Boson spectral function and the electron-phonon interaction (EPI) of high-temperature superconductors (HTSC)

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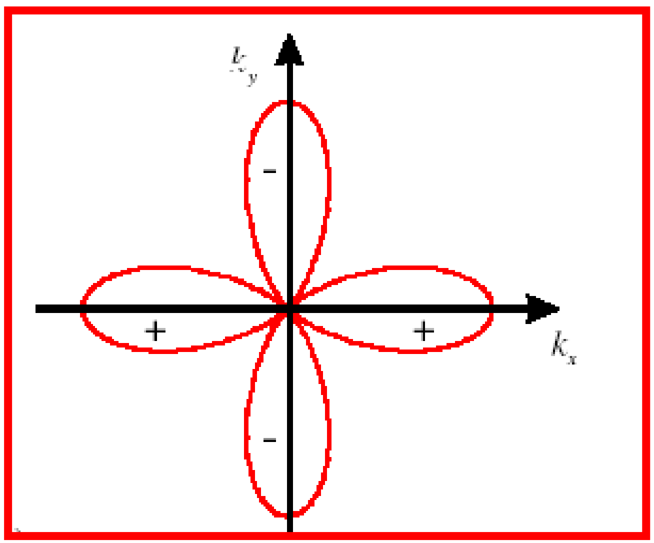
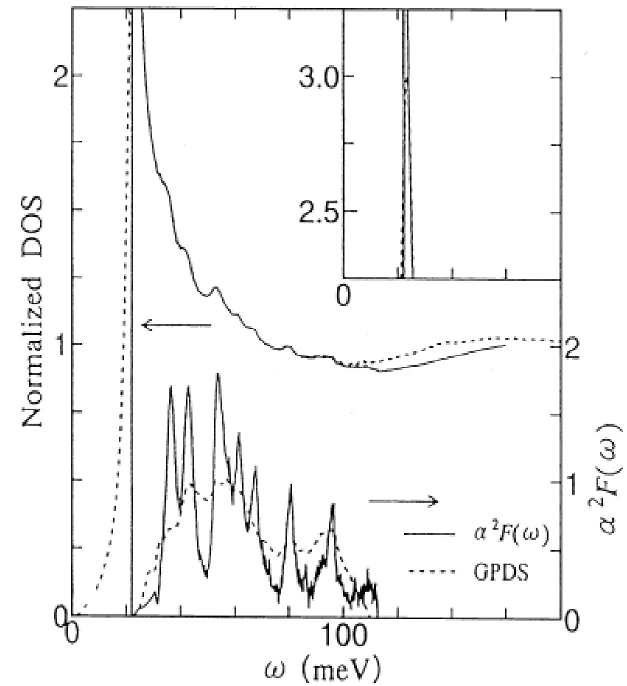


d-wave pairing

EPI
vs
Coulomb (SFI,...)



EPI spectral function

Review

E. G. Maksimov, M. L. Kulic′, O. V. Dolgov, arXiv:0810.3789

Content

I. *Phononic (EPI) vs non - phononic mechanisms*

- d-wave pairing $\rightarrow \Delta_{d_{x^2-y^2}}(\boldsymbol{k},\omega) \approx \Delta^0(\omega)(\cos k_x - \cos k_y)$

- bosonic spectral function $\alpha^2 F_B(\omega)$

II. *Experiments related to* $\alpha^2 F_B(\omega)$

- Reflectivity $R(\omega) \rightarrow \sigma(\omega) \rightarrow \Gamma_{\mathrm{tr}}(\omega) \neq \Gamma(\omega) \equiv -2\,\mathrm{Im}\,\Sigma(\omega)$
- ARPES $\rightarrow \mathrm{I}(\mathbf{k},\omega) \sim \mathrm{n_F}(\omega)\mathrm{A}(\mathbf{k},\omega) \rightarrow \Sigma(\mathbf{k},\omega)$
- tunneling $\rightarrow I(V) \rightarrow \alpha^2 F_B(\omega)$ (for phonons)

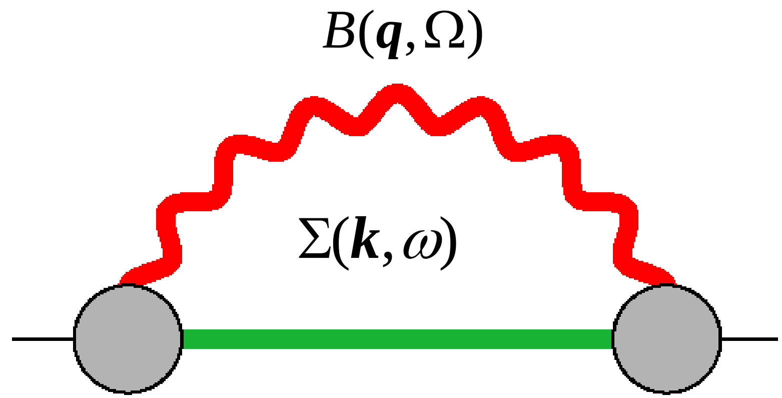
III. $\sigma(\omega), I(V), \mathrm{A}(\mathbf{k},\omega) \rightarrow$ EPI is important and *there is characteristic phononic energy scale*

$$\rightarrow \alpha^2 F_{ph}(\omega) \rightarrow \lambda_{ep} = 2\text{-}3$$

IV. Challenge for the theory $\rightarrow$ *EPI must be strongly momentum dependent*!

V. *Concusions*

Phononic vs non-phononic pairing – bosonic spectral function



$$\Sigma(\boldsymbol{k},\omega) = \int_0^\infty d\Omega \left\langle \alpha^2 F_B(\boldsymbol{k},\boldsymbol{k}',\Omega) \right\rangle_{\boldsymbol{k}'} R(\omega,\Omega)$$

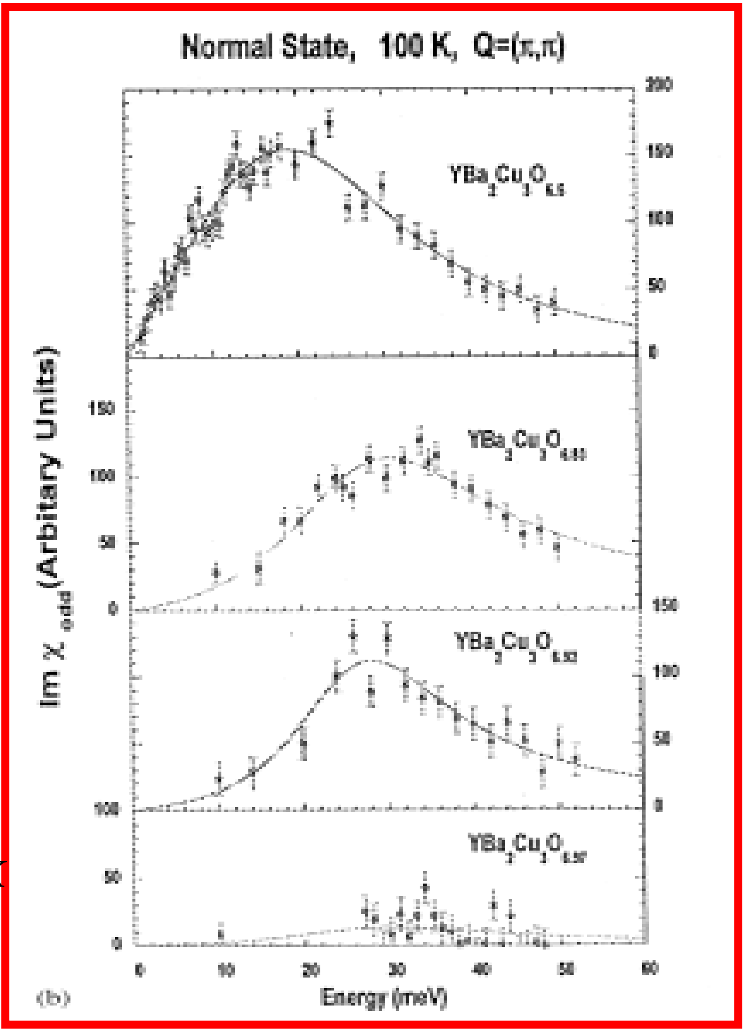
$$\alpha^2 F_B(\boldsymbol{k},\boldsymbol{k}',\Omega) = N(0) \left| g_{\boldsymbol{k},\boldsymbol{k}'} \right|^2 \operatorname{Im} B(\boldsymbol{k}-\boldsymbol{k}',\Omega)$$

$$B(\boldsymbol{q},\Omega) = D_{ph}(\boldsymbol{q},\Omega) \; or \; V_c(\boldsymbol{q},\Omega) \; or \; Im\chi(\boldsymbol{q},\Omega)$$

Inelastic magnetic neutron scattering and NMR hint against SFI mechanism

$T_c = 91\,K$

$T_c = 92.5\,K$

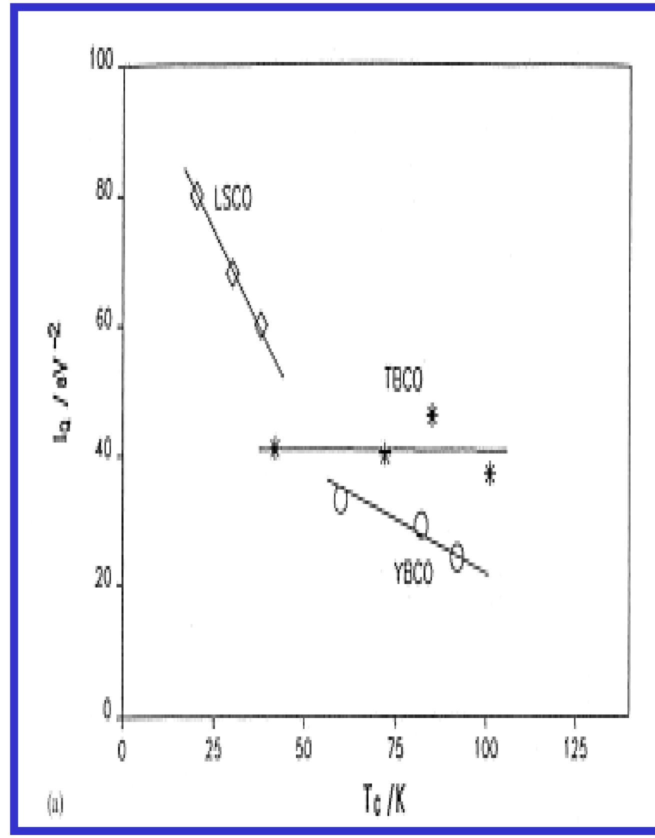


Big change in $Im\chi$ but **small change** in T_c !

Ph. Bourges et al. (1999)

$$I_Q = \lim_{\omega \to 0} \frac{Im\chi_Q(\omega)}{\omega}$$

$$\mathbf{Q} = (\pi, \pi)$$



Anticorrelation between I_Q and T_c !

M. Mehring (1993)

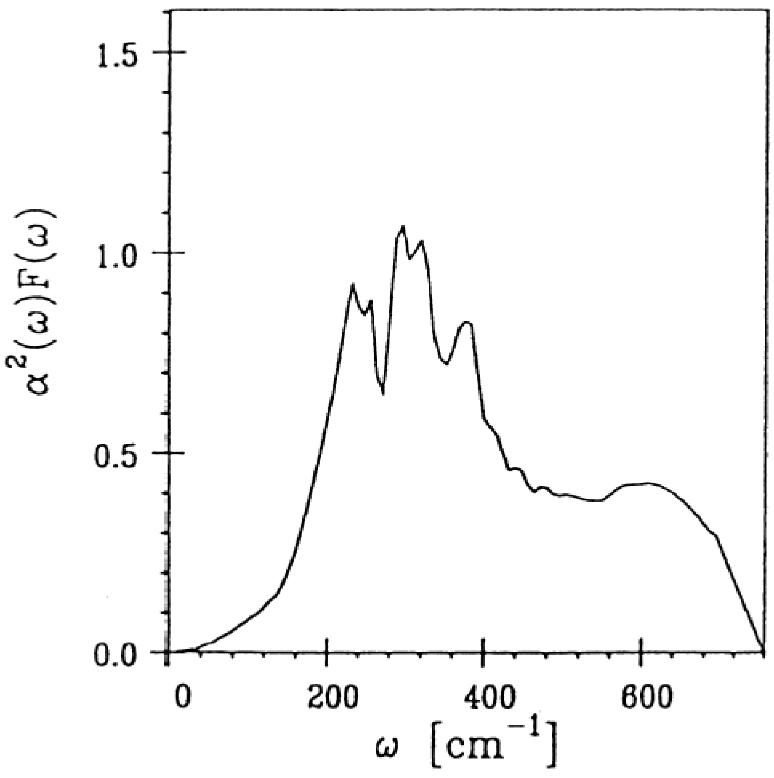
$\sigma(\omega)$ from reflectivity $R(\omega)$

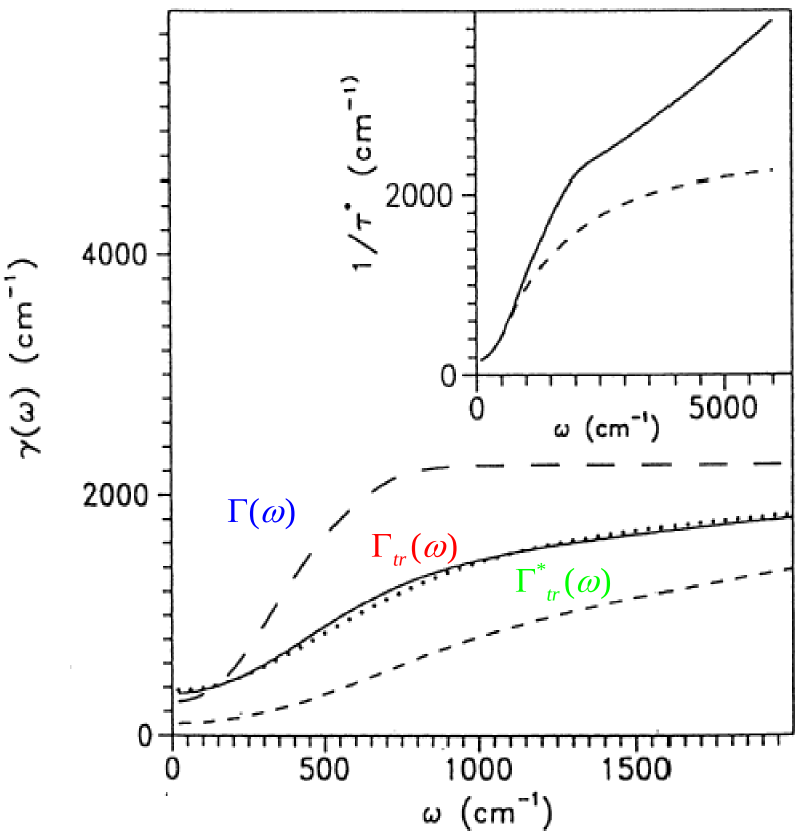
Optical data can be naturally explained by strong EPI !

$$\sigma(\omega) = \frac{\omega^2{}_{pl}}{4\pi} \frac{1}{\Gamma_{tr}(\omega) - i\omega m_{tr}(\omega)/m_\infty}$$

$$\Gamma_{tr}(\omega) \neq \Gamma(\omega) = -2\,\mathrm{Im}\,\Sigma(\omega)$$

$$\Gamma^*{}_{tr}(\omega) = \frac{m_{tr}(\omega)}{m_\infty}\Gamma_{tr}(\omega)$$



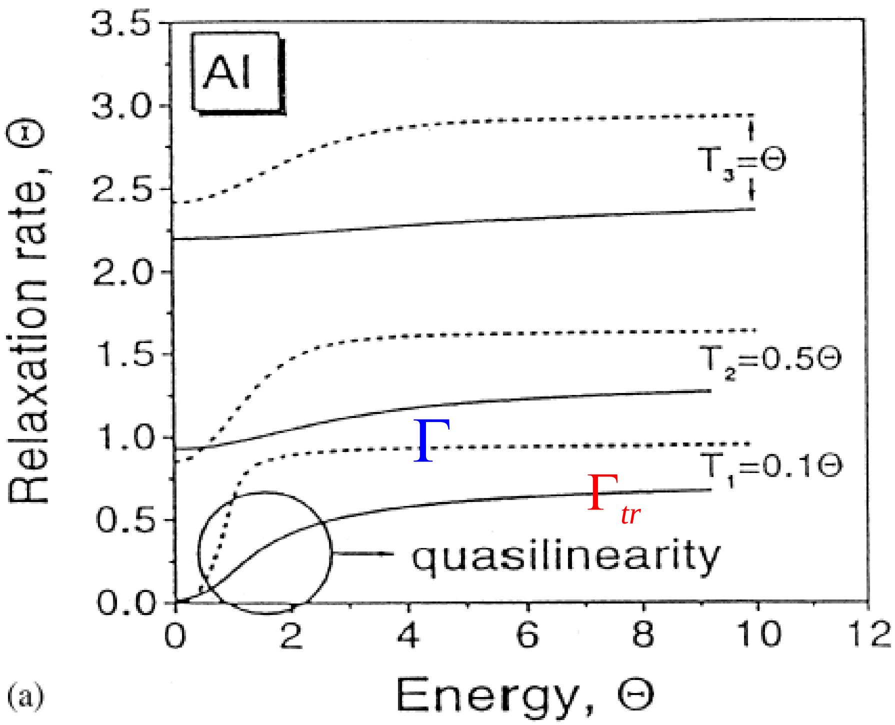


E. G. Maksimov group

Low-temperature superconductors

- linearity in $\Gamma_{tr}(\omega,T=0)=\frac{2\pi}{\omega}\int_0^{\omega} d\Omega(\omega-\Omega)\alpha_{tr}^2(\Omega)F(\Omega)$ for $\omega>\omega_{\mathrm{D}}$ is natural phenomenon !

For $\alpha_{tr}^2(\Omega)F(\Omega)=\frac{\lambda_{tr}\Omega_E}{2}\delta(\omega-\Omega_E) \;\Rightarrow\; \Gamma_{tr}(\omega,T=0)=\pi\lambda_{tr}\Omega_E\left(1-\frac{\Omega_E}{\omega}\right)$



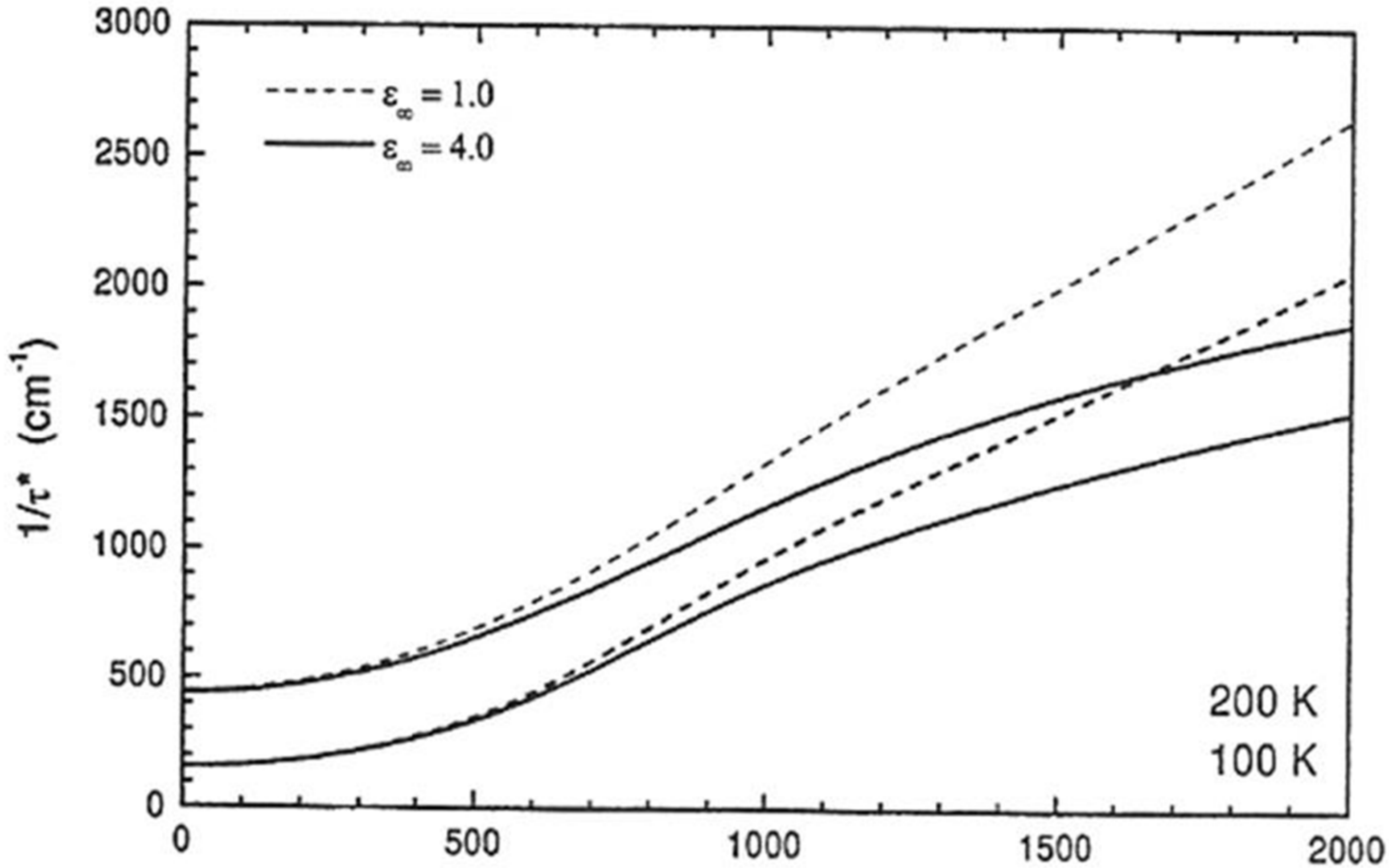
(a)

- only at $T=0$ it holds

$$\alpha_{tr}^2(\Omega)F(\Omega)=\frac{\omega_{pl}^2}{8\pi^2}\frac{\partial^2}{\partial\omega^2}\left[\omega\,\mathrm{Re}\frac{1}{\sigma(\omega,T=0)}\right]$$

$\Gamma^*_{tr}(\omega)(\equiv 1/\tau^*)$ strongly depends on ε_∞

- too small $\varepsilon_\infty \Rightarrow$ overestimated linearity of $\Gamma^*_{tr}(\omega)$!



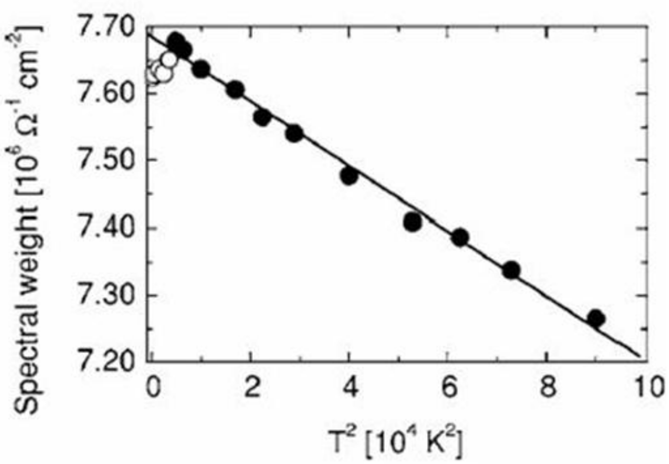
Restricted spectral weight W

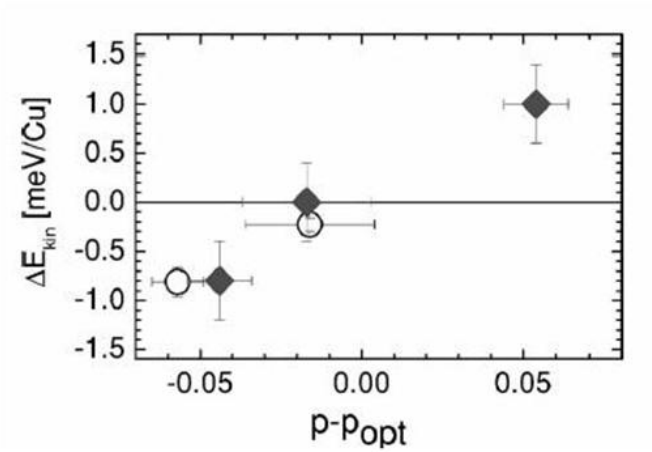
$W(\Omega_c, T)$ can be naturally explained by strong EPI !

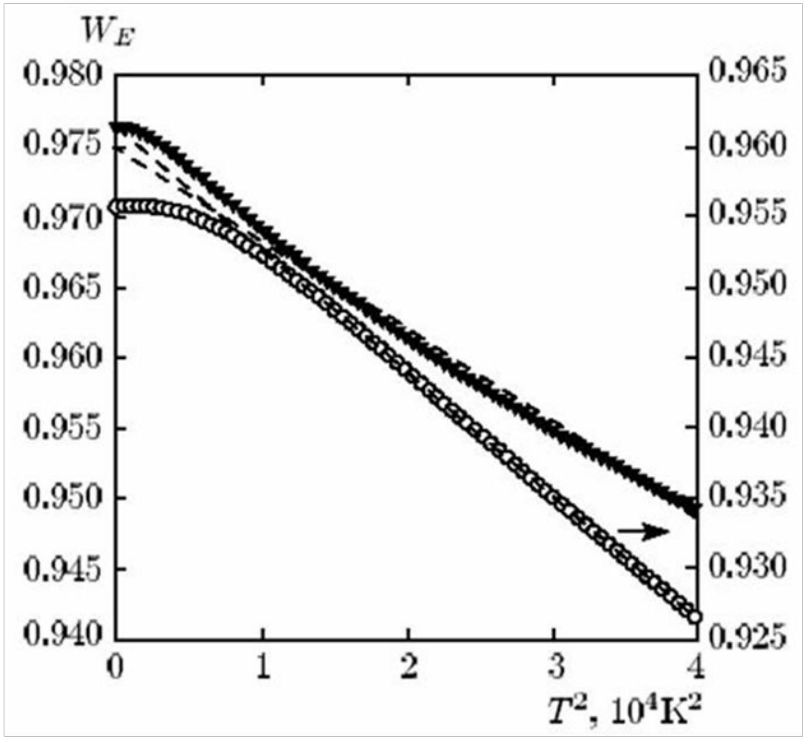
BCS-like $W(\Omega_c, T)$

$$W(\Omega_c, T) = \int_0^{\Omega_c} Re\sigma(\omega) d\omega \propto \left\langle -\hat{T} \right\rangle; \quad \hat{T} = \sum_{\boldsymbol{p}} \varepsilon_{\boldsymbol{p}} \hat{n}_{\boldsymbol{p}}$$

$$W(\Omega_c, T) \approx \frac{\omega_{pl}^2}{8} \left[1 - \frac{\Gamma(T)}{W_b} \right] \qquad \Gamma(T) = \Gamma_{ep}(T)$$







G. Deutscher et al. (2005)

A. E. Karakozov, E. G. Maksimov (2006)

ARPES kink at the nodal (N) -point

ARPES spectra can be explained by strong EPI !

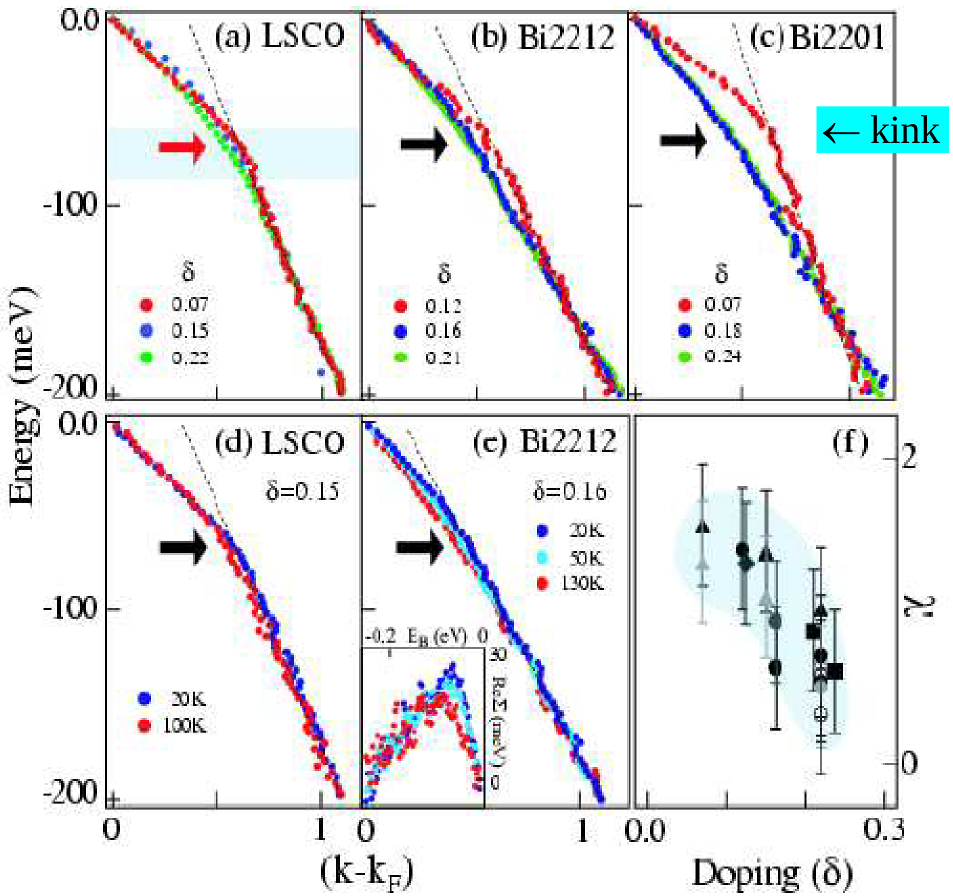
Puzzle: $\omega_{kink}^{(s)} = \omega_{kink}^{(n)}$

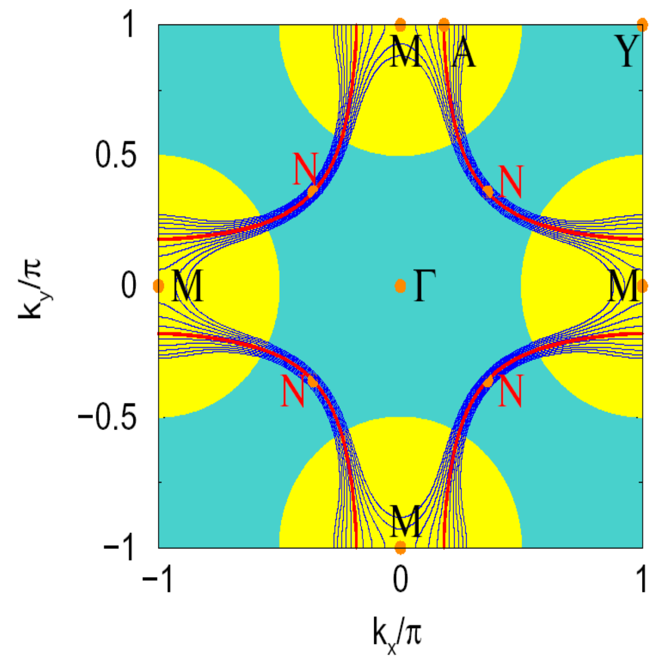
isotropic EPI theory predicts: $\omega_{kink}^{(s)} = \omega_{kink}^{(n)} + \Delta_{\max}$

$\rightarrow$ FSP in $\alpha^2 F(\boldsymbol{q}, \omega)$!

$$\Delta(k_N) = 0$$

$$\Delta(k_A) = \Delta_{\max}$$

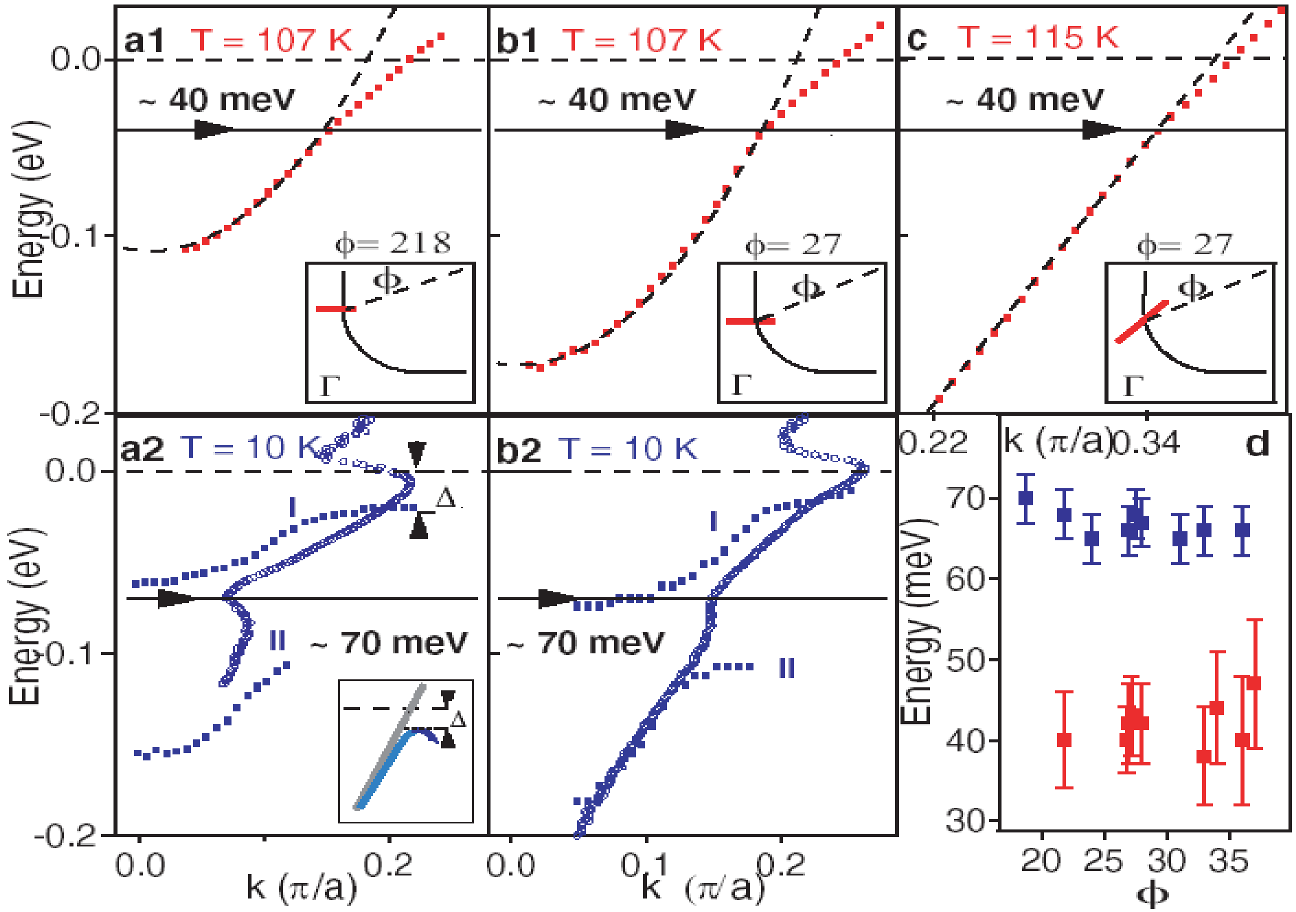




A. Lanzara et al. (2001)

ARPES kink at the anti-nodal (A) point

- shift at A-point $\rightarrow\ \omega_{kink}^{(s)} = \omega_{kink}^{(n)} + \Delta_{\max}\ ,\ \Delta_{\max} \approx 30\ meV$

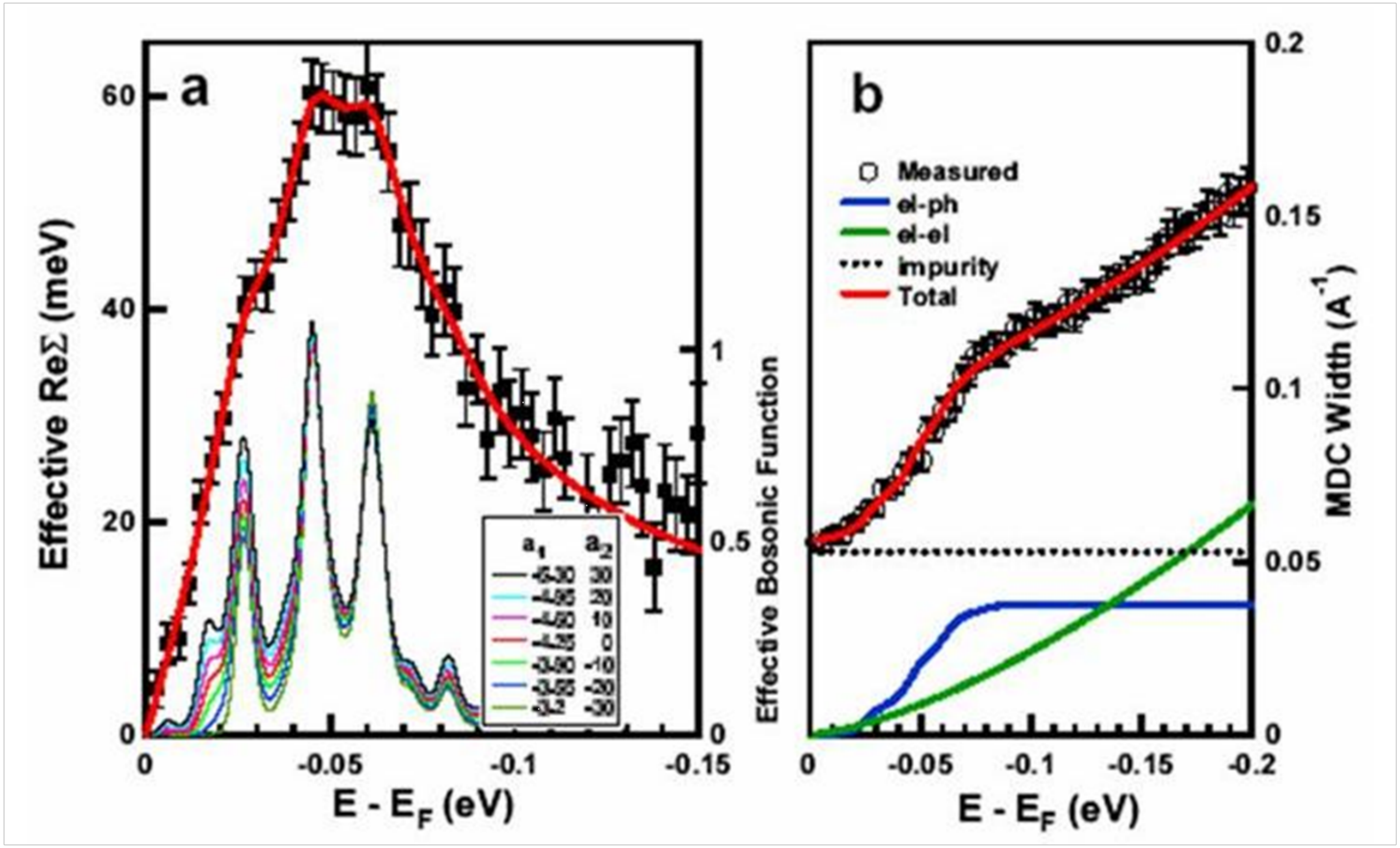


T. Cuk et al. (2004)

Strong *EPI* and phononic structure $\alpha^2 F(k_N, \omega)$
is seen in ARPES self-energy $\Sigma(k_N, \omega)$

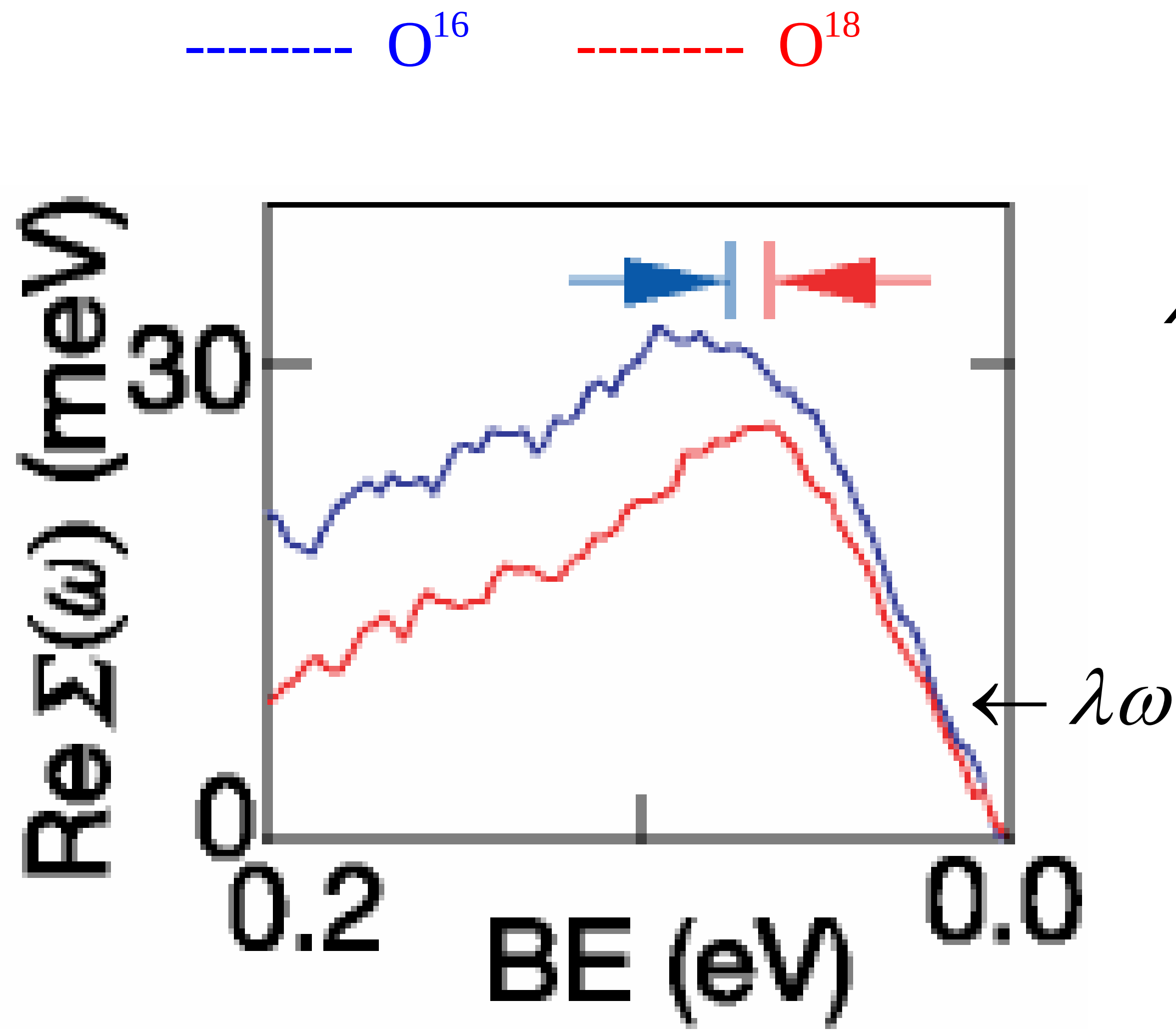
$Re\Sigma(k_N, \omega)$, $Im\Sigma(k_N, \omega)$ and $\alpha^2 F(k_N, \omega)$ in $La_{2-x}Sr_xCuO_4$, $x = 0.03$

$Re\Sigma(k_N, \omega)$ $Im\Sigma(k_N, \omega)$



X. J. Zhou et al. (2005)

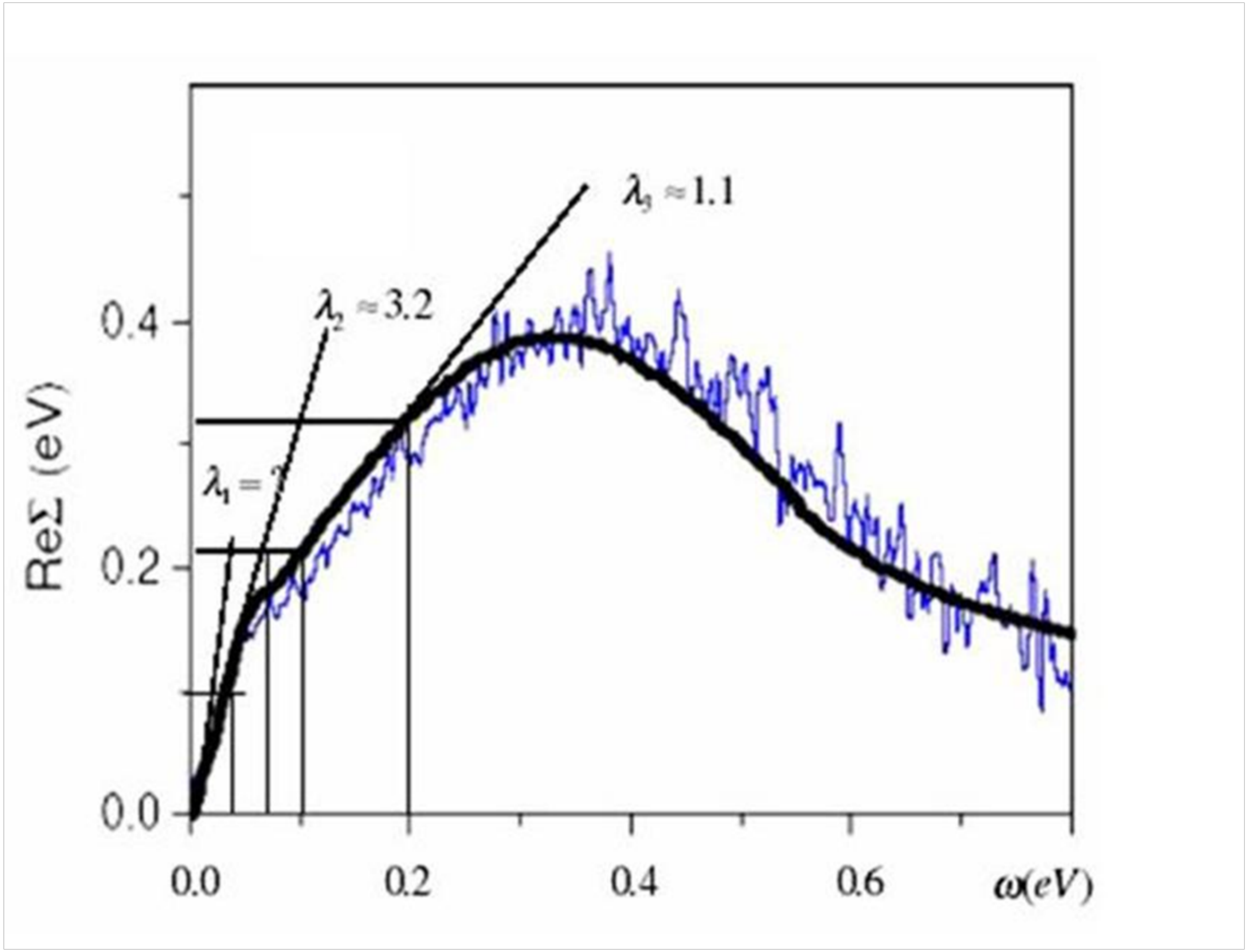
Isotope effect in ARPES at N-point



$$\lambda(\approx \lambda_{EP}) \approx 1$$

G.-H. Gweon et al., Nature, **430**, 187 (2004)

$Re\Sigma(k_N, \omega)$ obtained from ARPES in $Bi_2Sr_2CaCu_2O_8 \Rightarrow \lambda_{ep} = 2-3,\ \lambda_c \approx 1$

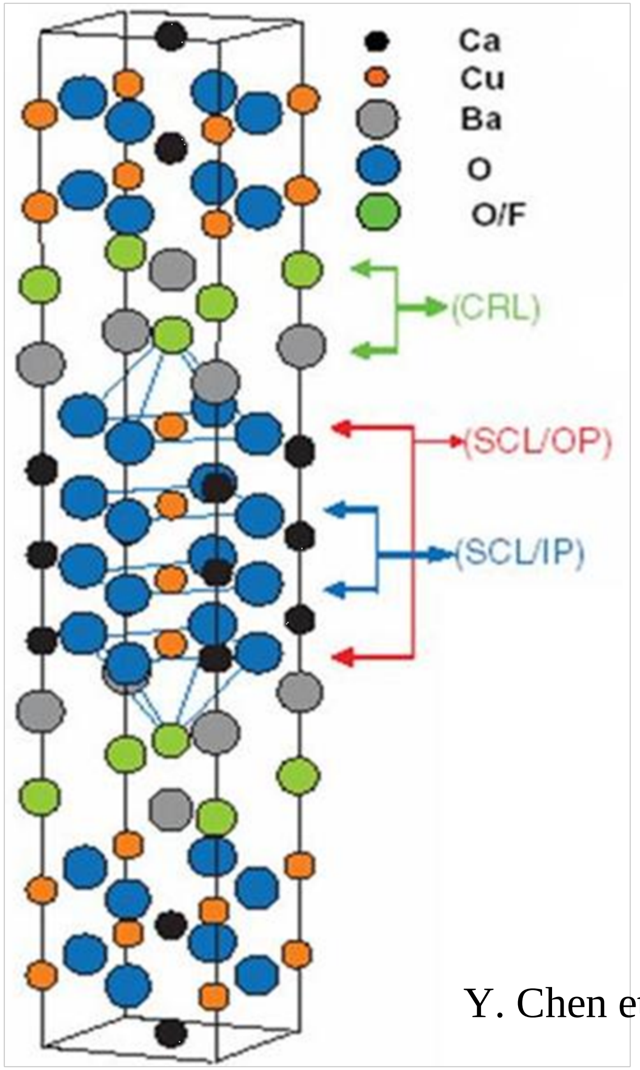


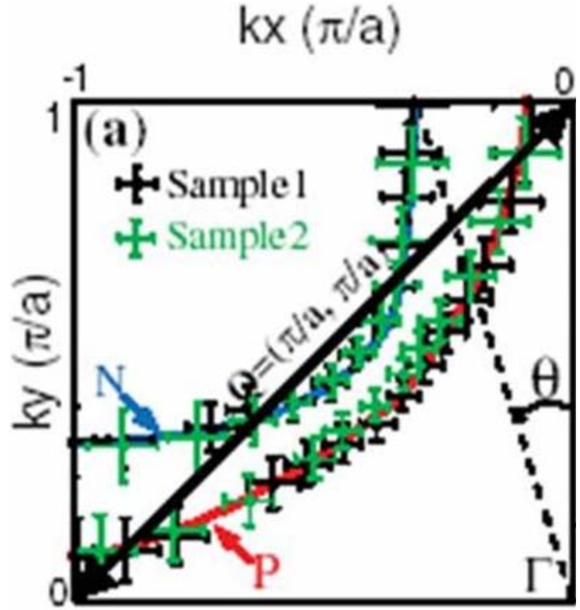
T. Valla et al. (2007)

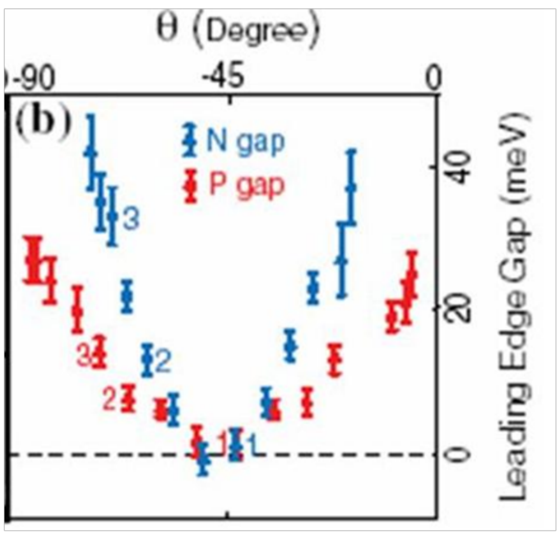
M.L.K., O. Dolgov (2007)

ARPES in 4-layered HTSC is against SFI

$Ba_2Ca_3Cu_4O_8(O_\delta F_{1-\delta})_2$







Y. Chen et al. (2006)

Theory of ARPES in the **FSP model**

FSP $\Rightarrow$ **on the Fermi surface:** $\alpha^2 F(\varphi,\varphi',\omega) \approx \alpha^2 F(,\omega)\delta(\varphi-\varphi')$

$\Downarrow$

- "local" Eliashberg equations due to FSP in charge scattering

$$\tilde{\omega}_{n,\varphi} = \omega_n + \pi T \sum_m \frac{\lambda_{1,\varphi}(n-m)\tilde{\omega}_{m,\varphi}}{\sqrt{\tilde{\omega}^2_{m,\varphi} + \tilde{\Delta}^2_{m,\varphi}}} + \Sigma^c_{n,\varphi}$$

$$\tilde{\Delta}_{n,\varphi} = \pi T \sum_m \frac{\lambda_{2,\varphi}(n-m)\tilde{\Delta}_{m,\varphi}}{\sqrt{\tilde{\omega}^2_{m,\varphi} + \tilde{\Delta}^2_{m,\varphi}}} + \tilde{\Delta}^c_{n,\varphi}$$

$$\lambda_{1(2),\varphi}(n-m) = \lambda_{epi,\varphi}(n-m) + \delta_{mn}\gamma^{imp}_{1(2),\varphi}$$

$$\lambda_{epi,\varphi}(n) = 2\int_0^\infty d\omega \frac{\omega\alpha^2_{epi,\varphi}F(\omega)}{\omega^2+\omega_n^2}$$

M. L. K. & O. V. Dolgov, Phys. Rev. **B 71** (2005)

$\alpha^2 F(\omega)$ and strong EPI from tunneling conductance

Extraction of $\alpha^2 F(\omega)$ from:

- planar junctions $\mathrm{Bi}_2 Sr_2 CaCu_2 O_8 - GaAs$ (and Au)

- break-junctions from $\mathrm{Bi}_2 Sr_2 CaCu_2 O_8$

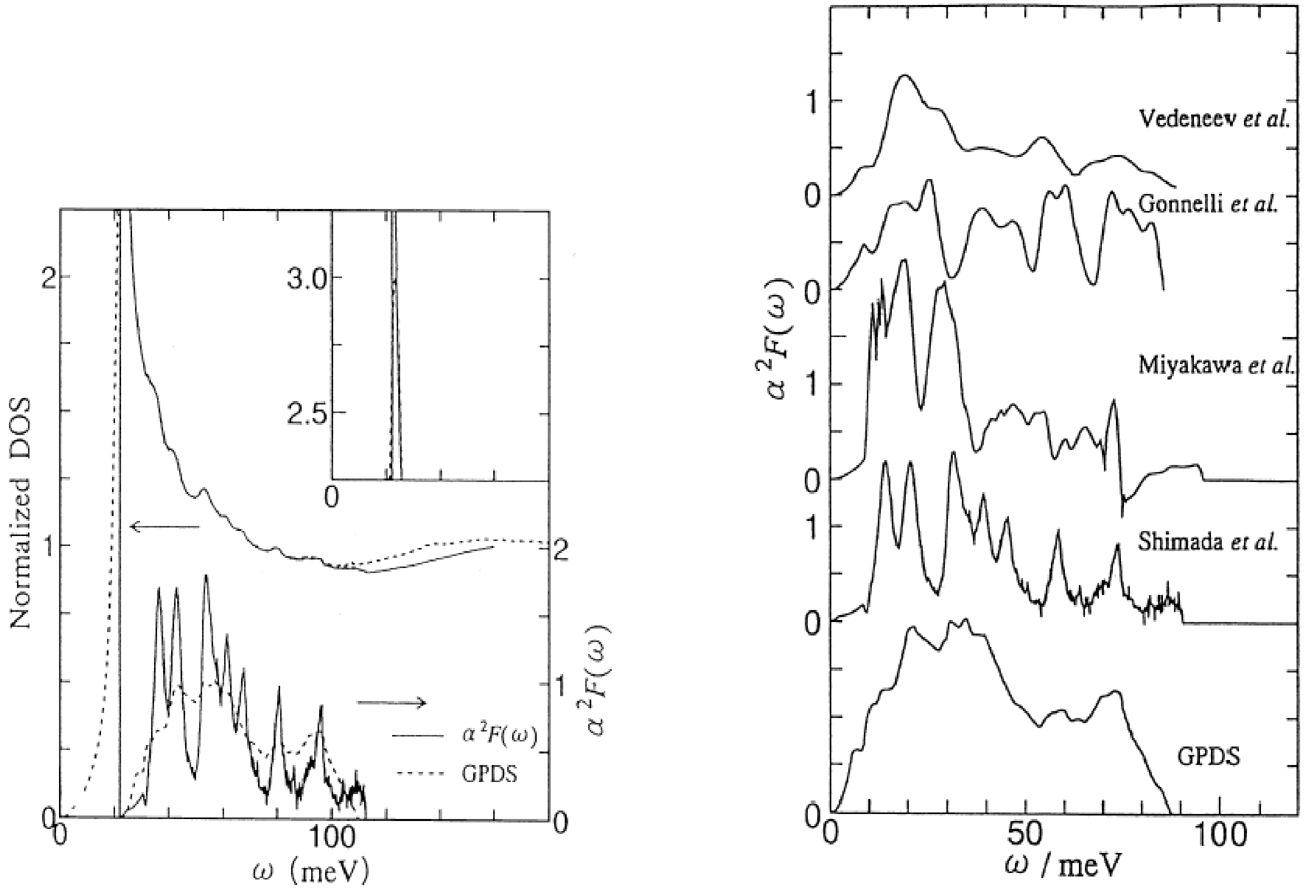
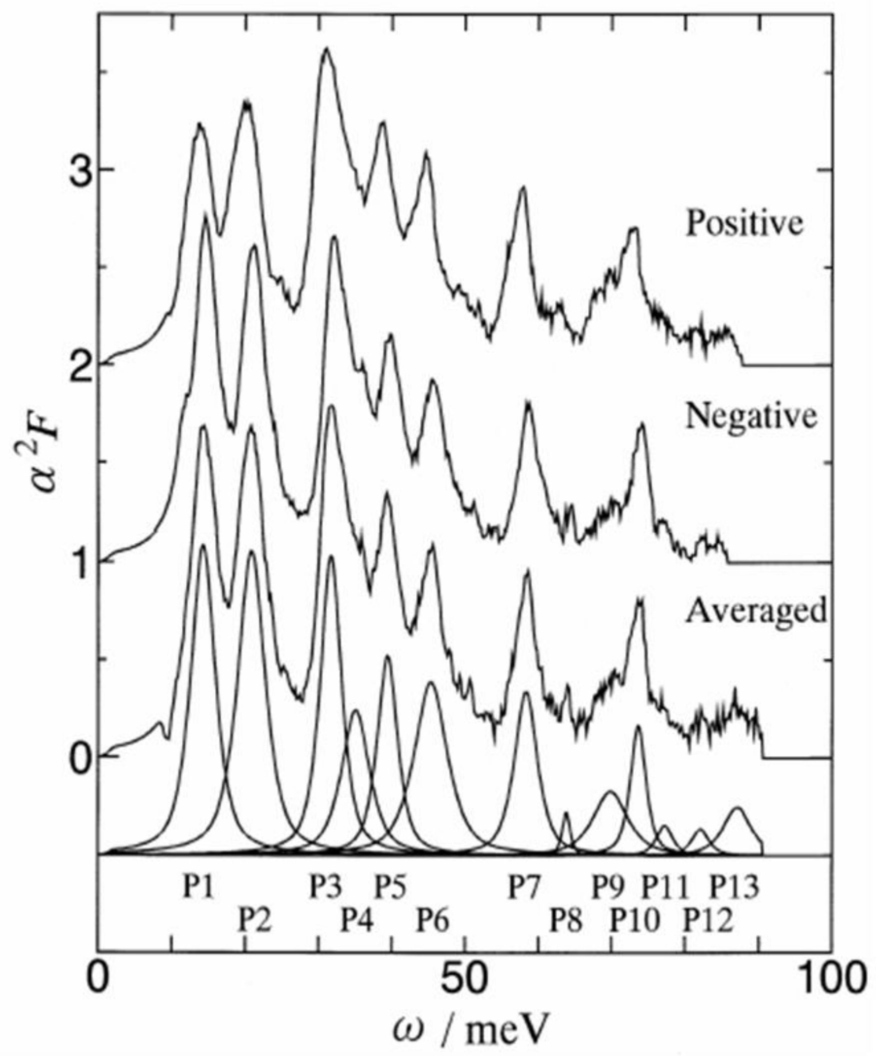


Fig. 58. The spectral function $\alpha^2 F(\omega)$ and the calculated density of states at 0 K (upper solid line) obtained from the conductance measurements on the Bi(2212)–Au planar tunneling junction; from [42].

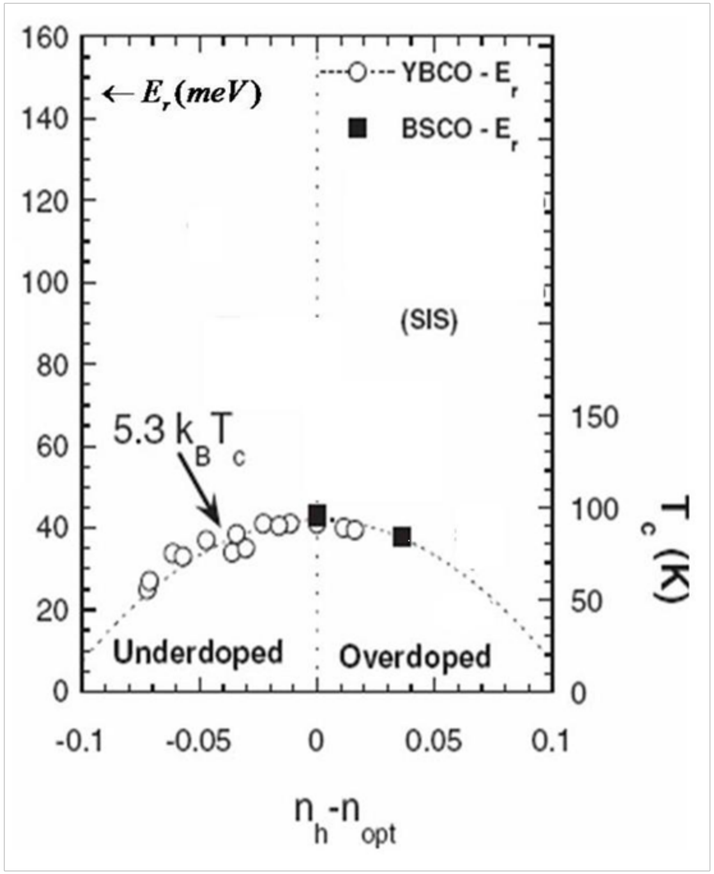
All phonons contribute to T_c

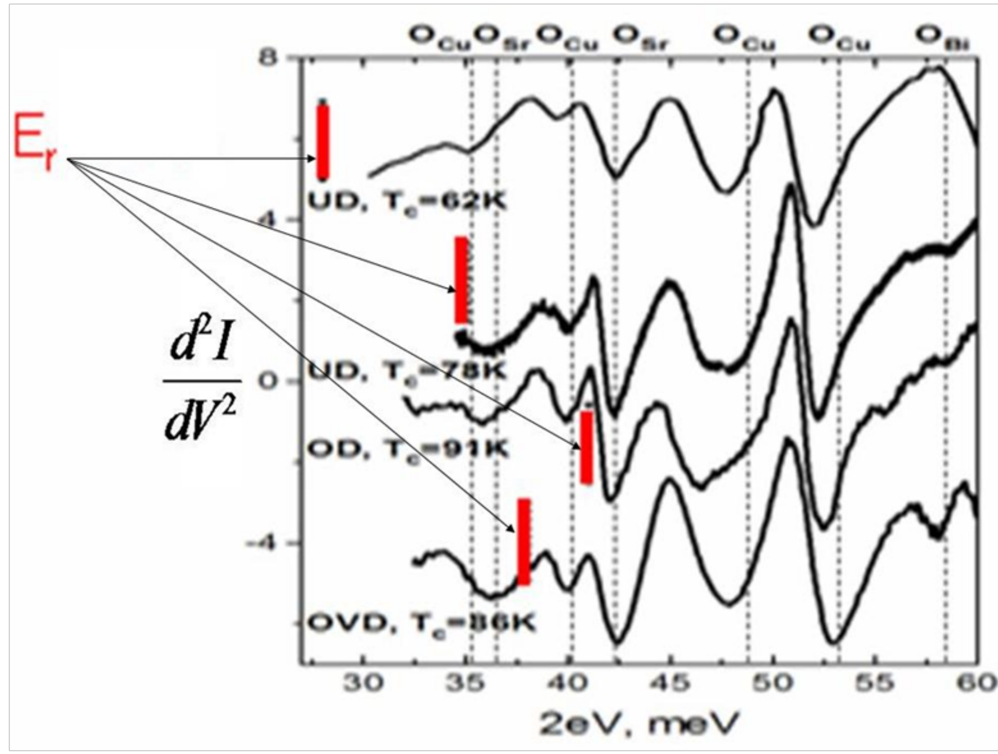
$No.peak$	$\omega[meV]$	λ_i	$\Delta T_c[K]$
$P1$	14.3	1.26	7.4
$P2$	20.8	0.95	11.0
$P3$	31.7	0.48	10.5
$P4$	35.1	0.28	6.7
$P5$	39.4	0.24	7.0
$P6$	45.3	0.30	10.0
$P7$	58.3	0.15	6.5
$P8$	63.9	0.01	0.6
$P9$	69.9	0.07	3.6
$P10$	73.7	0.06	3.3
$P11$	77.3	0.01	0.8
$P12$	82.1	0.01	0.7
$P13$	87.1	0.03	1.8



D. Shimada eta al. (1997, 2007)

Magnetic resonance mode is ineffective for pairing

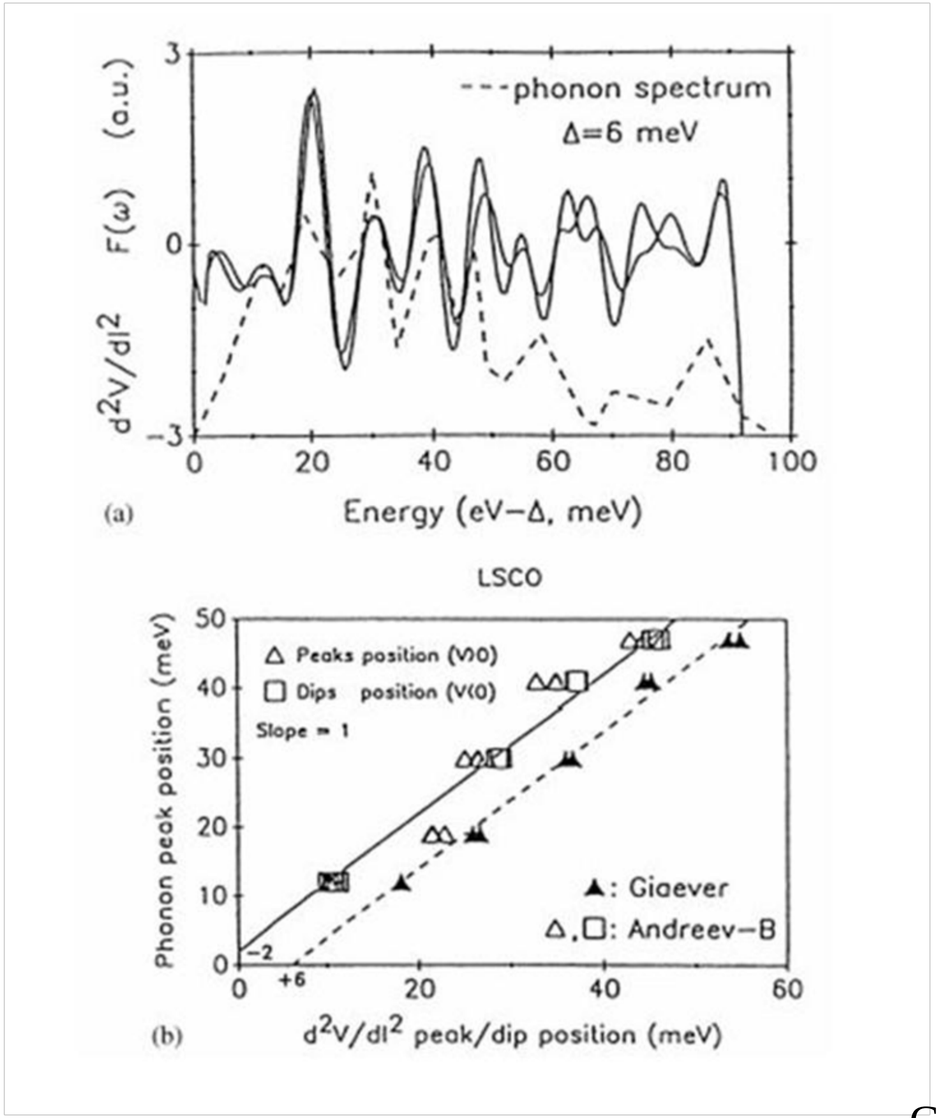




- Resonance energy E_r vs doping in Bi2212; Y. Sidis (2004)

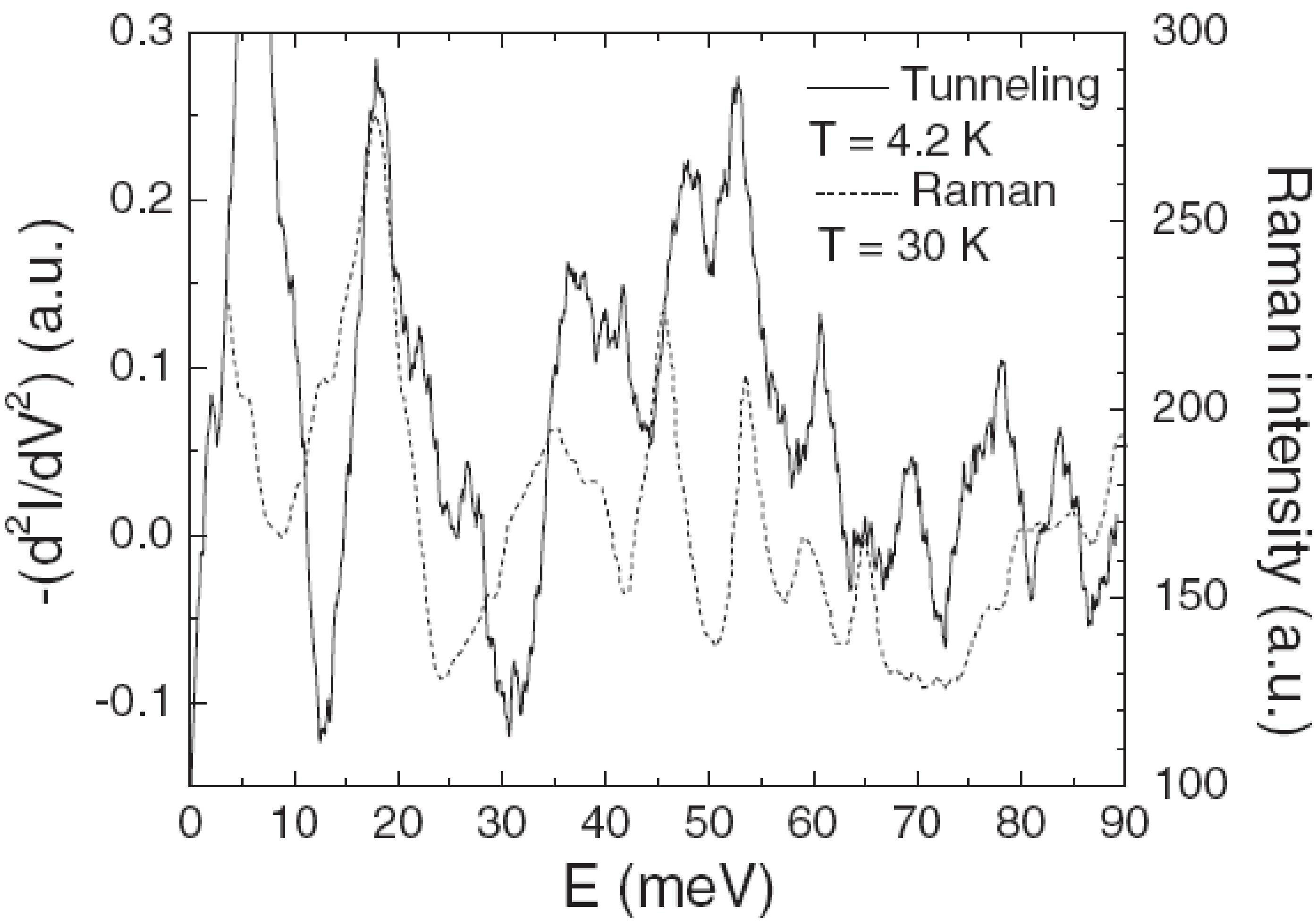
- tunneling junctions: $Bi_2Sr_2CaCu_2O_8$ Y. G. Ponomarev et al. (2008)

Tunneling in $La_{2-x}Sr_xCuO_4$ spectra reproduce phononic structure



G. Deutscher et al. (1994)

Tunneling vs phonon Raman spectra in LASCO films – evidence for strong EPI



H. Shim et al. (2008)

Constraints on EPI imply strong q-dependence

1. **d-wave** pairing $\Rightarrow$ $\Delta(\boldsymbol{k},\omega) \approx \Delta^0(\omega)(\cos k_x - \cos k_y)$

2. high $T_c \approx 160\ K$

3. rather **large EPI** coupling $\Rightarrow$ $\lambda_{epi} = 2-3$

4. **small** $\lambda_{tr} \sim 0.4-1$ $(\rho(\mathrm{T}) \sim \lambda_{tr} T)$

Asuumption: pairing is due to high-energy non-phononic boson $\Rightarrow$ EPI is pair-breaking

Question - how large is the **bare** T_{c0} due to the boson?

$$Z(\omega)\Delta(\boldsymbol{k},\omega) = \int d^3q \int \frac{d\Omega}{\Omega} V_{sfi}(\boldsymbol{k}-\boldsymbol{q},\Omega)\Delta(\boldsymbol{q},\omega)\, th\frac{\xi(\boldsymbol{q})}{2T_c}$$

$$\Delta(\boldsymbol{k},\omega) = \Delta(\omega)[\cos k_x - \cos k_y] \quad \text{and} \quad Z(\omega) \approx 1 + i\Gamma_{epi}$$

$\Rightarrow$

$$\ln\frac{T_c}{T_{c0}^{sfi}} = \Psi\left(\frac{1}{2}\right) - \Psi\left(\frac{1}{2} + \frac{\Gamma_{epi}}{2\pi T_c}\right)$$

$$\Gamma_{epi} \approx 2\pi\lambda_{epi} T$$

$\swarrow$

- for $T_c \approx 160\ K$ $\Rightarrow$ $T_{co}^{bos} \approx (400-1100)\ K$!

Way out $\Rightarrow$ forward scattering peak (**FSP**) in EPI $\lambda_{epi}(q)$

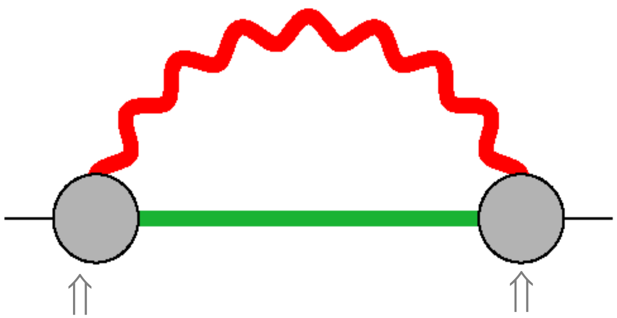
Theory: EPI must be strongly momentum dependent

Long range EPI (*forward scattering peak*) due to:

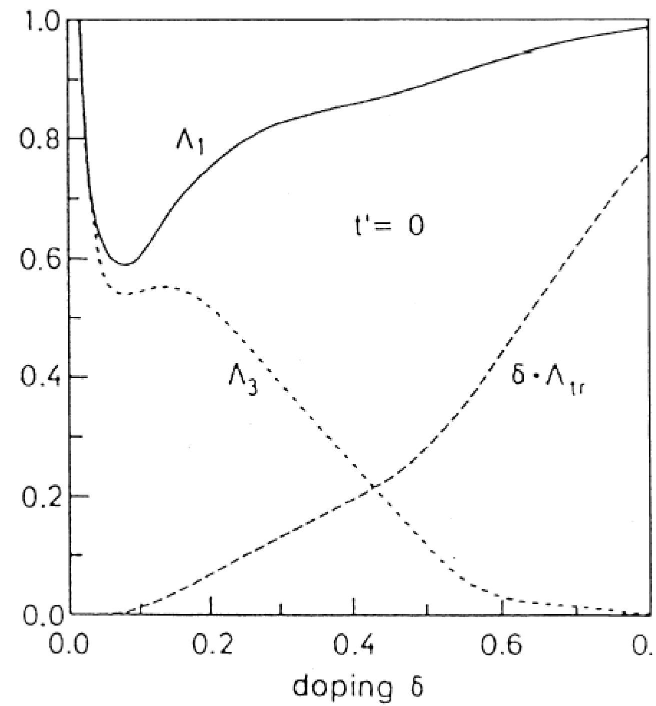
- long range due to *the Madelung energy* $\Rightarrow \dfrac{g_{ep}^{0}(\boldsymbol{k},\boldsymbol{q})}{\varepsilon(\boldsymbol{q})}$

- long range due to *strong correlations* $\Rightarrow \gamma_c(\boldsymbol{k},q)$ (vertex)

$\lambda_s \sim \Lambda_1 \qquad \lambda_d \sim \Lambda_3$



$\gamma_c(\boldsymbol{k},\boldsymbol{q})\dfrac{g_{ep}^{0}(\boldsymbol{k},\boldsymbol{q})}{\varepsilon(\boldsymbol{q})} \qquad \gamma_c(\boldsymbol{k},\boldsymbol{q})\dfrac{g_{ep}^{0}(\boldsymbol{k},\boldsymbol{q})}{\varepsilon(\boldsymbol{q})}$



$$T_c^{(i)} \sim \left\langle \omega_{ph} \right\rangle e^{-\frac{1+\lambda_i}{\lambda_i - \mu_i^*}}$$

- since $\lambda_d \approx \lambda_d$ and $\mu_d^* \ll \mu_s^* \Rightarrow T_c^{(d)} > T_c^{(s)}$

M.L.K., R.Zeyher, O. Dolgov...

Strong Correlations without Slave Bosons

Hubbard model: $\hat{H} = \sum_{ij,\sigma} \varepsilon_{a,i} n_{i,\sigma} - \sum_{ij,\sigma} t_{ij} c^{\dagger}_{i,\sigma} c_{j,\sigma} + U \sum_{i} n_{i,\uparrow} n_{i,\downarrow} + \hat{H}_{EP}$

$U \gg t \;\rightarrow$ no doubly occupancy $\rightarrow \mathrm{n_i} = 0,1$

Hubbard operators $\hat{\mathrm{X}}^{\alpha\beta}$

$$[\hat{X}_i^{\alpha\beta}, \hat{X}_j^{\gamma\lambda}]_{\pm} = \delta_{ij}[\delta_{\gamma\beta}\hat{X}_i^{\alpha\lambda} \pm \delta_{\alpha\lambda}\hat{X}_i^{\gamma\beta}]$$

$$\hat{X}_i^{00} + \sum_{\sigma=1} \hat{X}_i^{\sigma\sigma} = 1$$

$$\boxed{\begin{aligned}\hat{H} = &-\sum_{i,j,\sigma} t_{ij} \hat{X}_i^{\sigma 0} \hat{X}_j^{0\sigma} + \sum_{i,j} J_{ij} (\hat{\boldsymbol{S}}_i \cdot \hat{\boldsymbol{S}}_j - \frac{1}{4} \hat{n}_i \hat{n}_j) + \hat{V}_{LR-Coulomb} \\ &+ \sum_{i,\sigma} \varepsilon^0_{a,i} \hat{X}_i^{\sigma\sigma} + \hat{H}_{ph} + \hat{H}_{EPI}\end{aligned}}$$

Slave Boson method: $\hat{X}^{0\sigma} = c_{i,\sigma}(1 - n_{i,-\sigma}) = f_{\sigma} b^{\dagger}$

Results of X-method in O(1) order

1. **coherent quasiparticles** ($t \to \delta \cdot t$) (doping $\delta = 1-n$)

$$
\begin{aligned}
g_0(\mathbf{k},\omega) &\equiv G_0(\mathbf{k},\omega)/Q_0 = \frac{1}{\omega - (\epsilon_0(\mathbf{k}) - \mu}, \\
\epsilon_0(\mathbf{k}) &= \epsilon_c - q_0 t_0(\mathbf{k}) - \frac{1}{N_L}\sum_{\mathbf{p}} J_0(\mathbf{k}+\mathbf{p}) n_F(\mathbf{p}) \\
\epsilon_c &= \frac{1}{N_L}\sum_{\mathbf{p}} t_0(\mathbf{p}) n_F(\mathbf{p}), \\
Q_0 &= <\hat{X}_i^{00}> = N q_0 = N\frac{\delta}{2}.
\end{aligned}
$$

$q_0 = \delta/2 \;\Rightarrow\;$ **band narrowing!**

2. large Fermi surface ~ $1-\delta$ and QP residuum ~ δ

$$
\boxed{1 - \delta = 2\sum_{p} n_{\mathrm{F}}(p)\ .}
$$

3. sum rule $\int d\omega\sigma(\omega) \sim \delta \;\Rightarrow$ unusual Fermi liquid !

4. no spin-charge separation in the X-method !

$\Rightarrow$ in $O(1/N)$ SB and X-method are different

„Pseudogap“ phenomena due to internal Cooper pair fluctuations

- pairing Hamiltonian:

$$\boxed{\begin{aligned} H &= \sum_{\mathbf{k}\sigma} 2\xi_{\mathbf{k}} S^z_{\mathbf{k}\sigma} - \frac{1}{2}\sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'}(S^+_{\mathbf{k}} S^-_{\mathbf{k}'} + S^+_{\mathbf{k}'} S^-_{\mathbf{k}}) \\ &= \sum_{\mathbf{k}\sigma} 2\xi_{\mathbf{k}} S^z_{\mathbf{k}\sigma} - \sum_{\mathbf{k},\mathbf{k}'} V_{\mathbf{k}-\mathbf{k}'}(S^x_{\mathbf{k}} S^x_{\mathbf{k}'} + S^y_{\mathbf{k}'} S^y_{\mathbf{k}}), \end{aligned}}$$

$$\boxed{S^z_{\mathbf{k}\sigma} = \frac{1}{2}(c^\dagger_{\mathbf{k}\uparrow} c_{\mathbf{k}\uparrow} - c^\dagger_{-\mathbf{k}\downarrow} c_{-\mathbf{k}\downarrow} - 1),} \qquad \boxed{S^+_{\mathbf{k}\sigma} = c^\dagger_{\mathbf{k}\uparrow} c^\dagger_{-\mathbf{k}\downarrow}.}$$

BCS theory $\Rightarrow$ $V_{\mathbf{k}-\mathbf{k}'} = V_0$, $\Rightarrow$ "*longe – range*" in k-space $\Rightarrow$ MFA exact

FSP model $\Rightarrow$ $V_{\boldsymbol{k}-\boldsymbol{k}'} = V_0 \delta(\boldsymbol{k}-\boldsymbol{k}')$ $\Rightarrow$ $T_c^{MF} = V_0/4$ and $\Delta_0 = 2T_c^{MF}$

$\Rightarrow$ "*short – range*" in k-space $\Rightarrow$ low-energy bound states (spin-waves)

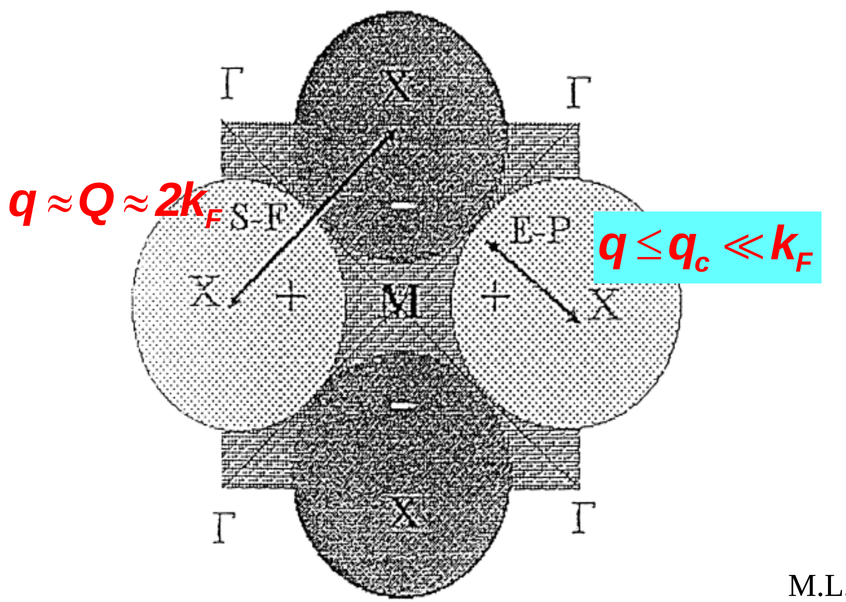
$$\boxed{T_c \sim \frac{T_c^{MF}}{(r_c/\xi)}.}$$

$$r_c = q_c^{-1} \sim a/2\delta$$

M. L. K., in Lectures of Physics of Highly Correlated Systems VIII, AIP Proceedings **715** (2004)

Conclusions

- Tunneling, ARPES, optics... $\Rightarrow$ EPI important for pairing in HTSC
- EPI strongly momentum dependent in order to be conform with d-wave
- Momentum dependence due to the Madelung energy and strong correlations
- EPI copupling constant $\lambda_{\mathrm{Z}} = 2-3$ and $\lambda_{\mathrm{d}} < \lambda_{\mathrm{Z}}$
- all phonons contribute to λ_{Z}
- T_c is dominated by EPI
- Coulomb interaction (and spin fluctuations) trigger d-wave pairing



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