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# Quantum plasmadynamics and quantum fluid theory

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# Overview

- Quantum plasmadynamics
- Quantum plasma effects
- Applications of QPD
- Quantum fluid theory (QFT)
- Quantum Zakharov equations
- Discussion & conclusions

# Questions

**How to synthesize kinetic theory of plasmas & QED?**

- both theories describe interaction of photons & electrons
- kinetic theory based on self-consistent field
  cooperative effects treated classically
- QED is a single-particle theory
- includes all relativistic quantum effects

**How to rewrite kinetic theory in covariant notation?**

- using formalism for the vacuum polarization tensor
- use forward-scattering to calculate response tensors

**How to include plasma responses using QED?**

- identify relevant Lagrangians
- generalization then almost trivial

**How to include magnetic field into the theory?**

- replace all wavefunctions and propagators with relevant
  exact solutions of Dirac's equation with $\mathbf{B} \neq 0$

# Classical covariant formulation

## Non-covariant forms of the response

- induced current expanded in powers of electric field
  $J_i(\omega, \mathbf{k}) = \sigma_{ij}(\omega, \mathbf{k}) E_j(\omega, \mathbf{k}) +$ nonlinear terms
  $K_{ij}(\omega, \mathbf{k}) = \delta_{ij} + i\sigma_{ij}(\omega, \mathbf{k})/\varepsilon_0\omega$

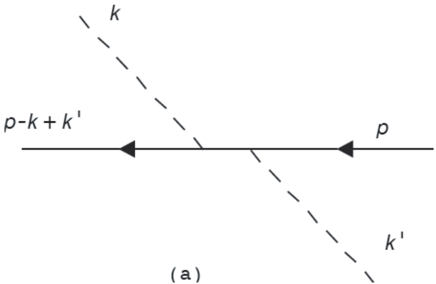
## Covariant formulation of kinetic theory of plasmas

- induced 4-current in proportional to 4-potential
- linear polarization tensor: $J^\mu(k) = \Pi^{\mu\nu}(k)\, A_\nu(k)$
- charge-continuity & gauge-invariance:
  $$k_\mu \Pi^{\mu\nu}(k) = 0 = k_\nu \Pi^{\mu\nu}(k)$$
- determines $\Pi^{\mu\nu}(k)$ in terms of response 3-tensor
  D.B. Melrose (1973)
- allows covariant formulation of wave dispersion
- extension to nonlinear responses straightforward
- quadratic response $\Pi^{\mu\nu\rho}(k, k_1, k_2)$, $k^\mu = k_1^\mu + k_2^\mu$
  $\Longrightarrow$ 3-wave coupling
- effective cubic response = cubic + 2 quadratic responses
  $\Pi^{\mu\nu\rho\sigma}_{\mathrm{eff}}(k, k_1, k_2, k_3) \Longrightarrow$ nonlinear wave equation

# Forward scattering method

**Linear response tensor**

- forward scattering: $k' = k$, $p' = p$
- all particles scatter in phase $\Longrightarrow$ 4-current $J^{\mu}(k)$
- allows covariant derivation of linear response tensor
- straightforward generalization to nonlinear responses



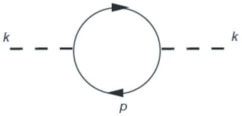

(a)

# Response tensors from QED

**Linear response tensor**

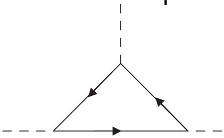
- forward scattering implicit in vacuum polarization tensor
- 'cut' diagram to include on-shell contribution Cutkovsjy (1960)
- statistical averages of Feynman propagators

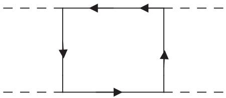
  generalization of thermal Green's function
- linear response from statistical average of bubble diagram




**Quadratic & cubic response tensors**

- quadratic & cubic responses from triangle & box diagrams
- includes vacuum & plasma contributions





# Three approaches to QPD

**Density-matrix approach** generalizes Harris' (1967) method

- evolution of density matrix for electrons
- operator-based evolution = Heisenberg picture
- no non-quantum counterpart

**Wigner-matrix approach** developed by Hakim & Sivak

- Wigner-Moyal function in Schrödinger theory
- generalize to Wigner matrix in Dirac theory
- wave-function-based evolution = Schrödinger picture
- counterpart of non-quantum Vlasov approach

**Green's function approach** Tsytovich (1961)

- $S$-matrix approach = interaction picture
- statistically averaged propagators (Green's functions)
- 'photon' propagator includes all plasma wave modes
- particles & waves described by occupation numbers
- closely analogous method from quark-gluon-plasma approach
  E. Braaten, D. Segel (1993)

# Quantum plasma effects

**Degeneracy**

- included in early theories of solid state plasmas

**Quantum recoil**

- classical resonance condition $\omega - \mathbf{k}\cdot\mathbf{v} = 0$
- conservation of energy & momentum in emission
- $\varepsilon \to \varepsilon' = \varepsilon - \hbar\omega$, $\mathbf{p} \to \mathbf{p}' = \mathbf{p} - \hbar\mathbf{k}$
- $(m^2c^4 + |\mathbf{p} - \hbar\mathbf{k}|^2c^2)^{1/2} = (m^2c^4 + \mathbf{p}^2c^2)^{1/2} - \hbar\omega$
- $\implies$ quantum recoil term in resonance condition

$$\omega - \mathbf{k}\cdot\mathbf{v} - \frac{\hbar(\omega^2 - \mathbf{k}^2c^2)}{2\varepsilon} = 0$$

**Nonrelativistic derivation of recoil**

- $\varepsilon \to mc^2 + \mathbf{p}^2/2m \implies |\mathbf{p} - \hbar\mathbf{k}|^2/2m = \mathbf{p}^2/2m - \hbar\omega$
- $\implies$ quantum recoil term in resonance condition

$$\omega - \mathbf{k}\cdot\mathbf{v} + \frac{\hbar\mathbf{k}^2}{2m} = 0 \qquad (\text{no } \omega^2\text{-term!})$$

- nonrelativistic treatment valid only for $\omega^2 \ll \mathbf{k}^2c^2$

**Spin**

- spin-polarized electrons modifies linear response to order $\hbar$
- unpolarized electrons $\implies$ average over spins
- different from spin-0 particles (boson plasma)

**Vacuum polarization & critical fields**

- vacuum birefringent for $B/B_c \neq 0$
- vacuum quadratic nonlinear response for $B/B_c \neq 0$

**One-photon pair creation (PC)**

- PC included in vacuum polarization tensor: $\mathrm{Im}\,\Pi^{\mu\nu}(k)$
- electron gas partially suppresses PC (Pauli exclusion)
- PC introduces an additional source of dispersion in RQ plasma
- $\implies$ existence intrinsically RQ 'pair' modes

**Quantum diffusion & tunneling**

- quantum phenomena in $(t, \mathbf{x})$
- must be included in $\omega, \mathbf{k}$ description
- where specifically?

# Applications of QPD

**Neutrino emission from compact stars**

- neutrino losses: cooling mechanism for compact stars
- ‘plasma process’—plasmons decay into neutrino pairs
- dispersion of plasmons in relativistic, degenerate plasms
- inclusion of $B/B_c$ in dispersion theory

**Early Universe**

- dispersion in hot dense plasma
- is one-photon pair creation possible?
- what role do pair modes play?

**Pulsars and magnetars**

- wave dispersion in pulsar magnetosphere
- interpretation of polarization of pulsar emission
- dispersion for $B \gg B_c$ relevant for magnetars

# Dispersion in RQ plasmas

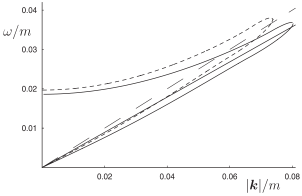
## Known results in relativistic degenerate plasma

- Friedel oscillations in relativistic degenerate plasma
  due to Kohn singularity in $K^L(0, \mathbf{k})$, at $|\mathbf{k}| = 2p_{\mathrm{F}}$
- Pauli spin paramagnetism : Landau diamagnetism = 3 : -1
  A.A. Rudkadze, V.P. Silin (1960)

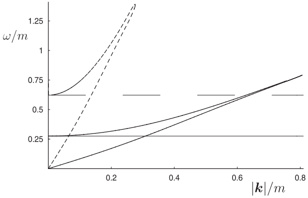
## Dispersion of plasmons in completely degenerate plasma

- approximate dispersion relation known since the 1950s:
  $$\omega^2 = \omega_p^2 + \frac{3}{5} v_F^2 + \hbar k^4 / 4m_e^2$$
- high-frequency turnover exists V.S. Krivitskii, S.V. Vladimirov (1991)
- relativistic case: Jancovici (solid) Lindhard (dashed):

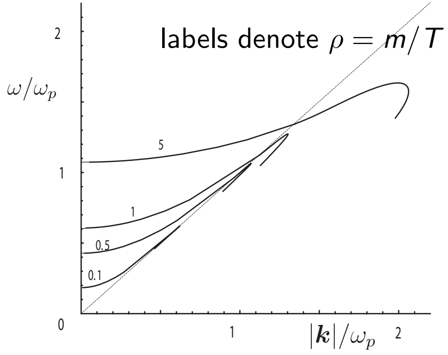
$p_F/m = 0.5$




$p_F/m = 5$



## Dispersion of L-waves in relativistic thermal plasmas



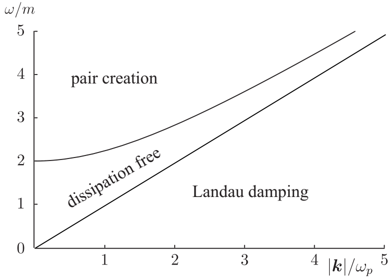
# Superdense plasmas

## Cutoff frequency

- cutoff frequency, $\omega_c$: same for L & T waves

$$\omega_c^2 = \frac{4\mu_0 e^2}{3\pi^2} \int d\varepsilon |\mathbf{p}|\, \bar{n}(\varepsilon)\, \frac{3\varepsilon^2 - |\mathbf{p}|^2 - 3\omega_c^2/4}{4\varepsilon^2 - \omega_c^2}$$

- superdense plasmas $\omega_p \gg m$: cutoff above PC threshold?
- relevant to the early Universe when $\omega_p \gg 2m$?
- effect of macroscopic mass renormalization?



# Pair modes

**Pair modes (B $= 0$)**

- pair modes have $\omega \gtrsim 2m$
- exist in degenerate spin 0 & spin 1 plasma V. Kowalenko, N.E. Frankel, K.C. Hines (1985); D.B Melrose, D.R.M.Williams (1989)
- do not exist in degenerate electron gas
- exist for transverse mode in spin 1 plasma

**Pair modes (B $\neq 0$)**

- pair modes exist in magnetized electron gas
- associated with thresholds for pair creation
- Landau quantum numbers $n, n'$ for $e^{\pm}$ P. Pulsifer, G. Kalman (1992)
- like gyromagnetic modes associated with singularities response

**Interpretation & implication of pair modes**

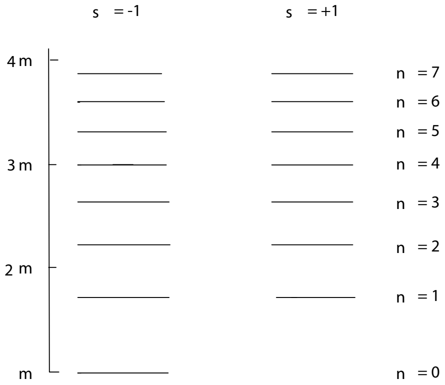
- analogy with Cooper pairs? No
- physical implications of pair modes?

# Generalization of QPD to B $\neq 0$

## Dirac electron for B $\neq 0$

- discrete Landau levels with energy
  $\varepsilon_n(p_z) = (m^2 + p_z^2 + 2neB)^{1/2}$
- critical field $B_c = mc^2/\hbar e = 4.4 \times 10^9$ T

energy diagram for $p_z = 0$, $B/B_c = 1$



## Choice of gauge and spin operator

- general forms for the response tensor long been available
  D.B. Melrose, A.J. Parle (1983)
- depends on choice of gauge (for **B**) & spin operator
- relevant spin operator identified by A.A. Sokolov, I.M. Ternov (1968)
- gauge & spin independent result using method of Ritus
  V.I. Ritus (1970); A.J. Parle (1985)

## Simplified form for pulsar magnetospheres

- Gyromagnetic losses $\Longrightarrow$ $n = 0$ for all electrons
- highly relativistic, streaming, 1D, pair plasma with $B \sim 0.1 B_c$
- relevant response tensor used to derive wave properties
- polarization properties of particular interest

# Quantum fluid theory (QFT)

**QFT approach** F. Haas, G. Manfredi, M. Feix (2000)

- derive fluid model incorporating quantum effects
- quantum effects included in Bohm potential
- QFT used to include quantum terms in linear waves
- QFT used to derive 1D quantum Zakharov equations

**Comparison QFT & QPD**

- QPD derivation establishes limits of QFT approach
- provides new physical insight
- QPD allows various generalizations

**QPD approach**

- derive relevant approximations for $\Pi^{\mu\nu}(k)$
- apply to 1D longitudinal case
- compare with QFT
- derive quantum Zakharov equations

# 1D Wigner-Poisson system

- 1D Schrödinger equation

$$i\hbar\frac{\partial\psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\,\frac{\partial^2\psi(x,t)}{\partial x^2} - e\phi(x,t)\,\psi(x,t) = 0$$

- 1D Wigner function defined by

$$f(x,p,t) = \int dy\,\psi^*(x-\tfrac{1}{2}y,t)\psi(x+\tfrac{1}{2}y,t)\,\exp\left(\frac{-ipy}{\hbar}\right)$$

- 1D Poisson equation

$$\frac{d^2\phi(x,t)}{dx^2} = \frac{e}{\varepsilon_0}\left[\int \frac{dp}{2\pi\hbar}\,f(x,p,t) - n_e\right]$$

- 1D Vlasov-like equation G. Manfredi, F. Haas (2001)

$$\left[\frac{\partial}{\partial t} + \frac{p}{m}\frac{\partial}{\partial x}\right] f(x,p,t) - \frac{ie}{\hbar}\int \frac{dp'dy'}{2\pi\hbar}\,[\phi(x-\tfrac{1}{2}y',t)$$
$$-\phi(x+\tfrac{1}{2}y',t)]\exp\left(\frac{i(p-p')y'}{\hbar}\right) f(x,p',t) = 0$$

## 1D quantum fluid equations

- zeroth and first moments $\Longrightarrow$
- continuity: $\partial n_e/\partial t + \partial(n_e u_e)/\partial x = 0$
- equation of fluid motion for electrons

$$\frac{du_e}{dt} = -\frac{e}{m_e}E - \frac{1}{n_e m_e}\frac{\partial P_e}{\partial x} + \frac{\hbar^2}{2m_e^2}\frac{\partial}{\partial x}\left[\frac{1}{\sqrt{n_e}}\frac{\partial^2\sqrt{n_e}}{\partial x^2}\right]$$

- $P_e = n_e m_e V_e^2$ ‘classical’ pressure term
- ‘quantum’ pressure term is the Bohm term

## Role of Bohm potential

- linearizing and Fourier transforming ($k$ is 1D wavenumber)
- Langmuir: $\omega^2 = \omega_p^2 + k^2 V_e^2 + \hbar^2 k^4/4m_e^2$
- ion sound: $\omega^2 = k^2 v_s^2 + \hbar^2 k^4/4m_e m_i$
- Bohm term corresponds to quantum recoil

# Linear response tensor in QPD

**Derivation of recoil terms from exact theory**

- QPD form for $\Pi^{\mu\nu}(k)$:

$$\Pi^{\mu\nu}(k) = -2e^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{\bar{n}(\mathbf{p})}{\varepsilon} \frac{(ku)^2}{(ku)^2 - (k^2/2m)^2} a^{\mu\nu}(k,u),$$

$$a^{\mu\nu}(k,u) = g^{\mu\nu} - \frac{k^\mu u^\nu + k^\nu u^\mu}{ku} + \frac{k^2 u^\mu u^\nu}{(ku)^2}.$$

- differs from relativistic classical form by

$$\frac{1}{\gamma^2(\omega - \mathbf{k}\cdot\mathbf{v})^2} \to \frac{1}{\gamma^2(\omega - \mathbf{k}\cdot\mathbf{v})^2 - \hbar^2(\omega^2 - \mathbf{k}^2c^2)^2/4m^2c^4}$$

- nonrelativistic approximation: $\gamma \to 1$
- "strictly" nonrelativistic approximation: $c \to \infty$
  $\Longrightarrow$ denominator $\to (\omega - \mathbf{k}\cdot\mathbf{v})^2 - \hbar^2\mathbf{k}^4/4m^2$

# Nonrelativistic thermal quantum plasma

## Quantum recoil in Langmuir

- including quantum recoil, susceptibilities become
$$\chi^L(\omega, \mathbf{k}) = -\frac{\omega_p^2}{\sqrt{2}|\mathbf{k}|V}\frac{1}{2\Delta}\left[\frac{\phi(y_-)}{y_-} - \frac{\phi(y_+)}{y_+}\right],$$
$$y_\pm = (\omega \pm \Delta)/\sqrt{2}|\mathbf{k}|V, \qquad \Delta = \hbar(\mathbf{k}^2 - \omega^2/c^2)/2m$$
- for $y^2 \gg 1$, $\phi(y) \approx 1 + 1/2y^2$
$$\chi^L(\omega, \mathbf{k}) = -\frac{\omega_p^2}{\omega^2 - \Delta^2}\left[1 + \frac{\left(3\omega^2 + \Delta^2\right)\mathbf{k}^2 V^2}{\left(\omega^2 - \Delta^2\right)^2}\right]$$
- $\Longrightarrow$ Langmuir waves with quantum recoil
$$\omega^2 \to \omega_L^2(\mathbf{k}) + \Delta_e^2 + \cdots,$$
- Landau damping ($H_e = \hbar\omega_L(k)/4m_e V_e^2$)
$$\gamma_L(k) \simeq \omega_L(k)\sqrt{\frac{\pi}{2}}\left(\frac{\omega_L(k)}{|\mathbf{k}|\,V_e}\right)^3 \frac{\sinh H_e}{H_e} \exp\left(-\frac{\omega_L^2(k) + \Delta_e^2}{2\,|\mathbf{k}|^2\,V_e^2}\right).$$

## Quantum recoil in ion sound waves

- for $y_\pm^2 \ll 1$, $\phi(y_\pm) \approx 2y_\pm^2 - 4y_\pm^4/3$
$$\chi^L(\omega, \mathbf{k}) = \frac{\omega_p^2}{\mathbf{k}^2 V^2}\left[1 - \frac{3\omega^2 + \Delta^2}{3\mathbf{k}^2 V^2}\right],$$
- assume $y_{e\pm}^2 \ll 1$, $y_{i\pm}^2 \gg 1$
$$K^L(\omega, \mathbf{k}) = 1 + \frac{1}{\mathbf{k}^2 \lambda_{De}^2}\left[1 - \frac{3\omega^2 + \Delta_e^2}{3\mathbf{k}^2 V_e^2}\right] - \frac{\omega_{pi}^2}{\omega^2 - \Delta_i^2}$$
- $\implies$ ion sound waves with quantum recoil
$$\omega^2 \to \omega_s^2(\mathbf{k})\left[1 + \frac{\Delta_e^2}{3\mathbf{k}^2 V_e^2(1 + \mathbf{k}^2\lambda_{De}^2)}\right] + \Delta_i^2,$$
- for $\mathbf{k}^2\lambda_{De}^2 \ll 1$ simplifies to
$$\omega^2 \to \mathbf{k}^2 v_s^2 + \frac{\Delta_{ei}^2}{3}, \qquad \Delta_{ei}^2 = \frac{\hbar^2(\mathbf{k}^2 - \omega^2/c^2)^2}{4m_e m_i}$$

# Zakharov equations (standard form)

**Nonlinear correction to Langmuir waves** (V. Zakharov 1972)

- fluctuation $\delta n_e(t, \mathbf{x})$ in electron density modifies $\omega_p$
- correction to dispersion relation for Langmuir waves
  $$\omega = \omega_p + \frac{3\mathbf{k}^2 V_e^2}{2\omega_p} + \frac{\delta n_e}{2n_e}\omega_p - i\frac{\gamma_L}{2}$$
- slowly varying envelope for Langmuir turbulence
  $$\mathbf{E}(t, \mathbf{x}) = \tilde{\mathbf{E}}(t, \mathbf{x})\, e^{-i\omega_p t} + \tilde{\mathbf{E}}^*(t, \mathbf{x})\, e^{i\omega_p t}$$
- equation for the envelope
  $$\left[i\frac{\partial}{\partial t} + \frac{3V_e^2}{2\omega_p}\nabla^2 + i\frac{\gamma_L}{2}\right]\tilde{\mathbf{E}}(t, \mathbf{x}) = \frac{\omega_p \delta n_e(t, \mathbf{x})}{2n_e}\,\tilde{\mathbf{E}}(t, \mathbf{x})$$

**Evolution of density fluctuations**

- $\delta n_e$ assumed ion-sound like $(\omega^2 - \mathbf{k}^2 v_s^2 + i\omega\gamma_s = 0)$
  $$\left[\frac{\partial^2}{\partial t^2} - v_s^2\nabla^2 + \gamma_s\frac{\partial}{\partial t}\right]\delta n_e(t, \mathbf{x}) = \frac{\varepsilon_0}{m_i}\,\nabla^2|\tilde{\mathbf{E}}(t, \mathbf{x})|^2$$
- driver = ponderomotive force due to Langmuir turbulence

# Zakharov equations from kinetic theory

## Nonlinear wave equation

- nonlinear wave equation involves effective cubic response
- QPD form for $\Pi^L_{\rm eff}(k, k_1, k_2, k_3)$ simplifies if beat at $k - k_1 = k_2 + k_3$ slow and longitudinal
- $\Pi^L_{\rm eff}(k, k_1, k_2, k_3)$ depends only on linear responses
- quantum recoil terms included in linear responses
- nonlinear wave equation becomes

$$K^L(k)\mathbf{E}(k) = \frac{e^2}{m_e^2\omega_p^4}\int d\lambda^{(3)}\,|\mathbf{k}-\mathbf{k}_1|^2\mathcal{D}^{-1}(k-k_1)\mathbf{E}(k_1)\,\mathbf{E}(k_2)\cdot\mathbf{E}(k_3)$$

$$\mathcal{D}^{-1}(k-k_1) = \frac{1+\chi^L_e(k-k_1)+\chi^L_i(k-k_1)}{\chi^L_e(k-k_1)[1+\chi^L_i(k-k_1)]}$$

## Factorization of nonlinear wave equation

- nonlinear wave equation becomes

$$K^L(k)\mathbf{E}(k) = \frac{e^2}{\varepsilon_0 m_e \omega_p^2} \int d\lambda^{(2)}\, \delta n_e(k_1)\mathbf{E}(k_2)$$

- with the identification

$$\delta n_e(k) = \frac{\varepsilon_0}{m_e \omega_p^2} \mathcal{D}(k)\, \mathbf{k}^2 \int d\lambda^{(2)}\, \mathbf{E}(k_1) \cdot \mathbf{E}(k_2)$$

- assume slowly varying envelope of Langmuir turbulence $\mathbf{E}(x) = \frac{1}{2}\left[\tilde{\mathbf{E}}(x)e^{-i\omega_p t} + \tilde{\mathbf{E}}^*(x)e^{i\omega_p t}\right]$
- invert Fourier transforms, introducing operators
- high frequency ($\omega \approx \omega_p$): $K^L(k) \rightarrow \hat{O}_h^L(x)$

$$\hat{O}_h^L(x) \approx \frac{1}{2\omega_p}\left[i\frac{\partial}{\partial t} + \frac{3V_e^2}{2\omega_p}\nabla^2 + i\frac{\gamma_L}{2} - \frac{\mathrm{QRT}_e}{2\omega_p}\right]$$

- $\mathrm{QRT}_e$ = quantum recoil term for electrons

## Ion sound approximation

- with $k' = k - k_1$, assuming $|\mathbf{k}'|V_i \ll \omega' \ll |\mathbf{k}'|V_e \implies$

$$\chi_e^L(k') \approx \frac{1}{\mathbf{k}'^2\lambda_{De}^2}, \qquad \chi_i^L(k') \approx -\frac{\omega_{pi}^2}{\omega'^2}$$

- ion-sound dispersion relation

$$\omega_s^2(\mathbf{k}') = \frac{\mathbf{k}'^2 v_s^2}{1 + \mathbf{k}'^2\lambda_{De}^2}, \quad v_s = \omega_{pi}\lambda_{De}$$

- beat is ion-sound-like disturbance

$$1 + \chi_e^L(k') + \chi_i^L(k') \approx \frac{1 + \mathbf{k}'^2\lambda_{De}^2}{\mathbf{k}'^2\lambda_{De}^2\omega'^2}[\omega'^2 - \omega_s^2(\mathbf{k}')]$$

- low frequency ($\omega' \ll \omega_{pi}$, $\mathbf{k}'^2\lambda_{De}^2 \ll 1$): $\mathcal{D}(k') \to -\hat{O}_i^L(x)$

$$\hat{O}_i^L \approx \frac{1}{\omega_{pi}^2}\left[\frac{\partial^2}{\partial t^2} - v_s^2\nabla^2 + \gamma_s\frac{\partial}{\partial t} + \mathrm{QRT}_{ei}\right]$$

- $\mathrm{QRT}_{ei}$ = quantum recoil correction for ion sound waves

## Quantum Zakharov equations

- nonlinear equations become

$$\left[i\frac{\partial}{\partial t}+\frac{3V_e^2}{2\omega_p}\nabla^2+i\frac{\gamma_L}{2}-\frac{\mathrm{QRT}_e}{2\omega_p}\right]\tilde{\mathbf{E}}(x)=\omega_p\frac{\delta n_e(x)}{n_e}\tilde{\mathbf{E}}(x)$$

$$\left[\frac{\partial^2}{\partial t^2}-v_s^2\nabla^2+\gamma_s\frac{\partial}{\partial t}+\mathrm{QRT}_i\right]\delta n_e(x)=\frac{\varepsilon_0\omega_{pi}^2}{4m_e\omega_p^2}\nabla^2|\tilde{\mathbf{E}}(x)|^2$$

- quantum Zakharov equations includes recoil terms

$$\mathrm{QRT}_e=\frac{\hbar^2}{4m_e^2}\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2}-\nabla^2\right)^2 \qquad \mathrm{QRT}_{ei}=\frac{m_e}{3m_i}\mathrm{QRT}_e$$

- $(1/c^2)(\partial^2/\partial t^2)$ absent in strictly nonrelativistic theory
- factors of 3 not correct in QFT counterpart

# Discussion & conclusions

## Discussion of QPD

- QPD synthesizes QED and plasma response theory
- provides a basis for treating all quantum plasma effects

## Applications of QPD

- interiors of compact stars
- early Universe
- pulsar & magnetar magnetospheres

## Application to QFT

- Bohm potential equivalent to quantum recoil for $\mathbf{k}^2 \gg \omega^2/c^2$
  Bohm potential wrong for $\omega^2 \gtrsim \mathbf{k}^2 c^2$
- quantum Zakharov equations in QFT?
  derived from nonlinear wave equation in QPD

## Further development of QPD

- inclusion macroscopic mass renormalization
- 'self-consistent' Dirac field
- different masses for electrons & positrons
- do new solutions ('plasmino') exist?