BFV-BRST operators and Fock space realizations of Verma modules for non-linear superalgebras underlying Lagrangian formulations for mixed-symmetry HS fields in AdS spaces

Alexander Reshetnyak

Institute of Strength Physics and Materials Science
Siberian Branch of RAS, Tomsk, Russia

4th Sakharov Conference, Moscow, 22 May, 2009

The talk is based on the research

1. I.L. Buchbinder, V.A. Krykhtin and A.A. Reshetnyak

BRST approach to Lagrangian construction for fermionic HS fields in AdS space,

Nucl. Phys. B787 (2007) 211. arXiv:hep-th/0703049

2. P.Yu. Moshin and A.A. Reshetnyak,

BRST approach to Lagrangian formulation for mixed-symmetry fermionic HS fields,

JHEP 10 (2007) 040, arXiv:0706.0386[hep-th]

3. A.A. Reshetnyak

Nonlinear Operator Superalgebras and BFV--BRST Operators for Lagrangian Description of Mixed-symmetry HS Fields in AdS Spaces, arXiv:0812.2329[hep-th]

4. A.Kuleshov and A. Reshetnyak,

Programming Realization of Symbolic Computations for Non-linear Commutator Superalgebras over the Heisenberg--Weyl Superalgebra: Data Structures and Processing Methods,

arXiv:0905.2705[hep-th]

LIST OF THE BASIC RESULTS:

- Non-linear operatorial superalgebras of half-integer and algebras of integer HS fields in AdS$_d$-spaces generated by the constraints $\{o_I\}$ in auxiliary Fock spaces, which are equivalent to conditions of (A)dS(d) group irreps subject to Young tableaux with 2 rows are derived;
- Representations of (super)algebras $\mathcal{A}'(\overset{\{o'_I\}}{Y(2)}, AdS_d)$ of additional parts $\{o'_I\}$ in order to convert (super)algebras $\mathcal{A}(\overset{\{o_I\}}{Y(2)}, AdS_d) \mapsto$ into ones with set of only 1st-class constraints, $\mathcal{A}_c(\overset{\{O_I\}}{Y(2)}, AdS_d)$: $\boxed{O_I = o_I + o'_I}$, with help of Verma modules constructions and its Fock space realizations are found;
- Exact nilpotent BFV–BRST operator: $\boxed{Q_b = \mathcal{C}^I O_I + Q_2 + Q_3}$, for non-linear algebra $\mathcal{A}_{cb}(Y(2), AdS_d)$ with non-vanishing terms Q_3 in 3rd power in C^I due to nontrivial Jacobi identity for $\mathcal{A}_{cb}(Y(2), AdS_d)$ is constructed;
- A gauge-invariant unconstrained Lagrangian formulation (LF) for integer HS fields subject to $YT(2)$ in AdS(d) space is developed;
- The computer program to verify within Symbolic Computations the correctness of Fock space realization of Verma module for $\mathcal{A}'(Y(2), AdS_d)$ is suggested.

Plan of the talk:

1. Motivations and setting of the problem:
   - **a)** Properly HS formulations on constant curvature spaces, String field theory, BRST approach for HS fields ;
   - **b)** Construction of an auxiliary representations for non-linear (super)algebras as the conversion procedure ;
   - **c)** Non-linear operator superalgebras and BFV-BRST operators;
2. Scheme of the solution the problem:
   - **a)** Derivation of massive (half-)integer HS symmetry (super)algebra $(\mathcal{A})\mathcal{A}_b(Y(2), AdS_d)$ in AdS_d space subject to Young tableaux with 2 rows ;
   - **b)** Additive conversion of (super)algebra $\mathcal{A}_{mod}(Y(2), AdS_d)$ with 1st and 2nd class constraints into one $\mathcal{A}(Y(2), AdS_d)$ with only 1-class constraints system;
   - **c)** Verma module for (super)algebra $\mathcal{A}'(Y(2), AdS_d)$ of additional parts to constraints and its oscillator realization in Fock space $\mathcal{H}'$;
   - **d)** (Super)algebra of converted constraints, exact BFV-BRST operator;
   - **e)** Unconstrained Lagrangian formulation;
   - **f)** New computer program to verify the validity of the oscillator Verma module realization.
3. Summary of the results; Outlook.

1. Motivations and setting of the problem:

Problems of HS field theory (starting to study by M. Fierz, W. Pauli; L. Singh, C. Hagen; C. Fronsdal) both for totally-symmetric ($k = 1$ row in Young tableau (YT)), and for mixed-symmetry ($k > 1$) half-integer $\mathbf{s} = (n_1 + 1/2, n_2 + 1/2, \ldots)$ and integer $\mathbf{s} = (n_1, n_2, \ldots)$ (massive and massless:$m = 0$) HS fields:

$$\Phi_{\mu_1 \ldots \mu_{n_1};\nu_1 \ldots \nu_{n_2};\ldots} \longleftrightarrow \begin{array}{|c|c|c|c|c|c|c|c|c|c|}\hline \mu_1 & \mu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{n_1} \\ \hline \nu_1 & \nu_2 & \cdot \quad \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdot & \nu_{n_2} \\ \cline{1-8} \cdots & \cdots & \cdots \quad \cdot \quad \cdot & \cdot & \cdot & \cdot & \cdots \\ \cline{1-7} \end{array} = Y(s_1, s_2, \ldots) \; .$$

in view of connection to SuperString Field Theory (SFT): (E. Witten (1986); C. Thorn(1989)) through special tensionless limit for intercept $(\alpha' \to 0)$: (A. Sagnotti, M. Tsulaia, (2004)).

$$\boxed{\Longrightarrow \; \mathrm{SFT} \stackrel{\alpha' \to 0}{\longrightarrow} \; \texttt{infinite set of HS fields in superstring spectrum}}$$

From cosmological research $\Longrightarrow$ an exceptional role of (Anti-)de-Sitter [(A)dS] space for consistent propagation of free (J. Fang, C. Fronsdal (1980); M. Vasiliev (1988)) and interacting (E. Fradkin, M. Vasiliev (1987, 2001), R. Metsaev (2005)) HS fields due to:

- natural dimensional parameter – square inverse radius r of d-dimensional (A)dS space,
- connection of HS fields on AdS(d) space to AdS/CFT correspondence between the conformal $\mathcal{N} = 4$ SYM theory and superstring theory on $AdS_5 \times S_5$ Ramond-Ramond background, justifying the study of dynamics of fermionic and bosonic HS fields on AdS(d) subject to $Y(s_1, s_2)$: $k \leq \left[\frac{d-1}{2}\right] = 2.$

Whereas the Lagrangian formulation (LF) for free bosonic and fermionic HS fields subject to $Y(s_1, s_2)$ within frame-like formulation (M. Vasiliev) was found (Yu. Zinoviev, Arxiv:0809.3287, Arxiv:0904.0549), the same problem in metric-like formulation WAS NOT SOLVED.

Within stringy-inspired BRST-BFV approach (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, J. Buchbinder, V. Krykhtin, A. Reshetnyak) this problem meets SIGNIFICANT OBSTACLES in constructing:

1) Verma module; 2) BFV-BRST operator

for non-linear (super)algebra underlying LF for (fermionic) bosonic HS fields on $AdS(d)$.

$\Longrightarrow$ Indeed, the standard BFV-BRST prescription to quantize in Hamiltonian formalism an initial degenerate field theory given in LF is described by the sequence:

$$\boxed{\begin{array}{c}\texttt{degenerate LF}\\ (S(q^i),\delta q^i)\end{array}} \xrightarrow{\texttt{Dirac-Bergmann}} \boxed{\begin{array}{c}\texttt{t-local HF}\ (H_0, o_a)(t)\\ \{o_a, o_b\} = f^c_{ab}(q,p)o_c + \Delta_{ab}\end{array}} \xrightarrow[\text{Batalin, Tyutin}]{\texttt{conversion}} \boxed{\begin{array}{c}\texttt{converted HF}(H_0, O_a)(t)\\ \{O_a, O_b\} = F^c_{ab}(q,p,\zeta)O_c\end{array}}$$

$$\xrightarrow{\texttt{BFV method}} \boxed{\begin{array}{c}\texttt{BFV-BRST charge}\ Q(t), Q^2 = 0\\ Q(t) = C^a O_a + \frac{1}{2}C^b C^a F^c_{ab}\mathcal{P}_c + \text{more}\end{array}} \xrightarrow[\texttt{Kugo, Ojima}]{\texttt{quantization}} \boxed{\begin{array}{c}|\Psi\rangle \in \mathcal{H}_{\texttt{phys}} : Q|\Psi\rangle = 0, \mathrm{gh}(|\Psi\rangle) = 0\\ \texttt{gauge transfs:}|\Psi'\rangle = |\Psi\rangle + Q|\Lambda\rangle\end{array}}$$

$\Longrightarrow$ BFV-BRST approach in applying to HS fields solves INVERSE PROBLEM: RECONSTRUCTION OF THE UNKNOWN LAGRANGIAN FORMULATION FOR HS FIELD WITH GIVEN MASS, SPIN BY MEANS OF HAMILTONIAN OBJECTS:

$$\boxed{\begin{array}{c}\texttt{Irreps conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(Super)algebra}\{o_I(x)\}:\mathcal{H}\\ [o_I,o_J\}=f^K_{IJ}(o)o_k+\Delta_{ab}(g_0)\end{array}} \xrightarrow[\text{Burdik, Pashnev}]{\texttt{conversion}} \boxed{\begin{array}{c}O_I=o_I+o'_I:\mathcal{H}\bigotimes\mathcal{H}'\\ [O_I,O_J\}=F^K_{IJ}(o',O)O_K\end{array}}$$

$$\xrightarrow[\text{Henneaux}]{\texttt{BFV}} \boxed{\begin{array}{c}\texttt{BRST operator for } \{O_I\}:Q'(x)\\ Q'=C^IO_I+\frac{1}{2}C^IC^JF^K_{IJ}\mathcal{P}_K+\text{more}\end{array}} \xrightarrow{\texttt{LF}} \boxed{\begin{array}{c}Q'=Q+(g_0+h+\text{more})C_g+\ldots:Q^2=0\\ \texttt{mass-shell}:Q|\Psi\rangle=0,\text{gh}(|\Psi\rangle)=0\\ \texttt{spin:}(g_0+\text{more})(|\Psi\rangle,|\Lambda\rangle,\ldots)=-h(|\Psi\rangle,|\Lambda\rangle,\ldots)\\ \texttt{gauge transfs:}\delta|\Psi\rangle=Q|\Lambda\rangle,\delta|\Lambda\rangle=Q|\Lambda^1\rangle,\ldots\end{array}}$$

Q is BFV-BRST operator for only 1st class constraints $\{O_\alpha\}\subset\{O_I\}$ without invertible operator g_0.

On the stages of conversion and BFV-BRST operator construction the auxiliary Stuckelberg and gauge fields are automatically introduced to obtain gauge-invariant LF for basic field from initial non-Lagrangian equations defining the irreducible representations of Poincare or AdS groups;

$\Longrightarrow$ IN TRANSITION FROM $Y(S_1)$ TO $Y(S_1,S_2)$ THE REALIZATION:

1. of the 2nd arrow for (super)algebra $\mathcal{A}'(Y(2),AdS_d)$ of the additional parts o'_I meets the obstacles earlier not arisen IN VERMA MODULE CONSTRUCTION for Lie (super)algebra in case of Poincare group and for non-linear (super)algebra for AdS-group with $Y(s_1)$ (C. Burdik, O. Navratil, A. Pashnev (2002); P. Moshin, A. Reshetnyak (2007) A. Kuleshov, A. Reshetnyak (2009));
2. of the 3rd arrow (BRST OPERATOR) for operator (super)algebra $\mathcal{A}_c(Y(2),AdS_d)$ of converted O_I is IMPOSSIBLLE, without explicit resolution of JACOBI IDENTITIES for O_I.

THE PURPOSE OF REPORT IS TO PRESENT THE RESULTS:

1. on Verma Modules construction for non-linear (super)algebras underlying (fermionic) bosonic HS fields on (A)dS$_d$ space subject to YT(s_1, s_2);
2. of BFV-BRST operators constructions and its applications to gauge-invariant unconstrained Lagrangian Formulation for free HS field ;
3. on the application of new computer program to verify the correctness of the Verma module realization in auxiliary Fock space as the formal power series in oscillator variables for the superalgebras $\mathcal{A}'(Y(2), AdS_d)$.

2. Scheme of the solution the problem:

Let us consider the scheme of obtaining the Lagrangian formulation

2a) Derivation of massive (half-)integer HS symmetry (super)algebra $(\mathcal{A})\mathcal{A}_b(Y(2), AdS_d)$ in AdS_d space subject to Young tableaux with 2 rows

The massive of generalized spin $s = (n_1+\frac{1}{2}, n_2+\frac{1}{2})$ AdS group irrep $D(E_0(m), s)$ are realized in the space of mixed-symmetry spin-tensor fields $\Phi_{(\mu^1)_{n_1},(\mu^2)_{n_2}}(x)$ $\left[E_0^{m=0} = \sqrt{r}(n_1 - \frac{3}{2} + k_1 + d)\right]$, with suppressed Dirac index, subject to YT being specific for 2 various cases

$$\begin{array}{|c|c|ccc|c|c|c|c|c|c|c|}\hline \mu^1_1 & \mu^1_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu^1_{n_1} \\ \hline \mu^2_1 & \mu^2_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu^2_{n_1} \\ \hline \end{array} \leftrightarrow k_1 = 2, \quad \begin{array}{|c|c|ccc|c|c|c|c|c|c|c|}\hline \mu^1_1 & \mu^1_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu^1_{n_1} \\ \hline \mu^2_1 & \mu^2_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu^2_{n_2} & \multicolumn{2}{c}{} \\ \cline{1-10} \end{array} \leftrightarrow k_1 = 1.$$

which satisfy to mass-shell Dirac equation, gamma-traceless equations for each type of the indices and to the mixed-symmetry equation (with conventional γ-matrix $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}(x)$)

$$\left[i\gamma^\mu \nabla_\mu - r^{\frac{1}{2}}(n_1 + \tfrac{d-2-2k_1}{2}) - m\right]\Phi_{(\mu^1)_{n_1},(\mu^2)_{n_2}}(x) = 0, \tag{1}$$

$$\gamma^{\mu^1_1}\Phi_{\mu^1_1\mu^1_2\ldots\mu^1_{n_1},(\mu^2)_{n_2}}(x) = \gamma^{\mu^2_1}\Phi_{(\mu^1)_{n_1},\mu^2_1\mu^2_2\ldots\mu^2_{n_2}}(x) = 0\,, \quad , \tag{2}$$

$$\Phi_{\{(\mu^1)_{n_1},\mu^2_1\}\mu^2_2\ldots\mu^2_{n_2}}(x) = 0\,, \tag{3}$$

We want to find the LF for given HS field on more wider configuration space $\mathcal{M}$:

$$\mathcal{S}_n : \mathcal{M} = \{(\Phi_{(\mu)_{n_1},(\mu)_{n_2}}, \Psi_{(\mu)_{n_1-1},(\nu)_{n_2}}, \ldots)\} \to \mathbb{R},$$

Following to SFT we introduce an auxiliary Fock space $\mathcal{H}$ for $i,j=1,2$, with bosonic creation and annihilation operators

$$[a^i_a, a^{j+}_b] = -\eta_{ab}\delta_{ij}, \Leftrightarrow [a^i_\mu, a^{j+}_\nu] = -g_{\mu\nu}\delta_{ij}, \quad \text{for } a^{(+)\mu}(x) = e^\mu_a(x)a^{(+)a},$$

with vielbein $e^\mu_a(x)$ satisfying to standard relation (of compatibility of the metric with connection [metric one $\Gamma^\lambda_{\mu\nu}$ and spin one $\omega_\mu{}^a{}_b$])

$$\nabla_\mu e^a_\nu = \partial_\mu e^a_\nu - \Gamma^\lambda_{\mu\nu} e^a_\lambda + \omega_\mu{}^a{}_b e^b_\nu = 0.$$

An arbitrary "string-like" vector $|\Phi\rangle \in \mathcal{H}$

$$\begin{aligned} |\Phi\rangle &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{n_1} \Phi_{(\mu)_{s_1},(\nu)_{n_2}}(x)\, a_1^{+\mu_1}\ldots a_1^{+\mu_{n_1}} a_2^{+\nu_1}\ldots a_2^{+\nu_{n_2}}|0\rangle \\ &= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{n_1} \Phi_{(a)_{n_1},(b)_{n_2}}(x)\, a_1^{+a_1}\ldots a_1^{+a_{n_1}} a_2^{+b_1}\ldots a_2^{+b_{n_2}}|0\rangle, \end{aligned} \tag{4}$$

permits to realize equivalently the conditions (1 - 3) as the constraints on $|\Phi\rangle$.

To this end, it is introduced the covariant derivative D_μ in $\mathcal{H}$: $D_\mu = \partial_\mu - \omega^{ab}_\mu \left(\sum_{i=1}^{2} a^+_{ia}a_{ib} - \frac{1}{8}\gamma_{[a}\gamma_{b]}\right)$

,

$$D_\mu|\Phi\rangle = \sum_{n_1\geq n_2=0}^{\infty} [\nabla_\mu \Phi_{(\mu)_{s_1},(\nu)_{n_2}}(x)]\, a_1^{+\mu_1}\ldots a_1^{+\mu_{n_1}} a_2^{+\nu_1}\ldots a_2^{+\nu_{n_2}}|0\rangle.$$

Then the constraints

$$\boxed{\tilde{t}_0|\Phi\rangle = \tilde{t}^i|\Phi\rangle = t|\Phi\rangle = 0} \Longleftrightarrow \texttt{irreps (1)- (3) for each } n_1, n_2 \tag{5}$$

and considered as the primary ones in terms of the operators $[\beta = (2,3) \Leftrightarrow k_1 = (1,2)]$

$$\tilde{t}_0 = i\gamma^\mu D_\mu - m - r^{\frac{1}{2}}(g_0^1 - \beta), \qquad \tilde{t}^i = \gamma^\mu a^i_\mu, \tag{6}$$

$$t = a^{1+}_\mu a^{2\mu}, \qquad g_0^i = -a^{i+\mu} a^i_\mu + \frac{d}{2}.$$

Because of the fermionic nature of Eqs.(1), (2) with respect to the standard Grassmann parity, and due to bosonic nature of $\tilde{t}_0, \tilde{t}^i$, we introduce equivalent representation for constraints by transition to $(d+1)$ Grassmann-odd "gamma-matrix-like objects": $\gamma^\mu \longrightarrow (\tilde{\gamma}^\mu, \tilde{\gamma})$ with nondegenerate odd Lorentz-scalar supermatrix $\tilde{\gamma} : \gamma^\mu \longrightarrow \tilde{\gamma}^\nu$

$$\{\tilde{\gamma}^\mu, \tilde{\gamma}^\nu\} = 2g^{\mu\nu}, \qquad \{\tilde{\gamma}^\mu, \tilde{\gamma}\} = 0, \qquad \tilde{\gamma}^2 = -1 : \quad \gamma^\mu = \tilde{\gamma}^\mu \tilde{\gamma} : \varepsilon(\tilde{\gamma}^\mu, \tilde{\gamma}) = 1.$$

The above relations on $|\Phi\rangle$ in terms of ε-odd constraints

$$\tilde{t}'_0 = -i\tilde{\gamma}^\mu D_\mu + \tilde{\gamma}\left(m + r^{\frac{1}{2}}(g_0 - \beta)\right), \qquad t^i = \tilde{\gamma}^\mu a^i_\mu,$$

takes the form $\tilde{t}'_0|\Phi\rangle = t^i|\Phi\rangle = 0.$

Hermiticity of BRST operator to be constructed through the total set of constraints will be determined through an odd scalar product:

$$\langle\tilde{\Psi}|\Phi\rangle \;=\; \int d^d x \sqrt{|g|} \sum_{n_1,k_1,n_2,k_2=0}^{\infty} \langle 0| a_1^{\rho_1} \ldots a_1^{\rho_{k_1}} a_2^{\sigma_1} \ldots a_2^{\sigma_{k_2}} \Psi^+_{(\rho)_{k_1},(\sigma)_{k_2}}(x) \times$$
$$\times \tilde{\gamma}_0 \Phi_{(\mu)_{n_1},(\nu)_{n_2}}(x) a_1^{+\mu_1} \ldots a_1^{+\mu_{n_1}} a_2^{+\nu_1} \ldots a_2^{+\nu_{n_2}} |0\rangle. \tag{7}$$

THEN THE REQUIREMENTS (being by the part of Dirac-Bergmann algorithm for formal gauge theory with vanishing Hamiltonian): $\{o_I\}^+ \subset \{o_I\}$ and $[o_I, o_J\} \in \{o_I\}$

generate an COMMUTATOR SUPERALGEBRA $\mathcal{A}(Y(2), AdS_d)$ with central charge $\boxed{\tilde{m} = (m - \beta r^{\frac{1}{2}})}$:

$$\tilde{t}'_0 = -\imath\tilde{\gamma}^\mu D_\mu + \overbrace{\tilde{\gamma}\Big(m + r^{\frac{1}{2}}(g^1_0 - \beta)\Big)}^{\texttt{to modified} \quad o_I}, \qquad g^i_0 = -a^{i+}_\mu a^{i\mu} + \tfrac{d}{2}, \tag{8}$$

$$t^i = \tilde{\gamma}^\mu a^i_\mu, \qquad t^{i+} = \tilde{\gamma}^\mu a^{i+}_\mu, \tag{9}$$

$$t = a^{1+}_\mu a^{2\mu}, \qquad t^+ = a^{2+}_\mu a^{1\mu}, \tag{10}$$

$$l^i = -ia^i_\mu D^\mu, \qquad l^{i+} = -ia^{i+}_\mu D^\mu, \tag{11}$$

$$l^{ij} = \tfrac{1}{2} a^{i\mu} a^j_\mu, \qquad l^{ij+} = \tfrac{1}{2} a^{i\mu+} a^{j+}_\mu, \tag{12}$$

$$\tilde{l}'_0 = \overbrace{g^{\mu\nu}(D_\nu D_\mu - \Gamma^\sigma_{\mu\nu} D_\sigma)}^{D^2} - r\Big(\Big(\sum_{i=1}^{2} g^i_0 + t^{i+} t^i\Big) + \tfrac{d(d-5)}{4}\Big) + \overbrace{\Big(m + r^{\frac{1}{2}}(g^1_0 - \beta)\Big)^2}^{\texttt{to modified } o'_I}, \tag{13}$$

For the aims of LF CONSTRUCTION AND ADDITIVE CONVERSION CONDITION for $o_I \to O_I = o_I + o'_I, : [o_I, o'_J\} = 0$ it is sufficient to have the MODIFIED SUPERALGEBRA without c.c. (and green-typed parts in $\tilde{t}'_0$, $\tilde{l}'_0$) which given by the TABLE

$[\downarrow,\rightarrow\}$	t_0	t^i	t^{i+}	t	t^+	l_0	l^i	l^{i+}	l^{ij}	l^{ij+}	g_0^i
t_0	$-2l_0$	$2l^i$	$2l^{i+}$	0	0	0	M^i	$-M^{i+}$	0	0	0
t^k	$2l^k$	$4l^{ki}$	A^{ki}	$-t^2\delta^{k1}$	$-t^1\delta^{k2}$	$2M^k$	0	$-t_0\delta^{ik}$	0	$B^{k,ij}$	$t^i\delta^{ki}$
t^{k+}	$2l^{k+}$	A^{ik}	$4l^{ki+}$	$t^{1+}\delta^{k2}$	$t^{2+}\delta^{k1}$	$-2M^{k+}$	$t_0\delta^{ik}$	0	$-B^{k,ij+}$	0	$-t^{i+}\delta^{ki}$
t	0	$t^2\delta^{i1}$	$-t^{1+}\delta^{i2}$	0	$g_0^1-g_0^2$	0	$l^2\delta^{i1}$	$-l^{1+}\delta^{i2}$	D^{ij}	$-G^{ij+}$	F^i
t^+	0	$t^1\delta^{i2}$	$-t^{2+}\delta^{i1}$	$g_0^2-g_0^1$	0	0	$l^1\delta^{i2}$	$-l^{2+}\delta^{i1}$	G^{ij}	$-D^{ij+}$	$-F^{i+}$
l_0	0	$-2M^i$	$2M^{i+}$	0	0	0	$r\mathcal{K}_1^{bi+}$	$-r\mathcal{K}_1^{bi}$	0	0	0
l^k	$-M^k$	0	$-t_0\delta^{ik}$	$-l^2\delta^{k1}$	$-l^1\delta^{k2}$	$-r\mathcal{K}_1^{bi+}$	W^{ki}	X^{ki}	0	$-K^{k,ij+}$	$l^i\delta^{ik}$
l^{k+}	M^{k+}	$t_0\delta^{ik}$	0	$l^{1+}\delta^{k2}$	$l^{2+}\delta^{k1}$	$-r\mathcal{K}_1^{bi}$	$-X^{ik}$	$-W^{ki+}$	$K^{k,ij}$	0	$-l^{i+}\delta^{ik}$
l^{kl}	0	0	$B^{i,kl+}$	$-D^{kl}$	$-G^{kl}$	0	0	$-K^{i,kl}$	0	$L^{kl,ij}$	$l^{i\{k}\delta^{l\}i}$
l^{kl+}	0	$-B^{i,kl}$	0	G^{kl+}	D^{kl+}	0	$K^{i,kl+}$	0	$-L^{ij,kl}$	0	$-l^{i\{k+}\delta^{l\}i}$
g_0^k	0	$-t_k\delta^{ik}$	$t^{k+}\delta^{ik}$	$-F^k$	F^{k+}	0	$-l^k\delta^{ik}$	$l^{k+}\delta^{ik}$	$-l^{k\{i}\delta^{j\}k}$	$l^{k\{i+}\delta^{j\}k}$	0

Table 1: **The superalgebra $\mathcal{A}(Y(2), AdS_d)$ of the modified initial operators.**

$$\texttt{where}\ \ \mathbf{A^{ik}} = -2g_0^i\delta^{ik} + 2t\delta^{i2}\delta^{k1} + 2t^+\delta^{i1}\delta^{k2}\,, \qquad F^i = t(\delta^{i2} - \delta^{i1}),$$

$$\mathbf{B^{k,ij}} = -\tfrac{1}{2}t^{\{i+}\delta^{j\}k}\,, \qquad D^{ij} = l^{\{i2}\delta^{j\}1}\,,\,,$$

$$\mathbf{K^{k,ij}} = \tfrac{1}{2}l^{\{i}\delta^{j\}k}, \qquad \mathbf{G^{ij}} = l^{1\{i}\delta^{j\}2}\,, \tag{14}$$

$$\begin{aligned}\mathbf{L^{kl,ij}} \ = \ & \tfrac{1}{4}\Big\{\delta^{ik}\delta^{lj}\Big[2g_0^k\delta^{kl} + g_0^k + g_0^l\Big] \\ & -\delta^{ik}\Big[t\Big(\delta^{l2}(\delta^{j1} + \delta^{k1}\delta^{kj}) + \delta^{k2}\delta^{j1}\delta^{lk}\Big) + t^+\Big(\delta^{l1}(\delta^{j2} + \delta^{k2}\delta^{kj}) + \delta^{k1}\delta^{j2}\delta^{lk}\Big)\Big] \\ & -\delta^{lj}\Big[t\Big(\delta^{k2}(\delta^{i1} + \delta^{l1}\delta^{li}) + \delta^{l2}\delta^{i1}\delta^{kl}\Big) + t^+\Big(\delta^{k1}(\delta^{i2} + \delta^{l2}\delta^{li}) + \delta^{l1}\delta^{i2}\delta^{lk}\Big)\Big]\Big\}\end{aligned} \tag{15}$$

and the independent non-linear (quadratic) terms of the supercommutators has the form

$$\frac{1}{2}[t^i\,, l_0\} \ = \ r\left(2\sum_{j=1}^{2}t^{j+}l^{ji} + (g_0^i - \tfrac{1}{2})t^i - tt^1\delta^{i2} - t^+t^2\delta^{i1}\right) \equiv \mathbf{M^i} \tag{16}$$

$$[l_0\,, l^i\} \ = \ -r\Big(4\sum_{k=1}^{2}l^{k+}l^{ik} + (2g_0^i - 1)l^i - 2t^+l^2\delta^{i1} - 2tl^1\delta^{i2}\Big) \equiv \mathbf{r}\mathcal{K}_{\mathbf{1}}^{\mathbf{bi+}} \tag{17}$$

$$[l^{i+}\,, l^{j+}\} \ = \ -2r\varepsilon^{ij}\left[l^{12+}(g_0^2 - g_0^1) - l^{11+}t^+ + l^{22+}t\right] + \frac{r}{4}t^{[j+}t^{i+]} \equiv -\mathbf{W^{ij+}} \tag{18}$$

$$\begin{aligned}[l^i\,, l^{j+}\} \ = \ & \Big\{l_0 + r\Big(\textstyle\sum_k K_0^{1k} + K_0^{0i} + \frac{1}{2}K_0^{1i} + \mathcal{K}_0^{12}\Big)\Big\}\delta^{ij} \\ & +r\Big\{\Big[4\sum_k l^{1k+}l^{k2} - \tfrac{1}{2}t^{1+}t^2 + (\textstyle\sum_k g_0^k - \frac{3}{2})t'\Big]\delta^{j1}\delta^{i2} \\ & +r\Big\{\Big[4\sum_k l^{k2+}l^{1k} - \tfrac{1}{2}t^{2+}t^1 + t^+(\textstyle\sum_k g_0^k - \frac{3}{2})\Big]\delta^{j2}\delta^{i1} \equiv \mathbf{X^{ij}}.\end{aligned} \tag{19}$$

Here, the quantities $K_0^{1i}, K_0^{0i}, \mathcal{K}_0^{12}$, $i=1,2$ compose CASIMIR OPERATORS $\mathcal{K}_0, \mathcal{K}_{0b}$ for maximal Lie superalgebra and Lie algebra ($so(3,2)$) in superalgebra $\mathcal{A}(Y(2), AdS_d)$

$$\mathcal{K}_0 = \overbrace{\sum_i \left(K_0^{0i} + K_0^{1i}\right)}^{\mathcal{K}_{0b} - so(3,2)} + 2\mathcal{K}_0^{12}$$

$$\left(K_0^{0i}; K_0^{1i}; \mathcal{K}_0^{12}\right) = \Big((g_0^i)^2 - 2g_0^i - 4l^{ii+}l^{ii};\ g_0^i + t^{i+}t^i;\ t^+t - g_0^2 - 4l^{12+}l^{12}\Big).$$

It follows obvious Consequences and Classification from the Constrained Dynamical Systems for the superalgebra $\mathcal{A}(Y(2), AdS_d)$

Consequences

- $\mathcal{A}(Y(2), AdS_d)|_{s_2=0} \to \mathcal{A}(Y(1), AdS_d)$ – (J.Buchbinder, V.Kryhktin, A. Reshetnyak, NPB 2007)
- $\mathcal{A}(Y(2), AdS_d)|_{r=0} \to \mathcal{A}(Y(2), R^{1,d-1})$ – (Moshin,A.R, JHEP (2007))

Hamiltonian Constraints Classification

- $\{t_0, l_0\}$ –2 1st-class constraints;
- $\{t^k, t^{k+}, l^{ij}, l^{ij+}, t, t^+, l^i, l^{i+}\} = \{o_a\}$ – 16 2nd-class constraints $\Longrightarrow$ we can not apply BFV-BRST approach;
- $\{g_0^i, \widetilde{m}_0\}$ – $\Delta_{ab} = \Delta_{ab}(g_0^i, \tilde{m})$, $[o_a, o_b\} = \Delta_{ab}(g_0^i, \tilde{m}) + f_{ab}^c o_c$, $\quad g_0^i|\Phi\rangle \neq 0 \Longrightarrow \exists \|\Delta_{ab}^{-1}\|$

In turn, for integer spin case the massive of generalized spin $s=(s_1,s_2)$ AdS group irrep $D_b(E_0(m),s)$ are realized in the space of mixed-symmetry tensor fields $\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2}}(x) \in D(E_0,s_1,s_2)$, $E_0^{m=0} = \sqrt{r}(n_1-\frac{3}{2}+k_1+d)$ subject to analogous $Y(s_1,s_2)$ as above for the spin-tensors.

$\Phi_{(\mu^1)_{s_1},(\mu^2)_{s_2}}$ satisfy to Klein-Gordon equation, divergentless and traceless equations for each type of the indices and to the mixed-symmetry equation:

$$
\begin{aligned}
&\left(\nabla^2 + r[(s_1-k_1-2+d)(s_1-k_1-1)-s_1-s_2]+m^2\right\}\Phi_{\mu_1\cdots\mu_{s_1},\nu_1\cdots\nu_{s_2}}(x)=0\,,\\
&\nabla^{\mu_1}\Phi_{\mu_1\cdots\mu_{s_1},\nu_1\cdots\nu_{s_2}}(x)=\nabla^{\nu_1}\Phi_{\mu_1\cdots\mu_{s_1},\nu_1\cdots\nu_{s_2}}(x)=0\\
&g^{\mu_1\mu_2}\Phi_{\mu_1\mu_2\cdots\mu_{s_1},\nu_1\cdots\nu_{s_2}}=g^{\nu_1\nu_2}\Phi_{\mu_1\cdots\mu_{s_1},\nu_1\nu_2\cdots\nu_{s_2}}=g^{\mu_1\nu_1}\Phi_{\mu_1\cdots\mu_{s_1},\nu_1\nu_2\cdots\nu_{s_2}}=0,\\
&\Phi_{\{(\mu^1)_{s_1},\mu^2_1\}\mu^2_2\cdots\mu^2_{s_2}}(x)=0.
\end{aligned}
\tag{20}
$$

Eqs. (20) maybe realized also in terms of the operator conditions for the general state $|\Phi\rangle \in \mathcal{H}_b$ to be now by LORENTZ SCALAR in contrast to the BISPINOR vector $|\Psi\rangle$.:

$$
\widetilde{l}_0^b|\Phi\rangle=0\,,\qquad l_i|\Phi\rangle=0\qquad t|\Phi\rangle=0\,,\qquad l_{ij}|\Phi\rangle=0\,. \tag{21}
$$

$$
\begin{aligned}
\texttt{with}\ \widetilde{l}_0^b &= D^2 - r\tfrac{d(d-6)}{4}+\widetilde{m}^2+r\Big((g_1^0-2\beta-2)g_0^1-g_0^2\Big)\equiv l_0^b+more,\\
l^{ij} &= \frac{1}{2}a^{i\mu}a_{j\mu}\,,\qquad t=a^{1+\mu}a^2_\mu,\qquad l^i=-ia^{i\mu}D_\mu
\end{aligned}
$$

These operators generate the non-linear integer HS symmetry algebra $\mathcal{A}_b(Y(2), AdS_d)$ with only bosonic operators $\{o_{I_b}\} = \{o_I\} \setminus \{t_0, t^i, t^{i+}\}$ with central charge $\widetilde{m}^2$ subject to the same multiplication table as for $\mathcal{A}_(Y(2), AdS_d)$ with restrictions:

- there are no 3 rows and columns with fermionic constraints: t_0, t^i, t^{i+};
- with change of the cells with non-linear with red-typed quantities W^{ij+}, X^{ij}:

$$
\begin{aligned}
[l^{i+}, l^{j+}] &= -2r\varepsilon^{ij}\left[l^{12+}(g_0^2 - g_0^1) - l^{11+}t^+ + l^{22+}t\right] \equiv -W_b^{ij+} \\
[l^i, l^{j+}] &= \Big\{l_{0b} + r\Big(K_0^{0i} + \mathcal{K}_0^{12}\Big)\Big\}\delta^{ij} + r\Big\{\Big[4\sum_k l^{1k+}l^{k2} + (\textstyle\sum_k g_0^k - 2)t\Big]\delta^{j1}\delta^{i2} \\
&\quad + r\Big\{\Big[4\sum_k l^{k2+}l^{1k} + t^+(\textstyle\sum_k g_0^k - 2)\Big]\delta^{j2}\delta^{i1} \equiv \mathbf{X_b^{ij}}.
\end{aligned}
$$

- removing matrix $2^{\left[\frac{d}{2}\right]} \times 2^{\left[\frac{d}{2}\right]}$ structure of o_I to scalar structure of o_{I_b}.

As in the fermionic case <u>for LF construction and additive conversion</u> of $o_I, o'_I : o_I \rightarrow O_I = o_I + o'_I, [o_I, o'_J\} = 0$ it is sufficient to have the modified algebra without c.c. $\mathcal{A}_{bmod}(Y(2), AdS_d)$ with l_{0b} instead of $\widetilde{l}_0^b$.

Consequences and Hamiltonian Constraints Classification

1) $\mathcal{A}_b(Y(2), AdS_d) \underset{s_2=0}{\longrightarrow} \mathcal{A}_b(Y(1), AdS_d)$ (J.Buchbinder, V.Kryhktin, P.Lavrov, NPB 2006),

2) $\mathcal{A}_b(Y(2), AdS_d) \underset{r=0,m=0}{\longrightarrow} so(3,2) \cup \{l_0, l^i, l^{i+}\}$ (C.Burdik, A.Pashnev, M.Tsulaia, 2000).

3) $\mathcal{A}_b \supset \{l_0\}$ - *1 1ST CLASS C.*, $\mathcal{A}_b \supset \{l^{ij}, l^{ij+}, t, t^+, l^i, l^{i+}\}$ - *12 2ND CLASS C.*, $\mathcal{A}_b \supset \{g_0^i, \widetilde{m}^2\}$.

2b) Additive conversion of (super)algebra $\mathcal{A}_{mod}(Y(2), AdS_d)$ with 1st and 2nd class constraints into one $\mathcal{A}_c(Y(2), AdS_d)$ with only 1-class constraints system;

The following statement plays main role in solving this problem (A.Reshetnyak arXiv:0812.2329)

Proposition: If the set of $\{\tilde{o}_I\}, \{\tilde{o}_I\} : \mathcal{H} \to \mathcal{H}$ satisfy to nth order polynomial relations

$$[o_I, o_J\} \;=\; f_{IJ}^{k_1} o_{k_1} + \sum_{m=2}^{n} f_{IJ}^{k_1\cdots k_m} o_{k_1} \prod_{l=2}^{m} o_{k_l},\;\; f_{IJ}^{k_1\cdots k_m} = -(-1)^{\varepsilon_i \varepsilon_j} f_{JI}^{k_1\cdots k_m},$$

then from the requirement

$$o_J \longrightarrow O_J = o_J + o'_J : \{o'_I\} : \mathcal{H}' \to \mathcal{H}', \quad [o_I, o'_J\} = 0 \quad \mathcal{H}' \cap \mathcal{H} = \varnothing,$$

such that $[O_I, O_J\} = F_{IJ}^{K}(\tilde{o}', O) O_K$, set of $\{\tilde{o}'_J\}$, $\{\tilde{O}_J\}$ form the non-linear superalgebras $\mathcal{A}'$ given in $\mathcal{H}'$ and $\mathcal{A}_{con}$ in $\mathcal{H} \otimes \mathcal{H}'$ with the respective multiplication laws:

$$[o'_I, o'_J\} \;=\; f_{IJ}^{k_1} o'_{k_1} + \sum_{l=2}^{n} (-1)^{l-1+\varepsilon_{k_{(l)}}} f_{ij}^{k_l\cdots k_1} \prod_{s=1}^{l} o'_{k_s},\;\; \varepsilon_{k_{(n)}} = \sum_{s=1}^{n-1} \varepsilon_{k_s} \Big(\sum_{l=s+1}^{n} \varepsilon_{k_l} \Big), \tag{22}$$

$$[\tilde{O}_I, \tilde{O}_J\} \;=\; \Big(f_{IJ}^{k} + \sum_{l=2}^{n} F_{IJ}^{(l)k}(o', \tilde{O}) \Big) \tilde{O}_k, \quad \texttt{with } F_{IJ}^{(l)k} \texttt{ explicitly constructed w.r.t. } \; f_{ij}^{k_1\cdots k_n}.$$

So, for $n = 2$ (I.Buchbider,V.Kryktin,P.Lavrov 2006, I.Buchbider,V.Kryktin,A. Reshetnyak 2007) it follows the (super)algebras of o'_I: $\mathcal{A}'(Y(2), AdS_d)$ and one of O_I: $\mathcal{A}_c(Y(2), AdS_d)$ of our problem:

$$[o'_I, o'_J\} = f_{IJ}^{k} o'_k - (-1)^{\varepsilon(o_k)\varepsilon(o_m)} f_{IJ}^{km} o'_m o'_k\,, \tag{23}$$

$$[O_I, O_J\} = f_{IJ}^{k} O_k - \Big(f_{IJ}^{mk} + (-1)^{\varepsilon(O_k)\varepsilon(O_m)} f_{IJ}^{km} \Big) o'_m O_k + f_{IJ}^{km} O_k O_m. \tag{24}$$

2c) Verma module for (super)algebra $\mathcal{A}'(Y(2), AdS_d)$ of additional parts to constraints and its oscillator realization in Fock space $\mathcal{H}'$;

1) require in constructing of Verma module for $\mathcal{A}_c(Y(2), AdS_d)$ the boundary condition on Cartan subsuperalgebra elements (Hermitian operators) which must contain linearly an arbitrary independent parameters to be determined later from the requirement of reproducing correct conditions on irrep within LF on its final stage of construction:

$$t_0\Big\{l_0\Big\} \to t_0'(m_0, r)\Big\{l_0'(m_0^2, r)\Big\} = \tilde{\gamma} m_0(m_0^2) + ..., \qquad g_0 \to g_0'^i(h^i) = h^i + ...,$$

. Considering as a basis the BOSONIC CASE we choose $\tilde{m}'^2 = -\tilde{m}^2 \Rightarrow \tilde{M}^2 = \tilde{m}^2 + \tilde{m}'^2 = 0$

To find $\{o_I'\} = \{o_I'(b_{ij}, b_{ij}^+, b_k, b_k^+, b, b^+)\}$, $i, j, k, l = 1, 2; i \leq j$ as formal power series in creation and annihilation variables whose number of pairs coincides with ones for 2-nd class constraints o_a we need:

- construct the Verma module $V_{\mathcal{A}'}$ for non-linear $\mathcal{A}_b'(Y(2), AdS_d)$:

  `being fundamental representation space for` $\mathcal{A}_b'(Y(2), AdS_d)| \quad T: \mathcal{A}' \to End(V_{\mathcal{A}'})$

- find $\mathcal{H}'$-realization for $V_{\mathcal{A}'}$

generalizing the procedures developed for totally-symmetric HS fields C. Burdik, O. Navratil, A. Pashnev, for $\mathcal{A}_b'(Y(1), AdS_d)$; A. Kuleshov, A. Reshetnyak arXiv:0905.2705 for superalgebra $\mathcal{A}'(Y(1), AdS_d)$.

The composition law (proposition) for $\mathcal{A}_b'(Y(2), AdS_d)$ is the same as for $\mathcal{A}_b(Y(2), AdS_d)$ with changes:

$$\boxed{o_I \to o_I' \text{ and } [r\mathcal{K}_1^{bi}, W_b^{ki}, X_b^{ki} - l_{0b}] \to -[r\mathcal{K}_1'^{bi}, W_b'^{ki}, X_b'^{ki} - l_{0b}']}$$

.

2) Introduction of Cartan-like decomposition:

$$\mathcal{A}'_b(Y(2), AdS_d) = \Big\{\overbrace{l'^{ij+}, t'^{+}}^{so(3,2)}, \frac{l'^{i+}}{m_i}\Big\} \oplus \Big\{\overbrace{g_0'^{i}}^{so(3,2)}, l'_0\Big\} \oplus \Big\{\overbrace{l'^{ij}, t'}^{so(3,2)}, \frac{l'^{i}}{m_i}\Big\} \equiv \mathcal{E}^- \oplus H \oplus \mathcal{E}^+$$

3) highest weight representation

$$E'^{\alpha}|0\rangle_V = 0\,, \alpha > 0\,, \qquad (g_0'^{i}, l'_0)|0\rangle_V = (h^i, m_0^2)|0\rangle_V\,,$$

with positive root vectors $E'^{\alpha} \equiv (l'^{ij}, \frac{l'^1}{m_1}, t', \frac{l'^2}{m_2})$ $(\alpha > 0)$ for $i \leq j$.

4) basis space of Verma module following generalization of POINCARE–BIRKHOFF–WITT THEOREM

$$|\vec{N}\rangle_V \equiv |\vec{n}_{ij}, \vec{n}_s\rangle_V = \big(E'^{-\alpha_1^1}\big)^{n_{11}}\big(E'^{-\alpha_1^2}\big)^{n_{12}}\big(E'^{-\alpha_1^3}\big)^{n_{22}}\big(E'^{-\alpha_1^4}\big)^{n_1}\big(E'^{-\alpha_1^5}\big)^{n}\big(E'^{-\alpha_1^6}\big)^{n_2}|0\rangle_V,$$

where $\vec{n}_{ij} = (n_{11}, n_{12}, n_{22})$, $\vec{n}_s = (n_1, n, n_2)$, $n_k, n_{ij}, n \in \mathbb{N}_0$.

Note, in opposite to Verma module for totally symmetric $\mathcal{A}'(Y(1), AdS_d)$ there exists **not supercommuting non-linearly triple**: **entanglement of negative root vectors**

$$\big(E'^{-\alpha_1^4}\big)^{n_1}\big(E'^{-\alpha_1^5}\big)^{n}\big(E'^{-\alpha_1^6}\big)^{n_2} \backsim \left(\frac{l'^{1+}}{m_1}\right)^{n_1}(t'^{+})^{n}\left(\frac{l'^{2+}}{m_2}\right)^{n_2} \Leftarrow [l'^{1+}, l'^{2+}] = -W'^{12+}$$

. does not considered earlier

5) Using the multiplication for $\mathcal{A}'_b(Y(2), AdS_d)$ and the formula for the product of graded operators

P. Moshin, A. Reshetnyak (2007),

$$AB^n = \sum_{k=0}^{n}(-1)^{\varepsilon(A)\varepsilon(B)(n-k)}C^{(s)n}_{k}B^{n-k}\mathrm{ad}_B^k A\,,\ n \geq 0\,, s = \varepsilon(B)\,, \mathrm{ad}_B^k A = \mathrm{ad}_B\left(\mathrm{ad}_B^{k-1}A\right)\,,\ \mathrm{ad}_B A = [A, B\}$$

one can calculate the explicit form of the Verma module.

1									
1	1								
1	0	1							
1	1	1	1						
1	0	2	0	1					
1	1	2	2	1	1				
1	0	3	0	3	0	1			
1	1	3	3	3	3	1	1		
1	0	4	0	6	0	4	0	1	
1	1	4	4	6	6	4	4	1	1
...	...	...	...	...	...	...	...		

Table 2: Odd Pascal triangle

6) Interestingly note, the generalized coefficients for number of graded combinations, $C^{(s)n}_{k}$, standard for $s=0$, for $s=1$ are determined by the sequence of sums (with $[(k+1)/2]$ being by integer part of $(k+1)/2)$):

$$C^{(1)n}_{k} = \sum_{i_k=1}^{n-k+1} \sum_{i_{k-1}=1}^{n-i_k-k+2} \ldots \sum_{i_2=1}^{n-\sum_{j=3}^{k} i_j -1} \sum_{i_1=1}^{n-\sum_{j=2}^{k} i_j} (-1)^{k(n+1)+\sum\limits_{j=1}^{[(k+1)/2]}(i_{2j-1}+1)}. \qquad (25)$$

and visually are presented by an ODD ANALOG OF THE STANDARD PASCAL TRIANGLE

$\Longrightarrow$ To find the explicit action of all o'_I on an arbitrary $|\vec{n}_{ij}, \vec{n}_s\rangle_V \equiv \Big|\vec{N}\rangle_V$ in opposite to the case of $\mathcal{A}'(Y(1), AdS_d)$ we need:

7) start from the relations considered as BASIC BLOCKS to construct o'_I

So for CARTAN GENERATORS, NEGATIVE ROOT VECTORS and part of the POSITIVE ROOT VECTORS we have:

$$
\begin{aligned}
l'^{i+}\left|\vec{N}\rangle_V\right. &= \delta^{i2}\prod_{j=1}^{3}(E'^{-\alpha_1^j})^{n_j}\boxed{l'^{2+}}|\vec{0}_{lm},\vec{n}_s\rangle_V + \delta^{i1}m_l\,|\vec{n}_{lm},\vec{n}_s+(1,0,0)\rangle_V \\
l'^{+}_{ij}\left|\vec{N}\rangle_V\right. &= |\vec{n}_{lm}+\delta_{il}\delta_{jm},\vec{n}_s\rangle_V \\
t'^{+}\left|\vec{N}\rangle_V\right. &= -2n_{11}\,|\vec{n}_{lm}-(1,-1,0),\vec{n}_s\rangle_V - n_{12}\,|\vec{n}_{lm}+(0,-1,1),\vec{n}_s\rangle_V \\
&\quad + \prod_{j=1}^{3}(E'^{-\alpha_1^j})^{n_j}\boxed{t'^{+}}|\vec{0}_{lm},\vec{n}_s\rangle_V \\
g_0'^{i}\left|\vec{N}\rangle_V\right. &= \left(\sum_{l\le m}n_{lm}\left(\delta^{il}+\delta^{im}\right)+n_k\delta^{ik}+n\left(\delta^{i2}-\delta^{i1}\right)+h^i\right)|\vec{n}_{lm},\vec{n}_s\rangle_V \\
t'\left|\vec{N}\rangle_V\right. &= -n_{12}|\vec{n}_{lm}+(1,-1,0),\vec{n}_s\rangle_V - 2n_{22}|\vec{n}_{lm}+(0,1,-1),\vec{n}_s\rangle_V \\
&\quad + n(h^1-h^2-n_2-n+1)\,|\vec{n}_{lm},n_1,n-1,n_2\rangle_V + \prod_{j=1}^{5}(E'^{-\alpha_1^j})^{n_j}\boxed{t'}\left|\vec{0}_{lm},0,0,n_2\rangle_V\right.
\end{aligned}
$$

$$
\begin{aligned}
l'^{12}\left|\vec{N}\rangle_V\right. &= \frac{n_{12}}{4}\Big(2n_{11}+n_{12}+2n_{22}+\sum_k(n_k+h^k)-1\Big)|\vec{n}_{lm}+(0,-1,0),\vec{n}_s\rangle_V \\
&+\frac{1}{2}nn_{11}(h^2-h^1+n_2+n-1)\,|\vec{n}_{lm}-(1,0,0),\vec{n}_s-(0,1,0)\rangle_V \\
&+n_{11}n_{22}\,|\vec{n}_{lm}-(1,-1,1),\vec{n}_s\rangle_V + \left\{\prod_{j=1}^{3}(E'^{-\alpha_1^j})^{n_j}\boxed{l'^{12}} - \frac{n_{22}}{2}\prod_{j=1}^{3}(E'^{-\alpha_1^j})^{n_j-\delta_{j3}}\boxed{t'^{+}}\right\}|\vec{0}_{lm},\vec{n}_s\rangle_V \\
&-\frac{n_{11}}{2}\prod_{j=1}^{5}(E'^{-\alpha_1^j})^{n_j-\delta_{j1}}\boxed{t'}|\vec{\mathbf{0}},n_2\rangle_V,
\end{aligned}
$$

from which it follows we have to know the action of: $\left(l'^{2+}, t'^{+}, l'^{12}\right)$ `on` $\left|\vec{0}_{lm}, \vec{n}_s\right\rangle_V$, t' `on` $\left|\vec{0}_{lm}, 0, 0, n_2\right\rangle_V$ and some others with less complicated structure.

8) To this end we with use of the Casimir operator for $so(3,2)$ Lie algebra

$$\mathcal{K}_{0b} \;=\; \sum_i (g_0'^{i\,2} - 2g_0'^{i} - 4l'^{ii+}l'^{ii}) + 2(t'^{+}t' - g_0'^{2} - 4l'^{12+}l'^{12})$$

calculate the 9 independent types of non-linear commutators, for instance 3 from them for above:

$$\varepsilon_{ij}\left[l'^{i+}, \left(\frac{l'^{j+}}{m_j}\right)^{n_j}\right] \;=\; -\sum_{m=0}^{[n_j-1/2]}\left(\frac{-2rl'^{jj+}}{m_j^2}\right)^{m}\left(\frac{l'^{j+}}{m_j}\right)^{n_j-2m-2}\left[\frac{l'^{j+}}{m_j}\frac{C^{n_j}_{2m+1}}{m_j}(\boxed{-W_b'^{12+}}) + \frac{C^{n_j}_{2m+2}}{m_j^2}(\boxed{-W_j'^{12+}})\right],$$

$$\left[l'^{ii}, \left(\frac{l'^{i+}}{m_i}\right)^{n_i}\right] \;=\; -\frac{n_i}{m_i}\left(\frac{l'^{i+}}{m_i}\right)^{n_i-1} l'^{i} - \frac{n_i(n_i-1)}{2m_i^2}\left(\frac{l'^{i+}}{m_i}\right)^{n_i-2}\boxed{K_2'^{0i}} + 2r\sum_{m=1}^{[(n_i-1)/2]}\left(-8rl'^{ii+}\right)^{m-1}$$
$$\times\left(\frac{l'^{i+}}{m_i}\right)^{n_i-2m-2}\left(\frac{1}{m_i}\right)^{2m+1}\left\{\frac{l'^{i+}}{m_i}C^{n_i}_{2m+1}\left[\boxed{\mathcal{K}_1^{i}} - \frac{2}{4^m}\boxed{\mathcal{K}_1^{12i}}\right] + \frac{1}{m_i}C^{n_i}_{2m+2}\left[\boxed{\mathcal{K}_2^{i}} - \frac{2}{4^m}\boxed{\mathcal{K}_2^{12i}}\right]\right\},$$

$$\left[l'^{12}, \left(\frac{l'^{i+}}{m_i}\right)^{n_i}\right] \;=\; -\frac{n_i}{2m_i}\left(\frac{l'^{i+}}{m_i}\right)^{n_i-1} l'^{\{1}\delta^{2\}i} - \delta^{i2}\frac{1}{2}\sum_{m=1}^{[n_2/2]}\left(-2rl'^{22+}\right)^{m-1}\left(\frac{l'^{2+}}{m_2}\right)^{n_2-2m-1}\left(\frac{1}{m_2}\right)^{2m}$$
$$\times\left[\frac{l'^{2+}}{m_2}C^{n_2}_{2m}\boxed{X_b'^{12}} + \frac{1}{m_2}C^{n_2}_{2m+1}\boxed{X_2'^{12}}\right] - \delta^{i1}\frac{1}{2}\sum_{m=1}^{[n_1/2]}\left(-2rl'^{11+}\right)^{m-1}\left(\frac{l'^{1+}}{m_1}\right)^{n_1-2m-1}$$
$$\times\left(\frac{1}{m_1}\right)^{2m}\left[\frac{l'^{1+}}{m_1}C^{n_1}_{2m}\boxed{X_b'^{21}} + \frac{1}{m_1}C^{n_1}_{2m+1}\boxed{X_1'^{21}}\right].$$

with completely definite operators derived from the components of $\mathcal{K}_{0b}$ and from the products of the superalgebra $\mathcal{A}'_b(Y(2), AdS_d)$: $W_b'^{12+}, X_b'^{ij}$: $\left\{\mathcal{K}_1^{bi}, \mathcal{K}_2^{i}, W_i'^{12+}, X_1'^{21}\right\} = \left\{[\mathcal{K}_{0b}, l'^{i+}], [\mathcal{K}_{1b}^{i}, l'^{i+}], \ldots\right\},$

It permits to express, for instance, the action of t'^{+} on vector $|\vec{0}_{lm},\vec{n}_s\rangle_V$ through the action of t':

$$\mathbf{t}'^{+}|\vec{0}_{lm},\vec{n}_s\rangle_V=\sum_{m=0}^{[n_1/2]}\left(\frac{-2r}{m_1^2}\right)^m\Big\{C^{n_1}_{2m}|m,0,0,n_1-2m,n+1,n_2\rangle_V-C^{n_1}_{2m+1}\frac{m_2}{m_1}|m,0,0,n_1-2m-1,n,n_2+1\rangle_V\Big\}$$

$$-\sum_{m=1}^{[n_1/2]}\left(\frac{-2r}{m_1^2}\right)^m\Big\{\Big(C^{n_1}_{2m}(h^2-h^1+2n+n_2)-C^{n_1}_{2m+1}\Big)|m-1,1,0,n_1-2m,n,n_2\rangle_V$$

$$+C^{n_1}_{2m}n(h^1-h^2-n_2-n+1)|m-1,0,1,n_1-2m,n-1,n_2\rangle_V\Big\}$$

$$-\sum_{m=1}^{[n_1/2]}\left(\frac{-2r}{m_1^2}\right)^m(l'^{11+})^{m-1}\left(\frac{l'^{1+}}{m_1}\right)^{n_1-2m}C^{n_1}_{2m}l'^{22+}t'^{+n}\boxed{\mathbf{t}'}|\vec{0},0,0,n_2\rangle_V,.$$

9) Then we find the RECURRENT RELATIONS:

$$\mathbf{t}'|\tilde{\mathbf{0}},\mathbf{0},\mathbf{0},\mathbf{n_2}\rangle_{\mathbf{V}}=|A_{n_2}\rangle_V-\sum_{{}^1m=0}^{[n_2/2]}\sum_{{}^1l=0}^{[n_2/2-{}^1m-1)/2]}\left(\frac{-2rl'^{22+}}{m_2^2}\right)^{{}^1m+{}^1l+1}C^{n_2}_{2^1m+1}C^{n_2-2^1m-1}_{2^1l+1}\boxed{t'|\vec{0},n_2-2({}^1m+{}^1l+1)\rangle_V.}$$

WITH VECTOR $|A_{n_2}\rangle_V=\sum_{m=1}^{[n_2/2]}\left(\frac{-2r}{m_2^2}\right)^m\Big\{(C^{n_2}_{2m}(h^2-h^1)+C^{n_2}_{2m+1})|0,1,m-1,0,0,n_2-2m\rangle_V$

$$-C^{n_2}_{2m}|1,0,m-1,0,1,n_2-2m\rangle_V\Big\}-\frac{m_1}{m_2}\sum_{m=0}^{[n_2/2]}\left(\frac{-2r}{m_2^2}\right)^m C^{n_2}_{2m+1}|0,0,m,1,0,n_2-2m-1\rangle_V$$

$$-\sum_{{}^1m=0,{}^1l=0}^{[n_2/2],[n_2/2-{}^1m-1]}\left(\frac{-2r}{m_2^2}\right)^{{}^1m+{}^1l+1}C^{n_2}_{2^1m+1}\Big\{\Big[C^{n_2-2^1m-1}_{2^1l+1}(h^2-h^1+n_2-2({}^1m+{}^1l+1))$$

$$-C^{n_2-2^1m-1}_{2^1l+2}\Big]|0,1,{}^1m+{}^1l,0,0,n_2-2({}^1m+{}^1l+1)\rangle_V-C^{n_2-2^1m-1}_{2^1l+1}|1,0,{}^1m+{}^1l,0,1,n_2-2({}^1m+{}^1l+1)\rangle_V$$

$$+\frac{m_1}{m_2}C^{n_2-2^1m-1}_{2^1l+2}|0,0,{}^1m+{}^1l+1,1,0,n_2-2({}^1m+{}^1l+1)-1\rangle_V\Big\}.$$

$\Longrightarrow$ the maximal degree of l'^{+}_2 decreases on 2 $\Rightarrow$ one may to find the direct action of t' resolving the recurrent relations:

$$\mathbf{t}'|\vec{\tilde{0}},0,0,n_2\rangle_V = \sum_{k=0}^{[(n_2-1)/2]}\Biggl\{\sum_{{}^1m=0}^{[n_2/2]}\sum_{{}^1l=0}^{[n_2/2-({}^1m+1)]}\cdots\cdots\sum_{{}^km=0}^{\left[n_2/2-\sum\limits_{i=1}^{k-1}({}^im+{}^il)-(k-1)\right]}\sum_{{}^kl=0}^{\left[n_2/2-\sum\limits_{i=1}^{k-1}({}^im+{}^il)-{}^km)-k\right]}(-1)^k$$

$$\times\left(\frac{-2r}{m_2^2}\right)^{\sum_{i=1}^k({}^im+{}^il)+k} C^{n_2}_{2{}^1m+1}C^{n_2-2{}^1m-1}_{2{}^1l+1}\ldots C^{n_2-2(\sum_{i=1}^{k-1}({}^im+{}^il)-k+1}_{2^km+1}C^{n_2-2(\sum_{i=1}^{k-1}({}^im+{}^il)-2^km-k}_{2^kl+1}$$

$$\times\boxed{|A_{0,0,\sum_{i=1}^k({}^im+{}^il)+k,0,0,n_2-2[\sum\limits_{i=1}^{k}({}^im+{}^il)+k]}\rangle_V}\Biggr\}, \quad |A_{0,0,m,0,0,n_2}\rangle_V \equiv (l'^{22+})^m|A_{n_2}\rangle_V$$

where the EXTERNAL SUM CONTAINS $[(n_2-1)/2]$ INTERNAL DOUBLE SUMS, and for $k=0$ we have only $|A_{n_2}\rangle_V$

10) $\Longrightarrow$ the action of o'_I on $|\vec{N}\rangle_V$ are found using $\mathbf{t}'|\vec{\mathbf{0}},0,0,n_2\rangle_V$ as the **BASIC BLOCK**. For instance, the action of t', l'^{2+}, l'^{12} has the exact form:

$$\mathbf{t}'|\tilde{\mathbf{N}}\rangle_\mathbf{V} = n(h^1-h^2-n_2-n+1)\,|\vec{n}_{lm},\vec{n}_s-(0,1,0)\rangle_V - n_{12}|\vec{n}_{lm}+(1,-1,0),\vec{n}_s\rangle_V$$

$$-2n_{22}|\vec{n}_{lm}+(0,1,-1),\vec{n}_s\rangle_V + \sum_{k=0}^{[(n_2-1)/2]}\Biggl\{\sum_{{}^1m=0}^{[n_2/2]}\sum_{{}^1l=0}^{[n_2/2-({}^1m+1)]}\cdots\cdots\sum_{{}^km=0}^{\left[n_2/2-\sum\limits_{i=1}^{k-1}({}^im+{}^il)-(k-1)\right]}\sum_{{}^kl=0}^{\left[n_2/2-\sum\limits_{i=1}^{k-1}({}^im+{}^il)-{}^km-k\right]}$$

$$\times(-1)^k\left(\frac{-2r}{m_2^2}\right)^{\sum_{i=1}^k({}^im+{}^il)+k} C^{n_2}_{2{}^1m+1}C^{n_2-2{}^1m-1}_{2{}^1l+1}\ldots C^{n_2-2(\sum_{i=1}^{k-1}({}^im+{}^il)-k+1}_{2^km+1}C^{n_2-2(\sum_{i=1}^{k-1}({}^im+{}^il)-2^km-k}_{2^kl+1}$$

$$\left|A_{\vec{n}_{lm}+(0,0,\sum_{i=1}^k({}^im+{}^il)+k),n_1,n,n_2-2[\sum\limits_{i=1}^{k}({}^im+{}^il)+k]}\right\rangle_V\Biggr\} \equiv \qquad (26)$$

$$\equiv \boxed{\begin{array}{c} n(h^1-h^2-n_2-n+1)\,|\vec{n}_{lm},\vec{n}_s-(0,1,0)\rangle_V - n_{12}|\vec{n}_{lm}+(1,-1,0),\vec{n}_s\rangle_V \\ -2n_{22}|n_{lm}+(0,1,-1),\vec{n}_s\rangle_V + \widehat{t'}|\vec{N}\rangle_V \end{array}},$$

with the vectors $\left|A_{\vec{n}_{lm}}+(0,0,q,n_1,n,n_2-2q]\right\rangle_V$ slightly modified as to $|A_{n_2}\rangle_V$.

$$\mathbf{l'^{2+}}|\vec{N}\rangle_V = m_2\sum_{m=0}^{[n_1+1/2]}\left(\frac{-2r}{m_1^2}\right)^m C^{n_1}_{2m}|\vec{n}_{lm}+(m,0,0),n_1-2m,n,n_2+1\rangle_V + m_1\sum_{m=0}^{[n_1-1/2]}\left(\frac{-2r}{m_1^2}\right)^{m+1}\Big\{\Big(C^{n_1}_{2m+1}(h^2-h^1+2n+n_2)$$
$$-C^{n_1}_{2m+2}\Big)|\vec{n}_{lm}+(m,1,0),n_1-2m-1,n,n_2\rangle_V + C^{n_1}_{2m+1}\Big(n(h^1-h^2-n_2-n+1)|\vec{n}_{lm}+(m,0,1),n_1-2m-1,n-1,n_2\rangle_V$$
$$-|\vec{n}_{lm}+(m+1,0,0),n_1-2m-1,n+1,n_2\rangle_V\Big)\Big\} + m_1\sum_{m=0}^{[n_1-1/2]}\left(\frac{-2r}{m_1^2}\right)^{m+1} C^{n_1}_{2m+1}\boxed{\widehat{t'}|\vec{n}_{lm}+(m,0,1),n_1-2m-1,n,n_2\rangle_V},$$
$$\mathbf{l'^{12}}|\vec{N}\rangle_V = \frac{n_{12}}{4}\Big(2n_{11}+n_{12}+2n_{22}+\sum_k(n_k+h^k)-1\Big)|\vec{n}_{lm}-(0,1,0),\vec{n}_s\rangle_V + \frac{1}{2}nn_{11}(h^2-h^1+n_2+n-1)$$
$$\times|\vec{n}_{lm}-(1,0,0),\vec{n}_s-(0,1,0)\rangle_V + n_{11}n_{22}\,|\vec{n}_{lm}-(1,-1,1),\vec{n}_s\rangle_V$$
$$+\mathbf{l'^{12}}|\vec{0}_{lm},\vec{n}_s\rangle_V\,\big|_{[\vec{0}_{lm}\to\vec{n}_{lm}]} - \frac{n_{22}}{2}\widehat{t'}^{+}\,|\vec{0}_{lm},\vec{n}_s\rangle_V\,\big|_{[\vec{0}_{lm}\to\vec{n}_{lm}-(0,0,1)]} - \frac{n_{11}}{2}\widehat{t'}\,|\vec{n}_{lm}-(1,0,0),\vec{n}_s\rangle_V, \qquad (27)$$

where to determine $\mathbf{l'^{12}|N\rangle_V}$ the ADDITIONAL BLOCKS (having the exact form and including the basic one with $\widehat{t'}|\vec{n}_{lm},\vec{n}_s\rangle_V$) are used.

$\Longrightarrow$ The actions of all other positive root vectors: l'^{i}, l'_0, l'^{ii} on $|\vec{N}\rangle_V$ are determined in the same form.

$\Longrightarrow$ Thus, we state that the VERMA MODULE $V_{\mathcal{A}'_b}$ FOR NON-LINEAR ALGEBRA $\mathcal{A}'_b(Y(2),AdS_d)$ IS CONSTRUCTED.

$\Longrightarrow$ In case of SUPERALGEBRA $\mathcal{A}'_(Y(2),AdS_d)$ the construction of the $V_{\mathcal{A}'_b}$ does not significantly complicate, but we have the enlarged Cartan-like decomposition extended by fermionic t'^{i+}, t'_0, t'^{i} for $\mathcal{E}^-\oplus H\oplus\mathcal{E}^+$.

The general vector $|\vec{N}\rangle_V \to |\vec{N}^f\rangle_V$:

$$|\vec{N}^f\rangle_V = \Big|\vec{n}^0_k,\vec{N}\rangle_V \equiv (t'^{1+})^{n^0_1}(t'^{2+})^{n^0_2}|\vec{N}\rangle_V,\, n^0_k=0,1, \qquad \text{and} \quad \boxed{(g'^{i}_0,t'_0)|0\rangle_V = (h^i,\tilde{\gamma}m_0)|0\rangle_V}$$

$\Longrightarrow$ for instance, the action of t' on $|\vec{N}\rangle_V$ is easily established through its action on $\left|\vec{0}^0_k, \vec{0}_{ij}, 0, 0, n_2\right\rangle_V$ and the same form as in bosonic case but with modified $|A^f_{n_2}\rangle_V$:

$$|A^f_{n_2}\rangle_V = \sum_{m=1}^{[n_2/2]}\left(\frac{-2r}{m_2^2}\right)^m \Big\{(C^{n_2}_{2m}(h^2-h^1-\frac{1}{2}) + C^{n_2}_{2m+1})|\vec{0}^0_k, 0, 1, m-1, 0, 0, n_2-2m\rangle_V$$
$$+C^{n_2}_{2m}\Big[\frac{1}{4}|\vec{1}^0_k, 0, 0, m-1, 0, 0, n_2-2m\rangle_V - |\vec{0}^0_k, 1, 0, m-1, 0, 1, n_2-2m\rangle_V\Big]\Big\}$$
$$-\frac{m_1}{m_2}\sum_{m=0}^{[n_2/2]}\left(\frac{-2r}{m_2^2}\right)^m C^{n_2}_{2m+1}|\vec{0}^0_k, 0, 0, m, 1, 0, n_2-2m-1\rangle_V - \sum_{{}^1m=0,{}^1l=0}^{[n_2/2],[n_2/2-{}^1m-1]}\left(\frac{-2r}{m_2^2}\right)^{{}^1m+{}^1l+1} C^{n_2}_{2{}^1m+1}\Big\{\Big[C^{n_2-2{}^1m-1}_{2{}^1l+1}(h^2$$
$$-h^1-\frac{1}{2}+n_2-2({}^1m+{}^1l+1)) - C^{n_2-2{}^1m-1}_{2{}^1l+2}\Big]|\vec{0}^0_k, 0, 1, {}^1m+{}^1l, 0, 0, n_2-2({}^1m+{}^1l+1)\rangle_V$$
$$+C^{n_2-2{}^1m-1}_{2{}^1l+1}\Big[\tfrac{1}{4}|\vec{1}^0_k, 0, 0, {}^1m+{}^1l, 0, 0, n_2-2({}^1m+{}^1l+1)\rangle_V - |\vec{0}^0_k, 1, 0, {}^1m+{}^1l, 0, 1, n_2-2({}^1m+{}^1l+1)\rangle_V\Big]\Big\}.$$

11) Then, making use of the mapping (C. Burdik, 1985)

$$|\vec{n}^0_k, \vec{n}_{ij}, \vec{n}_s\rangle_V \leftrightarrow |\vec{n}^0_k, \vec{n}_{ij}, \vec{n}_s\rangle = (f_1^+)^{n_1^0}(f_2^+)^{n_2^0}(b^+_{11})^{n_{11}}(b^+_{12})^{n_{12}}(b^+_{22})^{n_{22}}(b^+_1)^{n_1}(b^+)^{n}(b^+_2)^{n_2}|0\rangle\,,$$

where red-typed symbols correspond to only superalgebra $\mathcal{A}'_(Y(2), AdS_d)$, $|\vec{n}^0_k, \vec{n}_{ij}, n_1, n, n_2\rangle$ are THE BASIS VECTORS OF A FOCK SPACE $\mathcal{H}'$, for $n^0_k = 0, 1,\ n_k, n_{ij}, n \in \mathbb{N}_0$, and $(2+6)$ pairs of creation and annihilation operators

$$\{f_k\,, f_l^+\} = \delta_{kl}\,,\quad [b_k\,, b_l^+] = \delta_{kl}\,,\quad [b_{ij}, b^+_{lk}] = \delta_{il}\delta_{jk}\,,\ i \le j, k \le l\,,\quad [b\,, b^+] = 1\,,$$

$\Longrightarrow$ $V_{\mathcal{A}'}$ REPRESENTS AS FORMAL POWERS SERIES (DUE TO R) IN THE CREATION AND ANNIHILATION OPERATORS OF $\mathcal{H}'$ (for simplicity BOSONIC CASE) WITH HELP OF CORRESPONDENCE:

$$\boxed{C^{n_1}_{2l}\left(C^{n_2}_{2m} + C^{n_2}_{2m+1}\right)|\vec{n}_{lm} + (l, 0, m), \vec{n}_s - (2l, 0, 2m)\rangle_V \longleftrightarrow \frac{1}{(2l)!}\Big\{\frac{1}{(2m)!} + \frac{b_2^+b_2}{(2m+1)!}\Big\}(b^+_{11}b^2_1)^l(b^+_{22}b^2_2)^m\left|\vec{N}\right\rangle}$$

$\Longrightarrow$ The negative root vectors (with omitting nontrivial l'^{2+}, t'^{+})

$$l'^{1+} = m_1 b_1^+\,, \qquad l'^{+}_{ij} = b^+_{ij} \qquad g'^{i}_0 = \sum_{l\leq m} b^+_{lm} b_{lm}(\delta^{il} + \delta^{im}) + b^+ b(\delta^{i2} - \delta^{i1}) + b^+_i b_i + h^i.$$

$\Longrightarrow$ the BASIC INITIAL BLOCK:

$$\begin{aligned}
\boxed{t'} \;=\;& \left(h^1 - h^2 - b_2^+ b_2 - b^+ b\right) b - b^+_{11} b_{12} - 2 b^+_{12} b_{22} + \sum_{k=0}\Bigg[\sum_{{}^1m=0}\sum_{{}^1l=0}\cdots\sum_{{}^km=0}\sum_{{}^kl=0}(-1)^k\left(\frac{-2r}{m_2^2}\right)^{\sum_{i=1}^k({}^im+{}^il)+k} \\
& \times\prod_{i=1}^k \frac{1}{(2^im+1)!}\frac{1}{(2^il+1)!}(b^+_{22})^{\sum_{i=1}^k({}^im+{}^il)+k}\Bigg\{b_2^+ b b_2 - \frac{m_1}{m_2}\sum_{m=0}\left(\frac{-2r}{m_2^2}\right)^m \frac{(b^+_{22})^m}{(2m+1)!} b_1^+ b_2^{2m+1} \\
& + \sum_{m=1}\left(\frac{-2r}{m_2^2}\right)^m (b^+_{22})^{m-1}\left[b^+_{12}\left\{\frac{h^2 - h^1 + 2b^+ b}{(2m)!} + \frac{b_2^+ b_2}{(2m+1)!}\right\} - \frac{b^+_{11} b^+}{(2m)!} - \frac{b^+_{22}}{(2m)!}(h^2 - h^1 + b^+ b) b\right] b_2^{2m} \\
& - \sum_{m=0}\sum_{l=0}\left(\frac{-2r}{m_2^2}\right)^{m+l+1}\frac{1}{(2m+1)!}(b^+_{22})^{m+l}\left[b^+_{12}\left\{\frac{h^2 - h^1 + 2b^+ b + b_2^+ b_2}{(2l+1)!} - \frac{b_2^+ b_2}{(2l+2)!}\right\} - \frac{b^+_{11} b^+}{(2l+1)!}\right. \\
& \left. - \frac{b^+_{22}}{(2l+1)!}(h^2 - h^1 + b_2^+ b_2 + b^+ b) b + \frac{m_1}{m_2}\frac{b^+_{22}}{(2l+2)!} b_1^+ b_2\right] b_2^{2(m+l+1)}\Bigg\}(b_2)^{2(\sum_{i=1}^k({}^im+{}^il)+k)}\Bigg] \equiv \\
& \equiv \boxed{\left(h^1 - h^2 - b_2^+ b_2 - b^+ b\right) b - b^+_{11} b_{12} - 2 b^+_{12} b_{22} + \widehat{t'}}
\end{aligned}$$

$$\begin{aligned}
\Longrightarrow \boxed{l'^{2+}} \;=\;& m_2\sum_{m=0}\left(\frac{-2r}{m_1^2}\right)^m (b^+_{11})^m \frac{b_2^+}{(2m)!} b_1^{2m} + m_1\sum_{m=0}\left(\frac{-2r}{m_1^2}\right)^{m+1}(b^+_{11})^m\Bigg\{b^+_{12}\left[\frac{(h^2 - h^1 + 2b^+ b + b_2^+ b_2)}{(2m+1)!} - \frac{b_1^+ b_1}{(2m+2)!}\right] \\
& + b^+_{22}\frac{(h^1 - h^2 - b^+ b - b_2^+ b_2)}{(2m+1)!} b - b^+_{11}\frac{b^+}{(2m+1)!}\Bigg\} b_1^{2m+1} - + m_1\sum_{m=0}\left(\frac{-2r}{m_1^2}\right)^{m+1}\frac{(b^+_{11})^m b^+_{22}}{(2m+1)!}\boxed{\widehat{t'}}\, b_1^{2m+1},
\end{aligned}$$

$$\Longrightarrow \boxed{\mathbf{l}'^{12}} = \frac{1}{4}\Big(2b_{11}^+b_{11} + b_{12}^+b_{12} + 2b_{22}^+b_{22} + \sum_k (b_k^+b_k + h^k) - 1\Big)b_{12} + \frac{1}{2}(h^2 - h^1 + b_2^+b_2 + b^+b)b_{11}b$$

$$+b_{12}^+b_{11}b_{22} + \frac{1}{4}\sum_{m=0}\sum_{l=1}\left(\frac{-2r}{m_2^2}\right)^l\left(\frac{-2r}{m_1^2}\right)^m \frac{(b_{11}^+)^m}{(2m)!}b^+\Big(\frac{(b_{22}^+)^m}{(2l)!}(h^1 + h^2 - 2) + \frac{b_2^+b_2}{(2l+1)!}\Big)b_1^{2m}b_2^{2l}$$

$$-\frac{1}{2m_1}\sum_{m=0}\left(\frac{-2r}{m_1^2}\right)^m \frac{(b_{11}^+)^m}{(2m+1)!}\boxed{\widehat{l}'^2}\, b_1^{2m+1} - \frac{1}{2}\boxed{\widehat{t}'^+}\, b_{22} - \frac{1}{2}\boxed{\widehat{t}'}\, b_{11}$$

$$+\sum_{m=1}\left(\frac{-2r}{m_1^2}\right)^m (b_{11}^+)^{m-1}\Bigg[\frac{b_{12}^+}{(2m)!}\boxed{\widehat{l}'^{22}} + \frac{1}{4}\Big\{\frac{\boxed{\widehat{t}'}}{(2m)!}(h^1 + h^2 + b_2^+b_2 - 2) + \frac{b_1^+b_1}{(2m+1)!}\boxed{\widehat{t}'}\Big\}$$

$$+\frac{1}{4}\Big\{\frac{h^1 + h^2 + b_2^+b_2 - 2}{(2m)!} + \frac{b_1^+b_1}{(2m+1)!}\Big\}(h^1 - h^2 - b_2^+b_2 - b^+b)b\Big\}\Bigg]b_1^{2m},$$

with "BLOCK"-OPERATORS $\widehat{l}'^2, \widehat{t}'^+, \widehat{l}'^{22}$ having the same form as $\widehat{t}'$.

$\Longrightarrow$ $\mathcal{H}'$-realization of all other opeartors are determined in a similar way.

$\Longrightarrow$ $\mathcal{H}'$-realization of VERMA MODULE FOR NON-LINEAR ALGEBRA $\mathcal{A}'_b(Y(2), AdS_d)$ IS CONSTRUCTED.

The set of o'_I is invariant w.r.t. the new Hermitian conjugation defined by the operator K' in the

$$\langle\tilde{\Psi}_1|K'E^{-\prime\alpha}|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'E^{\prime\alpha}|\Psi_1\rangle^*, \qquad \langle\tilde{\Psi}_1|K'g_0^{\prime i}|\Psi_2\rangle = \langle\tilde{\Psi}_2|K'g_0^{\prime i}|\Psi_1\rangle^*. \qquad (28)$$

where $K' = Z^+Z, \qquad Z = \sum_{(\vec{n}_{lm},\vec{n}_s)=(\vec{0},\vec{0})}^{\infty}\ \sum_{\vec{n}_k^0=(0,0)}^{(1,1)} |\vec{n}_k^0, \vec{n}_{lm}, \vec{n}_s\rangle_V \frac{1}{(\vec{n}_{lm})!\vec{n}_s!}\langle 0|b^{n_2}b^nb^{n_1}b_{11}^{n_{11}}b_{12}^{n_{12}}b_{22}^{n_{22}}f_1^{n_1^0}f_2^{n_2^0},$

$\Longrightarrow$ for the HERMITICITY OF BFV-BRST OPERATOR $Q(Q_b)$ for $\mathcal{A}_c(Y(2), AdS_d), (\mathcal{A}_{cb}(Y(2), AdS_d))$.

2d)(Super)algebra of converted constraints, exact BFV-BRST operator;

To construct NILPOTENT BFV-BRST OPERATOR for non-linear algebra $\mathcal{A}_{cb}(Y(2), AdS_d)$ of O_I:

1. determine the multiplication table for O_I ;

2. choose the ordering of the constraints in r.h.s of non-linear commutators;

3. solve nontrivial (determined by non-linear commutators) Jacobi identities for O_I in $\mathcal{A}_{cb}(Y(2), AdS_d)$;

4. prove that there are not other higher order algebraic relations;

5. construct BRST operator Q_b following to general BFV prescriptions.

For the 1st, 2nd items we have the same multiplication table as for $A_b(Y(2), AdS_d)$ with choosing of WEYL-ORDERING PRESCRIPTION (following to experience for fermionic HS fields with $\mathcal{A}_c(Y(1), AdS_d)$ J.Buchbinder,V.Krykhtin,A.Reshetnyak, 2007)

$$O_I : O_I \ \rightarrow \ \frac{1}{2}\Big(O_I O_J + (-1)^{\varepsilon_I \varepsilon_J} O_i O_j + [O_I, O_J\}\Big), \tag{29}$$

where we have found that only this choice leads to exact Q which has nonvanishing terms of the 3rd degree in powers of ghosts $\mathcal{C}^I$.

The composition law (proposition) for $\mathcal{A}_{cb}(Y(2), AdS_d)$ is the same as for $\mathcal{A}_b(Y(2), AdS_d)$ with changes:

$$\boxed{o_I \rightarrow O_I \text{ and } [r\mathcal{K}^{bi}_1, W^{ki}_b, X^{ki}_b - l_{0b}] \rightarrow -[r\mathcal{K}'^{bi}_{1W}, W'^{ki}_{bW}, X^{ki}_{bW} - L_0]}$$

.

$$r\mathcal{K}^{bi+}_{1W} = -r\Big(2\sum_{k=1}^{2}\Big[(L^{k+}-2l'^{k+})L^{ik}+(L^{ik}-2l'^{ik})L^{k+}+(G^i_0-2g'^i_0)L^i+(L^i-2l'^i)G^i_0\Big]$$
$$-\Big[(T^+-2t'^+)L^2+(L^2-2l'^2)T^+\Big]\delta^{i1}-\Big[(T-2t')L^1+(L^1-2l'^1)T\Big]\delta^{i2}\Big),$$

$$W^{ij}_{bW} = r\varepsilon^{ij}\left\{[G^2_0-G^1_0-2(g'^2_0-g'^1_0)]L^{12}+(L^{12}-2l'^{12})(G^2_0-G^1_0)-[(T-2t')L^{11}+(L^{11}-2l'^{11})T]+[(T^+-2t'^+)L^{22}\right.$$
$$\left.+(L^{22}-2l'^{22})T^+]\right\},$$

$$X^{ij}_{bW} = \Big\{L_0+r\Big({G^i_0}^2+\frac{1}{2}[(T^+-2t'^+)T+(T-2t')T^+]\Big)\Big\}\delta^{ij}+r\Big\{-2\Big[\sum_{k=1}^{2}(L^{jk+}-2l'^{jk+})L^{ik}+(L^{ik}-2l^{ik})L^{jk+}\Big]$$
$$-\frac{1}{2}\Big[(G^1_0+G^2_0-2(g'^1_0+g'^2_0))T+(T-2t')(G^1_0+G^2_0)\Big]\delta^{j1}\delta^{i2}-\frac{1}{2}\Big[(G^1_0+G^2_0-2(g'^1_0+g'^2_0))T^++(T^+-2t'^+)(G^1_0+G^2_0)\Big]\delta^{j2}\delta^{i1}\Big\}.$$

Jacobi identities

$$(-1)^{\varepsilon_I\varepsilon_K}[[O_I,O_J\},O_K\}+cycl.perm.(I,J,K)=0$$

has the general solution (A. Reshetnyak arXiv:0812.2329) with 3RD-ORDER STRUCTURAL FUNCTIONS $F^{RP}_{IJK}(o',O)$

$$(-1)^{\varepsilon_I\varepsilon_K}\Big(\big(f^M_{IJ}+F^{(2)M}_{IJ}\big)\big(f^P_{MK}+F^{(2)P}_{MK}\big)+(-1)^{\varepsilon_P\varepsilon_K}[F^{(2)P}_{IJ},O_K\}\Big)+\texttt{cycl.perm.}(I,J,K)$$
$$-\tfrac{1}{2}F^{RS}_{IJK}(f^P_{RS}+F^{(2)P}_{RS})=F^{RP}_{IJK}O_R,$$

$$\texttt{Proposition}\implies\boxed{F^{(2)K}_{IJ}(o',\tilde{O})=-\big(f^{MK}_{IJ}+(-1)^{\varepsilon_K\varepsilon_M}f^{KM}_{IJ}\big)o'_M+f^{MK}_{IJ}O_M}$$

The search of Q for a nonlinear superalgebra is seeked on the standard principles of the BFV method in the form of the expansion in powers of ghosts with use of $(\mathcal{CP})$-ordering of the ghost coordinate $\mathcal{C}^I$ and momenta $\mathcal{P}_I$ operators:

$$Q'=Q'_1+Q'_2+Q'_3+\ldots,\quad deg_{C}Q'_n=n,\ Q'^2=0,\quad \mathrm{gh}(Q',\mathcal{C}^I,\mathcal{P}_J)=(1,1,-1),\quad Q'_1=O_I\mathcal{C}^I. \qquad (30)$$

Definition of Q_1', Q_2' is standard $\Longrightarrow$ from the multiplication table whereas the form of Q_3' is determined from the resolution of the Jacobi identities. For vanishing of 4th-, 5th- and 6th- order structure functions for formal 2nd-rank "gauge" theory (M.Henneaux 1985), BRST operator Q has an exact form:

$$\boxed{Q' = \mathcal{C}^I\big[\tilde{O}_I + \tfrac{1}{2}\mathcal{C}^J(f^P_{JI} + F^{(2)P}_{JI})\mathcal{P}_P(-1)^{\varepsilon_I+\varepsilon_P} + \tfrac{1}{12}\mathcal{C}^J\mathcal{C}^K F^{RP}_{KJI}\mathcal{P}_R\mathcal{P}_P(-1)^{\varepsilon_I\varepsilon_K+\varepsilon_J+\varepsilon_R}\big]}. \tag{31}$$

In case of algebra $\mathcal{A}_{cb}(Y(2), AdS_d)$ there are **15** ghost pairs correspondingly for $L_0, L^{+i}, L^i, L_{ij}, L^+_{ij}, T^+, T, G^i_0$:

$$(\eta_0, \mathcal{P}_0), (\eta^+_i, \mathcal{P}_i), (\eta_i, \mathcal{P}^+_i), (\eta^+_{ij}, \mathcal{P}_{ij}), (\eta_{ij}, \mathcal{P}^+_{ij}), (\eta, \mathcal{P}^+, (\eta^+, \mathcal{P}), (\eta^i_G, \mathcal{P}^i_G)$$

, There are 3 types of nontrivial Jacobi identities (JI) for 6 triplets (L_1, L_2, L_0), (L^+_1, L^+_2, L_0), (L_i, L^+_j, L_0) for $A_{cb}(Y(2), AdS_d)$, with the existence of 3rd-order structure functions. So, JI for (L_i, L^+_j, L_0) after reduction of L^+_{11} has the form

$$\begin{aligned}
&2\Big\{\delta^{i2}\delta^{j1}\Big[(L^{22}-2l'^{22})(T^+-2t'^+) + (G^i_0 - 2g'^i_0)(L^{12}-2l'^{12}) + r^{-1}(\hat{W}^{ij}_{bW} - 2W'^{ij}_b) - (T-2t')(L^{11}-2l'^{11})\\
&-(G^j_0 - 2g'^j_0)(L^{12}-2l'^{12})\Big] - \varepsilon^{\{1j}\delta^{2\}i}\Big[(L^{12}-2l'^{12})(T^+-2t'^+) - (T^+-2t'^+)(L^{12}-2l'^{12})\Big]\Big\}\\
=\ &\delta^{i2}\delta^{j1}\Big(\{L_{11},T\} - \{L_{22},T^+\} - \{L_{12}, G^2_0 - G^1_0\} - 4L^{12}\Big) + 2\varepsilon^{\{1j}\delta^{2\}i}L^{11}.
\end{aligned} \tag{32}$$

in view of the absence of higher-order structural functions, BRST operator Q' for $\mathcal{A}_{bc}(Y(2), AdS_d)$ has an exact form of the maximal 3rd degree in the powers of $\mathcal{C}^I$:

$$\begin{aligned}
Q' =\ &\frac{1}{2}\eta_0 L_0 + \eta^+_i L^i + \eta^+_{lm}L^{lm} + \eta^+ T + \frac{1}{2}\eta^i_G G_i + \frac{\imath}{2}\eta^+_i\eta^i\mathcal{P}_0 + \frac{\imath}{2}\eta^+_{ii}\eta^{ii}\mathcal{P}^i_G + \frac{\imath}{2}\eta^+_i\eta^i\mathcal{P}_0 + \frac{\imath}{2}\eta^+_{ii}\eta^{ii}\mathcal{P}^i_G + 2\eta^i_G\eta^+_{ii}\mathcal{P}_{ii}\\
&+ (\eta^i_G\eta^+_i + \eta^+_{ii}\eta^i)\mathcal{P}^i - \eta_{12}(\eta^+\mathcal{P}^+_{11} + \eta\mathcal{P}^+_{22}) - 2\left[\frac{1}{2}\sum_k \eta^k_G\eta_{12} - \eta^+\eta_{22} - \eta\eta_{11}\right]\mathcal{P}^+_{12} + \tfrac{\imath}{2}\eta\eta^+\textstyle\sum_k(-1)^k\mathcal{P}^k_G\\
&+ \frac{\imath}{8}\eta^+_{12}\eta_{12}\sum_k \mathcal{P}^k_G + \Big[\frac{1}{2}\eta^+_{12}\eta_{11} + \tfrac{1}{2}\eta^+_{22}\eta_{12} + \textstyle\sum_k(-1)^k\eta^k_G\eta^+\Big]\mathcal{P} + \Big[\tfrac{1}{2}\eta^+_{12}\eta_2 - \eta\eta^+_2\Big]\mathcal{P}_1 + \Big[\tfrac{1}{2}\eta^+_{12}\eta_1 - \eta^+\eta^+_1\Big]\mathcal{P}_2
\end{aligned}$$

$$+\mathbf{r}\Big\{\eta_0\eta_i^+\Big(2(L^{ii}-2l'^{ii})\mathcal{P}_i^+ + 2(L^{i+}-2l'^{i+})\mathcal{P}_{ii} - \imath(L^i-2l'^i)\mathcal{P}_G^i + (G_0^i-2g_0'^i)\mathcal{P}_i + 2\Big[\Big((L^{12}-2l'^{12})\mathcal{P}^{\{1+}$$

$$+(L^{\{1+}-2l'^{\{1+})\mathcal{P}_{12}\Big)\delta^{2\}i} - \frac{1}{2}\delta^{1i}\Big((L^2-2l'^2)\mathcal{P}^+ + (T^+-2t'^+)\mathcal{P}_2\Big) - \frac{1}{2}\delta^{2i}\Big((L^1-2l'^1)\mathcal{P} + (T-2t')\mathcal{P}_1\Big)\Big]\Big)$$

$$-\frac{1}{2}\eta_i^+\eta_j^+\varepsilon^{ij}\{\sum_k(-1)^k(G_0^k-2g_0'^k)\mathcal{P}_{12} - \imath(L^{12}-2l'^{12})\sum_k(-1)^k\mathcal{P}_G^k - [(T-2t')\mathcal{P}_{11} + (L^{11}-2l'^{11})\mathcal{P}]$$

$$+(T^+-2t'^+)\mathcal{P}_{22} + (L^{22}-2l'^{22})\mathcal{P}^+\} + 2\eta_i^+\eta_j\{\sum_k(L^{jk+}-2l'^{jk+})\mathcal{P}^{ik} + [\frac{\imath}{4}(G_0^i-2g_0'^i)\mathcal{P}_G^i - \frac{1}{8}(T^+-2t'^+)\mathcal{P}$$

$$-\frac{1}{8}(T-2t')\mathcal{P}^+]\delta^{ij} + \frac{1}{4}[\sum_k(G_0^k-2g_0'^k)\mathcal{P} - \imath(T-2t')\sum_k\mathcal{P}_G^k]\delta^{j1}\delta^{i2}\}\Big\}$$

$$\overbrace{\boxed{r^2}}^{Q_3'}\ \eta_0\eta_i\eta_j\varepsilon^{ij}\Big\{\frac{1}{2}(\sum_k G_0^k[\mathcal{P}\mathcal{P}^{22+} - \mathcal{P}^+\mathcal{P}^{11+}] - \frac{i}{2}(L^{11+}\mathcal{P}^+ - L^{22+}\mathcal{P})\sum_k\mathcal{P}_G^k + \frac{i}{2}\sum_k G_0^k\mathcal{P}^{12+}\sum_l(-1)^l\mathcal{P}_G^l$$

$$-L^{12+}\mathcal{P}_G^1\mathcal{P}_G^2 - \sum_l L^{1l}\mathcal{P}^{l2+}\mathcal{P}^{11+} + \sum_l L^{l2}\mathcal{P}^{1l+}\mathcal{P}^{22+} - \sum_l(-1)^l L^{ll}\mathcal{P}^{ll+}\mathcal{P}^{12+}\Big\}$$

$$+r^2\eta_0\eta_i^+\eta_j\Big\{\frac{i}{2}\sum_l(-1)^l G_0^l\sum_k\mathcal{P}_G^k\mathcal{P}\delta^{1j}\delta^{2i} + 2(L^{22+}\mathcal{P}^{22} - L^{11}\mathcal{P}^{11+})\mathcal{P}\delta^{1j}\delta^{2i} - 2T\mathcal{P}^{11}\mathcal{P}^{22+}\delta^{1i}\delta^{2j}$$

$$+\frac{1}{2}\varepsilon^{\{1j}\delta^{2\}i}\Big\{iT\mathcal{P}^+\sum_k\mathcal{P}_G^k - 4iL^{12}\mathcal{P}^{12+}\sum_l(-1)^l\mathcal{P}_G^l\Big\} - T\mathcal{P}_G^1\mathcal{P}_G^2\delta^{2i}\delta^{1j}$$

$$+2\varepsilon^{\{1j}\delta^{2\}i}\Big\{(T^+\mathcal{P}^{12} - L^{12}\mathcal{P}^+)\mathcal{P}^{11+} + (L^{12}\mathcal{P} - T\mathcal{P}^{12})\mathcal{P}^{22+}\Big\} + 2\sum_l(-1)^l G_0^l\mathcal{P}^{12+}(\mathcal{P}^{11}\delta^{1i}\delta^{2j} - \mathcal{P}^{22}\delta^{2i}\delta^{1j})$$

$$-2i(L^{22}\delta^{2i}\delta^{1j} - L^{11}\delta^{1i}\delta^{2j})\mathcal{P}^{12+}\sum_l(-1)^l\mathcal{P}_G^l\Big\} + h.c.$$

BRST OPERATOR Q' is Hermitian: $Q'^+K = KQ'$. with K defined in Fock space $\mathcal{H}_{tot} = \mathcal{H}'\otimes\mathcal{H}\otimes\mathcal{H}_{gh}$: $K = K'\otimes\hat{1}\otimes\hat{1}_{gh}$.

2e) **Unconstrained Lagrangian formulation;**

To construct LF FOR MASSIVE BOSONIC HS FIELDS IN ADS(D) SPACE we must extract dependence on ghost η^i_G for non-constraint G^i_0 from Q' and Hilbert space $\mathcal{H}_{tot}$, choose representation for $\mathcal{H}_{tot}$.

$$\Longrightarrow Q' = Q + \eta^i_G(\sigma^i + h^i) + \mathcal{B}^i\mathcal{P}^i_G, \ (\sigma^i + h^i) = G^i_0 + \left(\textstyle\sum_j(1+\delta_{ij})\eta^{ij+}\mathcal{P}_{ij} + (-1)^i\eta^+\mathcal{P} + h.c.\right).$$

The same applies to a scalar physical vector $|\chi\rangle \in \mathcal{H}_{tot}$, $\mathrm{gh}(|\chi\rangle) = 0$,

$$|\chi\rangle = |\Phi\rangle + |\Phi_A\rangle, |\Phi_A\rangle_{\{(b,b^+,)=\mathcal{C}=\mathcal{P}=0\}} = 0 \ \texttt{with}\ |\Phi\rangle - \texttt{basic initial HS field}$$

and with the use of the BFV–BRST EQUATION $Q'|\chi\rangle = 0$ that determines the physical states,

$$Q|\chi\rangle = 0, \quad (\sigma^i + h^i)|\chi\rangle = 0, \quad \delta|\chi\rangle = Q|\chi^1\rangle, \quad (\varepsilon, gh)\,[|\chi\rangle, |\chi^1\rangle] = [(0,0),(1,-1)]. \qquad (33)$$

the 2nd Eqs. determines the spectrum of spin values for $|\chi\rangle$ and gauge parameters $|\chi^k\rangle$, $k = 1, ..., 6$ and the corresponding proper eigenvalue,

$$h^i \ = \ -\left(s_i + \frac{d-5}{2} - 2\delta^{i2}\right), \qquad (s_1, s_2) \in (\mathbb{Z}, \mathbb{N}_0), \ |\chi\rangle_{(s_1,s_2)}, \qquad (34)$$

whereas the 1st equation is valid only in the subspace of $\mathcal{H}_{tot}$ with the zero ghost number.

Because of $[\sigma^i, Q\} = 0$, Q being subject to the substitution $h^i \to -\left(s_i + \frac{d-5}{2} - 2\delta^{i2}\right)$, i.e., $Q \to Q_{(s_1,s_2)}$, is nilpotent in each of the subspaces $H_{tot(s_1,s_2)}$ whose vectors obey Eqs. (33), (34).

$\Longrightarrow$ The equations of motion and the sequence of reducible gauge transformations:

$$Q_{(s_1,s_2)}|\chi^0\rangle_{(s_1,s_2)} = 0, \;\; \delta|\chi^l\rangle_{(s_1,s_2)} = Q_{(s_1,s_2)}|\chi^{1+1}\rangle_{(s_1,s_2)}, \;\; l = 0, ..., 6,$$

for $|\chi^0\rangle \equiv |\chi\rangle$, and can be obtained from the LAGRANGIAN ACTION

$$\mathcal{S}_{s_1,s_2} = \int d\eta_0 \;{}_{(s_1,s_2)}\langle\chi^0|K_{(s_1,s_2)}Q_{(s_1,s_2)}|\chi^0\rangle_{(s_1,s_2)} \;\; K_{(s_1,s_2)} = K|_{h^i \to -\left(s_i + \frac{d-5}{2} - 2\delta^{i2}\right)},$$

where the standard ε-even scalar product in $\mathcal{H}_{tot}$ is assumed.

The corresponding LF of a bosonic field with a specific value of spin $\mathbf{s}$ subject to $Y(s_1, s_2)$ is an UNCONSTRAINED REDUCIBLE GAUGE THEORY OF MAXIMALLY $L = 5$-TH STAGE OF REDUCIBILITY.

2f) New computer program to verify the validity of the oscillator Verma module realization

The oscillator realization of Verma module at least for the non-linear algebra $\mathcal{A}'_b(Y(2), AdS_d)$ has rather complicated form. Because of NON-DIRECT procedure of its derivation: by means of **1)** the Verma module construction then $2)$ its oscillator realization, the question arises:

Whether really the obtained formal power series (o'_I) in non(super)commuting variables will satisfy to a given multiplication table?

To resolve the problem we (A. Kuleshov, A. Reshetnyak arXiv:0905.2705) elaborate the computer program on $C\#$-language within SYMBOLIC COMPUTATIONAL APPROACH which realize the check of the coincidence of the left-hand-side of given supercommutator of operators o_I, o'_J with its right-hand-side given by the multiplication table in each fixed degree in r.

Given programm is applicable for the case of non-linear commutator superalgebras over Heisenberg-Weyl superalgebras $\Longrightarrow$ extending the properties of known programs, for instance, module PLURAL (used for polynomial algebra, so-called GR-algebra, over non-commuting variables)

We verify the validity of the $\mathcal{H}'$-realization for the Verma Module of superalgebra $A'(Y(1), AdS_d)$ (A. Kuleshov, A. Reshetnyak) as the particular (for totally-symmetric HS fields) case of $A'(Y(2), AdS_d)$.

$$A'(Y(1), AdS_d) = \{t'_0, t'^+_1, t'_1; l'_0, l'^+_1, l'_1, l'^+_2, l'_2\} - \texttt{3 fermionic and 6 bosonic operators}$$

`oscillator form:` $t_1'^{+} = f^+ + 2b_2^+ f,\qquad l_1'^{+} = m_1 b_1^+,$

$$g_0' = b_1^+ b_1 + 2b_2^+ b_2 + f^+ f + h,\qquad l_2'^{+} = b_2^+,$$

$$t_0' = 2m_1 b_1^+ f - \frac{m_1}{2}(f^+ - 2b_2^+ f)\, b_1^+ \sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1} b_1^{2k}}{(2k)!} + \tilde{\gamma} m_0 \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k b_1^{2k}}{(2k)!}$$
$$+ \frac{r(h-\frac{1}{2})}{m_1}(f^+ - 2b_2^+ f)\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!}$$

$$t_1' = -2g_0' f - (f^+ - 2b_2^+ f) b_2 + \frac{1}{2}(h - \tfrac{1}{2})(f^+ - 2b_2^+ f)\sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1} b_1^{2k}}{(2k)!}$$
$$+ \frac{1}{2}(f^+ - 2b_2^+ f)\, b_1^+ \sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1}\, b_1^{2k+1}}{(2k+1)!} - \frac{\tilde{\gamma} m_0}{m_1}\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!},$$

$$l_0' = m_0^2 - r\frac{\tilde{\gamma} m_0}{m_1}(f^+ - 2b_2^+ f)\sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!}(1 - 4^{-k}) - r b_1^+ \sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!}(2h - 4^{-k})$$
$$+ 4r\,\frac{\tilde{\gamma} m_0}{m_1}\, f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^{k+1}\, b_1^{2k+1}}{(2k+1)!} + r\left(h - \frac{1}{2}\right)\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^{k+1} \frac{(b_2^+)^{k+1}\, b_1^{2k+2}}{(2k+2)!}$$
$$-2r\,(b_1^+)^2 \sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+2}}{(2k+2)!} - 2r f^+ f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \left\{\frac{(h-\frac{1}{2})}{(2k)!} + \frac{b_1^+ b_1}{(2k+1)!}\right\}(b_2^+)^k b_1^{2k}$$
$$+ \frac{m_0^2 - r(h^2 - \frac{1}{4})}{2}\sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^{k+1} \frac{(b_2^+)^{k+1}\, b_1^{2k+2}}{(2k+2)!},$$

$$
\begin{aligned}
l_1' = &-m_1 b_1^+ b_2 + \frac{m_1}{4}\, b_1^+ \sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \left\{\frac{2h-4^{-k}}{(2k)!} + \frac{2b_1^+ b_1}{(2k+1)!}\right\}(b_2^+)^{k-1} b_1^{2k} + \\
&+ \frac{\tilde{\gamma} m_0}{4}\,(f^+ - 2b_2^+ f)\sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1}\, b_1^{2k}}{(2k)!}(1-4^{-k}) + \frac{r(h-\frac{1}{2})}{2m_1}\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!} \\
&+ \frac{m_1}{2}\, b_1^+ f^+ f \sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1} b_1^{2k}}{(2k)!} - \frac{r(h-\frac{1}{2})}{m_1} f^+ f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k b_1^{2k+1}}{(2k+1)!} \\
&- \tilde{\gamma} m_0 f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k b_1^{2k}}{(2k)!} + \frac{m_0^2 - r(h^2-\frac{1}{4})}{m_1}\sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+1}}{(2k+1)!}, \\
l_2' = &\; g_0' b_2 - b_2^+ b_2^2 - \frac{m_0^2 - r(h^2-\frac{1}{4})}{m_1^2}\sum_{k=0}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+2}}{(2k+2)!} - \frac{r(h-\frac{1}{2})}{2m_1^2}\sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k\, b_1^{2k+2}}{(2k+2)!} \\
&+ \frac{\tilde{\gamma} m_0}{m_1} f \sum_{k=0}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \frac{(b_2^+)^k b_1^{2k+1}}{(2k+1)!} - \frac{\tilde{\gamma} m_0}{4m_1}(f^+ - 2b_2^+ f)\sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \frac{(b_2^+)^{k-1} b_1^{2k+1}}{(2k+1)!}(1-4^{-k}) \\
&- \frac{1}{4}\, b_1^+ \sum_{k=1}^{\infty}\left(\frac{-8r}{m_1^2}\right)^k \left\{\frac{2h-4^{-k}}{(2k+1)!} + \frac{2b_1^+ b_1}{(2k+2)!}\right\}(b_2^+)^{k-1} b_1^{2k+1} \\
&- \frac{1}{2}\, f^+ f \sum_{k=1}^{\infty}\left(\frac{-2r}{m_1^2}\right)^k \left\{\frac{h-\frac{1}{2}}{(2k)!} + \frac{b_1^+ b_1}{(2k+1)!}\right\}(b_2^+)^{k-1} b_1^{2k}.
\end{aligned}
$$

The program with draft title "PhysProject" represent the separate application, calculates:

1) `L.H.S. of` $[o_I', o_J'\} : \left(o_I' o_J' - (-1)^{\varepsilon_I \varepsilon_J} o_J' o_I'\right) \equiv P^l_{IJ}(r; b_i, b_i^+, f, f^+)$

with given accuracy in powers of r: r^q and pass it to the ordering form, then calculates:

2) `R.H.S. of` $[o_I', o_J'\} : \equiv P^r_{IJ}(r; b_i, b_i^+, f, f^+)$

with the same accuracy in powers of r: r^q. Then we visually compare it in two windows both in SYMBOLIC

FORM and in the VECTOR FORM:

$$\tfrac{1}{5!}m_1^{-2}b_2^+f^+b_1^+fb_1^3 \mapsto \tfrac{1}{5!}m_1^{-2}(1,1,1,0,1,3). \tag{35}$$

Computer verification $\Longrightarrow$: oscillator realization for $A'(Y(1), AdS_d)$ is valid up to 4th order in r, and therefore due to restricted induction principle the $\mathcal{H}'$-realization of Verma Module for $A'(Y(1), AdS_d)$ is correct.

3. Summary of the results; Outlook

The basic results are listed on the 3rd slide, while the open problem look as follows:

- Explicit proof that Q-cohomologies in Hilbert subspace $H_{tot;(s_1,s_2)}$ with zero ghost number coincide with space of solutions for AdS-group irreps:$H^{(0,0)}_{(s_1,s_2)}(Q_b, \mathcal{H}_{tot}) \simeq D(E_0(m), s_1, s_2)$;
- Consideration the LFs for HS fields in AdS(d)-space both for $Y(s_1, s_2, ..., s_k), k > 2$, and with off-shell algebraic constraints l_{ij}, t^i when the Verma modules for reduced superalgebra $\mathcal{A}'_r(Y(2), AdS_d)$ are constructed without entanglement with lesser spectrum of auxiliary fields;
- Investigation the problem of LF for interacting HS fields on flat and AdS spaces;
- Development of the computer program properties to apply it for checking: 1) validity of the oscillator representation for $\mathcal{A}'(Y(2), AdS_d)$; 2) nilpotency of Q for superalgebra $\mathcal{A}_c(Y(2), AdS_d)$.

Thank you for attention