Gauge Fields in $(A)dS_d$ and Connections of $(A)dS_d$-space symmetry algebra

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Gravity as a Gauge Theory

$$g_{\mu\nu} \longrightarrow h^a_\mu,\ \varpi^{a,b}_\mu$$

Yang-Mills field $A_\mu = P_a h^a_\mu + L_{a,b}\varpi^{a,b}_\mu$

Algebra

$$[P_a, P_b] = \pm\lambda^2 L_{ab} \qquad \lambda^2 > 0 \ \ \mathfrak{so}(d,1)$$
$$[L_{ab}, P_c] = L_a\eta_{bc} - L_b\eta_{ac} \qquad \lambda^2 = 0 \ \ \mathfrak{iso}(d-1,1)$$
$$[L_{ab}, L_{cd}] = L_{ad}\eta_{bc} + \ldots \qquad \lambda^2 < 0 \ \ \mathfrak{so}(d-1,2)$$

Field strength $R = dA + [A, \wedge A] = P_a R^a + L_{a,b} R^{a,b}$

Torsion $R^a = dh^a + \varpi^{a,}{}_b \wedge h^b = 0$

Curvature $R^{a,b} = d\varpi^{a,b} + \varpi^{a,}{}_c \wedge \varpi^{c,b} \pm \lambda^2 h^a \wedge h^b = 0$

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Gravity as a Gauge Theory

$$d = 4$$

$$\frac{1}{\lambda^2}\int R^{a,b} \wedge R^{c,d} \epsilon_{abcd} \quad \sim \quad \int \sqrt{g}\left(R + \lambda^2\right) + \text{Gauss Bonnet}$$

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Getting $(A)dS_d$ symmetry manifest

(MacDowell, Mansouri, Stelle, West, Vasiliev)

$$\Omega_\mu^{A,B} = -\Omega_\mu^{B,A}$$

$$R^{A,B} = d\Omega^{A,B} + \Omega^{A,}{}_C \wedge \Omega^{C,B}$$

$$h^a = \lambda \Omega^{a,\bullet}$$

$$\varpi^{a,b} = \Omega^{a,b}$$

Goldstone field $V^A V^B \eta_{AB} = \pm 1$

Frame field $H^A = D_\Omega V^A = dV^A + \Omega^{A,}{}_C V^C$

Spin-connection $\Omega_L^{A,B} = \Omega^{A,B} \pm \lambda (H^A V^B - H^B V^A)$

Standard gauge $V^A = \delta^A_\bullet$

$$\frac{1}{\lambda^2} \int R^{A,B} \wedge R^{C,D} \epsilon_{ABCDE...UN} H^E ... H^U V^N$$

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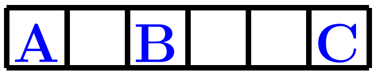
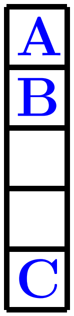
Tensors and Young Diagrams

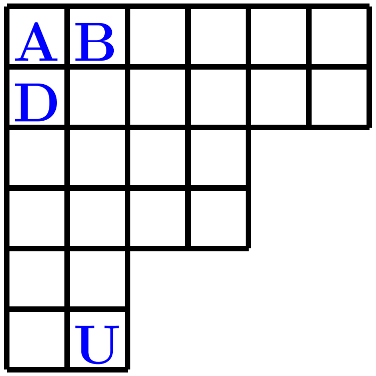
symmetric $T^{A...B...C} = T^{B...A...C}$



antisymmetric $T^{A,B,...,C} = -T^{B,A,...,C}$



mixed-symmetry $T^{AB...,D...,\ ....\ ,...U}$

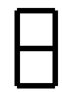


Generalized Yang-Mills Connections

(Alkalaev, Shaynkman, Vasiliev)

Yang-Mills Connection

$$W_\mu^{A,B} dx^\mu$$

one-form in adjoint= 

$$\Downarrow$$

Generalized Yang-Mills Connection

q-from in arbitrary module $\mathbf{A}$

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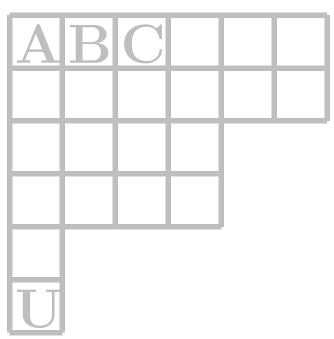
$\mathrm{A} =$ 

of $\mathfrak{so}(d,1)$ or $\mathfrak{so}(d-1,2)$

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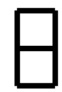
one-form in adjoint= 

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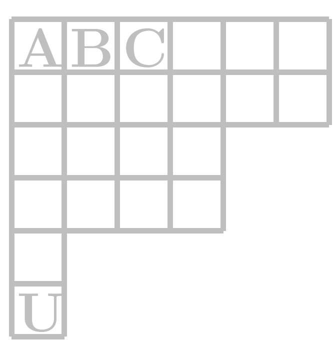
A = 

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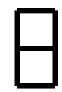
one-form in adjoint= 

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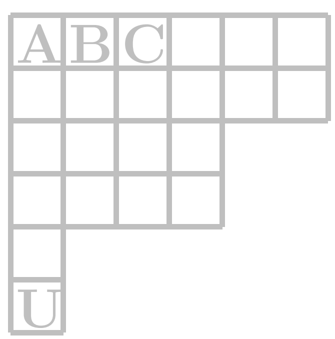
$\mathrm{A} =$ 

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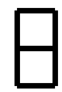
one-form in adjoint= 

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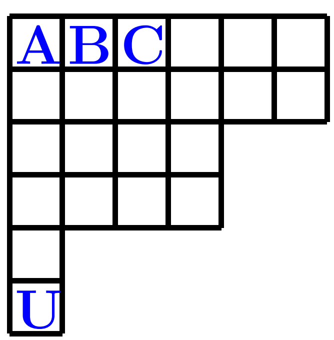
$\mathbf{A} =$ 

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Gauging with Generalized Connections

(Alkalaev, Shaynkman, Vasiliev)

$$d\Omega^{A,B} + \Omega^{A,}{}_{C} \wedge \Omega^{C,B} = 0 \; (A)dS_d$$

Covariant derivative

$$D_\Omega W_{\mathbf{q}}^{ABC\ldots} \equiv dW_{\mathbf{q}}^{ABC\ldots} + \Omega^{A,}{}_{M} \wedge W_{\mathbf{q}}^{MBC\ldots} + \ldots.$$

$$D_\Omega{}^2 = 0$$

Bianchi Identities	$D_\Omega R_{\mathbf{q+1}}^{\mathbf{A}}$	$= 0$
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What theory does $W_{\mathbf{q}}^{\mathbf{A}}$ describe?

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What should we expect?

massless spin-2

$$\delta\phi_{\mu\nu} = D_\mu\xi_\nu + D_\nu\xi_\mu$$
p.d.o.f.$= \pm 2, \pm\mathbf{1}, \mathbf{0}$

partially-massless spin-2
(Deser, Nepomechie)

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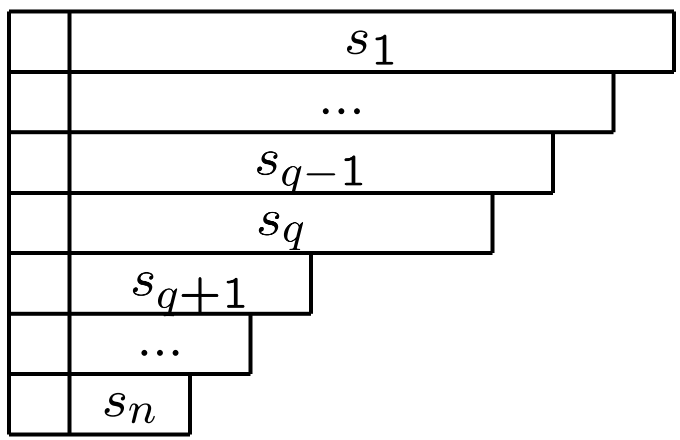
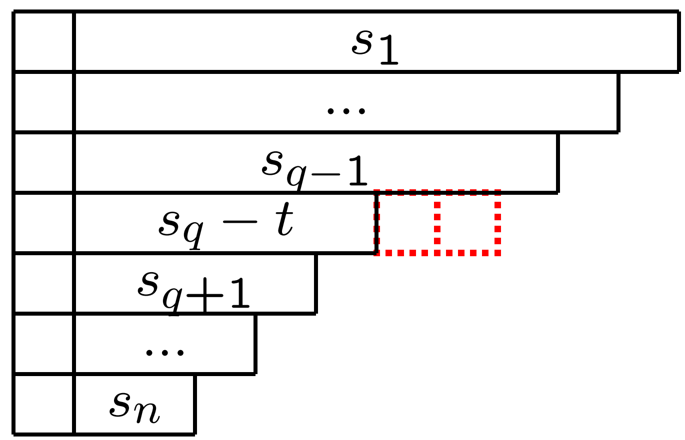
What should we expect? General case

(Metsaev,Brink,Vasiliev,Alkalaev,Skvortsov)

Field $\mathbf{s}_0$



Gauge parameter $\mathbf{s}_1$



$$\delta\phi^{\mathbf{s}_0} = \overbrace{D\ldots D}^{t}\,\xi^{\mathbf{s}_1} + \ldots$$

Gauge Fields in $(A)dS_d \Longleftrightarrow (S, q, t)$

The way it works

$$\boxed{\text{(A)}dS_d\text{-covariant frame-like}} \xrightarrow{V^A} \boxed{\text{Lorentz-covariant frame-like}} \xrightarrow[h^a_\mu]{h^\mu_a} \boxed{\text{Lorentz-covariant metric-like}}$$

$W^{\mathbf{A}}_{\mathbf{q}}$: $A \to A_k$ **of Lorentz algebra**

$$\delta W^{\mathrm{A}}_{\mathrm{q}} = D_\Omega \xi^{\mathrm{A}}_{\mathrm{q}-1}$$

$$\downarrow$$

$$\delta \omega^k_{\mathrm{q}} = D\xi^k_{\mathrm{q}-1} + \sigma_-(\xi^{k+1}_{\mathrm{q}-1}) + \sigma_+(\xi^{k-1}_{\mathrm{q}-1})$$

$$(\sigma_\pm)^2 = 0$$

The way it works

$(A)dS_d$-covariant frame-like $\xrightarrow{V^A}$ Lorentz-covariant frame-like $\xrightarrow[h^a_\mu]{h^\mu_a}$ Lorentz-covariant metric-like

$W^{\mathbf{A}}_{\mathbf{q}}$: $\quad A \to A_k$ **of Lorentz algebra**

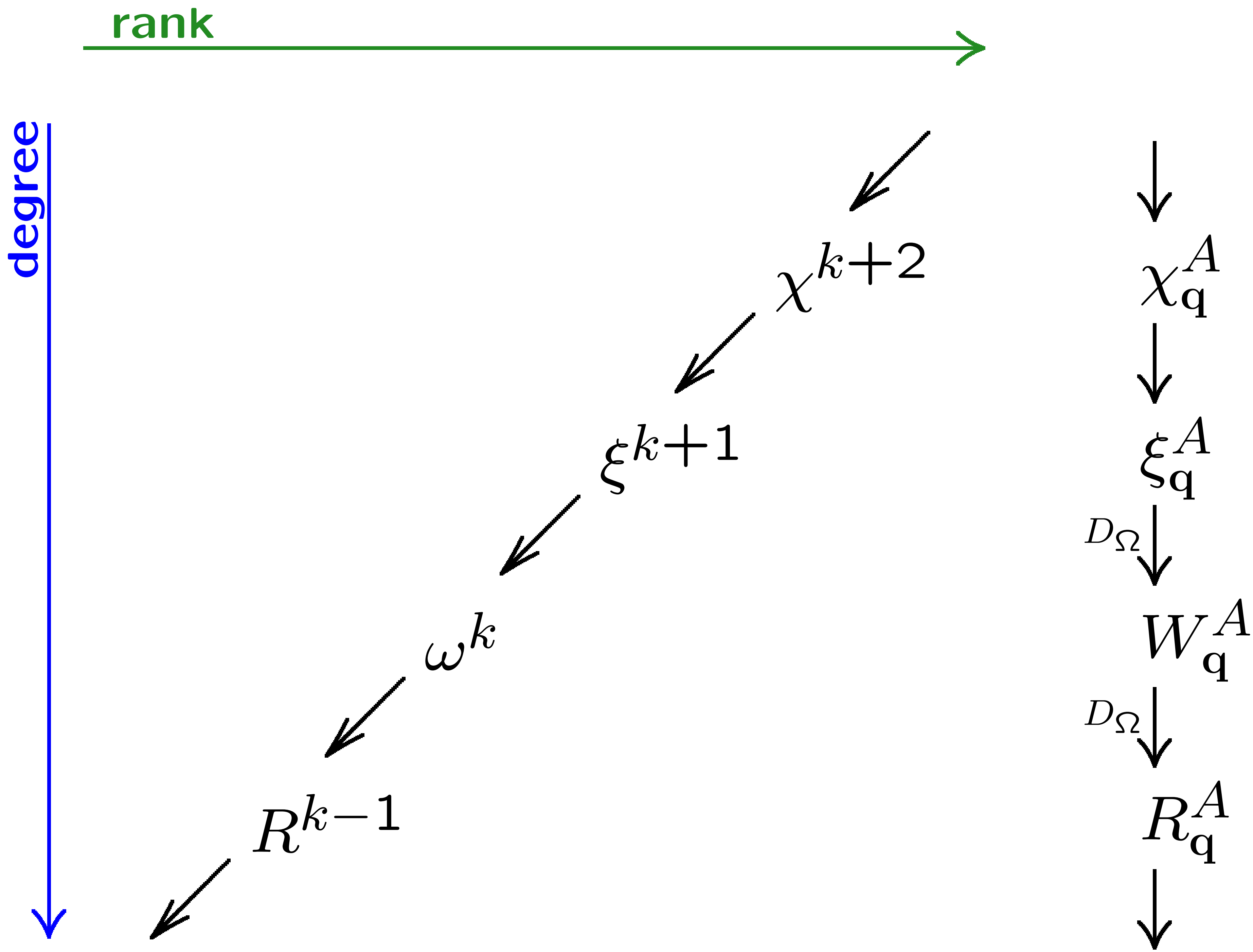
$$\delta W^{\mathbf{A}}_{\mathbf{q}} = D_\Omega \xi^{\mathbf{A}}_{\mathbf{q-1}}$$

$$\downarrow$$

$$\delta \omega^k_{\mathbf{q}} = D\xi^k_{\mathbf{q-1}} + \sigma_-(\xi^{k+1}_{\mathbf{q-1}}) + \sigma_+(\xi^{k-1}_{\mathbf{q-1}})$$

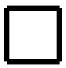
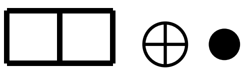
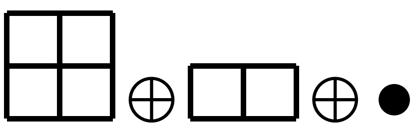
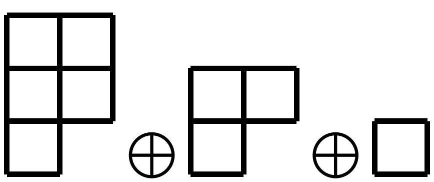
$$(\sigma_\pm)^2 = 0$$

σ_--Complex

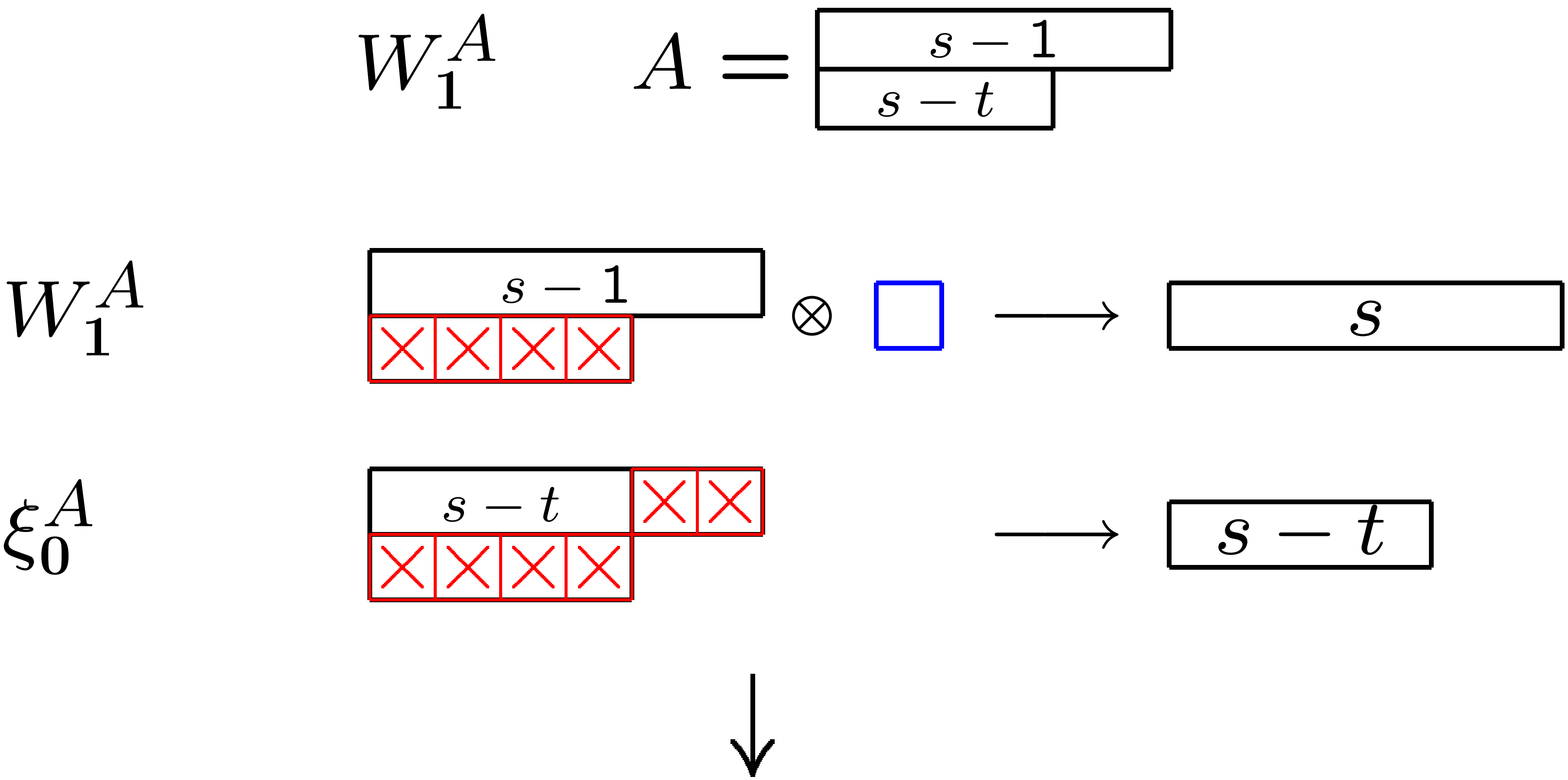


σ_--Cohomology

(Lopatin, Shaynkman, Vasiliev)

	H^{q-2}	**reducible gauge parameters**
	H^{q-1}	**gauge parameters**
	H^{q}	**dynamical fields**
	H^{q+1}	**gauge invariant equations**
	H^{q+2}	**Bianchi identities**

The way it works
Simplest cases



$$\delta\phi_{\underbrace{\mu\mu\ldots\mu}_{s}} = \underbrace{D_\mu\ldots D_\mu}_{t}\,\xi_{\underbrace{\mu\ldots\mu}_{s-t}}$$

Unfolded Field Equations

(Vasiliev)

Chevalley-Eilenberg cocycle

$\downarrow$

$$\begin{cases} R^A_{\mathbf{q+1}} = D_\Omega W^A_{\mathbf{q}} = H \ldots H C_{\mathbf{0}} \\ \widetilde{D}_\Omega C_{\mathbf{0}} = 0 \end{cases}$$

$W^A_{\mathbf{q}}$ - **Gauge module**

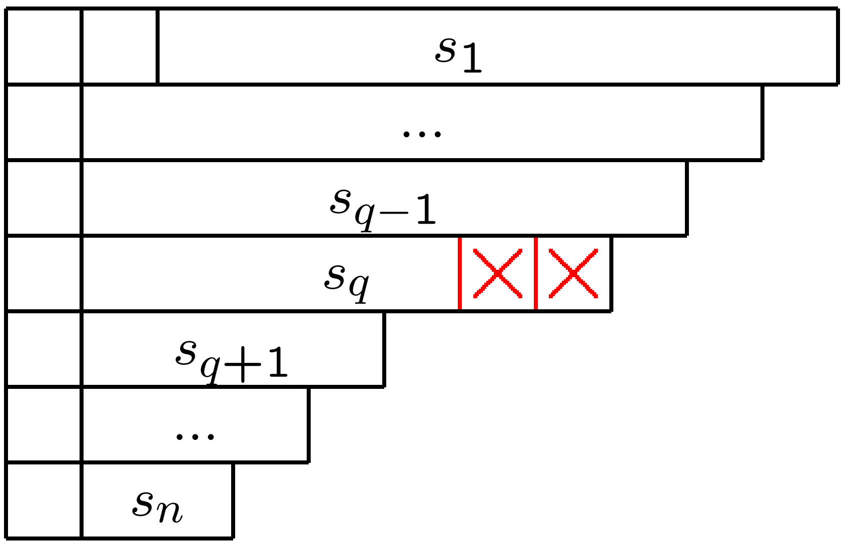
$C_{\mathbf{0}}$ - **Weyl module**

explicit realization: (Vasiliev), (Boulanger, Iazeolla, Sundell)

Gauge Fields vs. Gauge Connections

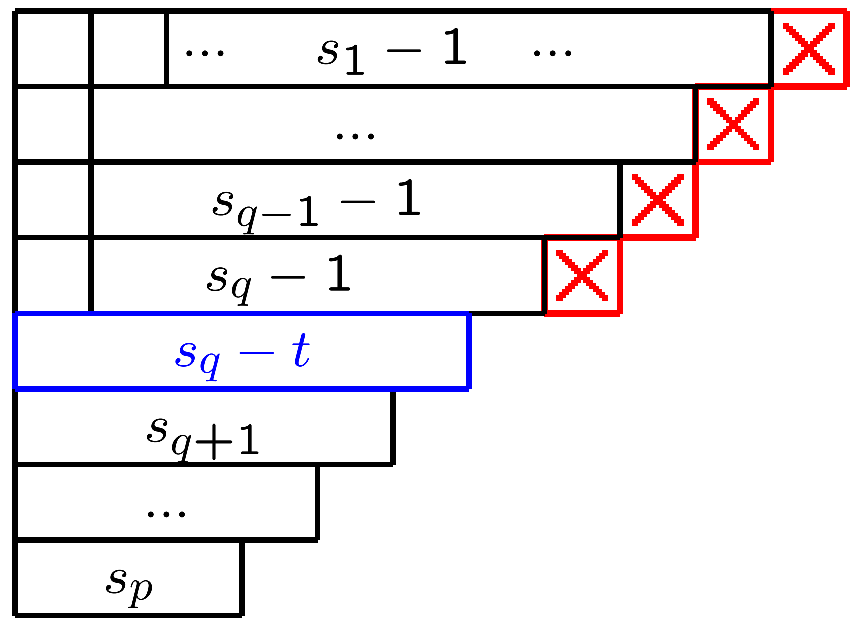
Lorentz metric-like

$\mathfrak{so}(d-1,1)$



Connection

$\mathfrak{so}(d,1)$ or $\mathfrak{so}(d-1,2)$



$$\delta\phi^{\mathbf{s}_0} = \overbrace{D\dots D}^{t}\,\xi^{\mathbf{s}_1} + \dots \qquad \delta W^{\mathbf{A}}_{\mathbf{q}} = D_\Omega \xi^{\mathbf{A}}_{\mathbf{q}-1}$$

$$R^{\mathbf{A}}_{\mathbf{q}+1} = D_\Omega W^{\mathbf{A}}_{\mathbf{q}}$$

Conclusions

$(A)dS_d$-Gauge Fields

$$\updownarrow$$

Yang-Mills Connections

- **Manifest gauge invariance**
- **Manifest $(A)dS_d$ covariance**
- **Manifest general covariance**
- **Nonlinear theory??** $\longrightarrow$ **String theory**

Thank you

Welcome Party
will be in 30 minutes