a A^\mu(x)\bar{\nu}\gamma_\mu\gamma^5\nu + \\ + \tfrac{g_t}{2} T^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\nu + \tfrac{g'_t}{2}\Pi^{\mu\nu}\bar{\nu}\sigma_{\mu\nu}\gamma_5\nu,\end{gathered}}$$

scalar, pseudoscalar, vector, axial-vector, tensor and pseudotensor fields:

$$s, \pi, V^\mu = (V^0, \vec{V}), A^\mu = (A^0, \vec{A}),$$

$$T_{\mu\nu} = (\vec{a}, \vec{b}), \Pi_{\mu\nu} = (\vec{c}, \vec{d})$$

Relativistic equation (quasiclassical) for ν *spin vector:*

$$\begin{aligned}\dot{\vec{\zeta}}_\nu = 2g_a \left\{A^0[\vec{\zeta}_\nu \times \vec{\beta}] - \tfrac{m_\nu}{E_\nu}[\vec{\zeta}_\nu \times \vec{A}] - \tfrac{E_\nu}{E_\nu + m_\nu}(\vec{A}\vec{\beta})[\vec{\zeta}_\nu \times \vec{\beta}]\right\} \\ +2g_t \left\{[\vec{\zeta}_\nu \times \vec{b}] - \tfrac{E_\nu}{E_\nu + m_\nu}(\vec{\beta}\vec{b})[\vec{\zeta}_\nu \times \vec{\beta}] + [\vec{\zeta}_\nu \times [\vec{a} \times \vec{\beta}]]\right\} + \\ +2ig'_t \left\{[\vec{\zeta}_\nu \times \vec{c}] - \tfrac{E_\nu}{E_\nu + m_\nu}(\vec{\beta}\vec{c})[\vec{\zeta}_\nu \times \vec{\beta}] - [\vec{\zeta}_\nu \times [\vec{d} \times \vec{\beta}]]\right\}.\end{aligned}$$

● *Neither* ***s*** *nor* ***π*** *nor* ***V*** *contributes to spin evolution*

- **Electromagnetic interaction**

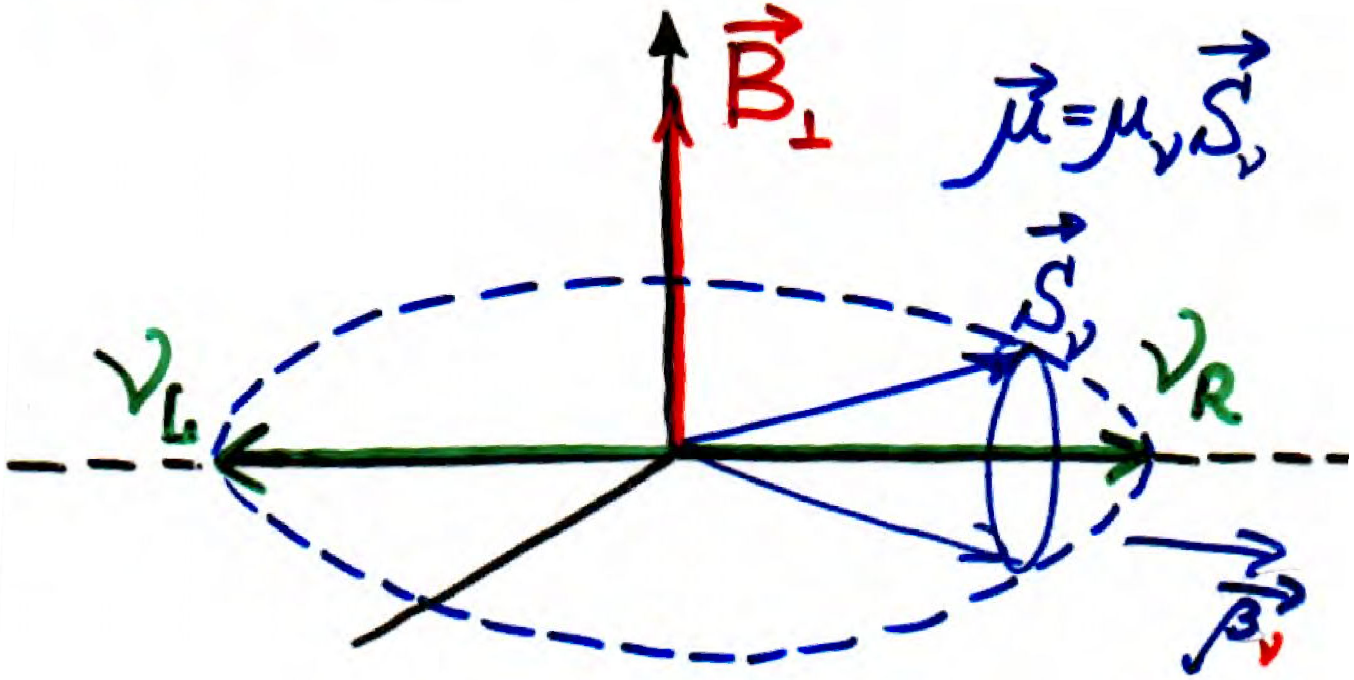
$$T_{\mu\nu} = F_{\mu\nu} = (\vec{E}, \vec{B})$$

- **SM weak interaction**

$$G_{\mu\nu} = (-\vec{P}, \vec{M})$$

$$\vec{M} = \gamma(A^0\vec{\beta} - \vec{A})$$

$$\vec{P} = -\gamma[\vec{\beta} \times \vec{A}],$$



$$\frac{d\vec{S}_\nu}{dt} = 2\mu_\nu[\vec{S}_\nu \times \underline{\underline{\vec{B}}}] + \boxed{2\mu_\nu[\vec{S}_\nu \times \vec{G}]}$$

electromagnetic interaction with e.m. field

weak interaction with matter

"Spin light of neutrino" in matter and electromagnetic fields

$SL\nu$

$$\boxed{\nu \xrightarrow{F_{\mu\nu},\; G_{\mu\nu}} \nu + \gamma}$$

Quasi-classical theory of spin light of neutrino in matter and gravitational field

$SL\nu$

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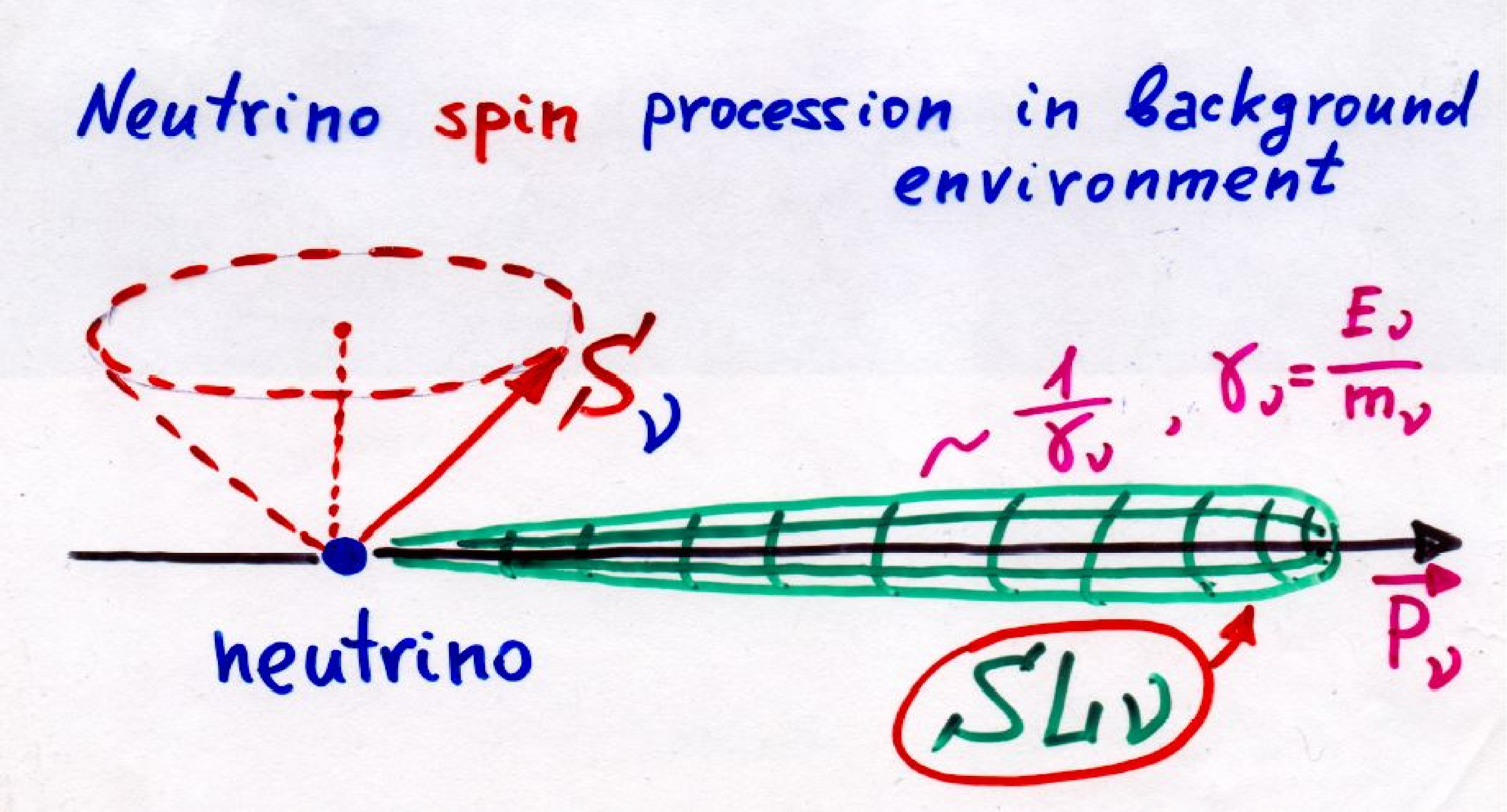

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JETP 99 (2004) 254; **JHEP 09** (2002) 016

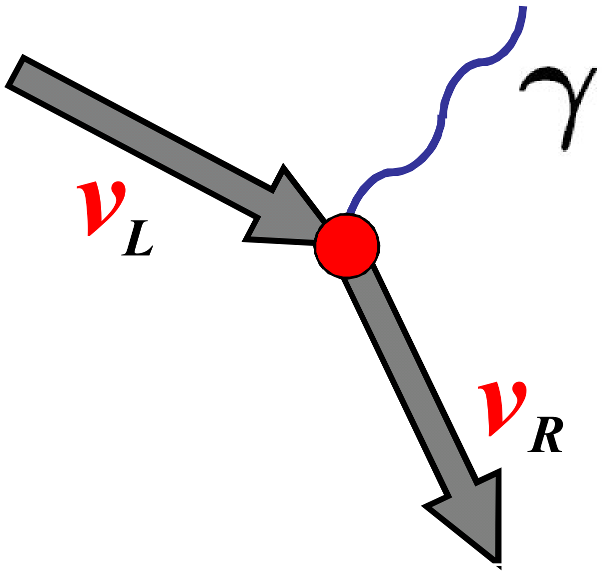
A.Lobanov, A.Studenikin, **Phys.Lett.B 601** (2004) 171; *ibid* **564** (2003) 27, **515** (2001) 94

A.Grigoriev, A.Lobanov, A.Studenikin, **Phys.Lett.B 535** (2002) 187

A.Egorov, A.Lobanov, A.Studenikin, **Phys.Lett.B 491** (2000) 137

$SL\nu$

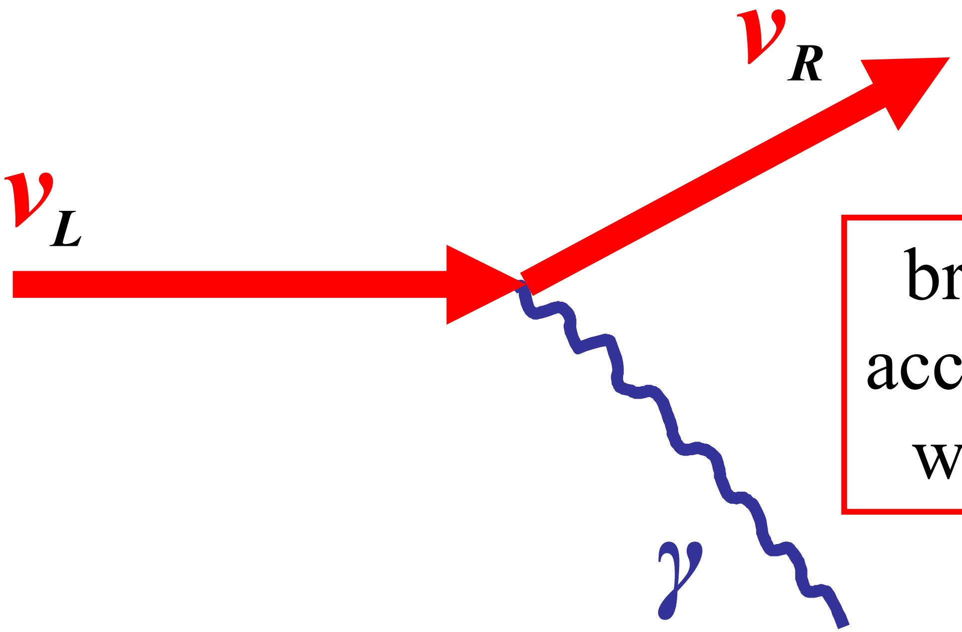
Spin light of neutrino ***in matter***



- We predict the existence of a **new mechanism** of the electromagnetic process stimulated by the presence of matter, in which a neutrino with **non-zero magnetic moment** emits light.

A.Lobanov, A.Studenikin, PLB 2003
A.Studenikin, A.Ternov, PLB 2004
A.Grigoriev, Studenikin, Ternov, PLB 2005
A.S., J.Phys.A: Math.Gen. 39 (2006) 6769
A.S., J.Phys.A: Math.Theor. 41 (2008) 16402

Neutrino – photon couplings (II)



broad neutrino lines account for interaction with environment

“Spin light of neutrino in matter”

… within the quantum treatment based on method of exact solutions …

«method of exact solutions »

Interaction of particles in external electromagnetic fields

(**Furry representation** in quantum electrodynamics)

$$e \xrightarrow{B_\perp} e + \gamma$$

synchrotron radiation

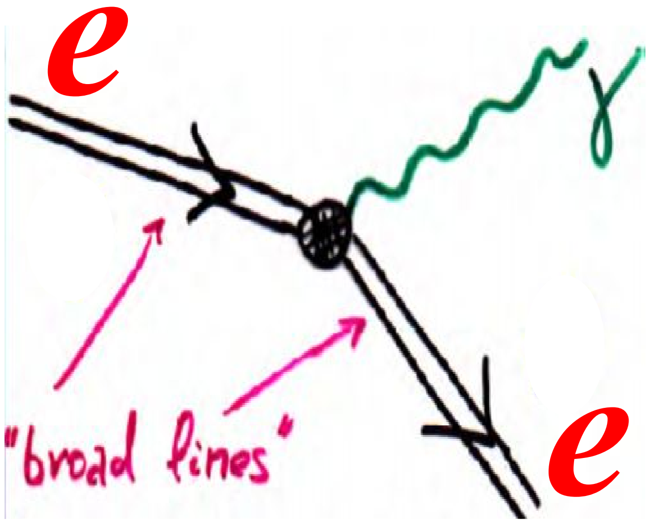
Potential of electromagnetic field

$$\boxed{A_\mu(x) = A^q_\mu(x) + A^{ext}_\mu(x)},$$

quantized part of potential

evolution **operator**

$$\boxed{U_F(t_1,t_2) = Texp\Big[-i\int_{t_1}^{t_2} j^\mu(x)A^q_\mu(x)dx\Big]},$$



charged particles **current**

$$\boxed{j_\mu(x) = \frac{e}{2}[\overline{\Psi}_F\gamma_\mu, \Psi_F]},$$

Dirac equation in **external classical** **(non-quantized)** **field** $A^{ext}_\mu(x)$

$$\boxed{\Big\{\gamma^\mu\Big(i\partial_\mu - eA^{ext}_\mu(x)\Big) - m_e\Big\}\Psi_F(x) = 0}$$

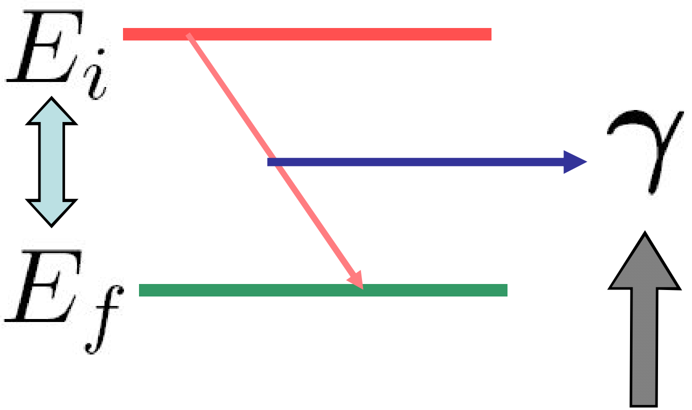
…beyond perturbation series expansion,
strong fields and non linear effects…

Quantum theory of spin light of neutrino (I)

Quantum treatment of *spin light of neutrino* in matter shows that this process originates from the **two subdivided phenomena:**

$SL\nu$

the **shift** of the neutrino **energy levels** in the presence of the background matter, which is different for the two opposite **neutrino helicity states**,



$$E = \sqrt{\mathbf{p}^2\left(1 - s\alpha\frac{m}{p}\right)^2 + m^2} + \alpha m$$

$$s = \pm 1$$

the radiation of the photon in the process of the neutrino transition from the **“excited” helicity state** to the **low-lying helicity state** in matter

A.Studenikin, A.Ternov, Phys.Lett.B 608 (2005) 107;

A.Grigoriev, A.Studenikin, A.Ternov, Phys.Lett.B 622 (2005) 199;
Grav. & Cosm. 14 (2005) 132;

hep-ph/0507200, hep-ph/0502210,

hep-ph/0502231

neutrino-spin self-polarization effect in the matter

A.Lobanov, A.Studenikin, Phys.Lett.B 564 (2003) 27;
Phys.Lett.B 601 (2004) 171

Direct and Indirect influence of electromagnetic fields on ν

through non-trivial neutrino electromagnetic properties (magnetic moment):

- neutrino spin
- spin-flavour oscillations…
- different $\nu\gamma$ processes

due to e.m. field influence on “charged” particles coupled to neutrinos

- neutron beta-decay in B
- change of ν oscillation pattern due to matter polarization under influence of external e.m. fields …

…remark on How can ν be affected by $\vec{B}$?

1) *"direct influence"*

non-trivial electromagnetic properties

- * $\mu_\nu \neq 0 \, (m_\nu \neq 0)$
- * spin and
- * spin-flavour oscillations in B

!

2) "indirect influence" of B on interacting with ν particles

- * $n \xrightarrow{B} p^+ + e^- + \tilde{\nu}_e$
- * $\nu + n \xrightarrow{B} p^+ + e^-$
- * flavour and spin ν oscillations in polarized (by B) matter (e, n, p, …)

3) *"direct-indirect influence"*

Spin light of ν in matter and e.m.fields

μ_ν is presently known to be in the range

$$\boxed{10^{-20}\mu_B \leq \mu_\nu \leq 10^{-10}\mu_B}$$

μ_ν provides a tool for exploration possible physics beyond the Standard Model

Due to smallness of neutrino-mass-induced magnetic moments,

$$\boxed{\mu_{ii} \approx 3.2 \times 10^{-19} \left(\frac{m_i}{1\ eV}\right) \mu_B}$$

any indication for non-trivial electromagnetic properties of ν , that could be obtained within reasonable time in the future, would give evidence for interactions beyond extended Standard Model

Model-independent upper bound on μ_ν

$$|\mu_\nu^D| = \frac{16\sqrt{2}G_F m_e \sin^4\theta_W}{9\alpha^2 |f| \ln(\Lambda/v)} \mu_B,$$

Bell, Cirigliano, Ramsey-Musolf, Vogel, Wise, 2005

$$f = 1 - r - \frac{2}{3}\tan^2\theta_W - \frac{1}{3}(1+r)\tan^4\theta_W,$$

$$\mu_\nu \leq 10^{-14}$$

$$\delta m_\nu \leq 1 \text{ eV} \qquad \Lambda \sim 1 \text{ TeV}$$

… situation with

ν electromagnetic properties

is better that it was in the case of W. Pauli, 1930

… once they will be observed experimentally

… are important in astrophysics

… there is a need for theoretical and experimental studies

Experimental and theoretical studies of

ν electromagnetic properties

is a tedious task

⬇

important impact on understanding of

fundamentals of particle physics

(Dirac ↔ Majorana etc) and

applications in astrophysics