$\mathcal{N} = 3$ Superfield Formulation of the ABJM and BLG Models

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References

I.L. Buchbinder, E.A. Ivanov, O. Lechtenfeld, N.G. Pletnev, I.B.S., B.M. Zupnik, JHEP 0903 (2009) 096, arXiv:0811.4774 [hep-th].

Very well known example:

- In the bulk of D3 brane we have four-dimensional gauge theory with 16 supersymmetries $\Rightarrow$ $\mathcal{N} = 4$ Abeian gauge theory.
- Stack of D3 branes $\Rightarrow$ $\mathcal{N} = 4$ SYM with gauge group $SU(n)$.
- AdS$_5$/CFT$_4$ correspondence: Correlation functions of composite operators in $\mathcal{N} = 4$ SYM are related to the corresponding functions of the IIB supergravity in $AdS_5 \times S^5$ background.

Motivations: A model for multiple M2 branes

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Analogous recent achievements for M2 brane

- In the bulk of M2 brane lives a three-dimensional superconformal gauge theory with 16 supersymmetries $\Rightarrow$ $\mathcal{N}=8$, $d=3$ Chern-Simons-matter gauge theory, or, Bagger-Lambert-Gustavsson (BGL) theory [J. Bagger, N. Lambert, Phys. Rev. D75 (2007) 045020; D77 (2008) 065008; JHEP 0802 (2008) 105; A. Gustavsson, JHEP 0804 (2008) 083; Nucl. Phys. B807 (2009) 315; Nucl. Phys. B811 (2009) 66].
- Stack of M2 branes $\Rightarrow$ $\mathcal{N}=6$, $d=3$ Chern-Simons-matter theory with gauge group $SU(n)\times SU(n)$, or, Aharony-Bergman-Jafferis-Maldacena (ABJM) theory [O. Aharony, O. Bergman, D.L. Jafferis, J. Maldacena, JHEP 0810 (2008) 091]
- $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence: Chern-Simons-matter model with $\mathcal{N}=6$ supersymmetry have dual gravitation description in terms of superstrings on $AdS_4 \times CP^3$ in low-energy limit.

Definition:

- ABJM model is a $d = 3$ $\mathcal{N} = 6$ superconformal Chern-Simons-matter theory with gauge group $SU(n)_L \times SU(n)_R$.
- BLG model is a $d = 3$ $\mathcal{N} = 8$ superconformal Chern-Simons-matter theory with gauge group $SO(4) = SU(2) \times SU(2)$.

Note: The ABJM model reduces to BLG theory when the gauge group is $SU(2) \times SU(2)$. $\Rightarrow$ BLG model is a particular case of ABJM theory.

Field content:

- 4 complex scalars: f^I, $\bar{f}_I$, $I = 1, 2, 3, 4$ (index of $SU(4)$) in bifundamental representation;
- 4 complex spinors: ψ^I_α, $\bar{\psi}_{I\alpha}$, $\alpha = 1, 2$ (index of $SU(2)$) in bifundamental representation;
- 2 vector fields: A^L_μ, A^R_μ in the adjoint representations of $SU(n)_L$ and $SU(n)_R$ respectively.

The ABJM action

$$
\begin{aligned}
S^{ABJM} &= S_{mat} + S_{CS} + S_{int}, \\
S_{mat} &= \frac{k}{2\pi}\mathrm{tr}\int d^3x(-\nabla^\mu f^I \nabla_\mu \bar{f}_I + i\bar{\psi}_I\gamma^\mu\nabla_\mu\psi^I), \\
S_{CS} &= \frac{k}{2\pi}\mathrm{tr}\int d^3x\varepsilon^{\mu\nu\rho}(\frac{1}{2}A^L_\mu\partial_\nu A^L_\rho + \frac{1}{3}A^L_\mu A^L_\nu A^L_\rho - \frac{1}{2}A^R_\mu\partial_\nu A^R_\rho - \frac{1}{3}A^R_\mu A^R_\nu A^R_\rho \\
S_{int} &\sim \mathrm{tr}\int d^3x(\sum\bar{\psi}\psi\bar{f}f + \sum f\bar{f}f\bar{f}f\bar{f}).
\end{aligned}
$$

$\mathcal{N}=6$ supersymmetry (on-shell)

$$
\begin{aligned}
\delta f^I &= -i\epsilon^{[IJ]\alpha}\bar{\psi}_{J\alpha} \\
\delta\psi^I &= \gamma^\mu\epsilon^{[IJ]}\nabla_\mu\bar{f}_J + \delta_3\psi, \qquad (\delta_3\psi\sim\epsilon fff) \\
\delta A^L_\mu &= \epsilon^{[IJ]}\gamma_\mu\bar{\psi}_I\bar{f}_J - \epsilon_{[IJ]}f^I\psi^J\gamma_\mu \\
\delta A^R_\mu &= \epsilon^{[IJ]}\bar{f}_I\gamma_\mu\bar{\psi}_J - \epsilon_{[IJ]}\psi^I\gamma_\mu f^J.
\end{aligned}
$$

Here $\nabla_\mu f^I = \partial_\mu f^I + iA^L_\mu f^I - if^I A^R_\mu$, k is the Chern-Simons level.

Our goal:

To develop an unconstrained $\mathcal{N}=3$ superfield formulation of the ABJM model. Such a formulation should

- make manifest the symmetries of the ABJM theory,
- explain the structure of the scalar potential.

Known superfield formulations of the ABJM theory

- In terms of $\mathcal{N}=1$ superfields: A. Mauri, A.C. Petkou, Phys. Lett. B666 (2008) 527;
- In terms of $\mathcal{N}=2$ superfields: M. Benna, I. Klebanov, T. Klose, M. Smedback, JHEP 0809 (2008) 027;
- In terms of on-shell $\mathcal{N}=6$ and $\mathcal{N}=8$ superfields: M. Cederwall, JHEP 0809 (2008) 116, JHEP 0810 (2008) 070;
  I.A. Bandos, Phys. Lett. B669 (2008) 105.

Standard $\mathcal{N}=3$ superspace:

$$\{x_\mu, \theta^A_\alpha\}, \qquad A=1,2,3 \text{ (index of } SO(3)).$$

Harmonic $\mathcal{N}=3$ superspace:

Coordinates: $\{x^\mu, \theta^{++}_\alpha, \theta^{--}_\alpha, \theta^0_\alpha, u^\pm_i\}$,

Harmonics: $u^\pm_i \in SU(2), \quad u^{+i}u^+_i = 0,\ u^{-i}u^-_i = 0,\ u^{+i}u^-_i = 1.$

Harmonic derivarives: $\mathcal{D}^{++}, \quad \mathcal{D}^{--}, \quad \mathcal{D}^0 = [\mathcal{D}^{++}, \mathcal{D}^{--}].$

Grassmann derivatives: $D^{++}_\alpha, \quad D^{--}_\alpha, \quad D^0_\alpha.$

Superfields

1. q-hypermultiplet: $q^+(x_\mu, \theta^{++}_\alpha, \theta^0_\alpha, u^\pm_i)$

$$q^+: \quad \{f^i, \bar f_i, \psi^i_\alpha, \bar\psi_{i\alpha}\}, \quad i=1,2.$$

2. Vector superfield: $V^{++}(x_\mu, \theta^{++}_\alpha, \theta^0_\alpha, u^\pm_i)$

$$V^{++}: \quad \{A_\mu, \phi^{(ij)}, \lambda_\alpha, \lambda^{(ij)}_\alpha\}.$$

For the ABJM model we need:

- 2 hypermultiplet superfields, q^{+a}, $a=1,2$
- 2 vector superfield, V_L^{++}, V_R^{++}.

The action in the Abelian case

$$
\begin{aligned}
S_{\mathcal{N}=6} &= S_{hyp} + S_{gauge}, \\
S_{hyp} &= \int d\zeta^{(-4)} \bar{q}^{+a}(\mathcal{D}^{++} + V_L^{++} - V_R^{++}) q_a^+, \\
S_{gauge} &= S_{CS}[V_L^{++}] - S_{CS}[V_R^{++}], \quad S_{CS}[V^{++}] = \frac{ik}{8\pi}\int d\zeta^{(-4)} V^{++} W^{++}.
\end{aligned}
$$

Here $W^{++} = -\frac{1}{4}(D^{++})^2 V^{--}$ is a superfield strength corresponding to the gauge superfield V^{++}; V^{--} is expressed through V^{++} from the equation $D^{++}V^{--} = D^{--}V^{++}$.

- No any superfield potential $S_{int} \sim g \int d\zeta^{(-4)} (\bar{q}^{+a} q_a^+)^2$ is admissible!

Symmetries of the action $S_{\mathcal{N}=6}$:

- Manifest $\mathcal{N}=3$ supersymmetry;
- Hidden $\mathcal{N}=3$ supersymmetry with parameters $\epsilon^{\alpha(ab)}$:

$$\begin{aligned} \delta_\epsilon q^{+a} &= i\epsilon^{\alpha(ab)}[\nabla^0_\alpha + \theta^{--\alpha}(W^{++}_L - W^{++}_R)]q^+_b , \\ \delta_\epsilon V^{++}_L &= \delta_\epsilon V^{++}_R = \epsilon^{\alpha(ab)}\theta^0_\alpha \bar{q}^+_a q^+_b . \end{aligned}$$

- $SO(6)\simeq SU(4)$ R-symmetry group: The $SU(2)\times SU(2)$ subgroup is manifest while the transformations from the coset $SU(4)/[SU(2)\times SU(2)]$ are given by

$$\begin{aligned} \delta_\lambda q^{+a} &= -i[\lambda^{0(ab)} - \lambda^{++(ab)}\hat{\nabla}^{--} - 2\lambda^{--(ab)}\theta^{++\alpha}\hat{\nabla}^0_\alpha + 4\lambda^{0(ab)}\theta^{0\alpha}\hat{\nabla}^0_\alpha]q^+_b \\ \delta_\lambda \bar{q}^+_a &= -i[\lambda^0_{(ab)} - \lambda^{++}_{(ab)}\hat{\nabla}^{--} - 2\lambda^{--}_{(ab)}\theta^{++\alpha}\hat{\nabla}^0_\alpha + 4\lambda^0_{(ab)}\theta^{0\alpha}\hat{\nabla}^0_\alpha]\bar{q}^{+b}, \\ \delta_\lambda V^{++}_L &= \frac{4\pi}{k}\kappa^{ab}q^+_a\bar{q}^+_b , \qquad \delta_\lambda V^{++}_R = \frac{4\pi}{k}\kappa^{ab}\bar{q}^+_a q^+_b , \end{aligned}$$

where $\kappa_{(ab)} = 4\lambda^{--}_{(ab)}(\theta^0\theta^{++}) - 8\lambda^0_{(ab)}(\theta^0)^2$ and $\hat{\nabla}^{--}$ and $\hat{\nabla}^0_\alpha$ are gauge-covariant analyticity-preserving derivatives:

$$\hat{\nabla}^0_\alpha = \nabla^0_\alpha + \theta^{--}_\alpha W^{++}, \quad \hat{\nabla}^{--} = \nabla^{--} + 2\theta^{\alpha--}\nabla^0_\alpha + (\theta^{--})^2 W^{++}.$$

Non-abelian generalization: $V_L^{++} \in SU(n)_L$, $V_R^{++} \in SU(n)_R$
q^{+a} are in the bifundamental representation,

$$\nabla^{++} q^{+a} = \mathcal{D}^{++} q^{+a} + V_L^{++} q_a^+ - q_a^+ V_R^{++}.$$

non-Abelian $\mathcal{N}=6$ supersymmetric action:

$$\begin{aligned}
S_{\mathcal{N}=6} &= S_{hyp} + S_{gauge}, \\
S_{hyp} &= \mathrm{tr} \int d\zeta^{(-4)} \bar{q}^{+a} \nabla^{++} q_a^+, \\
S_{gauge} &= S_{CS}[V_L^{++}] - S_{CS}[V_R^{++}], \\
S_{CS}[V^{++}] &= \frac{ik}{4\pi} \mathrm{tr} \sum_{n=2}^{\infty} \int d^9 z du_1 \dots du_n \frac{V^{++}(z,u_1)\dots V^{++}(z,u_n)}{(u_1^+ u_2^+)\dots(u_n^+ u_1^+)}.
\end{aligned}$$

Hidden $\mathcal{N}=3$ supersymmetry:

$$\begin{aligned}
\delta q^{+a} &= i\epsilon^{\alpha(ab)} [\nabla^0_\alpha q_b^+ + \theta_\alpha^{--}(W_L^{++} q_b^+ - q_b^+ W_R^{++})], \\
\delta V_L^{++} &= \epsilon^{\alpha(ab)} \theta^0_\alpha q_a^+ \bar{q}_b^+, \quad \delta V_R^{++} = \epsilon^{\alpha(ab)} \theta^0_\alpha \bar{q}_a^+ q_b^+.
\end{aligned}$$

Features of the $\mathcal{N}=3$ superfield formulation of the ABJM theory

- No any superfield potential in the model!
- Standard ABJM action is restored upon reduction to component fields.
- The scalar field potential appears solely owing to the elimination of auxiliary fields.
- The $SO(6) \simeq SU(4)$ R-symmetry explicitly demonstrated.
- It is checked that when the gauge group is $SU(2) \times SU(2)$ the supersymmetry is raised up to $\mathcal{N}=8$, reproducing the BLG model.
- Various generalizations of the ABJM model are analyzed within the $\mathcal{N}=3$ superfield formulation. In particular, the models with the gauge groups $U(m) \times U(n)$, $O(n) \times USp(2m)$ are shown to be admissible for this theory.

Higgs effect: "M2 to D2"

In components: [S. Mukhi, C. Papageorgakis, JHEP 0805 (2008) 085]

- There are 8 scalars f^I, $I = 1, \ldots, 8$.
- Give vev to f^8: $\langle f^8 \rangle = a = const$.
- One can gauge away this scalar f^8, leaving only 7 scalars f^i, $i = 1, \ldots, 7$.
- The gauge symmetry is partly fixed.
- The corresponding degree of freedom appears as a dynamical vector field:

$$
\begin{aligned}
\{A^L_\mu, A^R_\mu\} &\longrightarrow A_\mu \\
\varepsilon^{\mu\nu\rho}(A^L_\mu \partial_\nu A^L_\rho - A^R_\mu \partial_\nu A^R_\rho) &\longrightarrow F_{\mu\nu}F^{\mu\nu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .
\end{aligned}
$$

- As a result, the $\mathcal{N} = 8$, $d = 3$ SYM theory appears with 7 scalar fields, 8 fermions and 1 gauge superfield.
- In other words, the M2 brane turns into D2 brane.

Abelian Higgs effect in $\mathcal{N} = 3$ superspace

General procedure:

1. Convert two hypermultiplets q^{+a} into one complex ω-hypermultiplet.
2. Gauge away the imaginary part of ω, resulting in the real ω-hypermultiplet.
3. Give vev to the ω superfield: $\langle\omega\rangle = a$.
4. The $U(1) \times U(1)$ gauge symmetry is partly fixed to $U(1)$.
5. Two Chern-Simons vector superfields V_L^{++}, V_R^{++} turn into one dynamical V^{++}.
6. The resulting action is

$$S = \int d\zeta^{(-4)}[(D^{++}\omega)^2 - \frac{k^2}{16\pi^2}\frac{1}{(a+\omega)^2}W^{++}W^{++}]. \quad (1)$$

7. As a result, we have SYM action non-minimally interacting with ω-hypermultiplet.

Open problems:

- To study quantum aspects of the ABJM theory in $\mathcal{N}=3$ harmonic superspace (to construct the low-energy effective action, correlation functions, e.t.c).
- Non-abelian Higgs effect in $\mathcal{N}=3$ harmonic superspace.
- Are there $\mathcal{N}=4,5,6$ unconstrained superfield formulations of the ABJM theory?