The Squashed, Stretched and Warped Gets Perturbed

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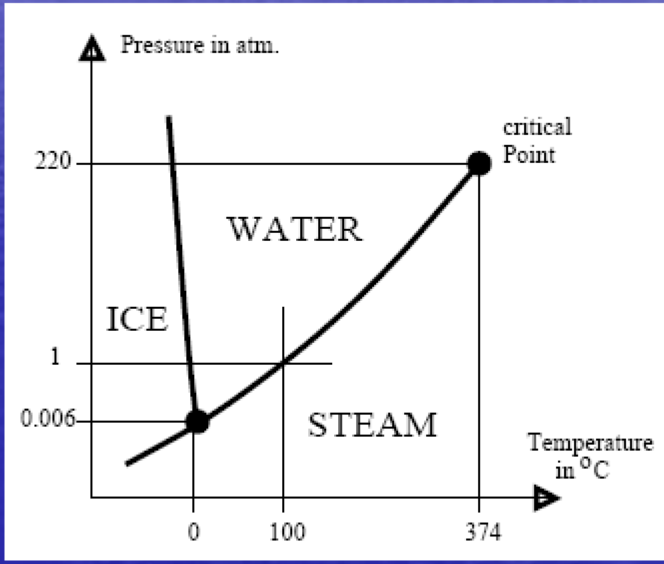
Introduction

- The gauge theory on coincident M2 branes has been a hot topic over the past year.
- This is a long-standing problem: how to find the world volume theory on coincident super-membranes in 11-dimensional M-theory. This is harder than the description of D-branes in string theory that is known explicitly at small string coupling.
- But M-theory is inherently strongly coupled: one can think of it as the strong coupling limit of a 10-dimensional superstring theory. What to do?

- The research on AdS_5/CFT_4 has rekindled interest in the maximally super-symmetric 4-d gauge theory and provided a host of information about its strongly coupled limit. See the January 2009 Physics Today article by I.K., J.Maldacena.
- This conformal gauge theory is becoming `The Harmonic Oscillator of 4-d Gauge Theory' in that it may be exactly solvable.
- It has provided a `hyperbolic cow' approximation to various phenomena at strong coupling.

AdS_4/CFT_3

- Besides describing all of known particle physics, Quantum Field Theory is important for understanding the vicinity of certain phase transitions, such as the all-important water/vapor transition.



- Here we are interested in a 3-d (Euclidean) QFT.

- This transition is in the 3-d Ising Model Universality Class.
- Other common transitions are described by 3-d QFT with O(N) symmetry.
- 3-d theories are also very important in describing 2-d quantum systems, such as those in the Quantum Hall effect, high-Tc superconductors, etc.
- Can we find a `Harmonic Oscillator' of 3-d Conformal Field Theory ?

O(N) Sigma Model

- Describes 2nd order phase transitions in statistical systems with O(N) symmetry.

$$S = \int d^3x \left[\frac{1}{2}(\partial_\mu \phi^a)^2 + \frac{\lambda}{2N}(\phi^a \phi^a)^2 \right]$$

- IR fixed point can be studied using the Wilson-Fisher expansion in ϵ=4-d.
- The model simplifies in the large N limit since it possesses conserved currents

$$J_{(\mu_1 \cdots \mu_s)} = \phi^a \partial_{(\mu_1} \cdots \partial_{\mu_s)} \phi^a + \ldots$$

Higher Spin Gauge Theory

- An AdS_4 dual of the large N sigma model was proposed. IK, Polyakov (2002)
- It is the Fradkin-Vasiliev gauge theory of an infinite number of interacting massless higher-spin gauge fields.
- There is no small AdS curvature limit. This makes the theory difficult to study in the dual AdS formulation. This is an interesting problem for the future.

M2 Brane Theory

- The theory on N coincident M2-branes has $\mathcal{N}=8$, the maximum possible supersymmetry in 3 dimensions.
- When N is large, its dual description is provided by the weakly curved $AdS_4 \times S^7$ background in 11-dimensional M-theory.
- This dual description is tractable and makes many non-trivial predictions.

- A general prediction of the AdS/CFT duality is that the number of degrees of freedom on a large number N of coincident M2-branes scales as $N^{3/2}$

  I.K., A. Tseytlin (1996)

- This is much smaller than the N^2 scaling found in the 4-d SYM theory on N coincident D3-branes (as described by the dual gravity). Gubser, I.K., Peet (1996)

What is the M2 Brane Theory?

- It is the Infrared limit of the D2-brane theory, the $\mathcal{N}=8$ supersymmetric Yang-Mills theory in 2+1 dimensions, i.e. it describes the degrees of freedom at energy much lower than $(g_{YM})^2$
- The number of such degrees of freedom $\sim N^{3/2}$ is much lower than the number of UV degrees of freedom $\sim N^2$.
- Is there a more direct way to characterize the Infrared Scale-Invariant Theory?

The BLG Theory

- In a remarkable development, Bagger and Lambert, and Gustavsson formulated an SO(4) Chern-Simons Gauge Theory with manifest $\mathcal{N}$=8 superconformal gauge theory. In Van Raamsdonk's SU(2)xSU(2) formulation,

$$X^* = -\varepsilon X \varepsilon$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{aligned}\mathcal{S} = \int d^3x \, \mathrm{tr}\Big[&-(\mathcal{D}^\mu X^I)^\dagger \mathcal{D}_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu \mathcal{D}_\mu \Psi \\ &- \frac{2if}{3}\bar{\Psi}^\dagger \Gamma^{IJ}(X^I X^{J\dagger}\Psi + X^J \Psi^\dagger X^I + \Psi X^{I\dagger} X^J) - \frac{8f^2}{3}\mathrm{tr}\, X^{[I} X^{\dagger J} X^{K]} X^{\dagger [K} X^J X^{\dagger I]} \\ &+ \frac{1}{2f}\epsilon^{\mu\nu\lambda}(A_\mu \partial_\nu A_\lambda + \frac{2i}{3}A_\mu A_\nu A_\lambda) - \frac{1}{2f}\epsilon^{\mu\nu\lambda}(\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3}\hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda)\Big]\end{aligned}$$

- X^I are the 8 fields transforming in (2,2), which is the 4 of SO(4)

$$X^I = \frac{1}{2}(x_4^I \mathbb{1} + i x_i^I \sigma^i)$$

$\mathcal{N}$=2 Superspace Formulation

- Define bi-fundamental superfields rotated by SU(4)$_{\text{flavor}}$ symmetry

$$\mathcal{Z} = Z(x_L) + \sqrt{2}\theta\zeta(x_L) + \theta^2 F(x_L) \ ,$$
$$\bar{\mathcal{Z}} = Z^\dagger(x_R) - \sqrt{2}\bar{\theta}\zeta^\dagger(x_R) - \bar{\theta}^2 F^\dagger(x_R)$$

$$Z^{\ddagger A} := -\varepsilon(Z^A)^{\mathrm{T}}\varepsilon = X^{\dagger A} + iX^{\dagger A+4}$$

- The superpotential is Benna, IK, Klose, Smedback,

$$\mathrm{W} = \frac{1}{4!}\epsilon_{ABCD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{Z}^{\ddagger B}\mathcal{Z}^C\mathcal{Z}^{\ddagger D}$$

- Using SO(4) gauge group notation,

$$\mathrm{W} = -\frac{1}{8\cdot 4!}\,\epsilon_{ABCD}\epsilon^{abcd}\,\mathcal{Z}_a^A\mathcal{Z}_b^B\mathcal{Z}_c^C\mathcal{Z}_d^D$$

The ABJM Theory

- Aharony, Bergman, Jafferis and Maldacena argued that the correct description of a pair of M2-branes is slightly different. It involves U(2) x U(2) gauge theory.
- The SU(4) flavor symmetry is not manifest because of the choice of complex combinations

$$Z^1 = X^1 + iX^5\,, \qquad W_1 = X^{3\dagger} + iX^{7\dagger}$$
$$Z^2 = X^2 + iX^6\,, \qquad W_2 = X^{4\dagger} + iX^{8\dagger}$$

- The manifest flavor symmetry is SU(2)xSU(2)

$$\mathrm{W} = \frac{1}{4}\epsilon_{AC}\epsilon^{BD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D$$

- For N M2-branes ABJM theory easily generalizes to U(N) x U(N). The theory with Chern-Simons coefficient k is then conjectured to be dual to $AdS_4 \times S_7/Z_k$ supported by N units of flux.
- For $k>2$ this theory has $\mathcal{N}=6$ supersymmetry, in agreement with this conjecture. In particular, the theory has manifest SU(4) R-symmetry.

$SU(4)_R$ Symmetry

- The global symmetry rotating the 6 supercharges is SO(6)~SU(4). The classical action of this theory indeed has this symmetry. Benna, IK, Klose, Smedback

$$V^{\rm bos} = -\frac{L^2}{48}\,{\rm tr}\Big[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4 Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6 Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big]$$

$$V^{\rm ferm} = \frac{iL}{4}\,{\rm tr}\Big[Y_A^\dagger Y^A \psi^{B\dagger}\psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2 Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2 Y_A^\dagger Y^B \psi^{A\dagger}\psi_B - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}\Big]\,.$$

Y^A, A=1,…4, are complex N x N matrices.

$$Y^A = \{Z^1, Z^2, W^{1\dagger}, W^{2\dagger}\}$$

Enhanced Symmetry

- For k=1 or 2 the global symmetry should enhance to SO(8) according to the ABJM conjecture. This is not seen in the classical lagrangian but should appear in the quantum theory.
- The key to it are probably the `monopole' operators that create singular monopole field configurations at a point. They create magnetic flux in a diagonal U(1) subgroup and are charged under the remaining gauge groups.
- For k=1 the singly-charged operator is $(e^{\tau})^{a}_{\hat{a}}$ and the doubly-charged one $(e^{2\tau})^{ab}_{\hat{a}\hat{b}} = (e^{2\tau})^{ba}_{\hat{b}\hat{a}}$

Relevant Deformations

- The M2-brane theory may be perturbed by relevant operators that cause it to flow to new fixed points with reduced supersymmetry. Benna, IK, Klose, Smedback; IK, Klose, Murugan; Ahn
- For example, a quadratic superpotential deformation, allowed for k=1, 2, may preserve SU(3) flavor symmetry

$$\Delta \mathrm{W} = m(\mathcal{Z}^4)^a{}_{\hat{a}}(\mathcal{Z}^4)^b{}_{\hat{b}}(e^{-2\tau})^{\hat{a}\hat{b}}_{ab}$$

Squashed, stretched and warped

- The dual AdS_4 background of M-theory should also preserve $\mathcal{N}=2$ SUSY and SU(3) flavor symmetry. Such an extremum of gauged SUGRA was found 25 years ago by Warner. Upon uplifting to 11-d Corrado, Pilch and Warner found a warped product of AdS_4 and of a `stretched and squashed' 7-sphere:

$$ds_{11}^2 = \Delta^{-1} ds_4^2 + 3^{3/2} L^2 \Delta^{\frac{1}{2}} ds_7^2(\rho, \chi)\,, \qquad \Delta \equiv (\xi \cosh \chi)^{-\frac{4}{3}}$$

- The squashing parameter is ρ; the stretching is χ

$$ds_8^2(\rho,\chi) = g_{IJ}dx^I dx^J = dx^I Q^{-1}_{IJ} dx^J + \frac{\sinh \chi^2}{\xi^2}(x^I J_{IJ} dx^J)^2$$

$$Q = \mathrm{diag}\left\{\rho^{-2},\rho^{-2},\rho^{-2},\rho^{-2},\rho^{-2},\rho^{-2},\rho^{6},\rho^{6}\right\}$$

$$\xi^2 \equiv x^I Q_{IJ} x^J$$

- The four complex coordinates

$$z^1 = x^1 + ix^2, \qquad z^2 = x^3 + ix^4, \qquad z^3 = x^5 + ix^6, \qquad w = x^7 - ix^8$$

$$|z^1|^2 + |z^2|^2 + |z^3|^2 + |w|^2 = 1$$

may be expressed in terms of the 7 angles.

- The equations of motion are satisfied with

$$\rho = 3^{\frac{1}{8}} , \qquad \chi = \frac{1}{2} \operatorname{arccosh} 2$$

$$A_{(3)} = \frac{3^{3/4}}{4} e^{3r/L} dx^0 \wedge dx^1 \wedge dx^2 + C_{(3)} + C^*_{(3)}$$

$$C_{(3)} = \frac{3^{11/4} L^3}{4 \left(z^i \bar{z}_i + 3 w \bar{w} \right)} \left[z^{[1} dz^2 \wedge dz^{3]} \wedge d\bar{w} - \bar{w} dz^1 \wedge dz^2 \wedge dz^3 \right]$$

- The internal components break parity (Englert). They preserve a flavor SU(3), and a U(1) R-symmetry

$$\frac{1}{3} \left(z^i \partial_{z^i} - \bar{z}_i \partial_{\bar{z}_i} \right) + w \partial_w - \bar{w} \partial_{\bar{w}}$$

The Spectrum via Group Theory

$$\begin{tikzcd}
\mathrm{Osp}(8|4) \arrow[rr, "\text{stretching and}", "\text{squashing of } S^7"'] \arrow[d, "\begin{array}{c}\text{decompose } \mathcal{N}=8\\ \text{supermultiplets}\end{array}"'] & & SU(3)\times \mathrm{Osp}(2|4) \\
SO(8)_R \times SO(3,2) \arrow[rr, "\text{RG flow}"] & & SU(3)\times U(1)_R \times SO(3,2) \arrow[u, "\begin{array}{c}\text{assemble } \mathcal{N}=2\\ \text{supermultiplets}\end{array}"']
\end{tikzcd}$$

- There are only two ways of breaking the SO(8) R-symmetry consistent with the Osp(2|4) symmetry in the IR:

$$[a,b,c,d] \to \begin{cases} [a,b]_{\left(\frac{a}{3}+\frac{2b}{3}+d\right)\varepsilon} & \text{Scenario I}\,, \\ [a,b]_{-\left(\frac{2a}{3}+\frac{4b}{3}+c+d\right)\varepsilon} & \text{Scenario II} \end{cases}$$

	Scenario I	Scenario II
Hyper	$[n+2,0]_{\frac{n+2}{3}},\ [0,n+2]_{-\frac{n+2}{3}}$	$[n+2,0]_{-\frac{2n+4}{3}},\ [0,n+2]_{\frac{2n+4}{3}}$
Vector	$[n+1,1]_{\frac{n}{3}},\ [1,n+1]_{-\frac{n}{3}}$	$[n+1,1]_{-\frac{2n}{3}},\ [1,n+1]_{\frac{2n}{3}}$
Gravitino	$[n+1,0]_{\frac{n+1}{3}},\ [0,n+1]_{-\frac{n+1}{3}}$	$[n+1,0]_{-\frac{2n-1}{3}},\ [0,n+1]_{\frac{2n-1}{3}}$
Graviton	$[0,0]_{n},\ [0,0]_{-n}$	$[0,0]_{0},\ [0,0]_{0}$

- We find that Scenario I gives $SU(3)xU(1)_R$ quantum numbers in agreement with the proposed gauge theory, where they are schematically given by

	Z^A	ζ^A	$Z_A^\dagger$	$\zeta_A^\dagger$	Z^4	ζ^4	$Z_4^\dagger$	$\zeta_4^\dagger$	x	θ	$\bar{\theta}$
SU(3)	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
Dimension	$\frac{1}{3}$	$\frac{5}{6}$	$\frac{1}{3}$	$\frac{5}{6}$	1	$\frac{3}{2}$	1	$\frac{3}{2}$	-1	$-\frac{1}{2}$	$-\frac{1}{2}$
R-charge	$+\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{1}{3}$	$+\frac{2}{3}$	$+1$	0	-1	0	0	$+1$	-1

Spin-2 Perturbations

- Consider graviton perturbations in AdS with $h^i{}_i = 0\,, \qquad \partial^i h_{ij} = 0$

$\phi = h^i{}_j$ satisfy the minimal scalar equation

$\Box\phi = 0$ $\qquad \phi = \Phi(x^i, r)Y(y^\alpha)$ $\qquad \Box_4\Phi(r,x^i) - m^2\Phi(r,x^i) = 0$

For the (p,q) irrep of SU(3), we find the angular dependence IK, Pufu, Rocha

$$Y(y^\alpha) = a^{j_1 j_2 \cdots j_q}_{i_1 i_2 \ldots i_p} \left(\prod_{k=1}^{p} z^{i_k}\right)\left(\prod_{l=1}^{q} \bar{z}_{j_l}\right) w^{n_r}$$
$$\times \begin{cases} {}_2F_1(-j, 3+p+q+j+n_r; 3+p+q; 1-w\bar{w}) & \text{if } n_r \geq 0 \\ {}_2F_1(-j+n_r, 3+p+q+j; 3+p+q; 1-w\bar{w}) & \text{if } n_r < 0\,. \end{cases}$$

- The R-charge is $R = \frac{1}{3}(p-q) + n_r$

- For the j-th KK mode the mass-squared is

$$m^2 = \frac{1}{L^2}\Big[2j^2 + 2j|n_r| + n_r^2 + 2j(p+q+3) + \frac{1}{3}n_r\,(p-q) + |n_r|(3+p+q) + \frac{1}{9}\left(p^2+q^2+4pq+15p+15q\right)\Big].$$

- The operator dimension is determined by

$$\Delta(\Delta-3) = m^2L^2$$

- For operators in the MGRAV and SGRAV multiplets $\Delta = |R| + 3$

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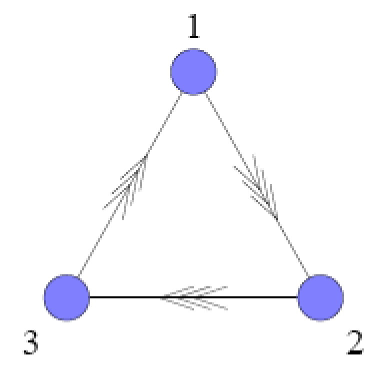
- Here are the low lying operators

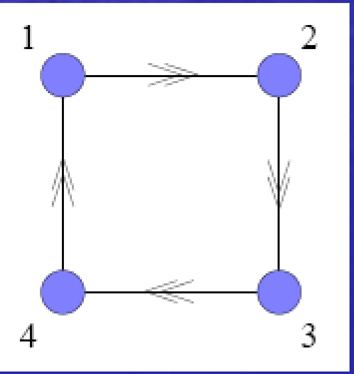
$$\mathcal{T}^{(0)}_{\alpha\beta} = \bar{D}_{(\alpha}\bar{\mathcal{Z}}_A D_{\beta)}\mathcal{Z}^A + i\bar{\mathcal{Z}}_A \overleftrightarrow{\partial}_{\alpha\beta}\mathcal{Z}^A$$

	$[p,q]_R$	j	n_r	Δ	m^2L^2	Operator
*	$[0,0]_0$	0	0	3	0	$\mathcal{T}^{(0)}_{\alpha\beta}$
*	$[0,0]_{\pm1}$	0	±1	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^4,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_4$
	$[0,1]_{-\frac{1}{3}},\ [1,0]_{\frac{1}{3}}$	0	0	$\frac{1}{6}\left(9+\sqrt{145}\right)$	$\frac{16}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A$
*	$[0,0]_{\pm2}$	0	±2	5	10	$\mathcal{T}^{(0)}_{\alpha\beta}(\mathcal{Z}^4)^2,\ \mathcal{T}^{(0)}_{\alpha\beta}(\bar{\mathcal{Z}}_4)^2$
	$[0,0]_0$	1	0	$\frac{1}{2}\left(3+\sqrt{41}\right)$	8	$\mathcal{T}^{(0)}_{\alpha\beta}\left(1-4a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4\right)$
	$[0,1]_{-\frac{4}{3}},\ [1,0]_{\frac{4}{3}}$	0	$-1,\ 1$	$\frac{1}{6}\left(9+\sqrt{337}\right)$	$\frac{64}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\bar{\mathcal{Z}}_4,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}_A\mathcal{Z}^4$
	$[0,1]_{\frac{2}{3}},\ [1,0]_{-\frac{2}{3}}$	0	$-1,\ 1$	$\frac{1}{6}\left(9+\sqrt{313}\right)$	$\frac{58}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\mathcal{Z}^4,\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}_A\bar{\mathcal{Z}}_4$
	$[0,2]_{-\frac{2}{3}},\ [2,0]_{\frac{2}{3}}$	0	0	$\frac{1}{6}\left(9+\sqrt{217}\right)$	$\frac{34}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)},\ \mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}$
	$[1,1]_0$	0	0	4	4	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^A\bar{\mathcal{Z}}_B-\frac{1}{3}\delta^A_B\mathcal{Z}^C\bar{\mathcal{Z}}_C\right)$
	$[0,0]_{\pm1}$	1	±1	$\frac{1}{2}(3+\sqrt{65})$	14	$\mathcal{T}^{(0)}_{\alpha\beta}\left(2-5a^2\mathcal{Z}^4\bar{\mathcal{Z}}_4\right)\mathcal{Z}^4$, c.c.
*	$[0,0]_{\pm3}$	0	±3	6	18	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^4\right)^3,\ \mathcal{T}^{(0)}_{\alpha\beta}\left(\bar{\mathcal{Z}}_4\right)^3$
	$[1,0]_{-\frac{5}{3}},\ [0,1]_{\frac{5}{3}}$	0	$-2,+2$	$\frac{1}{6}(9+\sqrt{553})$	$\frac{118}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(\bar{\mathcal{Z}}_4\right)^2,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\left(\mathcal{Z}^4\right)^2$
	$[1,0]_{\frac{1}{3}},\ [0,1]_{-\frac{1}{3}}$	1	0	$\frac{1}{6}(9+\sqrt{505})$	$\frac{106}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(1-5a^2\bar{\mathcal{Z}}_4\mathcal{Z}^4\right)$, c.c.
	$[1,0]_{\frac{7}{3}},\ [0,1]_{-\frac{7}{3}}$	0	$2,-2$	$\frac{1}{6}(9+\sqrt{601})$	$\frac{130}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^A\left(\mathcal{Z}^4\right)^2,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_A\left(\bar{\mathcal{Z}}_4\right)^2$
	$[1,1]_{\pm1}$	0	±1	5	10	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^A\bar{\mathcal{Z}}_B-\frac{1}{3}\delta^A_B\mathcal{Z}^C\bar{\mathcal{Z}}_C\right)\mathcal{Z}^4$, c.c.
	$[2,0]_{-\frac{1}{3}},\ [0,2]_{\frac{1}{3}}$	0	$-1,1$	$\frac{1}{6}(9+\sqrt{409})$	$\frac{82}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}\bar{\mathcal{Z}}_4,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)}\mathcal{Z}^4$
	$[2,0]_{\frac{5}{3}},\ [0,2]_{-\frac{5}{3}}$	0	$1,-1$	$\frac{1}{6}(9+\sqrt{457})$	$\frac{94}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^{B)}\mathcal{Z}^4,\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_{B)}\bar{\mathcal{Z}}_4$
	$[2,1]_{\frac{1}{3}},\ [1,2]_{-\frac{1}{3}}$	0	0	$\frac{1}{6}(9+\sqrt{313})$	$\frac{58}{9}$	$\mathcal{T}^{(0)}_{\alpha\beta}\left(\mathcal{Z}^{(A}\mathcal{Z}^{B)}\bar{\mathcal{Z}}_C-\frac{1}{3}\delta^{(A}_C\mathcal{Z}^{B)}\mathcal{Z}^D\bar{\mathcal{Z}}_D\right)$, c.c.
	$[3,0]_1,\ [0,3]_{-1}$	0	0	$\frac{1}{2}(3+\sqrt{33})$	6	$\mathcal{T}^{(0)}_{\alpha\beta}\mathcal{Z}^{(A}\mathcal{Z}^B\mathcal{Z}^{C)},\ \mathcal{T}^{(0)}_{\alpha\beta}\bar{\mathcal{Z}}_{(A}\bar{\mathcal{Z}}_B\bar{\mathcal{Z}}_{C)}$

Further Directions

- Other examples of AdS_4/CFT_3 dualities with $\mathcal{N}$=1,2,3,… supersymmetry are being studied by many groups.
- Various famous quivers assume new identitites: M^{111}, Q^{222}, etc.





- Ultimate Hope: to find a `simple' dual of a 3-d fixed point realized in Nature.