Keldysh Model in Time Domain

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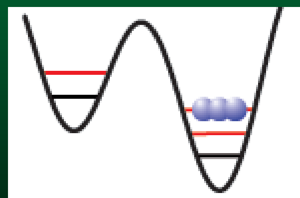


Motivation: <u>*TLS is as universal as an oscillator*</u>.

We will play with TLS ensemble in order to extend **to time domain** exactly solvable Keldysh model originally invented for description of disordered semiconductors (***L.V.Keldysh ‘65***) developed by ***A.Efros (’71)*** and exploited by ***M.V.Sadovskii (’74,’79 ff), Bartosh&Kopietz (’00) etc***

Applications:

- Time dependent Landau-Zener problem
- Well population redistribution in optical lattices
- Tunneling through quantum dots in presence of noise

Publications:

JETP Letters ***89,*** *133 (2009); arXiv:0901.2246*

arXiv:0803.2676; Phys. Stat. Solidi (c) (in press)

Time-dependent Landau-Zener model *(Kayanuma '84-85)*

$$H(t)=\frac{1}{2}\{vt+f(t)\}(|1\rangle\langle 1|-|2\rangle\langle 2|)+J(|1\rangle\langle 2|+|2\rangle\langle 1|),$$

The source of temporal fluctuations is the phonon bath

$$H_B=\sum_k\left(\frac{p_k^2}{2}+\frac{\omega_k^2 q_k^2}{2}\right),$$

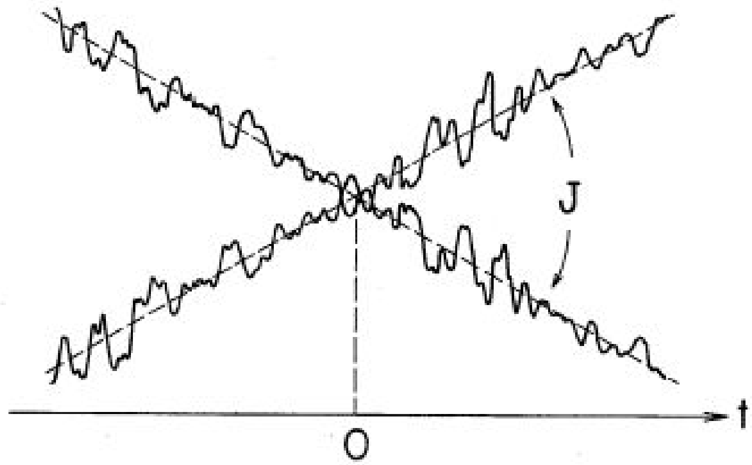
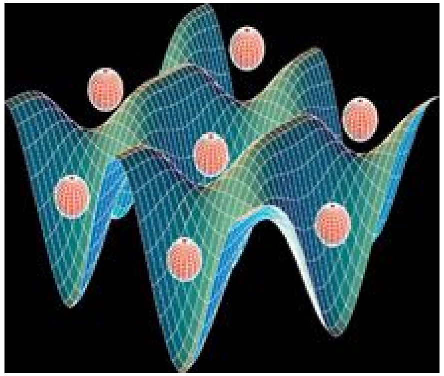


Fig. 1. Level crossing with energy fluctuation. The energy difference between the diabatic states changes with a constant velocity v in its mean value and with a fluctuation $f(t)$ around it.

Optical lattice in random laser field



The superlattice potential is given by:

$$V(x) = V_s \cos^2(4\pi x/\lambda_l - \phi) + V_l \cos^2(2\pi x/\lambda_l)\,, \qquad (1)$$

where ϕ is the relative phase between the short and long period lattices. $V_{s,l,t}$ denote the lattice depths of the short, long and transverse lattices

Counting atoms using interaction blockade in an optical superlattice (*PRL '08)*

P. Cheinet,[1] S. Trotzky,[1] M. Feld,[1,2] U. Schnorrberger,[1] M. Moreno-Cardoner,[1] S. Fölling,[1] I. Bloch[1*]

3D tetragonal optical lattice with split traps

Fluctuations *φ(t)* may be the source of noise

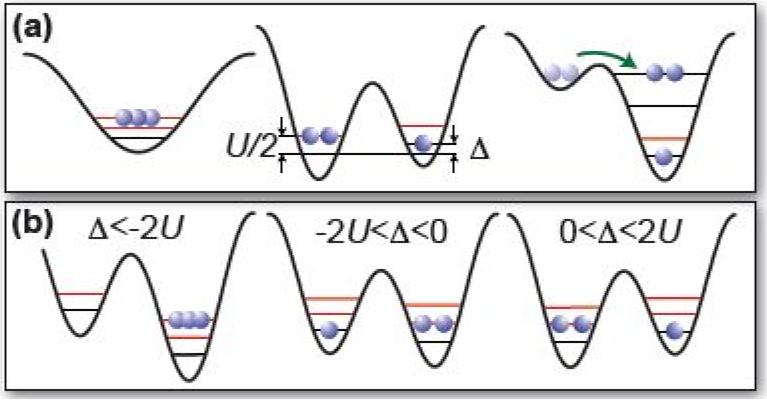
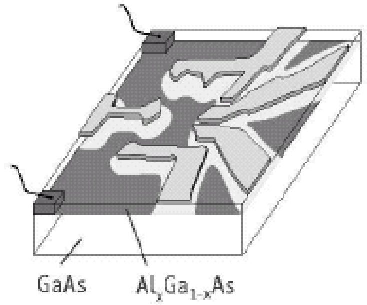
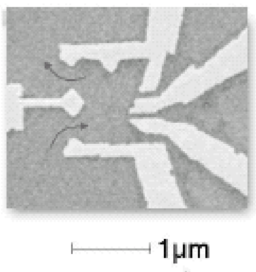
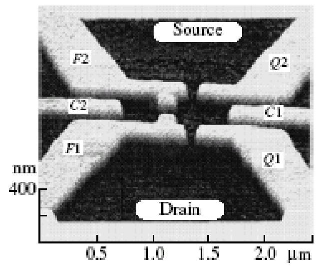


FIG. 2: (a) Experimental sequence. After loading the BEC into a 3D lattice, each site is split slowly into a biased double well.

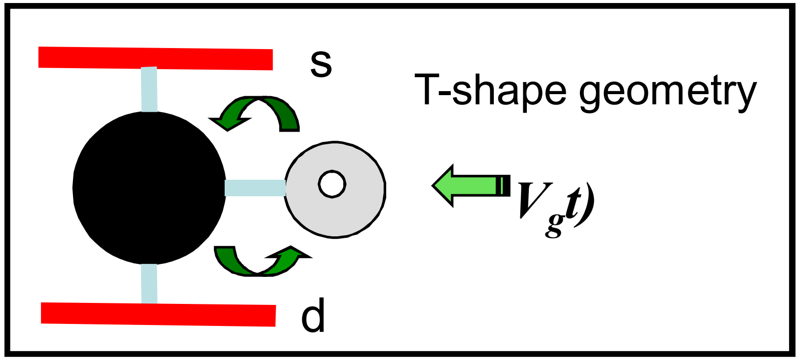
Double-well quantum dot
with time-dependent gate voltage



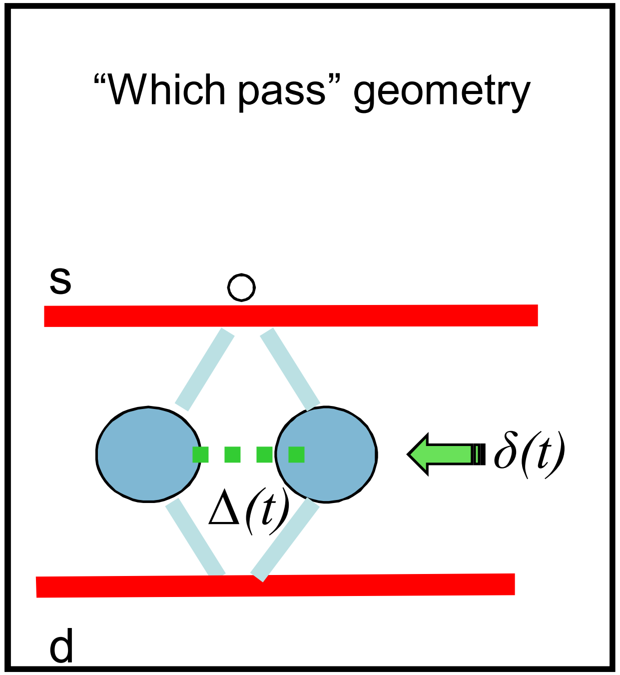




DQD



Electron possesses both spin ½ and pseudospin ½ corresponding to two positions in DQD.

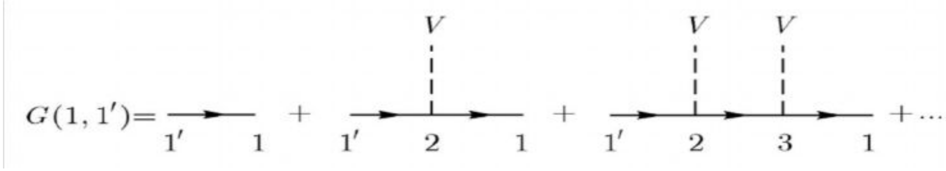


The symmetry of this objects is ***SU(4)***

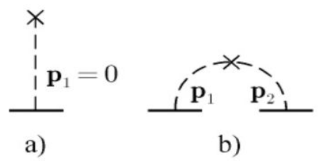
ORIGINAL KELDYSH MODEL

KM was proposed for description of non-interacting electrons in a random potential *V(**r**)* (disordered electrons in metals or semiconductors)

$$H_{int} = \int d\mathbf{r}\psi^{+}(\mathbf{r})V(\mathbf{r})\psi(\mathbf{r})$$

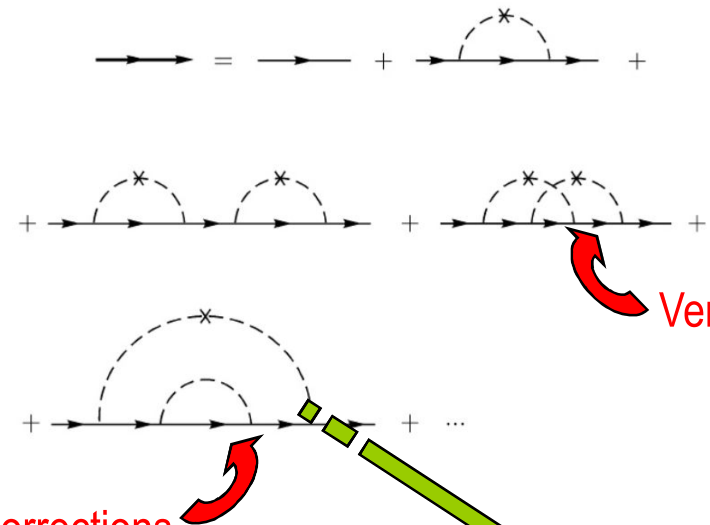


$$V(\mathbf{r}) = \sum_{\mathbf{p}}\sum_{j} v(\mathbf{p})e^{i\mathbf{p}(\mathbf{r}-\mathbf{R}_j)}$$



(cross technique)

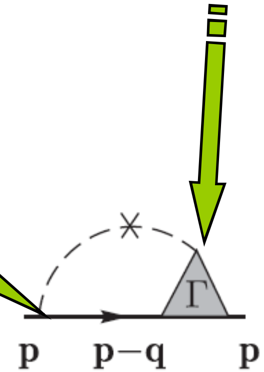
G(r,r') – bare electron propagator



Vertex corrections

Self energy corrections

$$G(\mathbf{p}\varepsilon_n) = \frac{1}{i\varepsilon_n - \xi(p) - \Sigma(\mathbf{p}\varepsilon_n)}$$



<u>Usual approach</u> - white noise (Gaussian) approximation

$$< V(\mathbf{r}_1)V(\mathbf{r}_2) >= \rho v^2 \delta(\mathbf{r}_1 - \mathbf{r}_2)$$

Short-range δ-like scattering

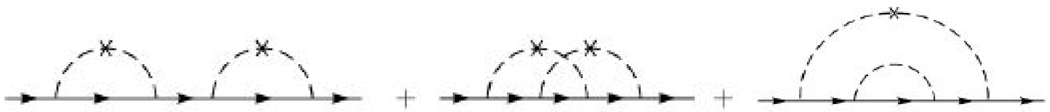
Keldysh model ("anti- white noise")

$$< V(\mathbf{r})V(\mathbf{r}') >= \int \frac{d^3q}{(2\pi)^3} e^{i\mathbf{q}(\mathbf{r}-\mathbf{r}')} (2\pi)^3 W^2 \delta(\mathbf{q}) = W^2$$

- infinite-range scattering

Impurity field is constant in each realization, but its magnitude changes randomly from one realization to another.

The perturbation series for the Green function can be summed **exactly** in approximation of IRS because all cross diagrams in a given order are **equivalent**



$$G(\varepsilon\mathbf{p}) = G_0(\varepsilon\mathbf{p})\left\{1 + \sum_{n=1}^{\infty} A_n W^{2n} G_0^{2n}(\varepsilon\mathbf{p})\right\}$$

A_n is the total number of diagrams in the order *2n* – purely combinatorial coefficient, $A_n = (2n-1)!!$

Then Keldysh uses the integral representation $(2n-1)!! = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt t^{2n} e^{-\frac{t^2}{2}}$

and changes the order of summation and integration (Borel summation procedure)

$$G(\varepsilon\mathbf{p}) = G_0(\varepsilon\mathbf{p})\left\{1 + \sum_{n=1}^{\infty} \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} dt t^{2n} e^{-\frac{t^2}{2}} W^{2n} G_0(\varepsilon\mathbf{p})^{2n}\right\}$$

Summation results in Gaussian distribution for G^R

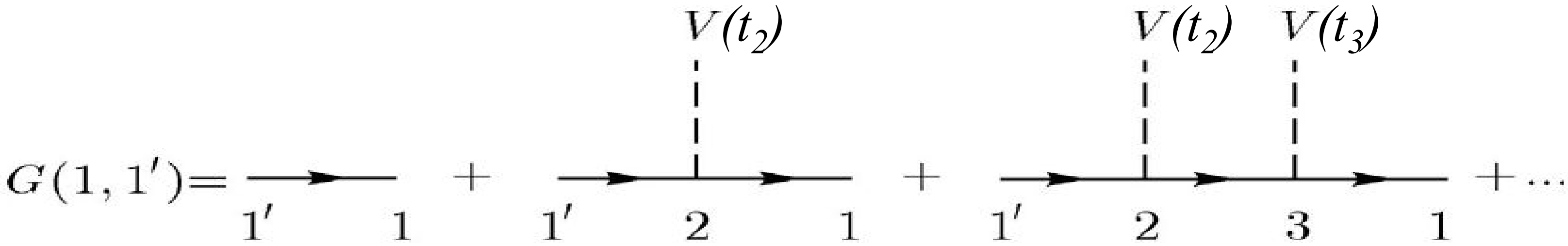
$$G^R(\varepsilon\mathbf{p}) = \frac{1}{\sqrt{2\pi}W}\int_{-\infty}^{\infty} dV e^{-\frac{V^2}{2W^2}} \frac{1}{\varepsilon - \frac{p^2}{2m} - V + i\delta}$$

This means that an electron moves in a spatially homogeneous field *V*, and the magnitude of this field obeys the Gaussian distribution with the variance *W*

Our idea is to convert the Keldysh model *into the* ***time domain***

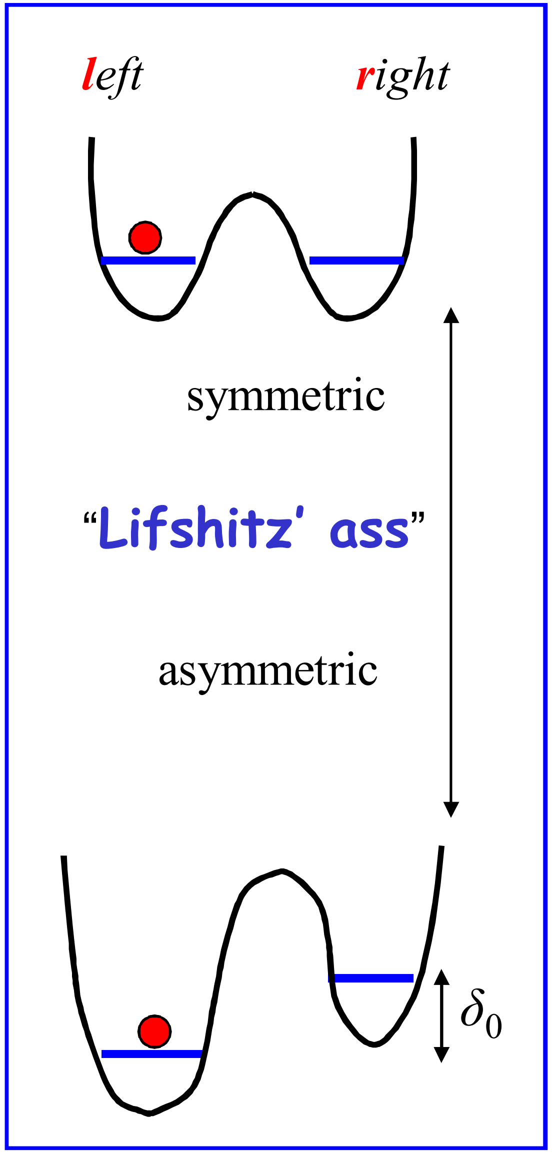
Time-Dependent Keldysh Model (TDKM)

Random field $V(t)$ fluctuating in time:



Then one may apply cross technique and Keldysh machinery to the ensemble of TLS,
Now disorder develops **on the 1D time axis** instead of spatial lattice or chain.

We apply this model to an ensemble of TLS consisting either of fermions or of bosons



Wide barrier limit: ***l*** – spin up, ***r*** – spin down

Original Hamiltonian (Fermi/Bose-Hubbard model):

$$H^{(0)}_{\rm TLS} = \sum_j (\varepsilon_j n_j + U n_j^2) - \Delta_0 (c_l^\dagger c_r + {\rm H.c.}).$$

Effective Hamiltonian for $N = 1$ rewritten in terms of Pauli matrices for two states $\{$***l,r***$\}$

$$\sigma_1 = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_3 = \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

$$H^{(0)}_{\rm TLS} = -\delta_0 \sigma_z - \Delta_0 \sigma_x - \mu_0 (N-1).$$

in the pseudospin subspace. Here the asymmetry parameter $\delta_0 = \varepsilon_r - \varepsilon_l$ play the role of effective "magnetic" field, the Lagrange parameter μ_0 controls constraint.

Both Δ and δ may slowly fluctuate in time

Time-dependent random fluctuations of asymmetry parameter: $\delta_\rho(t)$

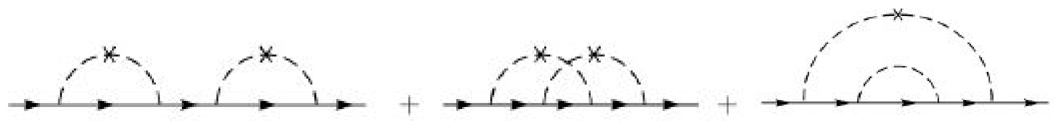
$$\overline{\delta(t)} = 0, \; \overline{\delta_\rho(t)\delta_\rho(t')} = D(t-t').$$

The analog of Keldysh conjecture: slowly varying random field ~ exp(- γt) . Then the Fourier transform of the noise correlation function is

$$D(\omega) = \lim_{\gamma \to 0} \frac{2\zeta^2\gamma}{\omega^2 + \gamma^2} = 2\pi\zeta^2\delta(\omega)$$

This means that we consider the ensemble of states with a field $\delta = const$ in a given state, but this constant is random in each realization .

Then we apply the cross technique in time domain to the single particle propagator



$$G^R_{j,s}(\varepsilon) = g_j(\varepsilon)\left[1 + \sum_{n=1}^{\infty} A_n \zeta^{2n} g_j^{2n}(\varepsilon)\right]$$

Here $g_l(\varepsilon) = (\varepsilon + i\eta)^{-1}$ and $g_r(\varepsilon + \delta_0 + i\eta)^{-1}$

are the bare propagators for a particle in the left and right valley.

Repeating Keldysh' calculations, we obtain

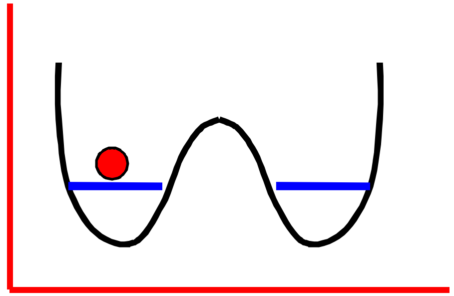
$$G^R_{l,s}(\varepsilon) = \frac{1}{\zeta\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2\zeta^2} \frac{dz}{\varepsilon - z + i\delta}$$

The same for $G_{r,s}(\varepsilon + \delta_0)$.

Remarkably, the Green function after Gaussian averaginig
has no poles, singularities or branch cuts.

"Vector" Keldysh model

for fluctuating transparency Δ



Let us consider symmetric TLS with nearly impenetrable barrier $\Delta_0 \to 0$
and allow intervalley tunneling only due to the random **transverse** perturbation

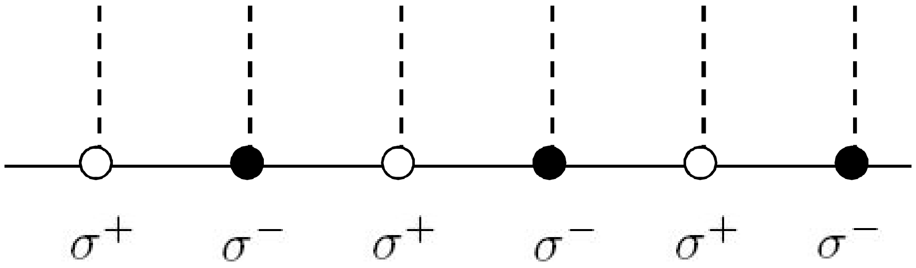
$$H_\rho(t) = \Delta_\rho(t)(\sigma^+ + \sigma^-) = 2\Delta_\rho(t)\sigma^x$$

and make Keldysh conjecture about correlation function $\overline{\Delta_\rho(t)\Delta_\rho(t')}$,
namely, approximate its Fourier transform by

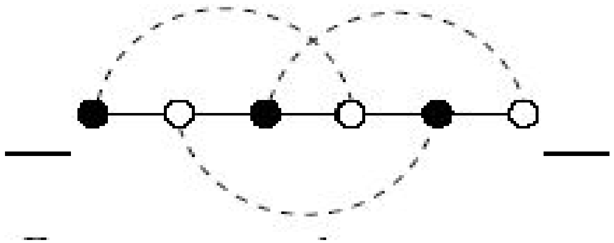
$$F(\omega) = \lim_{\gamma\to 0} \frac{4\xi^2\gamma}{\omega^2+\gamma^2} = 4\pi\xi^2\delta(\omega)$$

Now the cross technique is subject to kinematic restriction. Each site is associated ether with σ^+ or with σ^- but $\sigma^+\sigma^+ = \sigma^-\sigma^- = 0.$
and only diagrams with pseudospin operators ordered as $\ldots\sigma^+\sigma^-\sigma^+\sigma^-\ldots$

As a result the series becomes two-colored:



and the dashed lines connect only vertices of different colors



Keldysh summation is still possible but the combinatorial coefficients differ from those in “scalar” model

$$G^R_{j,v}(\varepsilon) = g_j(\varepsilon) + \sum_{n=1}^{\infty} B_n(\sqrt{2}\xi)^{2n} g_j^{2n+1}(\varepsilon). \text{ with } \quad B_n = n!$$

Similar expansion for “vector Keldysh model in real space was obtained by ***Sadovskii ('74)*** in a model of long-range fluctuations near the CDW instability.

Again we use the integral representation $n! = \int_0^{\infty} dz z^n e^{-z}$
and perform Borel summation.

The result:

$$G^R_{j,v}(\epsilon) = \frac{1}{2}\int_{-\infty}^{+\infty} \frac{dx e^{-x^2/2\xi^2}}{\xi\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{dy e^{-y^2/2\xi^2}}{\xi\sqrt{2\pi}} \left[\frac{1}{\epsilon - \sqrt{x^2+y^2} + i\delta} + \frac{1}{\epsilon + \sqrt{x^2+y^2} + i\delta}\right]$$

Two-dimensional Gaussian averaging of vector random field with transversal *(xy)* fluctuations.

Only the modulus of random field $r = \sqrt{x^2+y^2}$ is averaged

Like in the scalar Keldysh model, the Green functions have no poles. This means that the information about position of the particle in the left or right valley is partially lost due to stochastization.

More general result by simpler method

(path integral formalism)

Lagrangian action: $\mathcal{L}(t) = \sum_{j=l,r} \bar{c}_j i\partial_t c_j - H, \quad S_K = \int_K \mathcal{L}(t)dt.$

Here $\bar{c}_j, c_j$ are Grassmann variables describing the electron. The time-dependent gauge transformation $c_j(t) \rightarrow c_j(t)e^{i\varphi_j(t)}$, converts the fluctuation of the well depth to the fluctuation of the *phase* of the tunnel matrix element under the choice

$$\varphi_j(t) = \int_{-\infty}^{t} h_j(t')dt'. \qquad (8)$$

We therefore identify the longitudinal and transverse noise with phase fluctuations of the barrier transparency and fluctuations of the modulus of the tunnel matrix element, respectively and unify them in the path integral description.

The ensemble averaging

$$\langle ... \rangle_{noise} = \int dh_\rho P_l(h_\rho) \int d\Delta^*_\rho d\Delta_\rho P_{tr}(\Delta^*_\rho, \Delta_\rho) ...$$

is done with the help of probability distribution functions for longitudinal and transverse fluctuations

$$P_l = \frac{1}{\zeta\sqrt{2\pi}} \exp\left(-\int_K dt dt' h_\rho(t) D^{-1}(t-t') h_\rho(t')\right)$$

$$P_{tr} = \frac{1}{2\pi\xi^2} \exp\left(-\int_K dt dt' \Delta^*_\rho(t) F^{-1}(t-t') \Delta_\rho(t')\right).$$

Green function in a symmetric TLS

$$G^R_{j,v}(\epsilon) = \frac{1}{\zeta\xi^2(2\pi)^{3/2}} \int_{-\infty}^{\infty} dz e^{-z^2/2\zeta^2} \int dw^* dw e^{-|w|^2/2\xi^2} \frac{\epsilon \pm z}{(\epsilon + i\eta)^2 - z^2 - |w|^2}$$

From “Bloch ellipsoid” to “Bloch sphere”:

Turning to spherical cordinates and integrating over angles, we have

$$G^R_v(\epsilon) = \frac{1}{2\xi} \int_0^{\infty} d\rho \rho \exp\left(-\frac{\rho^2}{2\xi^2}\right) \frac{\operatorname{erf}\left(\rho\sqrt{\frac{\xi^2-\zeta^2}{2\xi^2\zeta^2}}\right)}{\sqrt{\xi^2-\zeta^2}} \left(\frac{1}{\epsilon - \rho + i\eta} + \frac{1}{\epsilon + \rho + i\eta}\right).$$

Ellipsoid transforms into sphere for $\zeta = \xi$

Scalar model: $\xi \rightarrow 0$

Planar model: $\zeta \rightarrow 0$

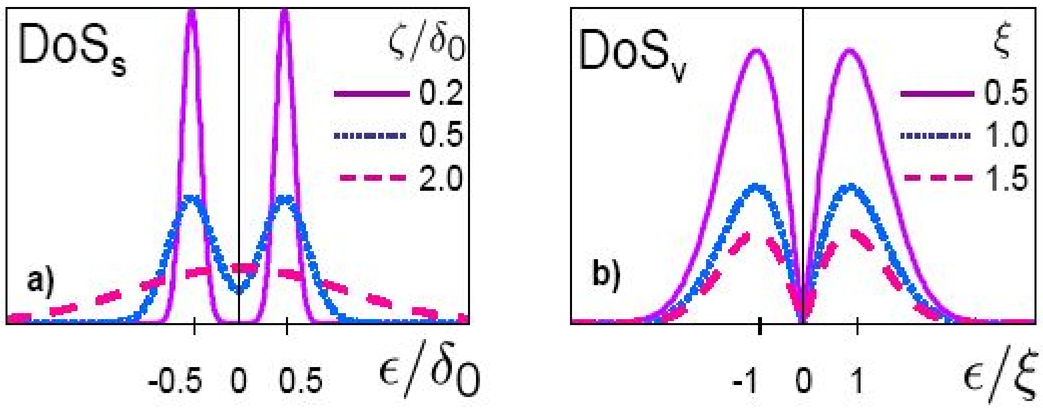
Density of States in stochastisized TLS

$$\nu_s(\varepsilon) = \frac{2}{\zeta\sqrt{2\pi}} \exp\left(-\frac{\varepsilon^2 + \delta_0^2}{2\zeta^2}\right) \cosh\left(\frac{\varepsilon\delta_0}{\zeta^2}\right),$$

$$\nu_v(\varepsilon) = \frac{1}{2\xi^2}|\varepsilon| \exp\left(-\frac{\varepsilon^2}{2\xi^2}\right).$$

Scalar TDKM: two superimposed Gaussians

Planar TDKM: single Gaussian with pseudogap around zero energy (*cf. Sadovskii*)



What about experimental manifestations?

What is already seen in optical lattice with split potental wells?

Repopulation of these wells

$|n, m> \rightarrow |n\text{-}1, m\text{+}1>$

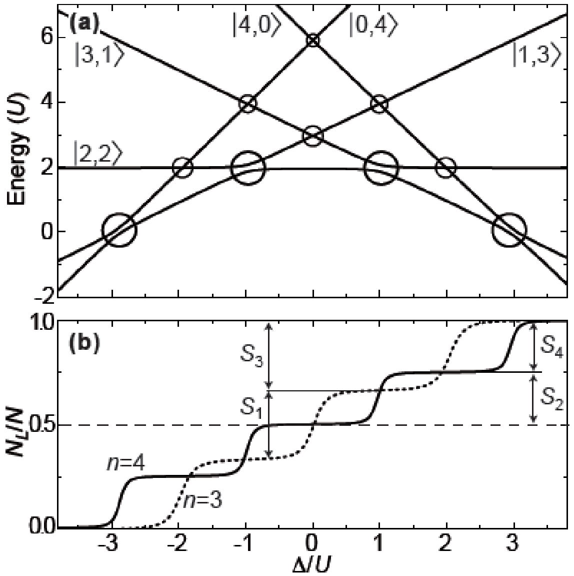


FIG. 1: (a) Eigenenergies in a double well filled with 4 atoms versus Δ for $U/J = 30$. The single particle resonances

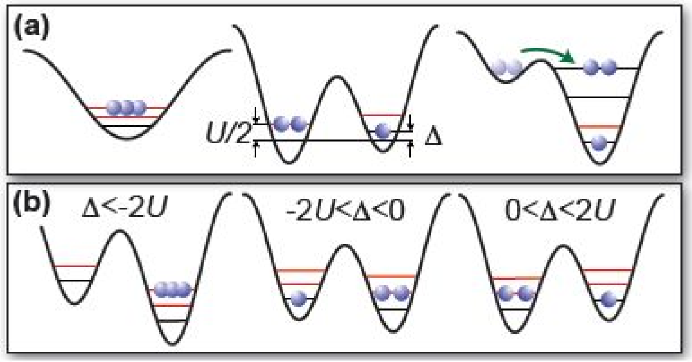


FIG. 2: (a) Experimental sequence. After loading the BEC into a 3D lattice, each site is split slowly into a biased double well. The atoms in the left well are transferred into a high

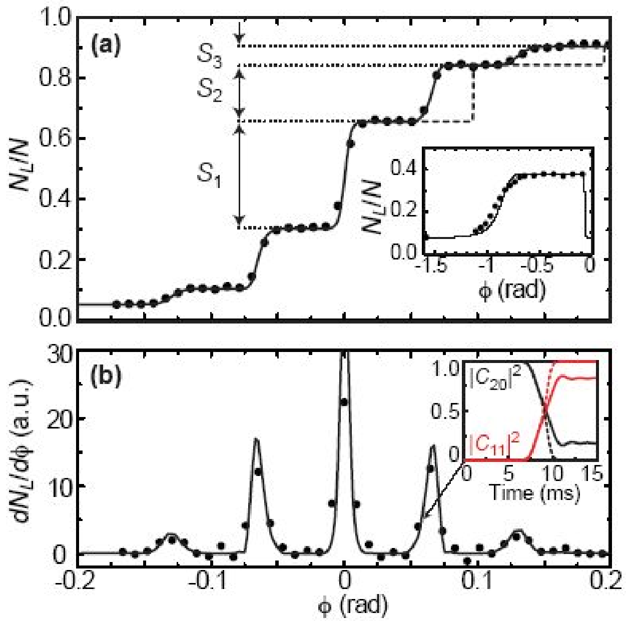


FIG. 3: (a) Left well population, measured as a function of the phase ϕ between the short and long lattices (points),

What we propose:

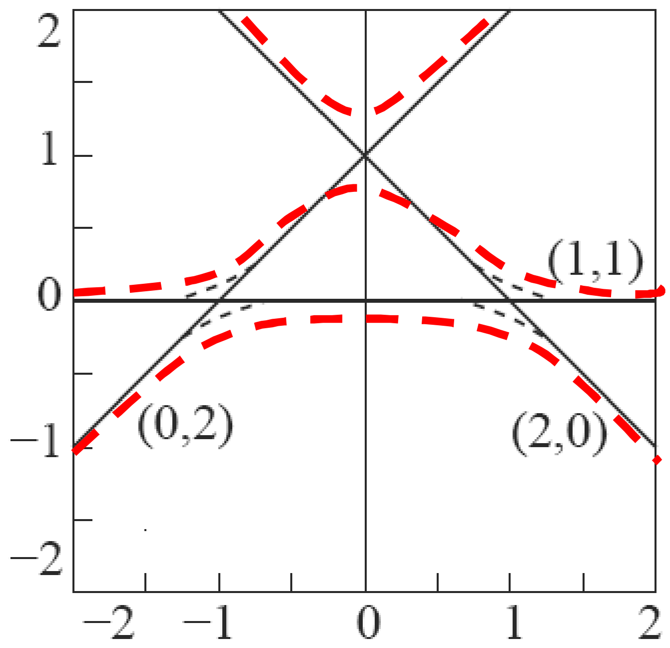
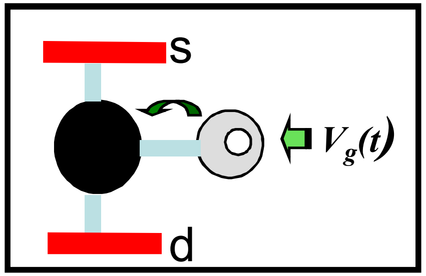
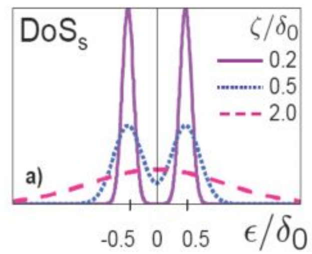


FIG. 1: Energy levels $E_\Lambda(N_l, N_r)$ for doubly occupied DW trap as a function of asymmetry parameter δ_0/U. Corrections resulting in ***noise-induced mixed valence states*** of the split optical trap

TDKM for Double Quantum Dots



N=1: Pseudospin is screened but spin survives:
Noise induced "local phase transition" $SU(4) \rightarrow SU(2)$



In more refined models with *SO(5)* symmetry spin degrees of freedom also may be dynamically stochastisized ***(K.K., M.K. and J. Richert ,'08, '09)***

Conclusions

- Keldysh model looks more realistic in time domain (long memory) than in real space (infinite range correlations)

- TDQM is useful in study of decoherence effects in nanosystems described in terms of TLS cells, including memory cells for quantum computers

The spin susceptibility of stochastisized TLS or QD may be calculated in a very elegant way via **"Ward identities"** (***Efros'71; K.K & K.M. '08)***

Direct differentiation of GF in Gaussian representation derived above shows that these GF obey differential equations

$$\varepsilon_\alpha G_{\alpha,s} - 1 = \zeta^2 G_{\alpha,s}^2 \frac{d}{d\varepsilon} G_{\alpha,s}^{-1}$$ for scalar TDKM ***(Efros)***

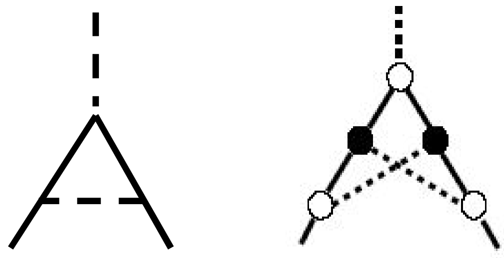
$$\varepsilon G_v - 1 = \xi^2 G_v^2 \left[\frac{1}{\varepsilon} \frac{d}{d\varepsilon} \left(\varepsilon G_v^{-1} \right) \right]$$ for vector TDKM ***(K.K & K.M)***

Now we may introduce the vertex parts Γ using the analogy with Ward identities

$$\Gamma_s = \frac{d}{d\varepsilon} G_s^{-1}, \quad \Gamma_v = \frac{1}{\varepsilon} \frac{d}{d\varepsilon} (\varepsilon G_v^{-1}).$$

First vertex corrections

for **scalar** and **vector** TDKM.

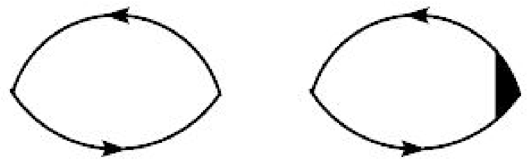


But these corrections may be found exactly by means of Ward identity

WI reads

$\zeta\, G^2 \Gamma = \varepsilon G - 1$

$$\chi_{\parallel}(0) = -\sum_{\alpha=\mp} \int_{-\infty}^{\infty} \frac{2y\,dy\,e^{-y^2/2}}{\sqrt{8\pi\zeta}}\, n_F\left(\frac{(2y-\alpha\delta_0)\zeta}{2T}\right)$$



Diagrams for bare and dressed susceptibility.

with asymptotical behavior

$$\chi(0) \sim \begin{cases} 1/T, & T \gg (\zeta, \delta_0) \\ 1/\zeta, & \zeta \gg (T, \delta_0) \end{cases}$$

Thus spin is indeed stochastisized at low temperature !