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Off-shell construction of some trilinear higher spin gauge field interactions

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Plan:

1. *Introduction and Motivation*
2. *Exercises on spin 1- field couplings with the higher spin gauge fields*
3. *Generalization to the 2-2-4 and 2-2-6 interactions*
4. *2S-S-S interaction Lagrangian*
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1. Introduction and Motivation

- The construction of interacting ***higher spin gauge field*** theories (HSF) has always been considered an important task.

- Particular attention caused the holographic duality between the *O(N)* sigma model in d=3 space and HSF gauge theory living in the AdS_4. This case of holography is singled out by the existence of two conformal points of the boundary theory and the possibility to describe them by the same HSF gauge theory with the help of *spontaneously breaking of higher spin gauge symmetry and mass generation by a corresponding Higgs mechanism*.

- *Does AdS/CFT works correctly on the level of loop diagrams in the general case and is it possible to use this correspondence for real reconstruction of unknown local interacting theories on the bulk from more or less well known conformal field theories on the boundary side?*

All these complicated physical tasks necessitate *quantum loop* calculations for HSF field theory and therefore information about manifest, off-shell and Lagrangian formulation of possible interactions for HSF.

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2. Exercises on spin 1- field coupling with the HS gauge fields

- We start this section constructing the well known interaction of the electromagnetic field in flat **D** -dimensional space-time with the linearized spin two field.
- Noether's procedure

$$L_0 = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = -\frac{1}{2} \partial_\mu A_\nu \partial^\mu A^\nu + \frac{1}{2} (\partial A)^2,$$

$$\delta^0_\varepsilon h^{(2)\mu\nu}(x) = 2\partial^{(\mu} \varepsilon^{\nu)}(x) = \partial^\mu \varepsilon^\nu(x) + \partial^\nu \varepsilon^\mu(x), \qquad \delta^1_\varepsilon A_\mu = -\varepsilon^\rho \partial_\rho A_\mu + C \varepsilon^\rho \partial_\mu A_\rho$$

$$\delta^1_\varepsilon L_0(A_\mu) + \delta^0_\varepsilon L_1(A_\mu, h^{(2)}_{\mu\nu}) = 0$$

Noether's procedure
regulates the relation between
gauge symmetries of different spin
fields

$$C = 1$$

- **Solution of Noether's Equation**

Field dependent Gauge Variation

Reparametrization

$-\mathcal{L}_\varepsilon A_\mu$

$$\delta^1_\varepsilon A_\mu = \varepsilon^\rho F_{\mu\rho}$$

$$= -\varepsilon^\rho \partial_\rho A_\mu - \partial_\mu \varepsilon^\rho A_\rho + \partial_\mu \left(\varepsilon^\rho(x) A_\rho(x) \right),$$

$$[\delta^1_\eta, \delta^1_\varepsilon] A_\mu = \delta^1_{[\eta,\varepsilon]} A_\rho + \partial_\mu \left(\varepsilon^\rho \eta^\sigma F_{\rho\sigma}(x) \right)$$

$$\delta^1_\varepsilon L_0(A_\mu) = \partial^{(\mu} \varepsilon^{\nu)} F_{\mu\rho} F^\rho_\nu - \frac{1}{4} \partial_\alpha \varepsilon^\alpha F_{\mu\nu} F^{\mu\nu},$$

Energy-Momentum Tensor

$$L_1(A_\mu, h^{(2)}_{\mu\nu}) = \frac{1}{2} h^{(2)\mu\nu} \Psi^{(2)}_{\mu\nu},$$

$$\Psi^{(2)}_{\mu\nu} = -F_{\mu\rho} F^\rho_\nu + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma},$$

- Now we turn to the first nontrivial case of the vector field interaction with a spin four gauge field
- Starting variation

$$\delta^1_\varepsilon A_\mu = \varepsilon^{\rho\lambda\sigma}(x)\partial_\rho\partial_\lambda F_{\mu\sigma}. \qquad \delta^0_\varepsilon h^{\mu\rho\lambda\sigma} = 4\partial^{(\mu}\varepsilon^{\rho\lambda\sigma)}, \quad \delta^0_\varepsilon h^{\rho\lambda\sigma}_\rho = 2\varepsilon^{\lambda\sigma}_{(1)}.$$

Notations

$$\varepsilon^{\mu\nu\dots}_{(1)} = \partial_\lambda\varepsilon^{\lambda\mu\nu\dots}, \quad \varepsilon^{\mu\dots}_{(2)} = \partial_\nu\partial_\lambda\varepsilon^{\nu\lambda\mu\dots}, \quad \dots$$

$$\delta^1_\varepsilon L_0 = -\partial^{(\mu}\varepsilon^{\rho\lambda\sigma)}\partial_{(\rho}F^\nu_\mu\partial_\lambda F_{\sigma)\nu} + \frac{1}{4}\varepsilon^{\lambda\sigma}_{(1)}\partial^\nu F_{\mu\lambda}\partial^\mu F_{\nu\sigma} + \frac{1}{4}\varepsilon^{\lambda\sigma}_{(1)}\partial_\lambda F_{\mu\nu}\partial_\sigma F^{\mu\nu}$$

$$+\partial^{(\mu}\varepsilon^{\nu)}_{(2)}F_{\mu\sigma}F^\sigma_\nu - \frac{1}{4}\varepsilon_{(3)}F_{\mu\nu}F^{\mu\nu} - \partial_\lambda(\varepsilon^{\lambda\sigma}_{(1)}F_{\mu\sigma})\partial_\nu F^{\nu\mu} - \frac{1}{4}\varepsilon^{\lambda\sigma}_{(1)}\partial^\mu F_{\mu\lambda}\partial^\nu F_{\nu\sigma} - \frac{1}{2}\partial^\rho\varepsilon^{\nu\lambda\sigma}\partial_\lambda F_{\sigma\rho}\partial^\mu F_{\mu\nu}$$

Field Redefinition

$$A_\mu \to A_\mu - \frac{1}{2}\partial_\lambda(h^{\alpha\lambda\sigma}_\alpha F_{\mu\sigma}) - \frac{1}{8}h^\alpha_{\alpha\mu\sigma}\partial_\beta F^{\beta\sigma}.$$

Variation Modification

$$\delta^1_\varepsilon A_\mu = \varepsilon^{\rho\lambda\sigma}\partial_\rho\partial_\lambda F_{\mu\sigma} + \frac{1}{2}\partial_\rho\varepsilon_{\mu\lambda\sigma}\partial^\lambda F^{\sigma\rho}.$$

Integrable Part

$$L_1(A_\mu, h^{(2)\mu\nu}, h^{(4)\mu\nu\alpha\beta}) = \frac{1}{4}h^{(4)\mu\nu\alpha\beta}\Psi^{(4)}_{\mu\nu\alpha\beta} + \frac{1}{2}h^{(2)\mu\nu}\Psi^{(2)}_{\mu\nu},$$

4-1-1 Interaction

$$\delta^1_\varepsilon L_0(A_\mu) + \delta^0_\varepsilon L_1(A_\mu, h^{(2)}_{\mu\nu}, h^{(4)}_{\mu\nu}) = 0$$

$$L_1(A_\mu, h^{(2)\mu\nu}, h^{(4)\mu\nu\alpha\beta}) = \frac{1}{4} h^{(4)\mu\nu\alpha\beta} \Psi^{(4)}_{\mu\nu\alpha\beta} + \frac{1}{2} h^{(2)\mu\nu} \Psi^{(2)}_{\mu\nu},$$

$$\Psi^{(4)}_{\mu\nu\alpha\beta} = \partial_{(\alpha} F^{\rho}_{\mu} \partial_\beta F_{\nu)\rho} - \frac{1}{2} g_{(\mu\nu} \partial^\lambda F_{\alpha\sigma} \partial^\sigma F_{\beta)\lambda} - \frac{1}{2} g_{(\mu\nu} \partial_\alpha F^{\sigma\rho} \partial_{\beta)} F_{\sigma\rho}.$$

$$\Psi^{(2)}_{\mu\nu} = -F_{\mu\rho} F^{\rho}_{\nu} + \frac{1}{4} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma},$$

$$\delta^1 A_\mu = \varepsilon^{\rho\lambda\sigma} \partial_\rho \partial_\lambda F_{\mu\sigma} + \frac{1}{2} \partial_\rho \varepsilon_{\mu\lambda\sigma} \partial^\lambda F^{\sigma\rho},$$

$$\delta^0 h^{(4)\mu\nu\alpha\beta} = 4\partial^{(\mu} \varepsilon^{\nu\alpha\beta)}, \delta^0_\varepsilon h^{\mu\alpha\beta}_{\mu} = 2\varepsilon^{\alpha\beta}_{(1)},$$

$$\delta^0 h^{(2)\mu\nu} = 2\partial^{(\mu} \varepsilon^{\nu)}_{(2)}, \delta^0 h^{(2)\mu}_{\mu} = 2\varepsilon_{(3)}.$$

an additional spin two field coupling !!

Spin one gauge field couplings to the higher spin gauge fields

$$\delta_{\varepsilon}^{1} A_{\mu} = \varepsilon_{\ell}^{\mu_1 \ldots \mu_{l-1}} \nabla_{\mu_1} \ldots \nabla_{\mu_{l-2}} F_{\mu_{l-1}\mu},$$

Starting variation

$$\delta_{\varepsilon}^{0} h^{(\ell)\mu_1 \ldots \mu_l} = l \nabla^{(\mu_l} \varepsilon_{\ell}^{\mu_1 \mu_2 \ldots \mu_{l-1})}, \quad \delta_{\varepsilon}^{0} h_{\alpha}^{(\ell)\alpha\mu_1 \ldots \mu_{l-2}} = 2 \varepsilon_{\ell(1)}^{\mu_1 \ldots \mu_{l-2}}$$

$$\delta_{\varepsilon}^{1} L_0(A_{\mu}) + \delta_{\varepsilon}^{0} L_1(A_{\mu}, h^{(2)\mu_1\mu_2}, \ldots., h^{(\ell)\mu_1 \ldots \mu_l}) = 0$$

$$\delta_\varepsilon^1 L_0 = \sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \left(-\nabla^{(\mu_{2m}}\varepsilon_{\ell(l-2m)}^{\mu_1\dots\mu_{2m-1})}\Psi_{\mu_1\dots\mu_{2m}}(A_\mu)\right)$$

$$+\sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{m} \nabla_{\mu_{m+1}}\dots\nabla_{\mu_{2m-2}} (\nabla_\nu \varepsilon_{\ell(l-2m)\mu}^{\mu_1\dots\mu_{2m-2}} \nabla_{\mu_1}\dots\nabla_{\mu_{m-1}} F^\nu_{\mu_m}) \nabla_\alpha F^{\alpha\mu}$$

$$+\sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{2m} \nabla_{\mu_m}\dots\nabla_{\mu_{2m-3}} (\varepsilon_{\ell(l-2m+1)\mu}^{\mu_1\dots\mu_{2m-3}} \nabla_{\mu_1}\dots\nabla_{\mu_{m-2}} \nabla^\nu F_{\nu\mu_{m-1}}) \nabla_\alpha F^{\alpha\mu}$$

$$-\sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{l-2m+1} \nabla_{\mu_m}\dots\nabla_{\mu_{2m-2}} (\varepsilon_{\ell(l-2m+1)}^{\mu_1\dots\mu_{2m-2}} \nabla_{\mu_1}\dots\nabla_{\mu_{m-2}} F_{\mu_{m-1}\mu}) \nabla_\alpha F^{\alpha\mu}$$

Field redefinition terms **Variation Modification**

Integrable terms

Interaction

$$L_1(A_\mu, h^{(2)}, h^{(4)}, \dots, h^{(\ell)}) = \sum_{m=1}^{\ell/2} \frac{1}{2m} h^{(2m)\mu_1\dots\mu_{2m}} \Psi^{(2m)}_{\mu_1\dots\mu_{2m}}(A_\mu)$$

$$\Psi_{\mu_1\dots\mu_{2m}}(A_\mu) = (-1)^m \Big(-\nabla_{\mu_1}\dots\nabla_{\mu_{m-1}} F^{\nu}_{\mu_m} \nabla_{\mu_{m+1}}\dots\nabla_{\mu_{2m-1}} F_{\mu_{2m}\nu}$$

$$+\frac{m-1}{2} g_{\mu_1\mu_2} \nabla_{\mu_3}\dots\nabla_{\mu_m} \nabla^{\alpha} F_{\mu_{m+1}\beta} \nabla_{\mu_{m+2}}\dots\nabla_{\mu_{2m-1}} \nabla^{\beta} F_{\mu_{2m}\alpha}$$

$$+\frac{m}{4} g_{\mu_1\mu_2} \nabla_{\mu_3}\dots\nabla_{\mu_{m+1}} F^{\rho\sigma} \nabla_{\mu_{m+2}}\dots\nabla_{\mu_{2m}} F_{\rho\sigma} \Big)$$

$$\delta^1_\varepsilon A_\mu = \varepsilon_\ell^{\mu_1\dots\mu_{l-1}} \nabla_{\mu_1}\dots\nabla_{\mu_{l-2}} F_{\mu_{l-1}\mu}$$

$$+\sum_{m=1}^{\ell/2} \binom{\ell-m-1}{m-1} \frac{m-1}{m} \nabla_{\mu_{m+1}}\dots\nabla_{\mu_{2m-2}} (\nabla_\nu \varepsilon^{\mu_1\dots\mu_{2m-2}}_{\ell(l-2m)\mu} \nabla_{\mu_1}\dots\nabla_{\mu_{m-1}} F^{\nu}_{\mu_m})$$

Final variation

Field Redefinition

$$A_\mu \to A_\mu - \sum_{m=1}^{\ell/2} \binom{\ell - m - 1}{m-1} \frac{m-1}{2(l-2m+1)} \nabla_{\mu_m} \dots \nabla_{\mu_{2m-2}} (h_\alpha^{(2m)\alpha\mu_1 \dots \mu_{2m-2}} \nabla_{\mu_1} \dots \nabla_{\mu_{m-2}} F_{\mu_{m-1}\mu})$$

$$+\sum_{m=1}^{\ell/2} \binom{\ell - m - 1}{m-1} \frac{m-1}{4m} \nabla_{\mu_m} \dots \nabla_{\mu_{2m-3}} (h_{\alpha\mu}^{(2m)\alpha\mu_1 \dots \mu_{2m-3}} \nabla_{\mu_1} \dots \nabla_{\mu_{m-2}} \nabla^\nu F_{\nu\mu_{m-1}})$$

The gauge invariant action for the spin ℓ gauge field coupled to the spin 1 gauge field includes gauge invariant actions of tower of all smaller even spin gauge fields coupled to the same vector field in the same way.

Generalization to the 2-2-4 and 2-2-6 interactions

Starting point transformation $$\delta^1_\varepsilon h^{(2)}_{\mu\nu} = \varepsilon^{\rho\lambda\sigma}\partial_\rho \Gamma_{\lambda\sigma,\mu\nu}$$

and spin two Pauli-Fierz Lagrangian

$$L_0(h^{(2)}_{\mu\nu}) = \frac{1}{2}\partial_\mu h^{(2)}_{\alpha\beta}\partial^\mu h^{(2)\alpha\beta} - \partial_\alpha h^{(2)\alpha\beta}\partial_\mu h^{(2)\mu}_\beta + \partial_\mu h^{(2)\alpha}_\alpha \partial_\beta h^{(2)\beta\mu} - \frac{1}{2}\partial_\mu h^{(2)\alpha}_\alpha \partial^\mu h^{(2)\beta}_\beta,$$

where $\Gamma_{\lambda\sigma,\mu\nu}$ is the spin two gauge invariant symmetrized linearized Riemann curvature

$$\Gamma_{\alpha\beta,\mu\nu} = \frac{1}{2}(R_{\alpha\mu,\beta\nu} + R_{\beta\mu,\alpha\nu}), \qquad \Gamma_{(\alpha\beta,\mu)\nu} = 0,$$

We solve the following Noether's equation

$$\delta^1_\varepsilon L_0(h^{(2)}_{\mu\nu}) + \delta^0_\varepsilon L_1(h^{(2)}_{\mu\nu}, h^{(4)\alpha\beta\lambda\rho}) = 0.$$

To integrate the Noether's equation we submit to the following strategy:

1) First we perform a partial integration and use the Bianchi identity

$$\partial_{\lambda} R_{\alpha\beta} = \partial^{\mu} \Gamma_{\mu\lambda,\alpha\beta} + \partial_{(\alpha} R_{\beta)\lambda},$$

to lift the variation to a curvature square term.

2) Then we make a partial integration again and rearrange indices using

$$\partial_{\lambda} \Gamma_{\mu\nu,\alpha\beta} = \partial_{(\mu} \Gamma_{\nu)\lambda,\alpha\beta} + \partial_{(\alpha} \Gamma_{\beta)\lambda,\mu\nu}$$

to extract an integrable part.

3) Symmetrizing expressions in this way we classify terms as

- **integrable**
- **integrable and subjected to field redefinition** (proportional to the free field equation of motion)
- **non integrable but reducible by deformation of the initial ansatz for the gauge transformation** (again proportional to the free field equation of motion)

Solution for 4-2-2

After field redefinition

$$h_{\mu\nu}^{(2)} \to h_{\mu\nu}^{(2)} - \frac{1}{2} h_{\alpha}^{(4)\alpha\lambda\sigma} \Gamma_{\lambda\sigma,\mu\nu} - \frac{1}{4} h_{\mu\nu}^{(4)\alpha\lambda} R_{\alpha\lambda} + \frac{1}{4} h_{\alpha(\mu}^{(4)\alpha\lambda} R_{\nu)\lambda}.$$

We arrive at the 4-2-2 gauge invariant interaction

$$L_1(h_{\mu\nu}^{(2)}, h_{\alpha\beta\mu\nu}^{(4)}) = \frac{1}{4} h^{(4)\alpha\beta\mu\nu} \Psi_{(\Gamma)\alpha\beta\mu\nu}^{(4)}(h_{\mu\nu}^{(2)})$$

Bell-Robinson current

$$\Psi_{(\Gamma)\alpha\beta\mu\nu}^{(4)}(h_{\mu\nu}^{(2)}) = \frac{1}{4} h^{(4)\alpha\beta\mu\nu} \Gamma_{\alpha\beta,\rho\sigma} \Gamma_{\mu\nu,}^{\rho\sigma} - \frac{1}{6} h_{\alpha}^{(4)\alpha\mu\nu} \Gamma_{\mu}^{\rho,\sigma\lambda} \Gamma_{\nu\rho,\sigma\lambda},$$

with the following gauge transformations

$$\delta_\varepsilon h_{\mu\nu}^{(2)} = \varepsilon^{\rho\lambda\sigma} \partial_\rho \Gamma_{\lambda\sigma,\mu\nu} - \partial_\rho \varepsilon_{\lambda\sigma(\mu} \Gamma_{\nu)}^{\rho,\lambda\sigma},$$

$$\delta_\varepsilon^0 h^{(4)\mu\rho\lambda\sigma} = 4\partial^{(\mu} \varepsilon^{\rho\lambda\sigma)}, \quad \delta_\varepsilon^0 h_\rho^{(4)\rho\lambda\sigma} = 2\varepsilon_{(1)}^{\lambda\sigma}.$$

Solution for 6-2-2

After field redefinition

$$h^{(2)\mu\nu} \to h^{(2)\mu\nu} + R^{\mu\nu}_{int}(\Gamma, R, h^{(6)}, h^{(4)}),$$

where

$$R^{\mu\nu}_{int}(\Gamma, R, \varepsilon) = \frac{1}{2} h^{(6)\mu\alpha\beta\lambda\delta}_{\mu} \partial_\alpha \partial_\beta \Gamma_{\lambda\delta,}^{\ \ \ \mu\nu} - \frac{1}{6} \partial^\lambda h^{(6)\gamma\alpha\beta\delta(\mu}_{\gamma} \partial_\alpha \Gamma^{\nu)}_{\ \lambda,\beta\delta} + \partial_\lambda \left[\tfrac{1}{6} h^{(6)\lambda\alpha\beta\delta\mu\nu} \partial_\alpha R_{\beta\delta} \right]$$

$$-\frac{1}{3} \partial_\lambda \left[h^{(6)\gamma\lambda\alpha\mu\nu}_{\gamma} \partial_\alpha R \right] + \frac{1}{12} h^{(6)\gamma\alpha\beta\mu\nu}_{\gamma} \partial_\alpha \partial_\beta F + \frac{1}{4} h^{(4)\alpha\beta\mu\nu} R_{\alpha\beta} + \frac{5}{6} \partial^\alpha h^{(6)\gamma\beta\lambda\mu\nu}_{\gamma} \partial_\lambda R_{\alpha\beta}$$

$$-\frac{5}{6} \partial_\lambda \left[h^{(6)\gamma\lambda\alpha\beta(\mu}_{\gamma} \partial_\alpha R^{\nu)}_{\beta} \right] + \frac{1}{12} h^{(6)\gamma\alpha\beta\mu\nu}_{\gamma} R_{\alpha\beta} - \frac{1}{12} \partial^\lambda h^{(6)\gamma\alpha\beta\mu\nu}_{\gamma} \partial_\lambda R_{\alpha\beta} - \frac{1}{4} h^{(4)\gamma\alpha(\mu}_{\gamma} R^{\nu)}_{\alpha}.$$

We arrive at the 6-2-2 gauge invariant interaction

$$L_1(h^{(2)},h^{(4)},h^{(6)}) = -\frac{1}{6}h^{(6)\alpha\beta\mu\nu\lambda\rho}\Psi^{(6)}_{(\Gamma)\alpha\beta\mu\nu\lambda\rho} + \frac{1}{4}h^{(4)\alpha\beta\mu\nu}\Psi^{(4)}_{(\Gamma)\alpha\beta\mu\nu}$$

$$\Psi^{(6)}_{(\Gamma)\alpha\beta\mu\nu\lambda\rho} = \partial_{(\alpha}\Gamma_{\beta\mu,}{}^{\sigma\delta}\partial_{\nu}\Gamma_{\lambda\rho),\sigma\delta} - g_{(\alpha\beta}\partial_{\mu}\Gamma^{\kappa,\sigma\delta}_{\nu}\partial_{\lambda}\Gamma_{\rho)\kappa,\sigma\delta}$$

$$-\frac{1}{2}g_{(\alpha\beta}\partial^{\kappa}\Gamma_{\mu\nu,}{}^{\sigma\delta}\partial_{\sigma}\Gamma_{\lambda\rho),\kappa\delta},$$

$$\Psi^{(4)}_{(\Gamma)\alpha\beta\mu\nu}(h^{(2)}_{\mu\nu}) = \frac{1}{4}h^{(4)\alpha\beta\mu\nu}\Gamma_{\alpha\beta,\rho\sigma}\Gamma^{\rho\sigma}_{\mu\nu,} - \frac{1}{6}h^{(4)\alpha\mu\nu}_{\alpha}\Gamma^{\rho,\sigma\lambda}_{\mu}\Gamma_{\nu\rho,\sigma\lambda},$$

with the following gauge transformations

$$\delta^1_\varepsilon h^{(2)}_{\alpha\beta} = \varepsilon^{\mu\nu\rho\lambda\sigma}\partial_\mu\partial_\nu\partial_\rho\Gamma_{\lambda\sigma,\alpha\beta} - \frac{4}{3}\partial^\rho\varepsilon_\alpha{}^{\mu\nu\lambda\sigma}\partial_\lambda\partial_\sigma\Gamma_{\beta\rho,\mu\nu} + \frac{1}{3}\partial^\rho\partial^\lambda\varepsilon_{\alpha\beta}{}^{\mu\nu\sigma}\partial_\sigma\Gamma_{\rho\lambda,\mu\nu}.$$

$$\delta^0_\varepsilon h^{(6)\mu\nu\alpha\beta\sigma\rho} = 6\partial^{(\mu}\varepsilon^{\nu\alpha\beta\sigma\rho)}(x),$$

2S-S-S Interaction

1) Formalism

$$h^{(s)}(z;a)=\sum_{\mu_i}(\prod_{i=1}^{s}a^{\mu_i})h^{(s)}_{\mu_1\mu_2\dots\mu_s}(z).$$

The most elegant and convenient way of handling symmetric tensors is by contracting it with the s'th tensorial power of a vector a^{μ_i} and star contraction

$$Tr: h^{(s)}(z;a)\Rightarrow Trh^{(s-2)}(z;a)=\frac{1}{s(s-1)}\Box_a h^{(s)}(z;a),$$

$$Grad: h^{(s)}(z;a)\Rightarrow Gradh^{(s+1)}(z;a)=(a\nabla)h^{(s)}(z;a),$$

$$*_a=\frac{1}{(s!)^2}\prod_{i=1}^{s}\overleftarrow{\partial}_a^{\mu_i}\overrightarrow{\partial}^a_{\mu_i}.$$

$$Div: h^{(s)}(z;a)\Rightarrow Divh^{(s-1)}(z;a)=\frac{1}{s}(\nabla\partial_a)h^{(s)}(z;a).$$

$$\Gamma^{(s)}_{(n)}(z;b,a)\equiv\Gamma^{(s)}_{(n)\rho_1\dots\rho_n,\mu_1\dots\mu_\ell}b^{\rho_1}\dots b^{\rho_n}a^{\mu_1}\dots a^{\mu_s}$$

De Witt-Freedman Christoffel Symbols- **bitensors**

$$\begin{matrix}(a\nabla)\\(b\nabla)\end{matrix}f^{(m-1,n-1)}(z;a,b)*_{a,b}g^{(m,n)}(z;a,b)=-f^{(m-1,n-1)}(z;a,b)*_{a,b}\begin{matrix}\frac{1}{m}(\nabla\partial_a)\\\frac{1}{n}(\nabla\partial_b)\end{matrix}g^{(m,n)}(z;a,b)$$

$$\begin{matrix}a^2\\b^2\end{matrix}f^{(m-2,n-2)}(a,b)*_{a,b}g^{(m,n)}(a^m,b^n)=f^{(m-2,n-2)}(a,b)*_{a,b}\begin{matrix}\frac{1}{m(m-1)}\Box_a\\\frac{1}{n(n-1)}\Box_b\end{matrix}g^{(m,n)}(a,b).$$

Duality relations

Fronsdal fields , Equation and Lagrangian

$$\delta h^{(s)}(z;a) = s(a\nabla)\varepsilon^{(s-1)}(z;a),$$

Gauge field

$$\Box_a^2 h^{(s)}(z;a) = 0, \qquad \Box_a \varepsilon^{(s-1)}(z;a) = 0$$

Field Equation

$$\mathcal{F}^{(s)}(z;a) = \Box h^{(s)}(z;a) - (a\nabla)(\nabla\partial_a)h^{(s)}(z;a) + \frac{1}{2}(a\nabla)^2\Box_a h^{(s)}(z;a) = 0$$

Lagrangian

$$L_0(h^{(s)}(a)) = -\frac{1}{2}h^{(s)}(a) *_a \mathcal{F}^{(s)}(a) + \frac{1}{8s(s-1)}\Box_a h^{(s)}(a) *_a \Box_a \mathcal{F}^{(s)}(a)$$

$$\delta L_0(h^{(s)}(a)) = -(\mathcal{F}^{(s)}(a) - \frac{a^2}{4}\Box_a \mathcal{F}^{(s)}(a)) *_a \delta h^{(s)}(a)$$

HS Curvature

$$\Gamma^{(s)}(z;b,a)=\prod_{n=1}^{s}[(b\nabla)-\frac{1}{n}(a\nabla)(b\partial_a)]h^{(s)}(z;a),$$

$$\Gamma^{(s)}(z;b,a)=\sum_{k=0}^{s}\frac{(-1)^k}{k!}(b\nabla)^{n-k}(a\nabla)^k(b\partial_a)^k h^{(s)}(z;a).$$

$$\Gamma^{(s)}(z;a,b)=\Gamma^{(s)}(z;b,a).$$

Primary Bianchi Identity

$$(a\partial_b)\Gamma^{(s)}(z;a,b)=(b\partial_a)\Gamma^{(s)}(z;a,b)=0.$$

Bianchi Identities **(Secondary)**

$$\frac{\partial}{\partial a^{[\mu}}\frac{\partial}{\partial b^{\nu}}\nabla_{\lambda]}\Gamma^{(s)}(z;a,b)=0,$$

$$s\nabla_{\mu}\Gamma^{(s)}(z;a,b)=(a\nabla)\partial_{\mu}^{a}\Gamma^{(s)}(z;a,b)+(b\nabla)\partial_{\mu}^{b}\Gamma^{(s)}(z;a,b)$$

From Primary Bianchi Identity

$$(\partial_a\partial_b)\Gamma^{(s)}(z;b,a)=-\frac{1}{2}(b\partial_a)\Box_b\Gamma^{(s)}(z;b,a)=-\frac{1}{2}(a\partial_b)\Box_a\Gamma^{(s)}(z;b,a)$$

$$(s-1)(\nabla\partial_b)\Gamma^{(s)}(z;a,b)=[(b\nabla)-\frac{1}{2}(a\nabla)(b\partial_a)]\Box_b\Gamma^{(s)}(z;a,b)$$

$$(\nabla\partial_a)\mathcal{F}^{(s)}(z;a)=\frac{1}{2}(a\nabla)\Box_a\mathcal{F}^{(s)}(z;a)$$

Similar to Ricci tensor Identity

Curvatute and Fronsdal Equation

$$\delta\Gamma^{(s)}(z;b,a)=0$$

$$\Box_b\Gamma^{(s)}(z;b,a)=s(s-1)U(a,b,3,s)\mathcal{F}^{(s)}(z;a)$$

Trace

$$U(a,b,3,s)=\prod_{k=3}^{s}[(b\nabla)-\frac{1}{k}(a\nabla)(b\partial_a)].$$

$$\delta L_0(h^{(s)}(a)) = -(\mathcal{F}^{(s)}(a) - \frac{a^2}{4}\Box_a \mathcal{F}^{(s)}(a)) *_a \delta h^{(s)}(a)$$

From double tracelessness of gauge field

$$\delta h^{(s)}(a) = \delta h^{(s)}_{(1)}(a) + a^2 \delta h^{(s-2)}(a) + (a^2)^2 \delta h^{(s-4)}(a) + \ldots$$

$$\delta h^{(s-2)}(a) = \frac{1}{2(D+2s-2)}\Box_a \delta h^{(s)}_{(1)}(a)$$

We choose the second term of our field variation in a form that simplifies Lagrangian variation.

$$\delta_{(1)} L_0(h^{(s)}(a)) = -\mathcal{F}^{(s)}(a) *_a \delta h^{(s)}_{(1)}(a)$$

To integrate the Noether's equation we submit to the following strategy:

1) First we perform a partial integration and use the Bianchi identity

$$(s-1)(\nabla\partial_b)\Gamma^{(s)}(z;a,b) = s(s-1)U(a,b,2,s)\mathcal{F}^{(s)}(z;a)$$

to lift the variation to a curvature square term.

2) Then we make a partial integration again and rearrange indices using

$$s\nabla_\mu\Gamma^{(s)}(z;a,b) = (a\nabla)\partial^a_\mu\Gamma^{(s)}(z;a,b) + (b\nabla)\partial^b_\mu\Gamma^{(s)}(z;a,b)$$

to extract an integrable part.

3) Symmetrizing expressions in this way we classify terms as

- **integrable**
- **integrable and subjected to field redefinition** (proportional to the free field equation of motion)
- **non integrable but reducible by deformation of the initial ansatz for the gauge transformation** (again proportional to the free field equation of motion)

Using this technique we can guess the right variation

$$\delta h_{(1)}^{(s)}(a) = \tilde{U}(b,a,2,s)[\varepsilon^{2s-1}(z;b) *_b \Gamma^{(s)}(z;b,a)]$$

where

$$\tilde{U}(b,a,2,s) = \frac{(-1)^s}{(s-1)!}\prod_{k=2}^{s}\left[(\nabla\partial_b) - \frac{1}{k}(a\partial_b)(\nabla\partial_a)\right]$$

Is dual to

$$[(b\nabla) - \frac{1}{2}(a\nabla)(b\partial_a)]U(b,a,3,s) = \prod_{k=2}^{s}[(b\nabla) - \frac{1}{k}(a\nabla)(b\partial_a)],$$

Then we bring variation of the Lagrangian to the following form

$$\delta_{(1)}L_0(h^{(s)}(a)) = -(b\nabla)\varepsilon^{2s-1}(b) *_b \Gamma^{(s)}(b,a) *_a \Gamma^{(s)}(b,a) + \frac{1}{2s(s+1)(2s-1)}(\nabla\partial_b)\varepsilon^{2s-1}(b) *_b \partial_\mu^b \Gamma^{(s)}(b,a) *_a \partial_b^\mu \Gamma^{(s)}(b,a)$$

Result for 2S-S-S Interaction

$$\delta_{(1)} L_0(h^{(s)}(a)) + \delta_0 L_1(h^{(s)}(a), h^{(2s)}(b)) = 0$$

$$L_1(h^{(s)}(a), h^{(2s)}(b)) = \frac{1}{2s} h^{(2s)}(z;b) *_b \Psi^{(2s)}_{(\Gamma)}(z;b)$$

generalized Bell-Robinson current

$$\Psi^{(2s)}_{(\Gamma)}(z;b) = \Gamma^{(s)}(b,a) *_a \Gamma^{(s)}(b,a) - \frac{b^2}{2(s+1)} \partial^b_\mu \Gamma^{(s)}(b,a) *_a \partial^\mu_b \Gamma^{(s)}(b,a)$$

$$\delta h^{(s)}_{(1)}(a) = \frac{(-1)^s}{(s-1)!} \prod_{k=2}^{s} \left[(\nabla\partial_b) - \frac{1}{k}(a\partial_b)(\nabla\partial_a) \right] [\varepsilon^{2s-1}(z;b) *_b \Gamma^{(s)}(z;b,a)]$$

$$\delta_0 h^{(2s)}(z;b) = 2s(b\nabla)\varepsilon^{(2s-1)}(z;b), \;\; \delta_0 \Box_b h^{(2s)}(z;b) = 4s(\nabla\partial_b)\varepsilon^{(2s-1)}(z;b)$$

Conclusions

We presented interaction Lagrangians for triplets of higher spin fields, a pair of which has equal spin S_1 whereas the third has spin $S_2 \geq 2S_1$. Besides the Lagrangians the next-to-leading order of the gauge transformations is given. The fields of smaller spins appear combined into currents of the Bell-Robinson form .

Remarkable is that for one such spin the interaction implies the existence of a whole ladder of interactions for smaller spins $S_2 - 2n \geq 2S_1$.

Outlook

- **Constraction of Self Interactions using language of deWit-Freedman curvature and Christoffel symbols**
- **Several one loop calculations with these interactions actual for AdS/CFT**

Thank You