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Non-Gaussian Curvature Perturbations from Multi-brid Inflation

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Atsushi Naruko & MS, PTP 121:193(2009) arXiv:0807.0180 [astro-ph]

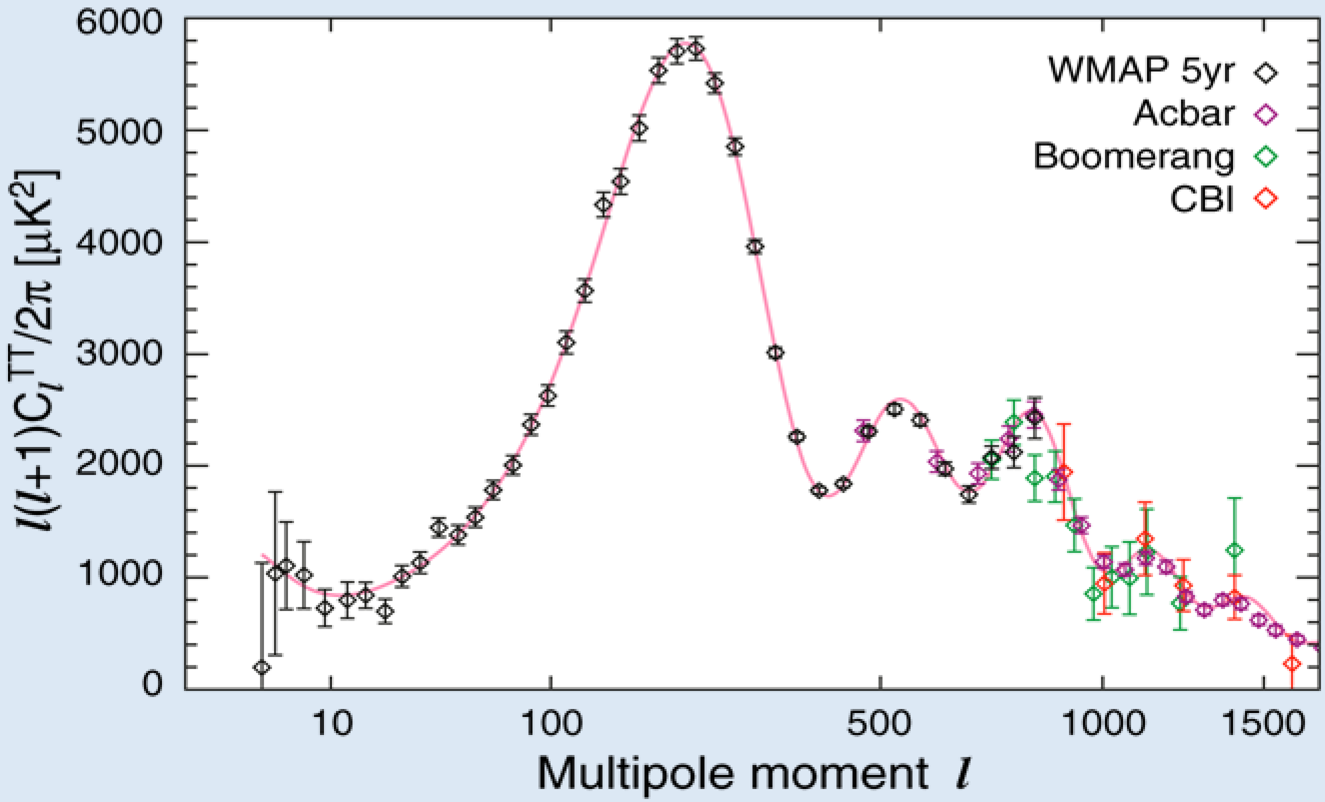
MS, PTP 120:159(2008) arXiv:0805.0974 [astro-ph]

MS, CQG 24:2433(2007) [astro-ph/0702182]

contents

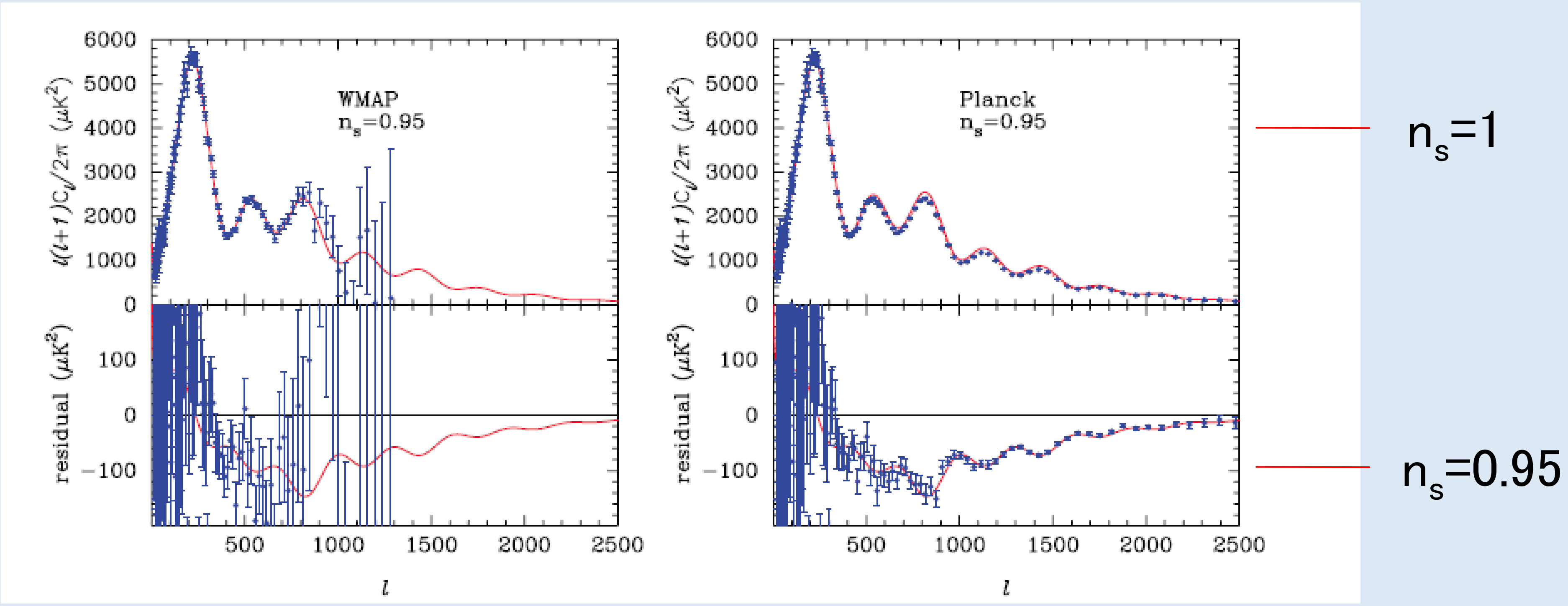
1. Introduction

- Prediction of the standard single-field slow-roll inflation:
  almost scale-invariant Gaussian random fluctuations



- perfectly consistent with CMB experiments

Scalar spectral index: $P_S(k) \propto k^{n_s}$



WMAP 4 years PLANCK 1 year

Planck determes spectral index to within 1 % accuracy

However, nature may not be so simple…

- Tensor (gravitational wave) perturbations?

  tensor perturbations induce B-mode polarization of CMB

  tensor-scalar ratio: **r<0.2 (95%CL)** WMAP+BAO+SN (‘08)

  $\Updownarrow$

  cf. chaotic inflation: r~0.15

- **Non-Gaussianity?** Komatsu & Spergel (‘01)

  (-)gravitational pot.: $\Phi = \Phi_{\mathrm{Gauss}} + f_{\mathrm{NL}} \Phi^2_{\mathrm{Gauss}} + g_{\mathrm{NL}} \Phi^3_{\mathrm{Gauss}} + \cdots$

  $-9 < f_{\mathrm{NL}} < 111$ (95%CL) WMAP (‘08)

  ($f_{\mathrm{NL}} = 51 \pm 30$ at 1σ)

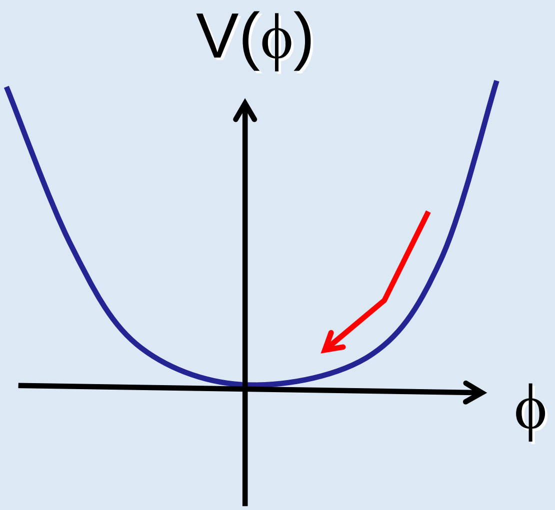
What does the presence of non-Gaussianity mean ?

2. Slow-roll inflation and δN formalism

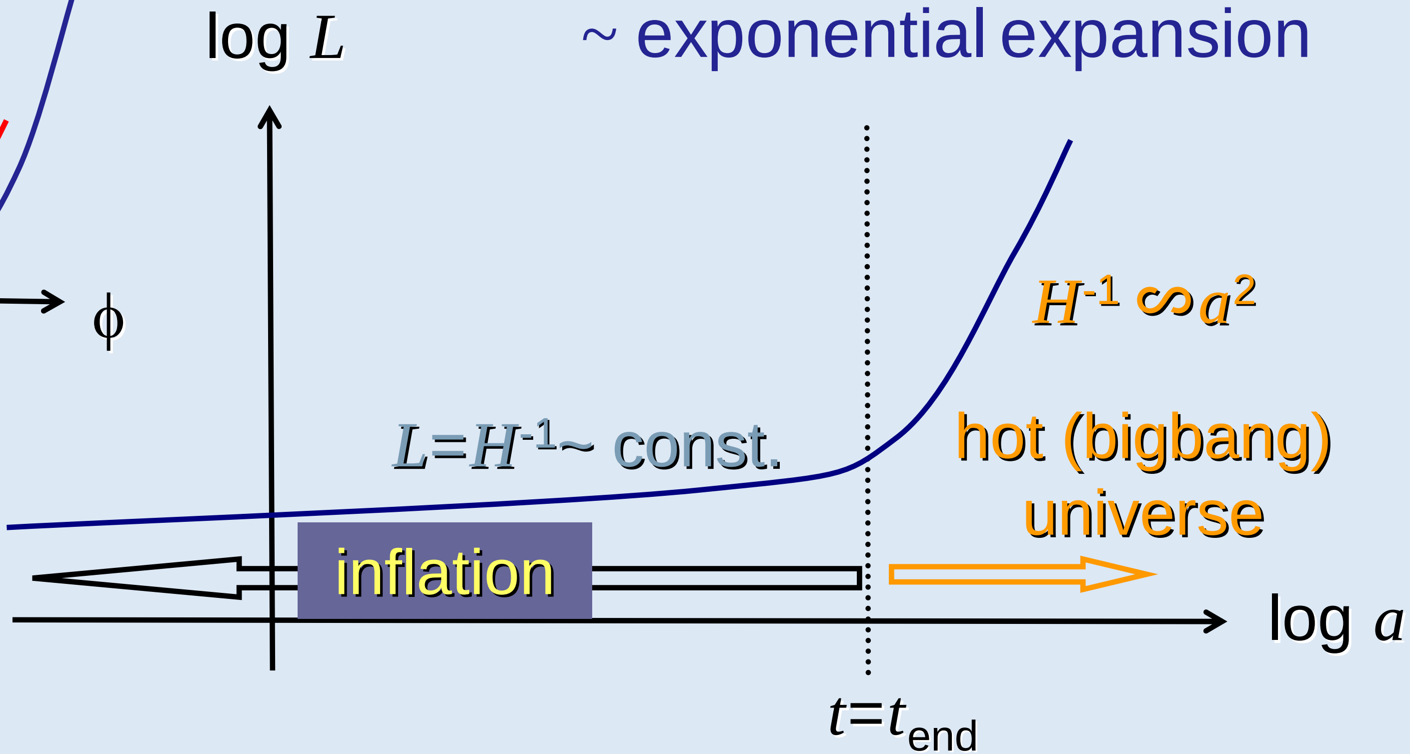
• single-field inflation, no other degree of freedom Linde '82, ...

$$H^2 = \frac{8\pi G}{3}\left[\cancel{\frac{1}{2}\dot{\phi}^2} + V(\phi)\right], \quad \cancel{\ddot{\phi}} + 3H\dot{\phi} + V'(\phi) = 0 \quad \Rightarrow \quad t = t(\phi)$$

(friction-dominated) over-damped system



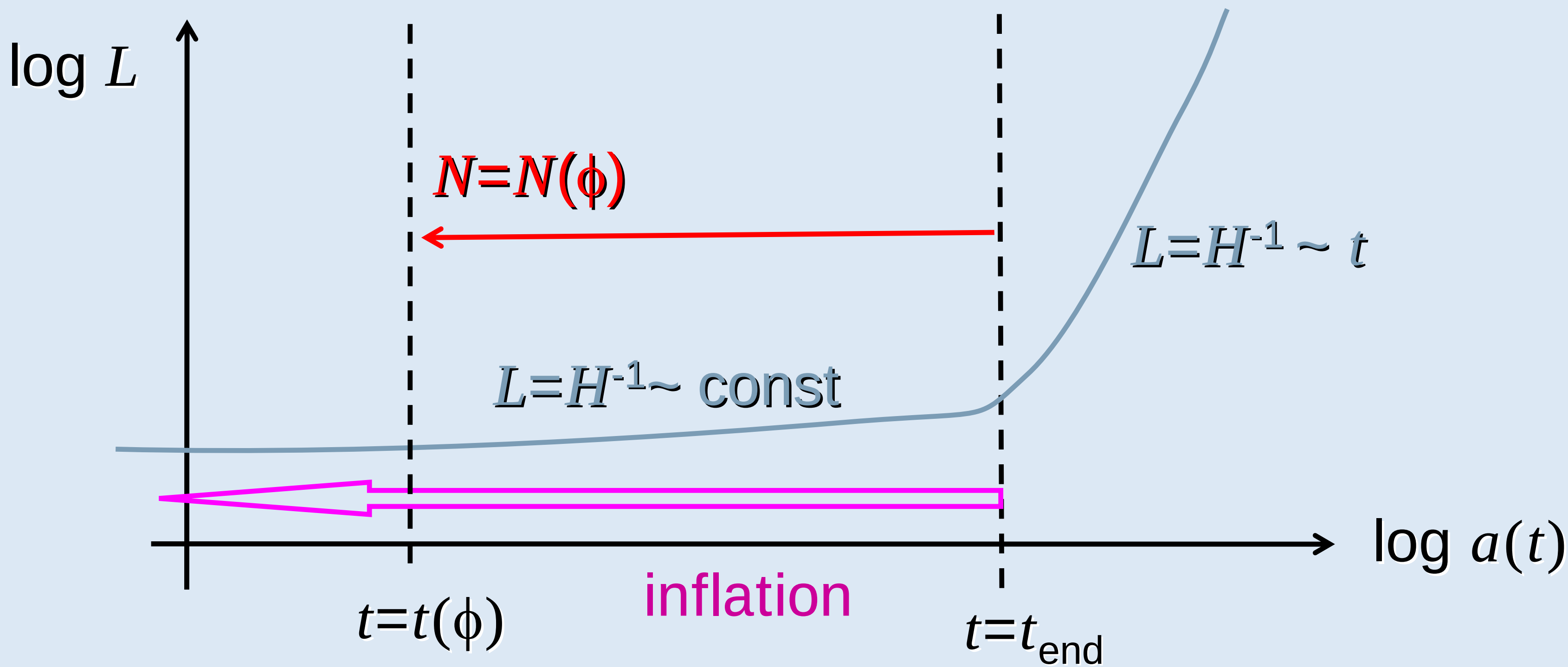
$H \approx$ const.
~ exponential expansion



Number of e-folds of inflation

- number of e-folds counted *backward in time* from the end of inflation ~ log(redshift)

$$\frac{a(t_{\rm end})}{a(t)} = \exp[N(t \to t_{\rm end})] \Rightarrow \quad N = N(\phi) = \int_{t(\phi)}^{t_{\rm end}} H dt = \ln\left[\frac{1+z(\phi)}{1+z(\phi_{\rm end})}\right]$$



Cosmological curvature perturbations

- standard single-field slow-roll case -

Inflaton fluctuations (vacuum fluctuations=Gaussian)

$$\left|\left\langle \phi \middle| \vec{k} \right\rangle\right|^2 = \left|\varphi_k\right|^2, \quad \varphi_k \sim \frac{1}{\sqrt{2w_k}} e^{-iw_k t} \; ; \quad w_k = \frac{k}{a} \gg H$$

Oscillation freezes out at $k/a < H$

(~ classical Gaussian fluctuations on superhorizon scales)

$$\varphi_k \sim \frac{H}{\sqrt{2k^3}} \; ; \quad \frac{k}{a} \ll H \quad \Rightarrow \quad \left\langle \delta\phi_k^2 \right\rangle = \left(\frac{H}{2\pi}\right)^2_{k/a \sim H}$$

Curvature perturbations

$$\mathcal{R}_c = -\frac{H}{\dot{\phi}} \delta\phi$$ ··· conserved on superhorizon scales

$$\left(\overset{(3)}{R} \sim -\frac{2}{3a^2} \overset{(3)}{\Delta} \mathcal{R}_c \right)$$

δN formalism

- spectrum of $\mathcal{R}_c$: $P_{\mathcal{R}}(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{k/a=H}$ ~ scale-invariant

- curvature perturbation = e-folding number perturbation (δN)

$$N(\phi) = \int_{t(\phi)}^{t_{end}} H dt = \int_{\phi}^{\phi_{end}} \frac{H}{\dot{\phi}} d\phi$$

$$\Rightarrow \delta N(\phi) = \left[\frac{\partial N}{\partial \phi}\delta\phi\right]_{k/a=H} = \left[-\frac{H}{\dot{\phi}}\delta\phi\right]_{k/a=H} = \mathcal{R}_c$$

Starobinsky ('85)

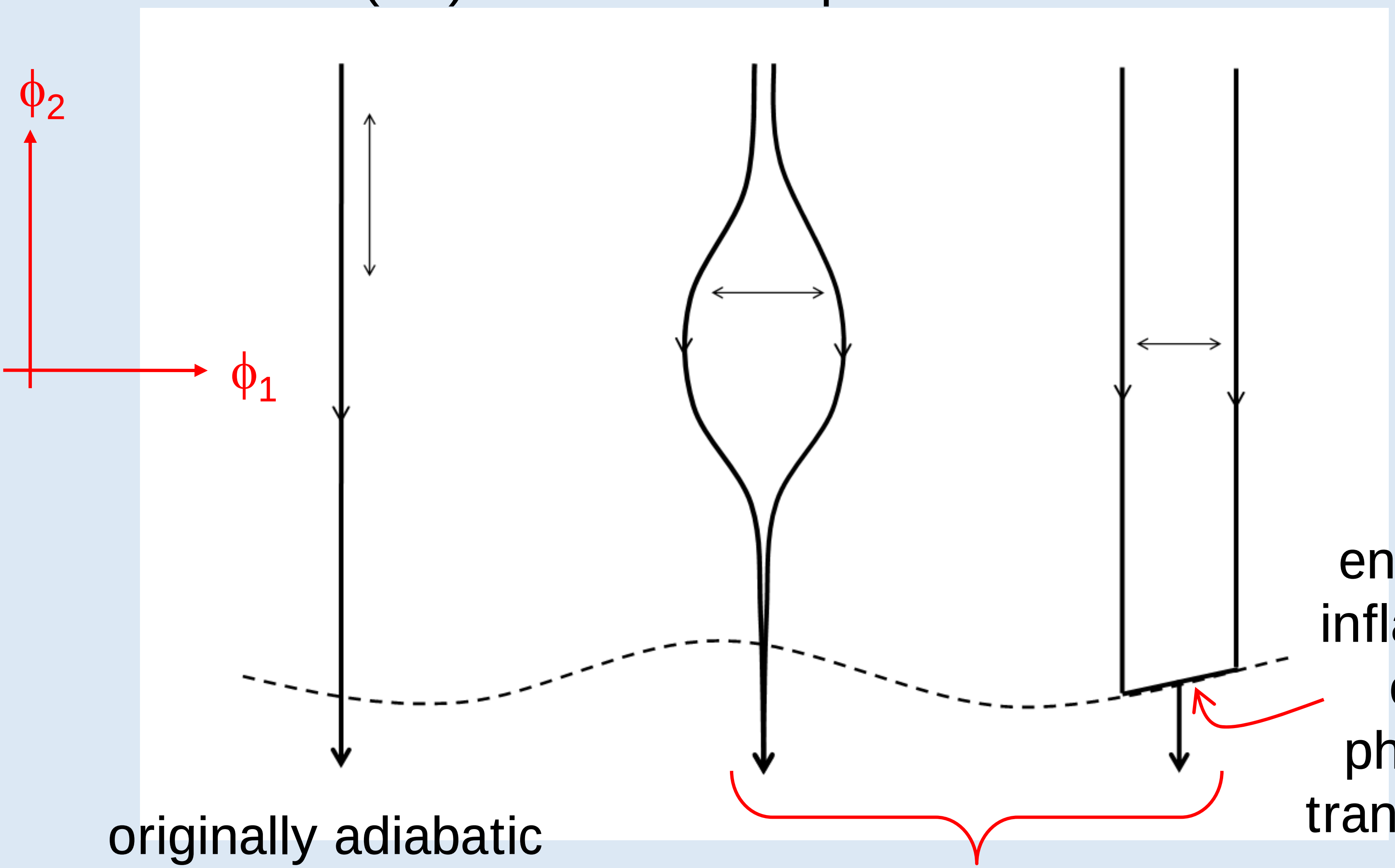
$$P_{\mathcal{R}}(k) = \left(\frac{H^2}{2\pi\dot{\phi}}\right)^2_{k/a=H} = \left(\frac{\partial N}{\partial \phi}\right)^2 \langle \delta\phi_k^2 \rangle \quad \langle \delta\phi_k^2 \rangle = \left(\frac{H}{2\pi}\right)^2_{k/a=H}$$

general slow-roll inflation $\delta N = \sum_A \frac{\partial N}{\partial \phi^A}\delta\phi^A$ MS & Stewart ('96)

nonlinear generalization Lyth, Malik & MS ('04)

Three types of δN

($\longleftrightarrow$) indicates field perturbations



3. Non-Gaussianity from inflation

- three origins:

Self-interaction of inflaton field

quantum physics, subhorizon scale during inflation

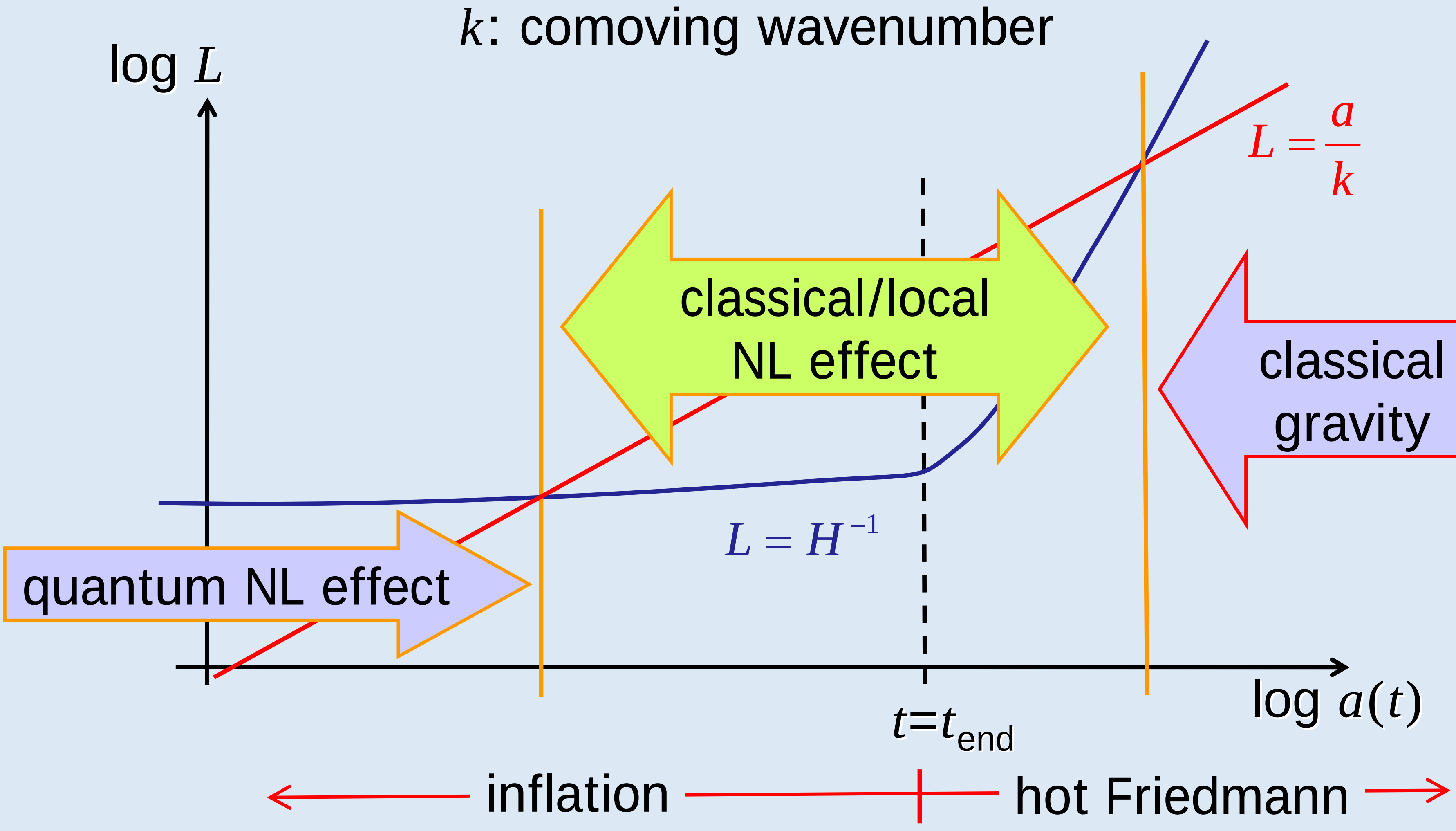
Multi-component field

classical physics, nonlinear coupling to gravity
superhorizon scale during and after inflation

Nonlinearity of gravity

classical general relativistic effect,
subhorizon scale after inflation

Origin of non-Gaussianity and cosmic scales



Origin of NG1: self-interaction of inflaton field

Non-Gaussianity from subhorizon scales

(QFT effect)

- interaction is very small for potential-type self-couplings

Maldacena ('03)

ex. chaotic inflation

$$V = \frac{1}{2} m^2 \phi^2 \quad \cdots \text{ free field!}$$

(gravitational interaction is Planck-suppressed)

$$V = \lambda \phi^4 \quad \to \lambda \sim 10^{-15}$$

- non-canonical kinetic term
  → strong self-interaction → large non-Gaussianity

example : DBI inflation

Silverstein & Tong (2004)

kinetic term: $K \sim f^{-1}(\phi)\boxed{\sqrt{1 - f(\phi) g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi}} \equiv f^{-1} \gamma^{-1}$

~ (Lorenz factor)$^{-1}$

perturbation expansion:

$$K = K_0 + \delta_1 K + \delta_2 K + \delta_3 K + \cdots$$

$$\begin{array}{cccc} K_0 & \delta_1 K & \delta_2 K & \delta_3 K \\ \propto & = & \propto & \propto \\ \gamma^{-1} & 0 & \gamma^{3} & \gamma^{5} \end{array}$$

$$\Rightarrow \quad \delta\phi \sim \delta\phi_0 + \gamma^2 \delta\phi_0^2 + \cdots$$

large non-Gaussianity for large γ

bispectrum (3-pt function) of curvature perturbation from DBI inflation

Alishahiha et al. ('04)

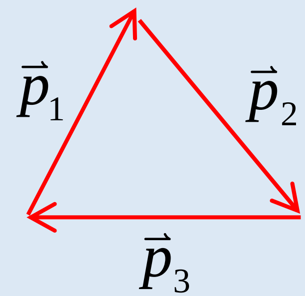
$$\left\langle \mathcal{R}_c(p_1)\mathcal{R}_c(p_2)\mathcal{R}_c(p_3)\right\rangle$$

$$\sim \delta\Big(\sum_j p_j\Big) f_{NL}(p_1,p_2,p_3)\Big(\mathcal{P}_{\mathcal{R}}(p_1)\mathcal{P}_{\mathcal{R}}(p_2)+\text{cyclic}\Big)$$

$$\Downarrow \qquad f_{NL} \sim \gamma^2$$

f_{NL} for equilateral configuration is large

$$|\vec{p}_1| \sim |\vec{p}_2| \sim |\vec{p}_3|$$



$$(\vec{p}_1+\vec{p}_2+\vec{p}_3=0)$$

$$f_{NL} \;\Rightarrow\; f_{NL}^{\text{equil}}$$

WMAP 5yr constraint: $-151 < f_{NL}^{\text{equil}} < 253$ (95% CL)

Origin of NG2 : nonlinearity of gravity

ex. post-Newton metric in harmonic coordinates

$$ds^2 = -\left(1 + 2\Psi - 2\Psi^2 + \cdots\right)dt^2 + \left(1 + 2\Psi + 2\Psi^2 + \cdots\right)dr^2 + \cdots$$

Newton potential

NL terms

important after the perturbation scale re-enters the Hubble horizon

Effect on CMB bispectrum seems small (but non-negligible?)

$$f_{NL} \sim O(1 \sim 10)$$

Bartolo et al. (2007)

Origin of NG3: superhorizon scales

Even if $\delta\phi$ is Gaussian, $\delta T^{\mu\nu}$ may be non-Gaussian due to its nonlinear dependence on $\delta\phi$

This effect is small for a single-field slow-roll model (⇔ linear approximation is extremely good)

Salopek & Bond ('90), ...

But it may be large for multi-field models

Non-Gaussianity in this case is local:

$$f_{NL}(p_1, p_2, p_3) \to f_{NL}^{\text{local}} = \text{const.}$$

Two typical models for f_{NL}^{local}

- curvaton model

Linde & Mukhanov (796),
Lyth & Wands ('01), Moroi & Takahashi ('01), …

- multi-brid inflation model

MS ('08), Naruko & MS ('08)

both may give large f_{NL}^{local}

but in the case of curvaton scenario
tensor-scalar ratio r will be very small.

Here we consider a multi-field model
and investigate the non-Gaussianity
generated at the end of inflation,

that is,

Multi-brid inflation

(multi-brid = multi-component hybrid)

4. Multi-brid inflation

- hybrid inflation:
  inflation ends by a sudden destabilization of vacuum

- multi-brid inflation = multi-field hybrid inflation:

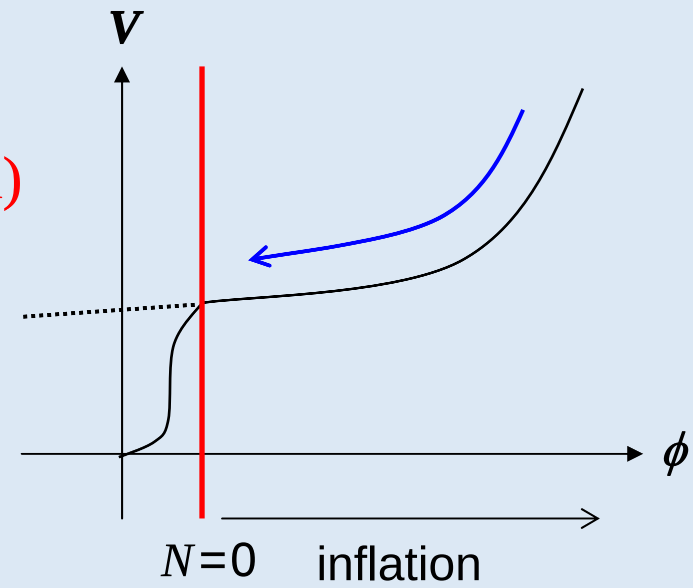
$$L_\phi = -\frac{1}{2}\sum_A g^{\mu\nu}\partial_\mu\phi_A\partial_\nu\phi_A - V(\phi)$$

Slow-roll eom

$$\dot{\phi}_A = -\frac{1}{3H}\frac{\partial V}{\partial\phi_A}\,, \qquad 3H^2 = V \qquad (8\pi G = M_{\text{Planck}}^{-2} = 1)$$

N as a time: $dN = -Hdt$

$$\Rightarrow \quad \frac{d\phi_A}{dN} = \frac{1}{3V}\frac{\partial V}{\partial\phi_A} = \frac{1}{3}\frac{\partial \log V}{\partial\phi_A}$$



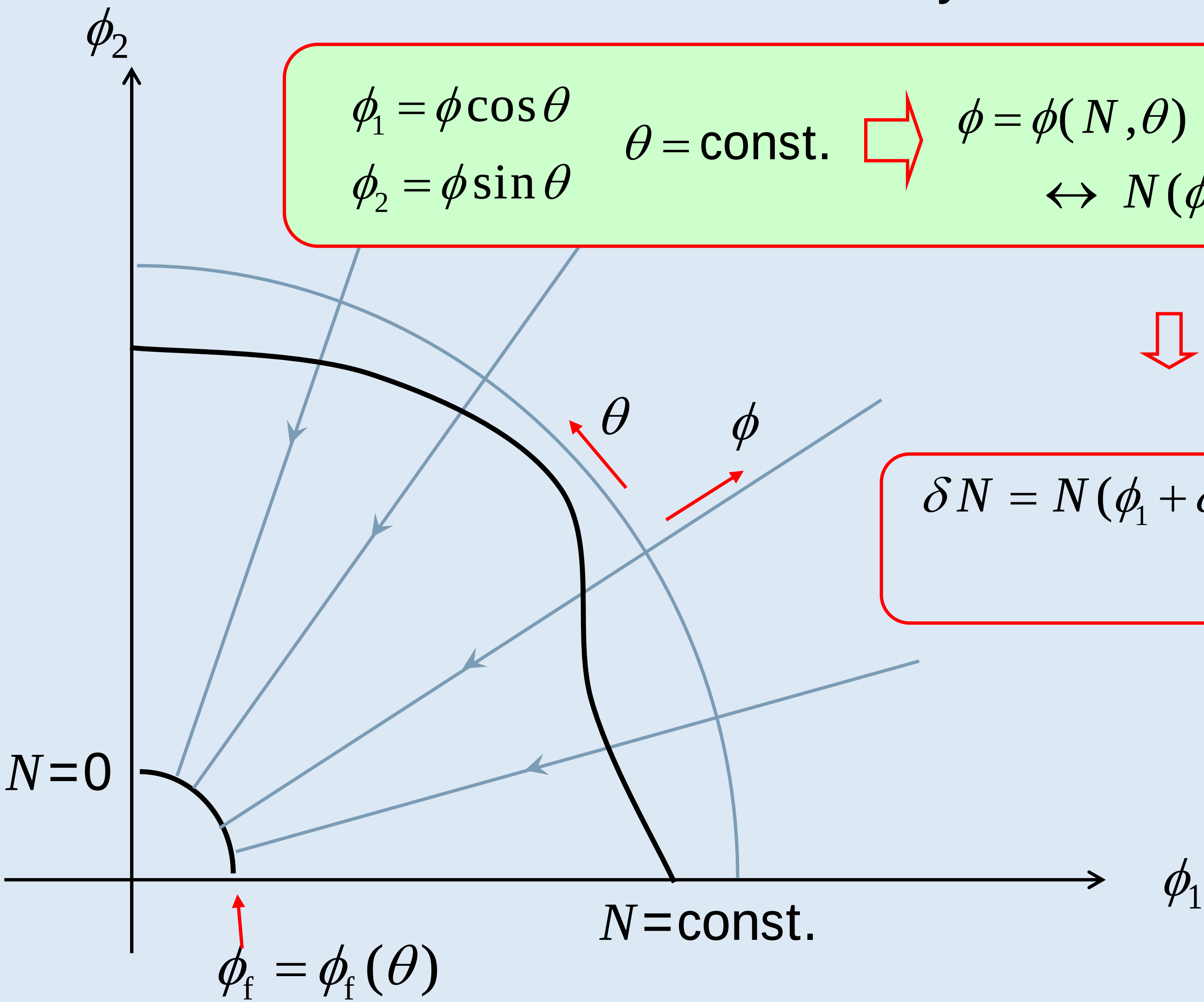
simple 2-brid example:
the case of radial inflationary orbits on (ϕ_1,ϕ_2)

$$\phi_1 = \phi\cos\theta \qquad \phi_2 = \phi\sin\theta$$

$$\theta = \text{const.} \;\Rightarrow\; \phi = \phi(N,\theta) \;\leftrightarrow\; N(\phi,\theta) = N(\phi_1,\phi_2)$$

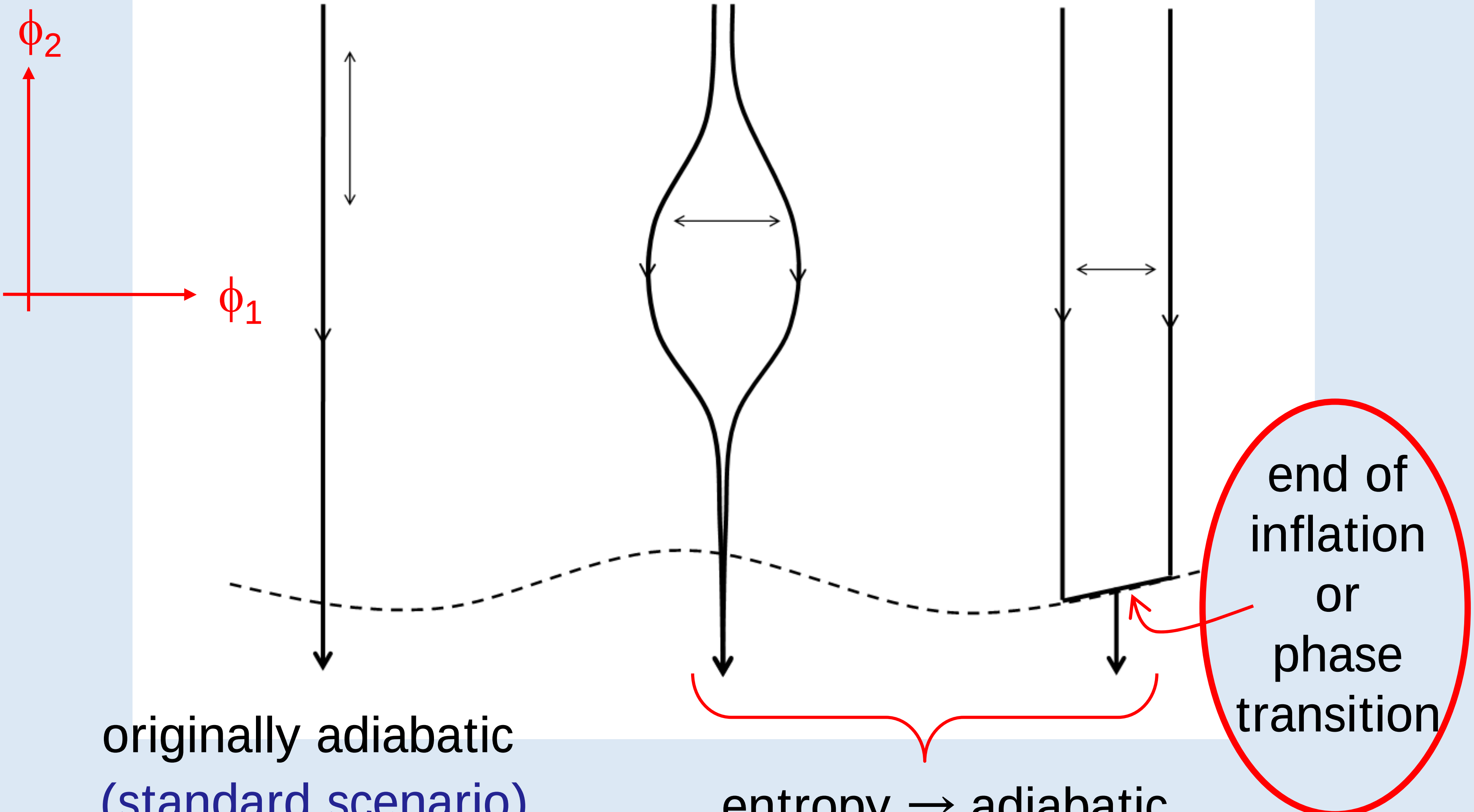
$$\Downarrow$$

$$\delta N = N(\phi_1+\delta\phi_1,\phi_2+\delta\phi_2) - N(\phi_1,\phi_2)$$



Three types of δN

(⟷) indicates field perturbations



originally adiabatic
(standard scenario)

entropy → adiabatic

condition for the end of inflation

ϕ_1, ϕ_2 ··· inflaton fields (2-brid inflation)

χ ··· waterfall field

$$V = V_0(\chi;\phi)U(\phi);\quad V_0 = \frac{1}{2}\left(g_1^2\phi_1^2 + g_2^2\phi_2^2\right)\chi^2 + \frac{\lambda}{4}\left(\chi^2 - \frac{\sigma^2}{\lambda}\right)^2$$

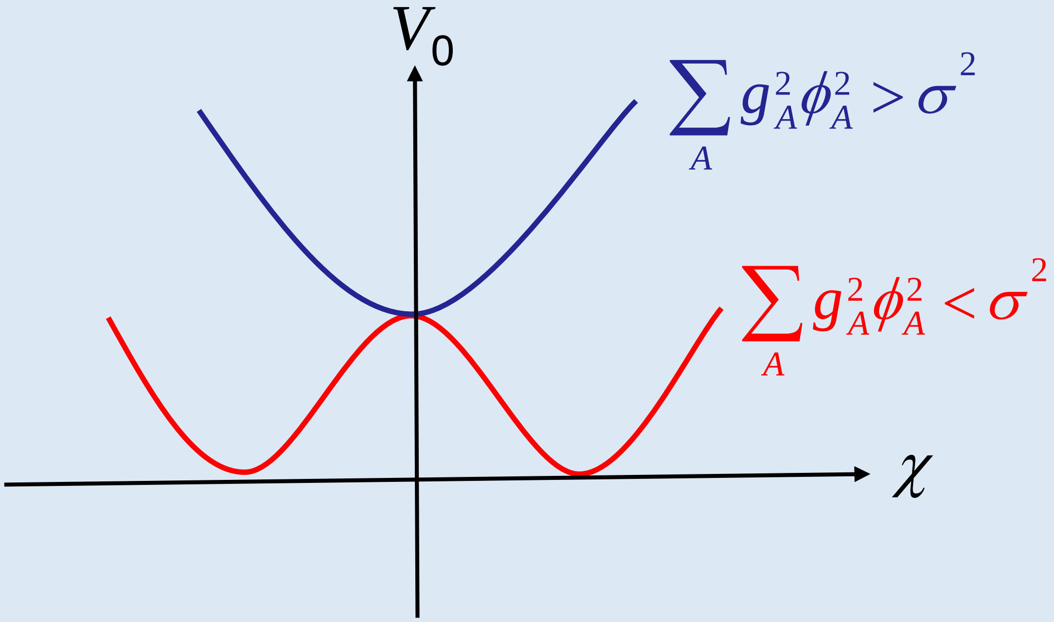
during inflation

$$g_1^2\phi_1^2 + g_2^2\phi_2^2 > \sigma^2$$

$$V_0(\chi=0) = \frac{\sigma^4}{4\lambda}$$

inflation ends when

$$g_1^2\phi_1^2 + g_2^2\phi_2^2 = \sigma^2$$



Simple analytically soluble model

MS ('08)

- exponential potential: $V = V_0 \exp\left[m_1\phi_1 + m_2\phi_2\right]$

  parametrize the end of inflation $\phi_{1,\mathrm{f}} = \dfrac{\sigma}{g_1}\cos\gamma, \quad \phi_{2,\mathrm{f}} = \dfrac{\sigma}{g_2}\sin\gamma$

- δN to 2nd order in $\delta\phi$:

$$\delta N = \frac{\delta\phi_1 g_1 \cos\gamma + \delta\phi_2 g_2 \sin\gamma}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma} + \frac{g_1^2 g_2^2}{2\sigma}\frac{(m_2\delta\phi_1 - m_1\delta\phi_2)^2}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^3}$$

- Spectrum of curvature perturbation

$$P_{\mathcal{R}}(k) = \frac{g_1^2\cos^2\gamma + g_2^2\sin^2\gamma}{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^2}\left(\frac{H}{2\pi}\right)^2\Bigg|_{k=Ha}$$

spectral index: $n_s = 1 - (m_1^2 + m_2^2)$

tensor/scalar: $r = \dfrac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 8\dfrac{(m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma)^2}{g_1^2\cos^2\gamma + g_2^2\sin^2\gamma}$

$$\delta_L N \equiv \frac{\delta\phi_1 g_1 \cos\gamma + \delta\phi_2 g_2 \sin\gamma}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma}, \quad S \equiv \frac{\delta\phi_1 g_2 \sin\gamma - \delta\phi_2 g_1 \cos\gamma}{m_2 g_1 \cos\gamma - m_1 g_2 \sin\gamma}$$

isocurvature perturbation

$$\left[\quad \langle \delta_L N \cdot S \rangle = 0 \;\text{ for }\; \langle \delta\phi^A \delta\phi^B \rangle = \left(\frac{H}{2\pi}\right)^2 \delta^{AB} \quad \right]$$

$$\Rightarrow \quad \delta N = \delta_L N + \frac{3}{5} f_{NL}^{\text{local}} (\delta_L N + S)^2$$

isocurvature contributes at 2nd order

$$f_{NL}^{\text{local}} = \frac{5 g_1^2 g_2^2}{6\sigma(g_1^2 \cos^2\gamma + g_2^2 \sin^2\gamma)^2} \frac{(m_2 g_1 \cos\gamma - m_1 g_2 \sin\gamma)^2}{m_1 g_1 \cos\gamma + m_2 g_2 \sin\gamma}$$

$$\Rightarrow \quad f_{NL}^{\text{local}} = O\left(gm/\sigma\right) \;\text{ for }\; m_1, m_2 \sim O(m),\; g_1, g_2 \sim O(g).$$

possibility of large non-Gaussianity

$$(N.B.,\; f_{NL}^{\text{local}} > 0)$$

Just an example ...

$$1 = M_{Pl} = (8\pi G)^{-1/2} = 2.43 \times 10^{18}\,\mathrm{GeV}$$

input parameters: $m_1^2 \sim 0.005, \quad m_2^2 \sim 0.035$

$$m_1 \cos\gamma \gg m_2 \sin\gamma$$

$$g_1{}^2 = g_2{}^2 \equiv g^2$$

outputs:

$$\left.\begin{aligned} n_s &= 1 - (m_1^2 + m_2^2) \sim 0.96 \\ r &\approx 8 m_1^2 \sim 0.04 \end{aligned}\right\} \text{indep. of waterfall } \chi$$

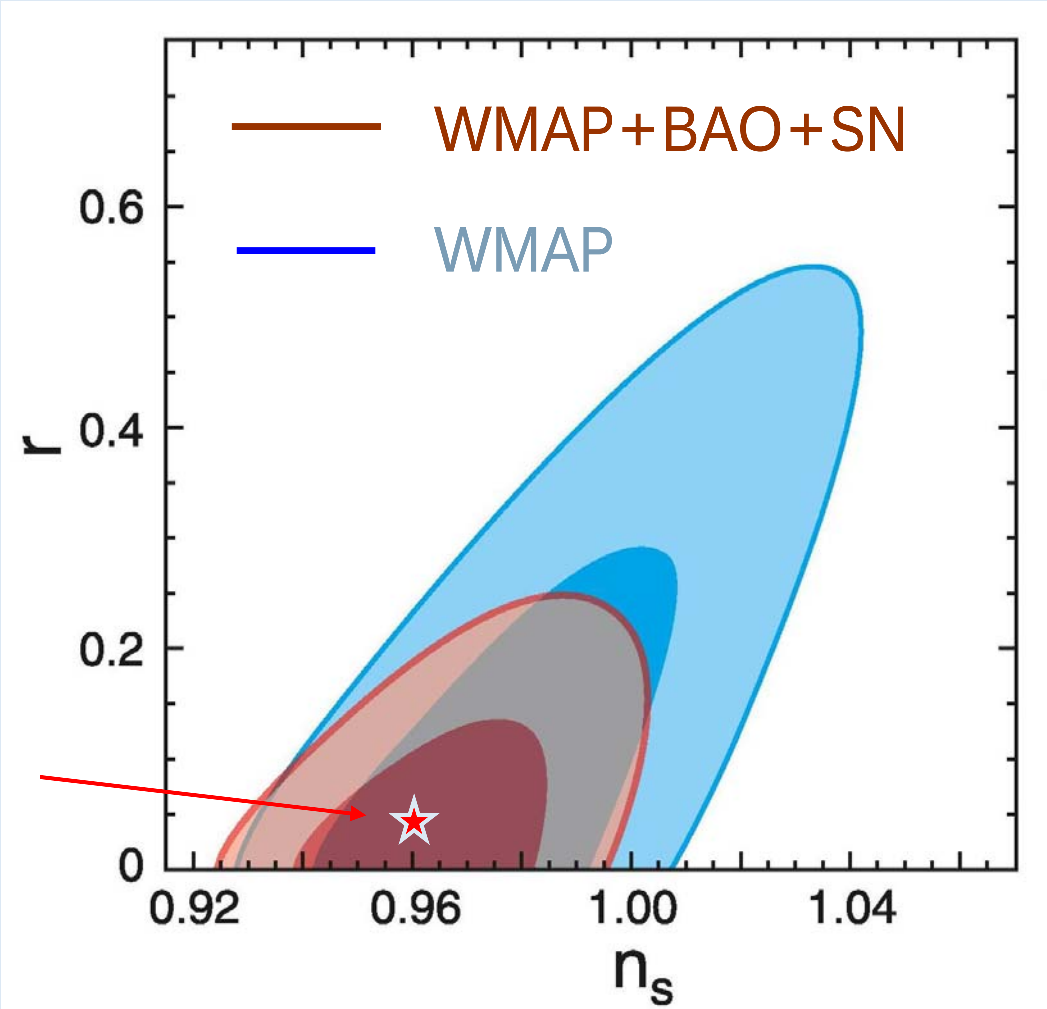
$$3H^2 = \sigma^4 / 4\lambda \sim 1.5 \times 10^{-9} \quad \left(\Leftrightarrow P_{\mathcal{R}}(k) \sim 2.5 \times 10^{-9} \right)$$

$$\Rightarrow \quad \sigma^2 \sim \lambda^{1/2} \times 10^{-4}$$

$$\Rightarrow \quad \boxed{f_{NL}^{\mathrm{local}} \approx \frac{5 g m_2^2}{6 m_1 \sigma} \sim 40 \frac{g}{\lambda^{1/4}}}$$

WMAP 5yr

Komatsu et al. '08



present example

tensor-scalar ratio *r* is not suppressed

AND f_{NL}^{local} ~ 50 (positive and large)

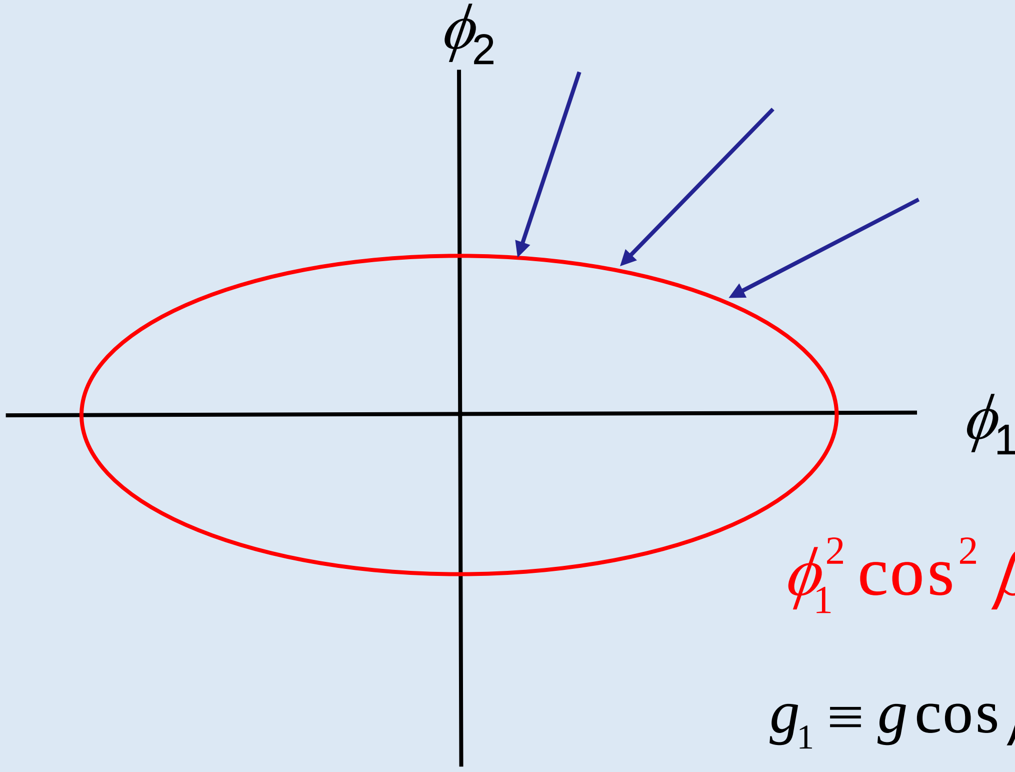
Another example: O(2) SSB model

$$V = V_0 \exp\left[\frac{1}{2} M^2(\phi_1^2 + \phi_2^2)\right]$$

Inflation ends at $g_1^2\phi_1^2 + g_2^2\phi_2^2 = \sigma^2; \quad g_1 \neq g_2$

symmetry breakdown at the end of inflation

Alabidi & Lyth '06



$$\phi_1^2 \cos^2\beta + \phi_2^2 \sin^2\beta = \frac{\sigma^2}{g^2}$$

$$g_1 \equiv g\cos\beta, \quad g_2 \equiv g\sin\beta$$

$$\phi_{1,f}^2 \cos^2\beta + \phi_{2,f}^2 \sin^2\beta = \frac{\sigma^2}{g^2} \quad \Rightarrow \quad \phi_{1,f} = \frac{\sigma}{g}\frac{\sin\delta}{\cos\beta}, \quad \phi_{2,f} = \frac{\sigma}{g}\frac{\cos\delta}{\sin\beta}$$

- curvature perturbation spectrum: $\mathcal{P}_{\mathcal{R}}(k) = \frac{8}{r}\left(\frac{H}{2\pi}\right)^2 = \frac{1}{r}\frac{\sigma^4}{6\pi^2\lambda}$
- spectral index: $n_s = 1 + 2M^2 - \frac{r}{8}\frac{1-\cos^2 2\beta\cos^2 2\delta}{\sin^2 2\beta}$
- tensor/scalar: $r = \frac{\mathcal{P}_T(k)}{\mathcal{P}_{\mathcal{R}}(k)} = 8\left(\frac{\sigma M^2 e^{M^2 N}}{g}\right)^2 \frac{2}{1-\cos 2\beta\cos 2\delta}$
- non-gaussianity: $f_{NL}^{\mathrm{local}} = \frac{5M^2}{6}\left\{\left(\frac{\cos 2\beta\sin 2\delta}{1-\cos 2\beta\cos 2\delta}\right)^2 - 1\right\}$

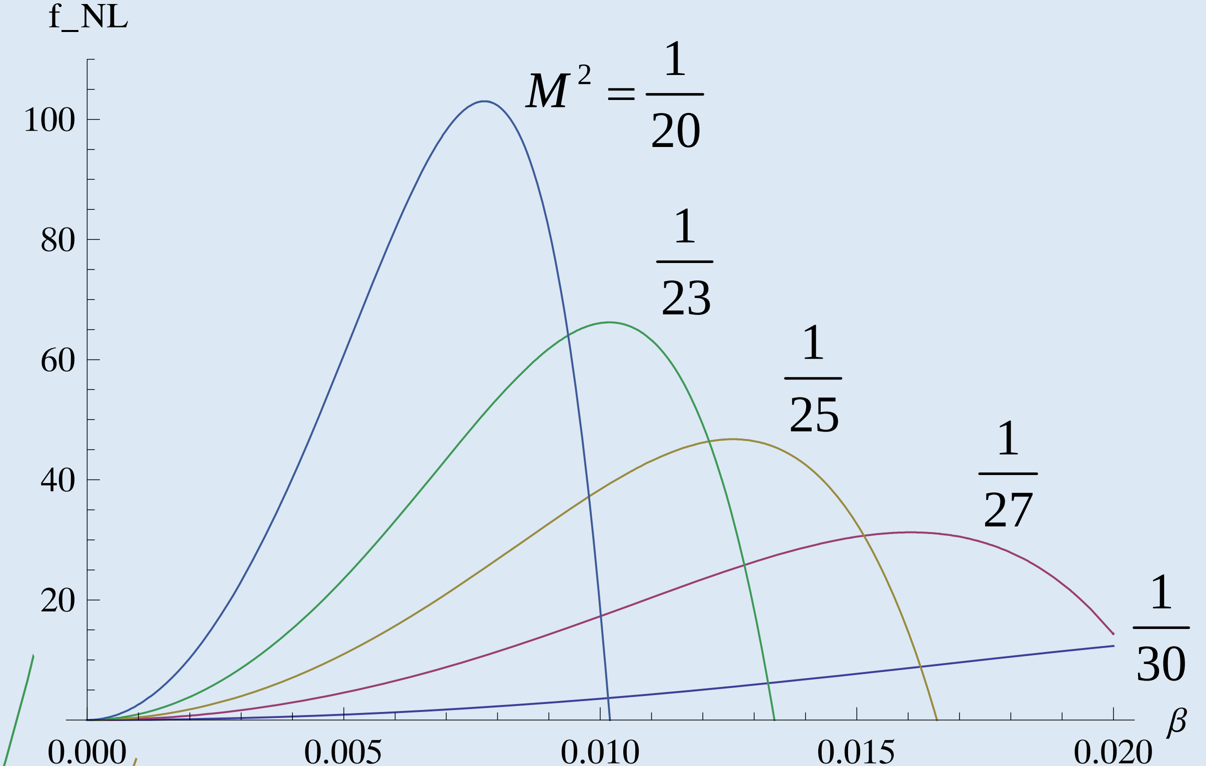
$\Rightarrow$ $f_{NL}^{\mathrm{local}} \sim 25\left(\frac{r}{0.1}\right)\left(\frac{10^{-2}}{\beta}\right)^2$ for $M^2 \gtrsim 0.02, \quad 1 \gg \delta^2 \gg \beta^2$

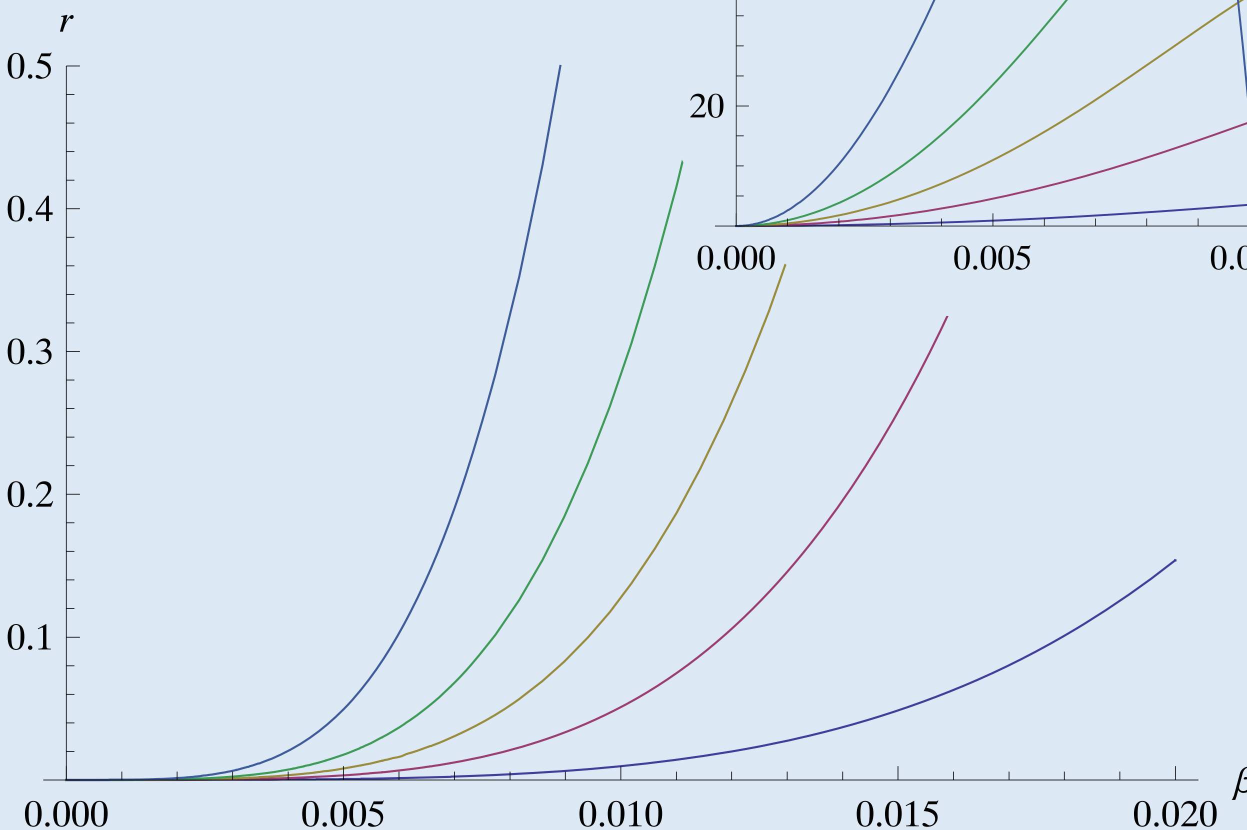
Again, f_{NL}^{local} may be large and positive.

f_{NL}^{local} and r

$$1 \gg \delta^2 \gg \beta^2$$

$$\frac{g^4}{\lambda} = 1$$





$$n_s = 0.96$$

$$\mathcal{P}_{\mathcal{R}}(k) = 2.5 \times 10^{-9}$$

The result is:

$$y_f = \sigma \cos\gamma, \quad x_f = \sigma \sin\gamma \quad (\varepsilon = +1)$$

$$\Rightarrow \quad f_{NL}^{\varepsilon=+1} = \frac{5m^2}{6} \frac{e^{-4m^2N} + \tan^2\gamma \left(\frac{1}{\cos^2\gamma} + \tan^2\gamma \right)}{\left(e^{-2m^2N} + \tan^2\gamma \right)^2}$$

$$m^2 \ll 1$$

$$y_f = \sigma \cosh\gamma, \quad x_f = \sigma \sinh\gamma \quad (\varepsilon = -1)$$

$$\Rightarrow \quad f_{NL}^{\varepsilon=-1} = -\frac{5m^2}{6} \frac{e^{-4m^2N} + \tanh^2\gamma \left(\frac{1}{\cosh^2\gamma} - \tanh^2\gamma \right)}{\left(e^{-2m^2N} + \tanh^2\gamma \right)^2}$$

If the end surface is concave (~hyperbolic), f_{NL} can become negative.

5. Summary

- 3 types of non-Gaussianity

  1. subhorizon ··· quantum origin
  2. superhorizon ··· classical (local) origin

  These are important

  3. gravitational dynamics ··· classical origin

- DBI inflation --- type 1.

  f_{NL}^{equil} may be large

- curvaton scenario, multi-brid inflation --- type 2.

  f_{NL}^{local} may be large

  but curvaton scenario predicts $r \ll 1$ if f_{NL} is large.

multi-brid inflation may give $r \sim 0.1$ as well

Non-Gaussianity plays an important role in determining (constraining) models of inflation

4-pt function (trispectrum) may be detected in addition to 3-pt function (bispectrum)

NG may be detected in the very near future

PLANCK launched on 14 May (4 days ago!)