

Gluelumps and Confinement

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1. Introduction: Mass scales in QCD

QCD is the selfconsistent quantum field theory which is defined by the QCD Lagrangian, not containing any dimensionful parameters (except for quark masses), and one needs one additional mass scale (\mathcal{M}) to fix the theory.

In gluodynamics one can choose $\Lambda_{QCD} \approx 0.3$ GeV or string tension $\sigma = 0.18$ GeV². They should be connected to one scale \mathcal{M} .

But there are other scales in QCD, which are very different

Glueball mass: $m_G = O(2$ Gev).

Deconfinement temperature $T_c = 0.27$ GeV \div 0.17 GeV for $n_f = 0 - 2$.

Gluonic condensate $G_2 = \frac{\alpha_3}{\pi} \langle F_{\mu\nu} F_{\mu\nu} \rangle$

$$G_2(\text{standard}) = 0.012 \pm 0.006 \text{ GeV}^4.$$

We aim at explaining all different scales in term of the only one, say \mathcal{M} . This is done in the framework of Field Correlator method (FCM).

Talk is based on many papers including recent V.I.Shevchenko, Yu.A.Simonov, arXiv:0902.1405.

2. Perturbative and Nonperturbative

The very concept of finite G_2 introduced by SVZ implies possibility of separation of Perturbative and Nonperturbative contributions in QCD. On general grounds for every physical amplitude of dimension m^2 the finite sum of Perturbative terms yields $\sim L^{-2} f\left(\ln\frac{1}{L\Lambda_{QCD}}\right)$, while Nonperturbative $\sim \mathcal{M}^2$. In principle there can be mixed terms $O\left(\frac{\mathcal{M}}{L}, L\mathcal{M}^3, \dots\right)$.

We shall prove that for Field correlator one can write $(x \rightarrow y)$

$$\begin{aligned} & \frac{g^2}{4\pi^2} \langle \text{tr} F_{\mu\nu}(x) \Phi(x, y) F_{\mu\nu}(y) \Phi(y, x) \rangle = \\ & = \text{Pert.} \left(O\left(\frac{\ln(x-y)}{(x-y)^4}\right) \right) + G_2 + \dots \end{aligned}$$

Infinite Pert. series are not defined due to IR renormalons, and perturbation theory in QCD has sense when background vacuum fields are taken into account. Confining background fields eliminate IR renormalons and define IR stable perturbative theory (Yu.S. 1993)

$$\alpha_s(q) = \frac{4\pi \left(1 + O\left(\frac{\ln \ln}{\ln}\right)\right)}{\beta \ln\left(\frac{q^2 + M_b^2}{\Lambda_{QCD}^2}\right)}.$$

The new scale $M_b \cong 1$ GeV is expressed via σ and is related to the hybrid masses.

3. Basics of Field Correlator Method

As will be shown below, Green's function of any white system is proportional to the path integral of the Wilson loop.

For $q\bar{q}$, $G_{q\bar{q}} \sim \int (Dz) \langle \text{tr} W(C) \rangle \dots$. Therefore Wilson loop defines the dynamics (pert. and nonpert.) of light and heavy quarks.

Building blocks: Wegner-Wilson loops

$$W(C) = \text{P exp } ig \oint_C A_\mu^a(z) t^a dz_\mu \quad (1)$$

Parallel transporter

$$\Phi(x; y) = \text{P exp } ig \int_x^y A_\mu^a(z) t^a dz_\mu \quad (2)$$

Field strength

$$\begin{aligned}
 F_{\mu\nu}(x) &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu] \\
 D_{\mu_1\nu_1\dots\mu_n\nu_n}^{(n)}(x_1, \dots, x_n) &= \\
 &= \left(\frac{g}{\sqrt{N_c}} \right)^n \langle \text{Tr } F_{\mu_1\nu_1}(x_1) \Phi(x_1, x_2) F_{\mu_2\nu_2}(x_2) \dots F_{\mu_n\nu_n}(x_n) \Phi(x_n, x_1) \rangle
 \end{aligned} \tag{3}$$

Nonabelian Stokes Theorem and Cluster Expansion

$$\begin{aligned}
 \langle \text{Tr } W(C) \rangle &= \left\langle \text{Tr } \mathcal{P} \exp ig \int_S \Phi F_{\mu\nu}(z) \Phi d\sigma_{\mu\nu}(z) \right\rangle = \\
 &= \exp \sum_{n=2}^{\infty} (i)^n \Delta^{(n)}[S] = \exp(-V(R)T)
 \end{aligned} \tag{4}$$

The basic element of Nonperturbative QCD – the correlator $D_{\mu\nu\rho\sigma}^{(2)}$.

$$\Delta^{(2)}[S] = \frac{1}{2} \int_S d\sigma_{\mu\nu}(z_1) \int_S d\sigma_{\rho\sigma}(z_2) D_{\mu\nu\rho\sigma}^{(2)}(z_1, z_2) \quad (5)$$

$$\Delta^{(2)}[S] = \sigma S$$

Gauge-invariant Field Correlators

$$D_{\mu\nu\rho\sigma}^{(2)}(z) = \frac{g^2}{N_c} \langle \text{Tr} F_{\mu\nu}(x) \Phi F_{\rho\sigma}(y) \Phi \rangle \quad (6)$$

Two basic scalars: D and D_1 (Dosch+ Yu.S., ('88)).

$$D_{\mu\nu\rho\sigma}^{(2)}(z) = (\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})D(z) + \frac{1}{2} \left(\frac{\partial}{\partial z_\mu} (z_\rho\delta_{\nu\sigma} - z_\sigma\delta_{\nu\rho}) - \frac{\partial}{\partial z_\nu} (z_\rho\delta_{\mu\sigma} - z_\sigma\delta_{\mu\rho}) \right) D_1(z) \quad (7)$$

$D(x)$ is purely nonperturbative (pert. cancel-Shevchenko+Yu.S.'98).

Important: Dominance of Gaussian correlator $D^{(2)}(z) \rightarrow$ the QCD vacuum is almost ($> 95\%$) Gaussian (Bali '99, Shevchenko and Yu.S.'00). Check: Casimir scaling $-\Delta^{(2)} \sim C_2$, hence all $Q\bar{Q}$ potentials in different representations (j) are proportional to $C_2(j)$. Odd n correlations vanish on flat surfaces).

$$\Delta^{(2)}[S] \gg \sum_{n=3}^{\infty} \Delta^{(n)}[S] \quad (8)$$

If (connected) average $D^{(n)}(x_1 - x_2, \dots) \sim \exp(-\frac{|x_i - x_j|}{\lambda})$ for large $|x_i - x_j|$, then

$$\frac{\Delta^{(n+2)}[S]}{\Delta^{(n)}[S]} \approx \lambda^4 \langle F^2 \rangle \approx \sigma \lambda^2$$

It will be shown, that $\lambda \sim 0.1\text{fm}$, and expansion parameter is $\sigma \lambda^2 \sim 0.05$. Therefore all $\Delta^{(n)}$ with $n > 2$ contribute few percent.

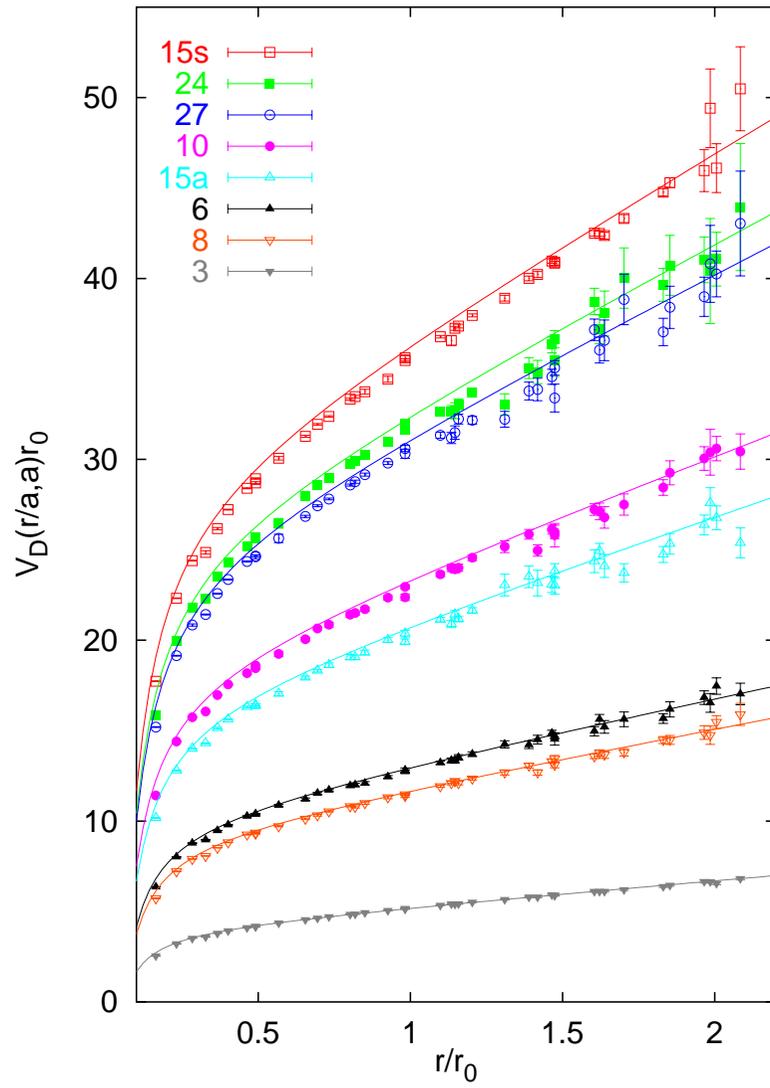


Figure 1:

From lattice and analytic data

$$D(x) \sim \exp(-|x|/\lambda),$$

Important feature of QCD vacuum! Vacuum correlator length λ
Campostrini, Di Giacomo, Olejnik ('86).

Di Giacomo et al. $\lambda \approx 0.2 \div 0.3$ fm

Bali, Brambilla, Vairo $\lambda \lesssim 0.2$ fm

Dosch et al. $\lambda \lesssim 0.2$ fm

Yu.S. $\lambda \approx 0.15$ fm.

Recently $D(x), D_1(x)$ were computed on lattice (Koma and Koma) in evaluating spin-dependent potentials. Results are compatible with $\lambda \lesssim 0.1$ fm.

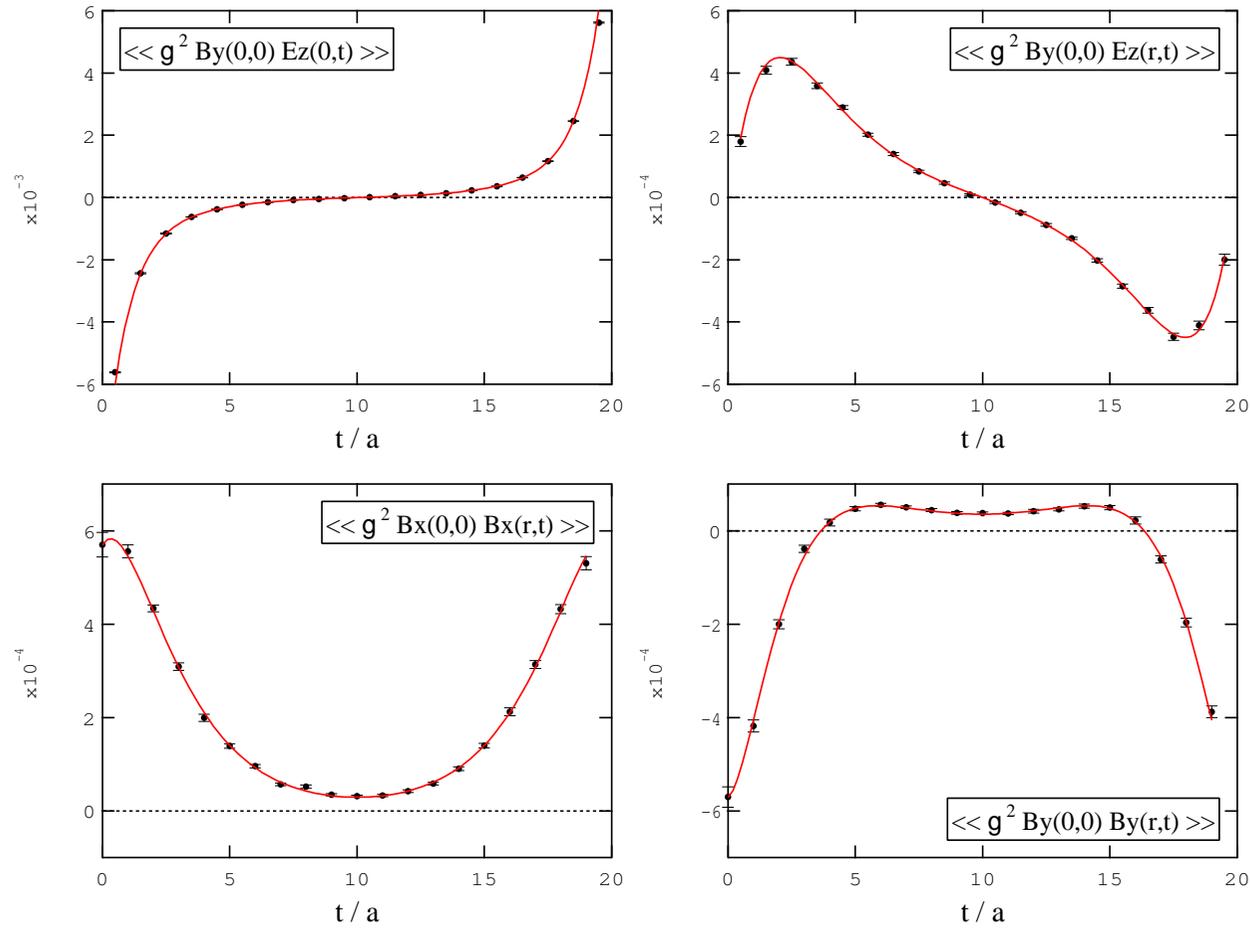


Figure 2: Field strength correlators at $\beta = 6.0$ on the 20^4 lattice for $r/a = 5$ as a function of t/a . The solid lines are the fit curves corresponding to Eqs. (??)-(??).

Static potentials from rectangular WW loop ($R \times T$),

$$\langle \text{tr} W(C) \rangle = \exp(-TV(R))$$

$$V(R) = V_D(R) + V_1(R)$$

$$V_D(R) = 2 \int_0^R (R - \rho) d\rho \int_0^\infty d\nu D(\sqrt{\rho^2 + \nu^2}), \quad (9)$$

$$V_1(R) = \int_0^R \rho d\rho \int_0^\infty d\nu D_1(\sqrt{\rho^2 + \nu^2}). \quad (10)$$

D ensures confinement

$$V_D(R) = \sigma R + \mathcal{O}(R^0) \quad ; \quad \sigma = \frac{1}{2} \int d^2 z D(z), R \rightarrow \infty \quad (11)$$

$$V_D(R) = cR^2 + \mathcal{O}(R^4), \quad R \lesssim \lambda \quad (12)$$

D_1 contains all (but not confinement), $V_1(R) = V_1^{(pert)} + V_1^{(nonpert)}$

$$V_1^{(nonpert)}(R \rightarrow \infty) = const \sim 0.5 GeV \quad (13)$$

V_1 supports bound states $Q\bar{Q}$ in quark-gluon plasma (Yu.S.'91, '05)

$$V_1^{(pert)} = -\frac{4(\alpha_s + \mathcal{O}(\alpha_s^2))}{3R}. \quad (14)$$

	α_S	σ, GeV^2	T_g, fm	T'_g, fm
set 1	0.16	0.22	0.2	0.2
set 2	0.16	0.22	0.1	0.1
set 3	0.16	0.22	0.07	0.1
set 4	0.16	—	0	0
set 5	0.32	0.17	—	—

Table 1: The sets of the FCM parameters for the spin-dependent potentials taken from Ref. [?]. Eqs. (??) and (??) are used for the sets 1-4 and set 5, respectively.

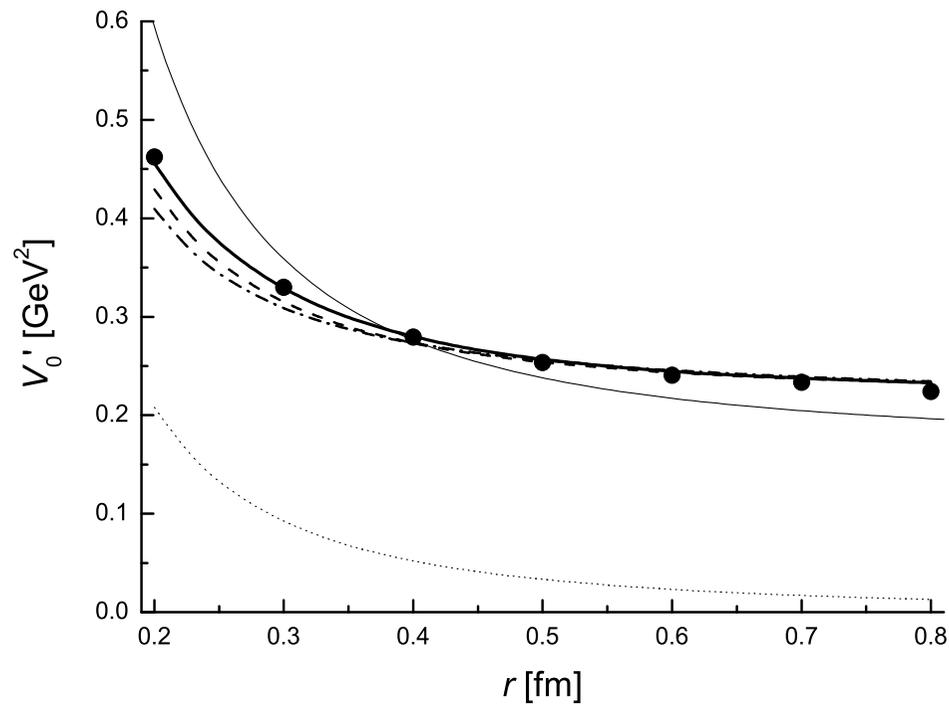


Figure 3: The profile of the $V_0'(r)$ for the set 1 (dash-dotted line), set 2 (dashed line), set 3 (fat solid line), set 4 (dotted line), and set 5 (thin solid line). Lattice data are given by dots.

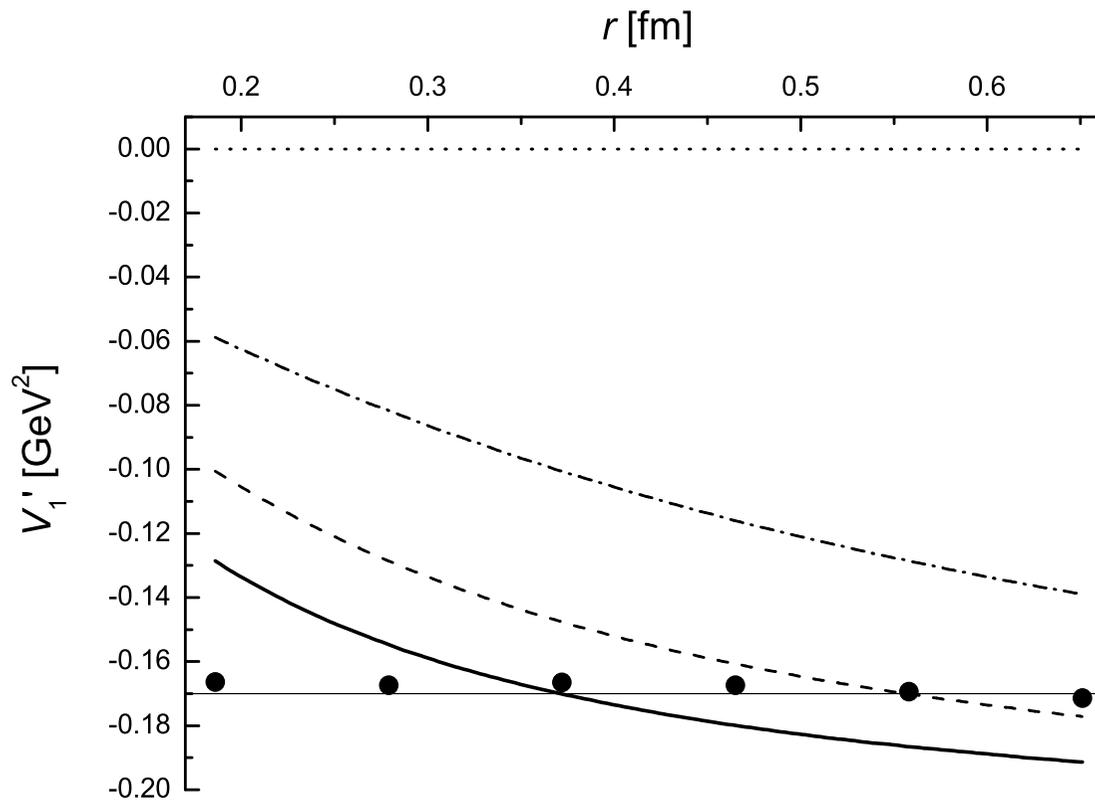


Figure 4: The same as in Fig. 3 but for $V_1'(r)$.

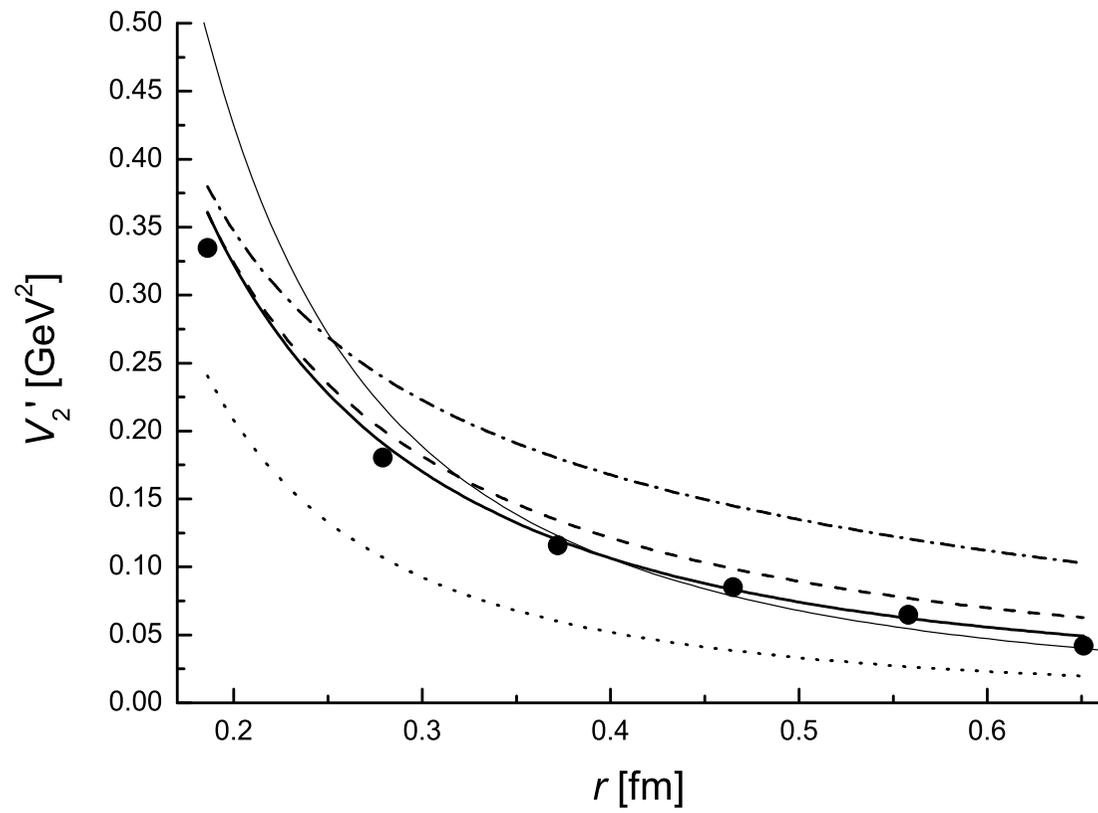


Figure 5: The same as in Fig. 3 but for $V_2'(r)$.

4. Expaining meson and glueball scales via string tension

Quark Green's function (Euclidean)

$$S_q(x, y) = (m + \hat{D})^{-1} = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K} \Phi_F(x, y)$$

where all dependence on field A_μ is in

$$\Phi_F(x, y) = (P \exp ig \int_y^x A_\mu dz_\mu) (P \exp g \int_0^s d\tau \sigma_{\mu\nu} F_{\mu\nu}) \equiv \Phi \Sigma$$

Φ charge factor, Σ spin factor.

Green's function for $q\bar{q}$ (mesons) or gg (glueballs)

$$G_M, G_{Gl} = \int \int \text{integral measure} \langle W_\sigma \rangle$$

Thus all dynamics is defined by the Wilson loop (with spin factor insertions).

Wilson loop with spin factors

$$\langle \text{tr} W_\sigma(C) \rangle = \exp(-\sigma \text{Area}) \text{ (spin factors)}$$

$$\text{Area} = \int_0^T dt \int_0^1 d\beta \sqrt{\dot{w}^2 w'^2 - (\dot{w} w')^2};$$

Note: no DOF on the area after vacuum averaging. Minimal area \rightarrow **minimal strings** without DOF except at the ends.



Figure 6:

Hamiltonian of minimal strings with quarks (gluons) at the ends

Last step: from path integral to Hamiltonian

$$G_{q\bar{q}}(x, y) = \langle x | \exp(-HT) | y \rangle \quad (15)$$

For equal current masses $m_q = m_{\bar{q}} = m$, $\mu_1 = \mu_2 = \mu$

$$H_0 = \frac{m^2 + \mathbf{p}^2}{\mu} + \mu + \frac{\hat{L}^2/r^2}{\mu + 2 \int_0^1 d\beta (\beta - \frac{1}{2})^2 \nu(\beta)} + \frac{\sigma^2 r^2}{2} \int_0^1 \frac{d\beta}{\nu(\beta)} + \int_0^1 \frac{\nu(\beta)}{2} d\beta. \quad (16)$$

$$\frac{\partial H_0}{\partial \mu_i} \Big|_{\mu_i = \mu_i^{(0)}} = 0, \quad \frac{\partial H_0}{\partial \nu} \Big|_{\nu = \nu^{(0)}} = 0. \quad (17)$$

$\mu_i^{(0)}$ play role of constituent mass of particle i , $\mu_i^{(0)} = \langle \sqrt{m_i^2 + \mathbf{p}^2} \rangle$

$$H_0(L=0) = \sum_{i=1}^2 \sqrt{m_i^2 + \mathbf{p}^2} + \sigma r. \quad (18)$$

For large L , $L \rightarrow \infty$ one obtains a free bosonic string.

$$H_0^2 \approx 2\pi\sigma \sqrt{L(L+1)}, \quad \nu^{(0)}(\beta) = \sqrt{\frac{8\sigma L}{\pi}} \frac{1}{\sqrt{1 - 4(\beta - \frac{1}{2})^2}}. \quad (19)$$

Constituent masses $\mu_i^{(0)}$ are calculated through σ and m_i .

For quarks, $m = 0$ $\mu_q = c_n \sqrt{\sigma} = 0.34$ GeV(ground state).

For gluons $\mu_g = \sqrt{C_2} \mu_q = \frac{3}{2} \mu_q = 0.5$ GeV. (Note: This mass is not connected with IR freezing of α_s .)

Total Hamiltonian

$$H = H_0 + H_{self} + H_{spin} + H_{Coul} + H_{rad} + H_{mix}. \quad (20)$$

For H_0 only, $m = 0$

$$M_0^2 \approx 8\sigma L + 4\pi\sigma \left(n + \frac{3}{4} \right), \quad n = 0, 1, 2, \dots$$

The input is minimal:

1. Quark current masses m_1, m_2 (pole masses if H_{pert} is used).
2. String tension σ .
3. Background strong coupling $\alpha_B(r)$.

In momentum space in one loop appr.

$$\alpha_B^{(1)}(Q) = \frac{4\pi}{\beta_0} \frac{1}{\ln \frac{(M_0^2 + Q^2)}{\Lambda_{QCD}^2}}$$

To be derived later.

Resulting spectra of light mesons are shown.

Orbital excitations (Regge trajectories) *vs* experiment (Badalian, Bakker).

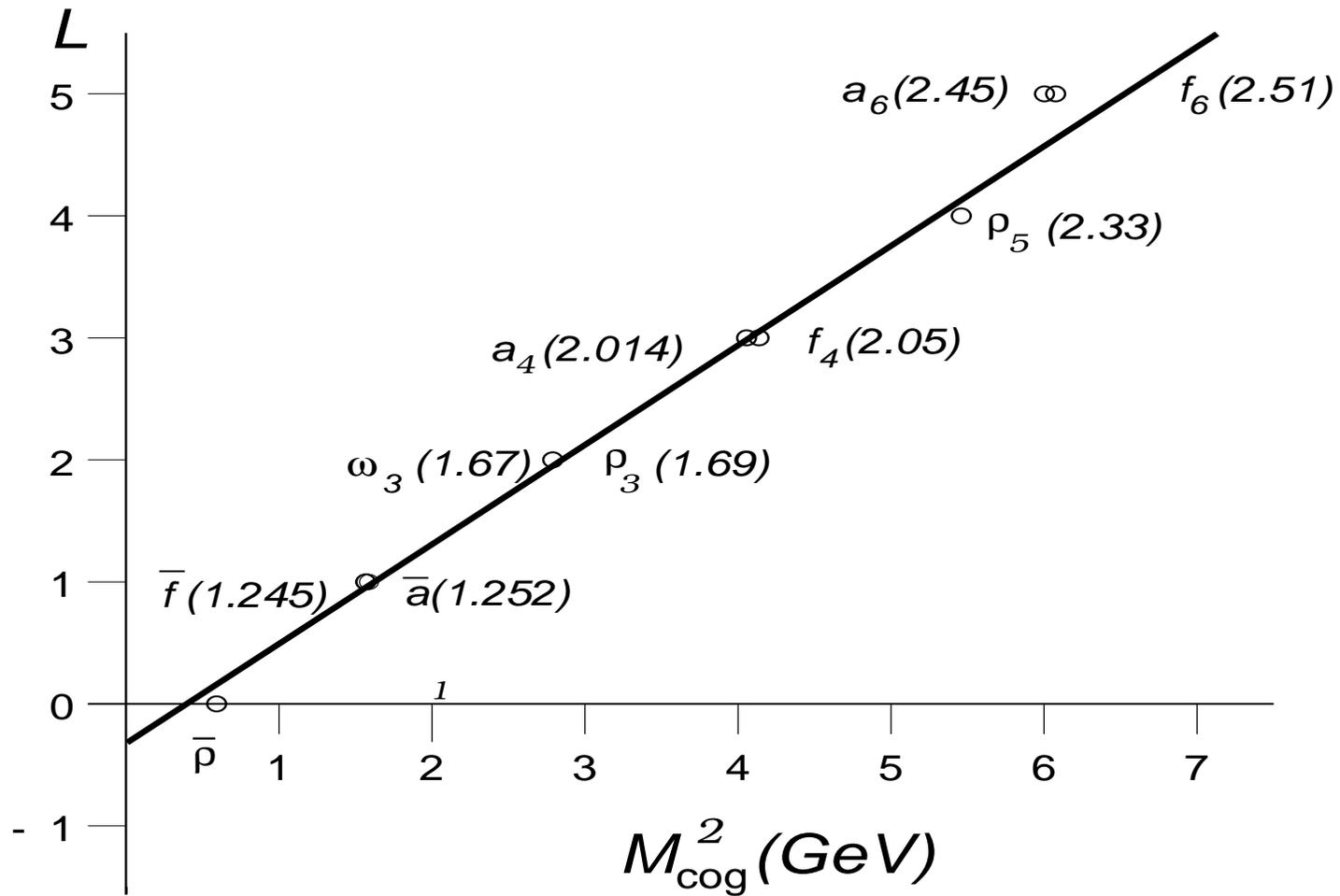


Figure 7: The Regge L -trajectory for the light mesons.

Table 2

Comparison of calculated glueball masses (in GeV) with lattice data
 ($\sigma_f = 0.18 \text{ GeV}^2$, $\alpha_s = 0.3$ ($\alpha_s = 0.2$ in parentheses))

J^{PC}	M_{theory} this work	M_{lat}		
		[22]	[23]	[24]
0^{++}	(1.61) 1.41	1.53 ± 0.10	1.53 ± 0.04	1.52 ± 0.13
2^{++}	(2.21) 2.30	2.13 ± 0.12	2.20 ± 0.07	2.12 ± 0.15
0^{++*}	(2.72) 2.41	2.38 ± 0.25	2.79 ± 0.09	
2^{++*}	(3.13) 3.32	2.93 ± 0.14	2.85 ± 0.28	
0^{-+}	2.28	2.30 ± 0.15	2.11 ± 0.24	2.27 ± 0.15
0^{-+*}	3.35	3.24 ± 0.2		
2^{-+}	2.70	2.76 ± 0.16	3.0 ± 0.28	2.70 ± 0.19
2^{-+*}	3.73	3.46 ± 0.21		

Glueballs: Kaidalov+Yu.S.('00,'05).

5. Explaining T_c via gluonic condensate

$$SVZ \quad \varepsilon_{vac} = 1/4\theta_{\mu\mu} = \frac{\beta(\alpha_s)}{16\alpha_s} \langle (F_{\mu\nu}^a)^2 \rangle \cong -\frac{(11 - \frac{2}{3}n_f)}{32} G_2^{(n_f)} \quad (21)$$

$$(NSVZ) \quad G_2^{(n_f=2)} \approx \left(\frac{1}{3} \div \frac{1}{4} \right) G_2^{(n_f=0)} \quad (22)$$

$$G_2(0.02 \pm 0.005) \text{ GeV}^4 \text{ S.Narison}$$

$$G_2(0.01 \pm 0.002) \text{ GeV}^4 \text{ Andreev, Zakharov} \quad (23)$$

$$P_1(T) = |\varepsilon_{vac}| + \frac{\pi^2}{30} T^4 + T \sum_k \frac{(2m_k T)^{3/2}}{8\pi^{3/2}} e^{-m_k/T} \equiv |\varepsilon_{vac}| + T^4 \chi_1(T). \quad (24)$$

In the deconfined phase one can assume (later confirmed by lattice) (Yu.S. JETP Lett.'92), that

$$D^E(x) = 0 = \sigma_E; \quad D^H(x), D_1^H, D_1^E \neq 0. \quad (25)$$

$$P_2(T) = |\varepsilon_{vac}^{dec}| + T^4(p_{gl} + p_q) \quad (26)$$

Critical line $T_c(\mu)$

$$P_I = |\varepsilon_{vac}| + \chi_1(T) \rightarrow \frac{11}{32}G_2$$

$$P_{II} = \frac{11}{32}G_2^{dec} + (p_{gl} + p_q)T^2;$$

$$P_I(T_c) = P_{II}(T_c)$$

$$T_c(\mu) = \left(\frac{\frac{11}{32}\Delta G_2}{p_{gl} + p_q} \right)^{1/4},$$

within 10% $\Delta G_2 \approx \frac{1}{2}G_2$.

$\frac{\Delta G_2}{0.01 \text{ GeV}^4}$		0.191	0.341	0.57	1
$T_c(\text{ GeV})$	$n_f = 0$	0.246	0.273	0.298	0.328
$T_c(\text{ GeV})$	$n_f = 2$	0.168	0.19	0.21	0.236
$T_c(\text{ GeV})$	$n_f = 3$	0.154	0.172	0.191	0.214
$\mu_c(\text{ GeV})$	$n_f = 2$	0.576	0.626	0.68	0.742
$\mu_c(\text{ GeV})$	$n_f = 3$	0.539	0.581	0.629	0.686

6. Field correlators via gluelumps

In this section we calculate D, D_1 analytically via gluelump Green's functions. Physical idea: Nonabelian mean field approach yields confining background field B_μ , with a_μ^a -quanta of gluonic field – propagating in vacuum with a fixed color index a , while $B_\mu \sim A_\mu^b, b \neq a$.

$$A_\mu = B_\mu + a_\mu$$

When averaging over B_μ one obtains confining string for a_μ^a .
As a result (Yu.S. '05, Antonov '05) to the lowest order in α_s

$$D_1(x) = -\frac{2g^2}{N_c^2} \frac{dG^{(1)}(x)}{dx^2}$$

$$D(x) = \frac{g^4(N_c^2 - 1)}{2} G^{(2)}(x)$$

and $G^{(1)}(x)$ is the one-gluon gluelump Green's function, $G^{(2)}$ - the same for two gluons.

For one-gluon-gluelump Green's function $G^{(1)}$ one can write.

$$G_{\mu\nu}^{ab}(x, y) = \left\{ \int_0^\infty ds (Dz)_{xy} e^{-K} P_a \exp\left(ig \int_y^x \hat{A}_\mu dz_\mu\right) P_\Sigma(x, y, s) \right\}_{\mu\nu}^{ab}, \quad (27)$$

where

$$P_\Sigma(x, y, s) = P_F \exp\left(2ig \int_0^s \hat{F}_{\lambda\sigma}(z(\tau)) d\tau\right).$$

For two-gluon-gluelump $G^{(2)} \sim \langle tr(G_{\mu\nu}^{ab} G_{\mu\nu}^{ba}) \rangle$.

As was shown in [V.Shevchenko, Yu.S., PLB 437 (1998) 146] perturbative terms cancel in $D(x)$ and not in $D_1(x)$.

$$D(z) = D^{np}(z) ; \quad D_1(z) = D_1^p(z) + D_1^{np}(z) \quad (28)$$

and general asymptotics for $D_1(z)$ is

$$D_1(z) = \frac{c}{z^4} + \frac{a_2}{z^2} + O(z^0). \quad (29)$$

Finally, for G_2

$$G_2 = \frac{6N_c}{\pi^2} (D^{np}(0) + D_1^{np}(0)) \quad (30)$$

For $G^{(1)}$ and $G^{(2)}$ one has path integrals

$$G_{\mu\nu}^{(1gl)}(x, y) = \text{Tr}_a \int_0^\infty ds (Dz)_{xy} \exp(-K) \langle W_{\mu\nu}^F(C_{xy}) \rangle, \quad (31)$$

$$G^{(2gl)}(z) = \int_0^\infty ds_1 \int_0^\infty ds_2 (Dz_1)_{0x} (Dz_2)_{0x} \text{Tr} W_\Sigma(C_1, C_2). \quad (32)$$

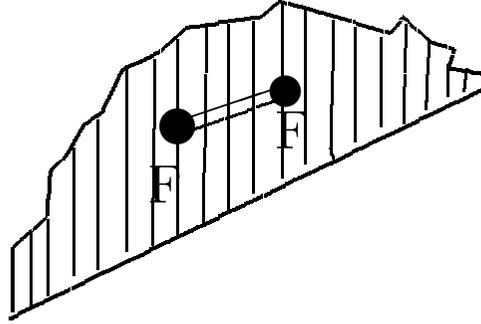


Fig. 8 One-gluon gluelump for $D_1(x)$

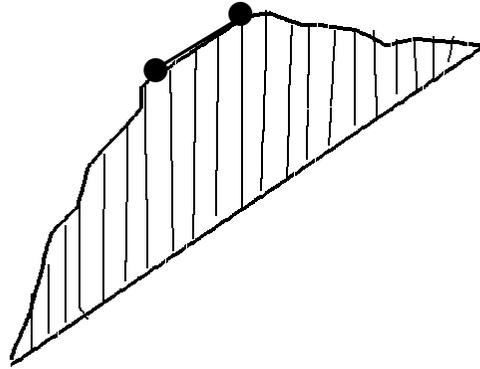


Fig.9

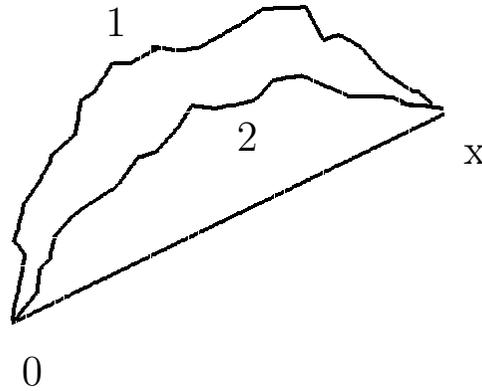


Fig. 10 Two-gluon gluelump for $D(x)$

Using Hamiltonian formalism for gluelumps, one has asymptotics,

$$D(x) \sim \exp(-|x|/\lambda), \quad D_1(x) \sim \exp(-|x|/\lambda_1)$$

$$\lambda = \frac{1}{M_0^{(2)}}, \quad \lambda_1 = \frac{1}{M_0^{(1)}},$$

where $M_0^{(2)}$ –lowest 2g gluelump mass,

$M_0^{(1)}$ – lowest 1g gluelump mass.

Specifically

$$D_1(z) = \frac{2C_2(f)\alpha_s M_0^{(1)} \sigma_{adj}}{|z|} e^{-M_0^{(1)}|z|}, \quad |z|M_0^{(1)} \gg 1. \quad (33)$$

where $M_0^{(1)} = (1.2 \div 1.4)$ GeV for $\sigma_f = 0.18$ GeV² [?, ?].

$$D(z) = \frac{g^4(N_c^2 - 1)}{2} 0.1\sigma_f^2 e^{-M_0^{(2)}|z|}, \quad M_0^{(2)}|z| \gg 1 \quad (34)$$

where $M_0 = (2.5 \div 2.6)$ GeV.

Check of selfconsistency at small and large distances for $D(x), D_1(x)$

Since np parts of $D(x), D_1(x)$ are calculated in $G^{(1)}, G^{(2)}$ through correlator $\langle F(x)F(y) \rangle$, i.e. via $D(x), D_1(x)$, one should check selfconsistency.

At small distances: there are corrections to $D_1(x), D(x)$ from diagrams

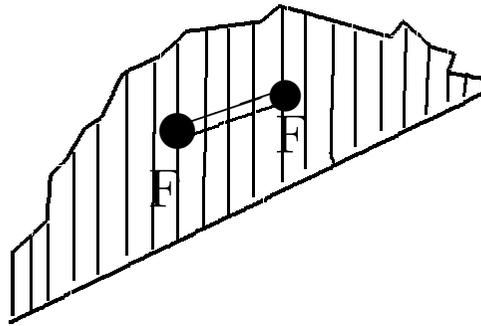


Fig. 8

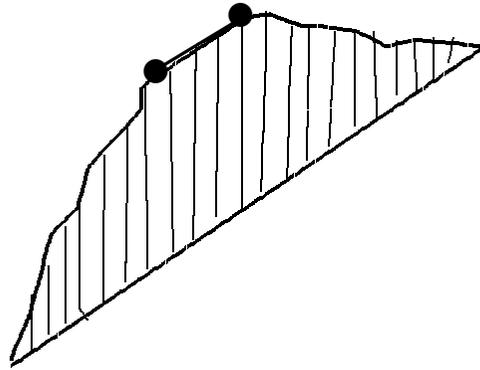


Fig. 9

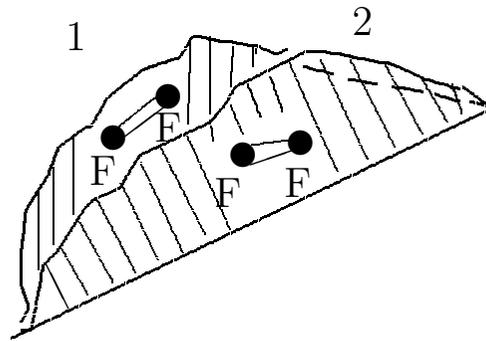


Fig. 11

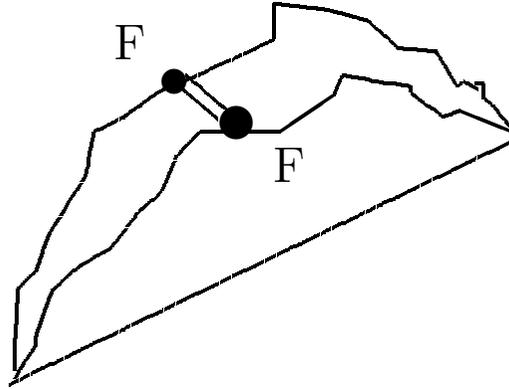


Fig. 12

which yields for $D_1(x)$

$$D_1(z) = \frac{4C_2(f)\alpha_s}{\pi} \frac{1}{z^4} + \frac{g^2}{12} G_2. \quad (35)$$

It is remarkable that the sign of the np correction is positive.

For $D(z)$ Fig.11, Fig.12 yield.

$$D(z) \approx -4N_c \alpha_s^2(\mu(z)) G_2 + N_c^2 \frac{\alpha_s^2(\mu(z))}{2\pi^2} D(\lambda_0) \log^2 \left(\frac{\lambda_0 \sqrt{e}}{z} \right) \quad (36)$$

$$D(0) = \frac{N_c^2}{2\pi^2} D(\lambda_0) \left(\frac{2\pi}{\beta_0} \right)^2 \quad (37)$$

Since from (37) one can infer, that $D(0) \approx 0.15D(\lambda_0)$ for $N_c = 3$, where $\lambda_0 \gtrsim \lambda^E$. So $D(z)$ is an increasing function of z at small z , $z \lesssim \lambda_0$, and for $z \gg \lambda$ one observes exponential falloff. The qualitative picture illustrating this solution for $D(z)$ is shown in Fig. 12.

This pattern may solve qualitatively the contradiction between the values of $D(0)$ estimated from the string tension $D_\sigma(0) \simeq \frac{\sigma}{\pi\lambda^2} \approx 0.35 \text{ GeV}^4$ and the value obtained in naive way from the gluon condensate $D_{G_2}(0) = \frac{\pi^2}{18} G_2 \approx (0.007 \div 0.012) \text{ GeV}^4$. One can see that $D_\sigma(0) \approx (30 \div 54) D_{G_2}(0)$. This seems to be a reasonable explanation of the mismatch discussed in the introduction.

Now we shall show that in our approach of gluelumps as field correlators one can establish connection between perturbative scale Λ_{QCD} and nonperturbative scale, say, σ . Indeed, following [?] we shall write the large distance behaviour (??) of $D(z)$ as

$$D(z) \equiv D_\sigma(0) \exp(-M_0^{(2)}|z|), \quad D_\sigma(0) = g^4 \frac{(N_c^2 - 1)}{2} 0.1\sigma_f^2. \quad (38)$$

On the other hand, from (33) one has integrating $D(z)$ (and taking into account that at small z , $D(z)$ is smaller than (38), see Fig.).

$$\sigma_f \leq \frac{\pi D_\sigma(0)}{(M_0^{(2)})^2} = 0.17\pi^3 \alpha_s^2(\mu) \frac{(N_c^2 - 1)\sigma_f^2}{(M_0^{(2)})^2} \quad (39)$$

Here μ corresponds to the average momentum (inverse radius) of the two-gluon gluelump with mass $M_0^{(2)}$. The latter was computed in [?] in terms of σ_f $M_0^{(2)} \cong 5.6\sqrt{\sigma_f}$, and one has from (39)

$$\alpha_s^2(\mu) \geq 0.16, \quad \mu = \sqrt{\langle k_{gl}^2 \rangle} \approx 2\sqrt{\sigma_f}.$$

Or to the lowest order $\Lambda_{QCD} \geq 0.17\mu = 0.16 \text{ GeV}$.

This is in the correct ballpark, since realistic Λ_{QCD} in \overline{MS} scheme for $n_f = 2$ is around 0.25 GeV [?, ?], however for better accuracy one needs to take into account NLO terms and nonasymptotic behaviour of $D(z)$ at small z , which will increase estimate of Λ_{QCD} .

On general grounds, one may preview, that any connection of Λ_{QCD} with np scale will have the form: $\alpha_s(\mu_{np}) = C$, where μ_{np} is defined by np effects and scale, and C is a fixed number.

8. Conclusions

1. Field correlator Method provides the explicit dynamical theory for Large-Distance QCD. The confinement is due to nonperturbative correlators of colorelectric fields, and for a flat (minimal) surface the lowest Gaussian correlator $D^E(x)$ plays the dominant role. Cluster expansion in n -th order correlators behaves as $\sim (\sigma\lambda^2)^n = (0.05)^n$.
2. Correlation length λ and correlators are calculated selfconsistently via gluelumps, $\lambda_D^E \approx 0.1$ fm. Thus one has a theory defined by the only parameter say σ (in addition to current quark masses).
3. The leading pert. and np terms enter additively at small distances in field correlators and selfconsistency is maintained both at small and large distances. In particular, Λ_{QCD} is connected to σ .
4. Within our method one can explain quantitatively all the mass scales in QCD.