Cosmic microwave background, structure, backreaction

V.G.Gurzadyan

Yerevan Physics Institute, Yerevan State University

References

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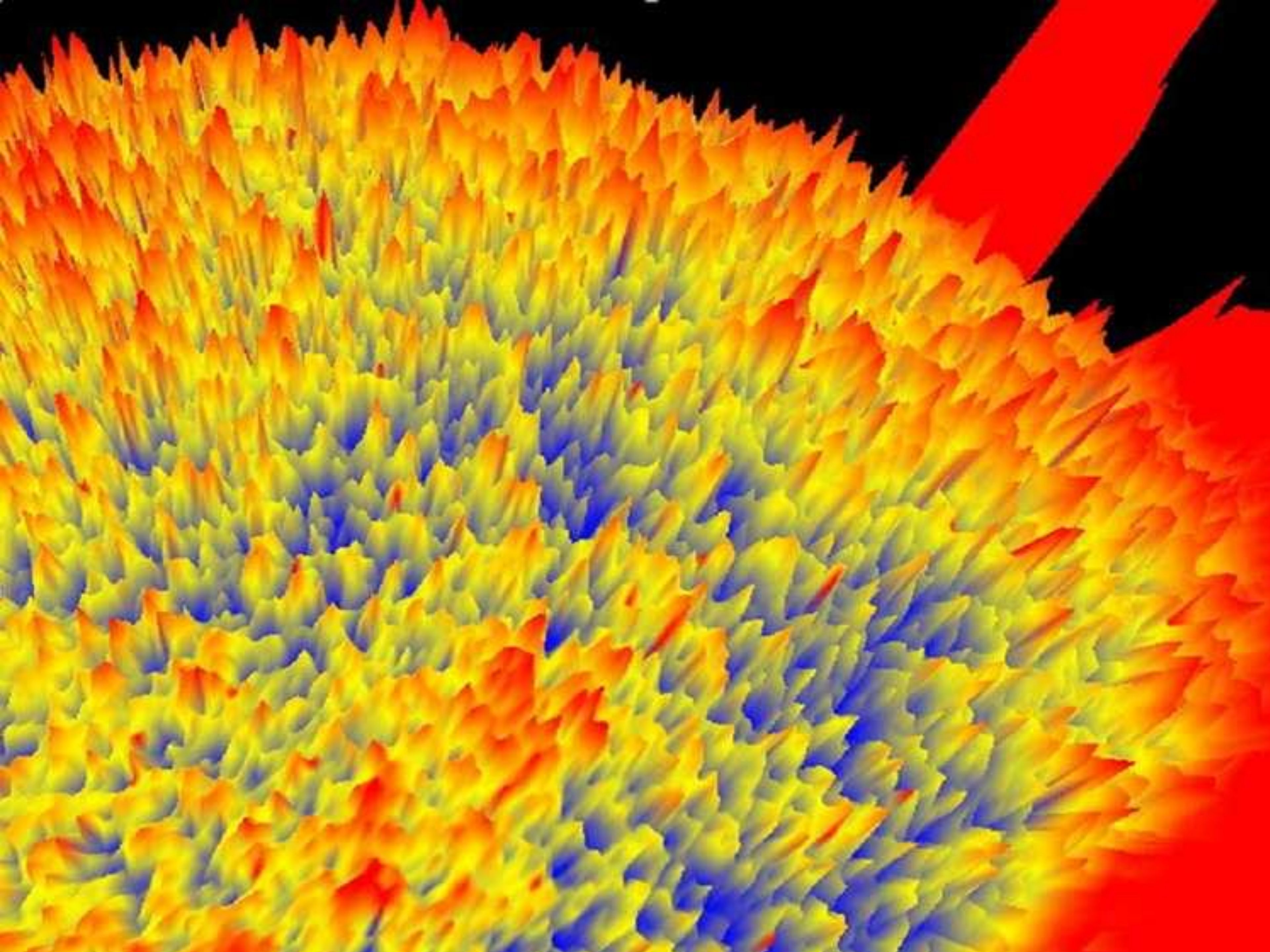
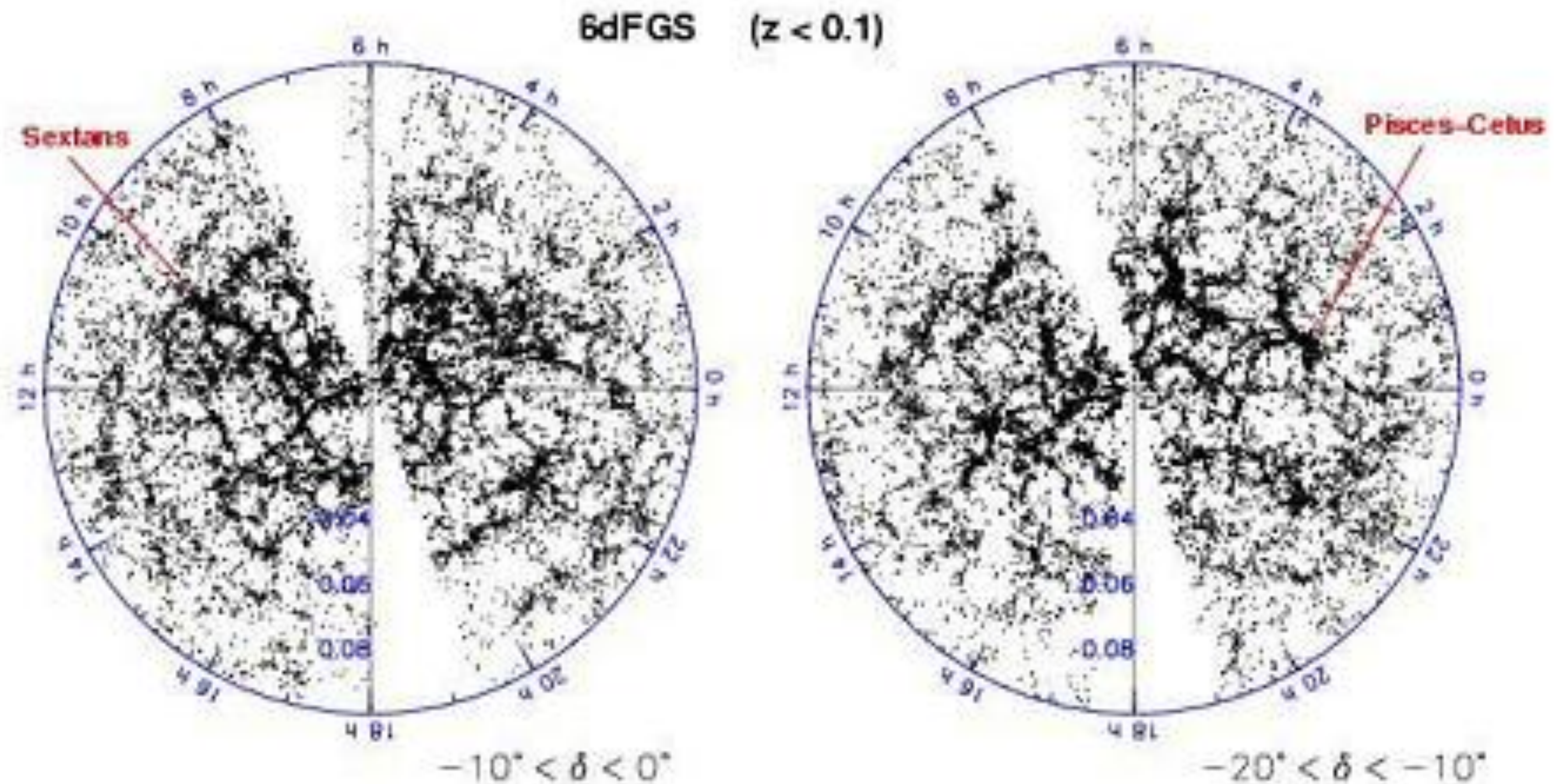




D.Heath Jones et al, 2009; 136,304 galaxies , z_mid = 0.053.


Ophiuchus Supercluster
Hercules Superclusters
Capricornus Void
100 million ly
Corona Borealis Void
Microscopium Void
Pavo-Indus Supercluster
Coma Wall
A3627
A4038
Centaurus Wall
Centaurus Supercluster
Sculptor Void
Sculptor Wall
Coma Supercluster
Virgo Supercluster
Phoenix Supercluster
Hydra Supercluster
Fornax Void
Perseus-Pisces Supercluster
Leo Supercluster
Canes-Major Void
Columba Void
Columba Supercluster

A. Fairall

Filamentary Universe: Observations

Voids: SDSS, Las Campanas, CfA, 2dF, etc.

mean scale: 30 Mpc
density contrast: -0.93, -0.96
40 % of volume
(e.g. Hoyle and Vogeley, 2004; Ceccarelli et al 2008).

Larger scale voids, of about 100 Mpc (?) (Einasto et al, 1997, 2006), even larger: 300-500 Mpc (?) (e.g. Alexander and Biswas et al, 2008).

Inhomogeneous Universe: Friedmann-Robertson-Walker approximation

Different approaches, averaging schemes:

Averaging tensors on curves of observers (Zalaletdinov 1992);
Averaging scalars, density, pressure, etc (Buchert 2008).

Dark energy without dark energy;
Lensing on inhomogeneities to mimic curvature effects;
(Wiltshire 2007, 2008; Mattsson 2007; Larena et al 2008…).

Copernican principle (e.g. Caldwell, Stebbins 2008).

Instability of dynamical systems

One can derive an equation for the length of n from the Jacobi equation

$$\nabla_u^2 n + \Re_u(n) = 0 \ .$$

Let $n = \ell\hat{n}$, where $g(n,n) = \ell^2$, $g(\hat{n},\hat{n}) = 1$, $g(\hat{n},u) = 0$, then we get

$$\ddot{\ell} + [K(u,\hat{n}) - g(\nabla_u\hat{n}, \nabla_u\hat{n})]\ell = 0 \ ,$$

where the sectional curvature has the following form

$$K(u,\hat{n}) = g(\Re_u(\hat{n}),\hat{n}) = g(\Re(\hat{n},u)u,\hat{n}) \ .$$

Let $V = M \times R^1$ be a (3+1)D smooth manifold with the following metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = a^2(\eta)\ e^{2\psi(x)}\left[-d\eta^2 + e^{-2f(x)}d\sigma^2\right] \ ,$$

Reduction of (3+1)D null geodesic flow to 3D geodesic flow

A geodesic flow on M x R can be described by the Hamiltonian

$$\mathcal{H}(p,x) = \frac{1}{2} g^{\mu\nu}(x) p_\mu p_\nu = \frac{1}{2}\, a^{-2}(\eta)\, e^{-2\psi} \left[-p_0^2 + e^{2f(x)} \gamma^{mn} p_m p_n \right]$$

and Hamiltonian equations

$$\frac{dx^\mu}{ds} = \frac{\partial \mathcal{H}}{\partial p_\mu}$$

$$\frac{dp_\mu}{ds} = -\frac{\partial \mathcal{H}}{\partial x^\mu}\,.$$

One can prove that the projections of the geodesics on M x R onto M are geodesics if the metric there is

$$\tilde{\gamma} = e^{-2f} \gamma$$

For the null geodesics

$$\frac{\partial \mathcal{H}}{\partial x^0} = \frac{\partial \mathcal{H}}{\partial \eta} = -2\frac{\dot{a}}{a}\mathcal{H} = 0 \ ,$$

therefore

$$\frac{dp_0}{ds} = 0 \ ,$$

and

$$\frac{d\eta}{ds} = -a^{-2}e^{-2\psi}p_0 \ ,$$

thus, p_0 is a constant ($E = -p_0$). For the "space" coordinates we obtain

$$\begin{aligned} \frac{dx^m}{ds} &= [a^{-2}e^{-2\psi}]\ \frac{\partial H}{\partial p_m} \\ \frac{dp_m}{ds} &= -[a^{-2}e^{-2\psi}]\ \frac{\partial H}{\partial x^m} \ , \end{aligned}$$

where

$$H = \tfrac{1}{2}\tilde{\gamma}^{mn}p_m p_n = \tfrac{1}{2}e^{2f}\gamma^{mn}p_m p_n = \tfrac{1}{2}p_0^2 \ .$$

If we define $d\tau = a^{-2}e^{-2\psi}ds$, then we get

$$\begin{aligned} \frac{dx^m}{d\tau} &= \frac{\partial H}{\partial p_m} \\ \frac{dp_m}{d\tau} &= -\frac{\partial H}{\partial x^m} \end{aligned}$$

The rate of decay of correlations (also called the rate of mixing) is proportional to

$$e^{-\chi\hat{\tau}}$$

where the Lyapunov exponent is

$$\chi \sim \sqrt{R(\hat{g})} = |p_0|^{-1}\sqrt{R(\tilde{\gamma})}$$

$$\hat{\tau} = p_0^2\tau = |p_0|\eta$$

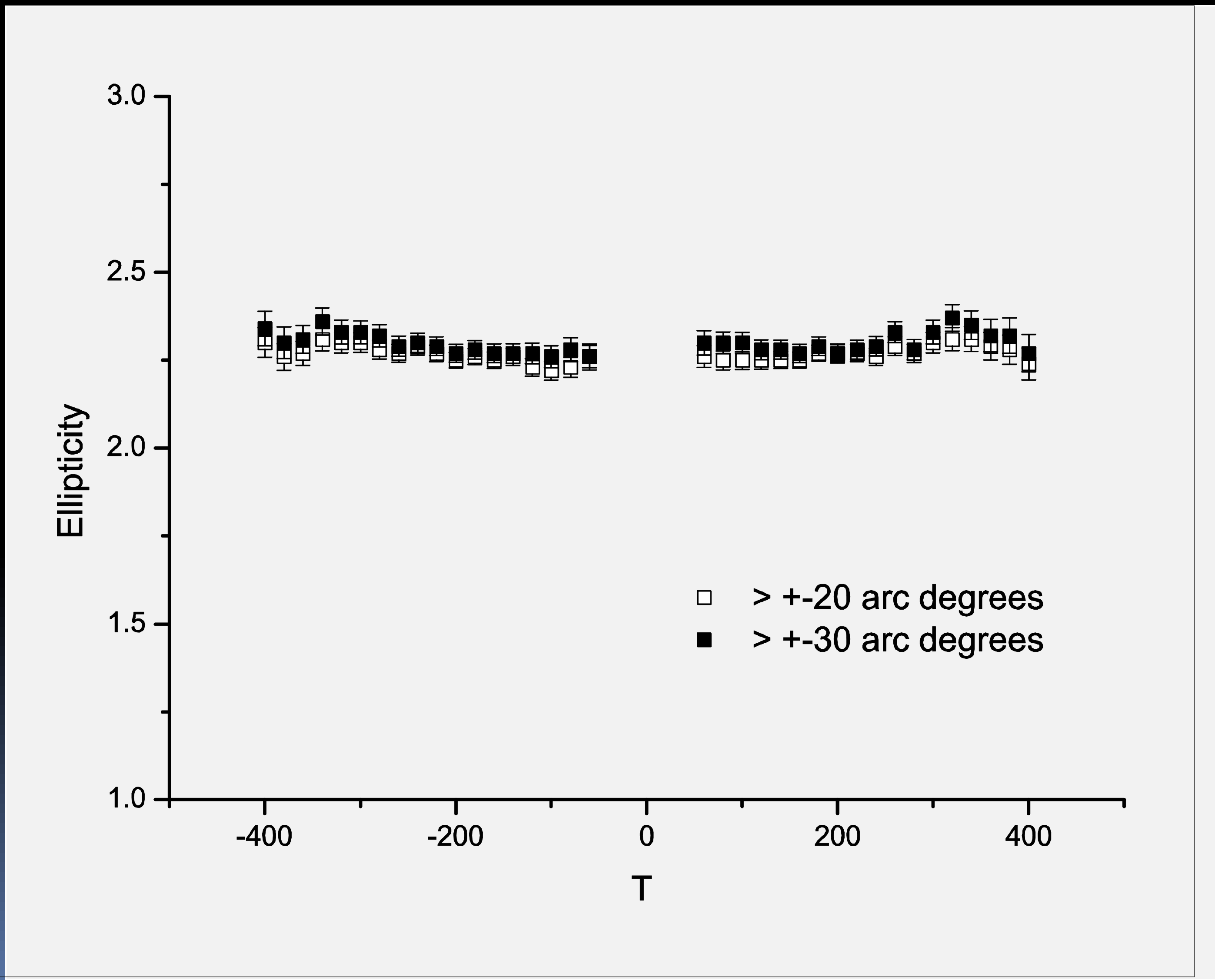
and

$$\chi\hat{\tau} \sim \eta\sqrt{R(\tilde{\gamma})}$$

i.e. the mixing rate does not depend on the energy of photons E=-p_0.

Relevant observable effects such as the ellipticity of excursion sets have to be independent on photon's energy, temperature.

Ellipticity of excursion sets, WMAP 94 GHz CMB map (Gurzadyan et al, PL A, 2007).



Perturbed Robertson-Walker metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -(1 + 2\phi)\ dt^2 + (1 - 2\phi)\ a^2(t)\ \gamma_{mn}(x) dx^m dx^n\ ,$$

where

$$\gamma_{mn} = \left(1 + \tfrac{k}{4}\left[(x^1)^2 + (x^2)^2 + (x^3)^2\right]\right)^{-2}\ \delta_{mn}$$

$$|\phi| \ll 1$$

$$\eta(t_0) - \eta(t) = \int_t^{t_0} \frac{d\tau}{a(\tau)}$$

$$1 = \Omega_k + \Omega_\Lambda + \Omega_m$$

$$\eta(t_0) - \eta(t) = (a_0 H_0)^{-1}\ \lambda(z, \Omega_\Lambda, \Omega_m)$$

$$\frac{d^2\ell}{d\lambda^2} + r\,\ell = 0\,,$$

where

$$\lambda(z, \Omega_\Lambda, \Omega_m) = \int_0^z \frac{d\xi}{\sqrt{\Omega_\Lambda + [1 - \Omega_\Lambda + \Omega_m \xi]\,(1+\xi)^2}}$$

and

$$r = -\Omega_k + 2\delta_0 \Omega_m\ .$$

$$\delta_0 \equiv \frac{\delta\rho_0}{\rho_0}$$

The underdense regions, the voids, have to contribute to the hyperbolicity, even if the Universe is globally flat or slightly positively curved.

Porosity criterion for hyperbolic voids

Adopting, for simplicity, periodicity in the line-of-sight distribution of voids, periodic, $\delta_0(\tau + \tau_\kappa + \tau_\omega) = \delta_0(\tau)$ and

$$\delta_0 = \begin{cases} +\kappa^2 & 0 < \tau < \tau_k\ , \\ -\omega^2 & \tau_\kappa < \tau < \tau_\kappa + \tau_\omega\ , \end{cases}$$

The solution of Jacobi equation is unstable if

$$\mu = \nu - 2 > 0, \text{ where}$$

$$\nu = \left| 2\cos(\omega\tau_\omega)\cosh(\kappa\tau_\kappa) + \left(\frac{\kappa}{\omega} - \frac{\omega}{\kappa}\right)\sin(\omega\tau_\omega)\sinh(\kappa\tau_\kappa)\right| .$$

the solutions are stable, if $\bar{\delta}_0 \leq 0$ and unstable, if $\bar{\delta}_0 > 0$, where

$$\bar{\delta}_0 \sim -\delta_{void} L_{void} - \delta_{wall} L_{wall};$$

$L_{void}, L_{wall}, \delta_{void}, \delta_{wall}$ being the distance scales and the density contrasts of the voids and the walls, respectively.

for the voids $\delta_{void} \simeq -1$, then the instability condition

$$p > \frac{\delta_{wall}}{\delta_{wall}+1},$$

Porosity

$$p = \frac{L_{void}}{L_{wall}+L_{void}} \leq 1$$

The Lyapunov exponent

$$\chi = \log\left(1 + \frac{\mu}{2} + \sqrt{\left(1+\frac{\mu}{2}\right)^2 - 1}\right)$$

The mixing rate for null geodesics

$$b \approx e^{\tau \sqrt{p - \delta_{wall}(1-p)}}.$$

The porosity depending on the ellipticity of excursion sets in CMB maps

$$p \approx \frac{1}{1+\delta_{wall}} \left[\delta_{wall} + \left(\frac{\log \epsilon}{2\tau} \right)^2 \right].$$

For Boomerang, WMAP maps

$$\epsilon \simeq 2.2 - 2.5$$

The porosity criterion for hyperbolicity is fulfilled for 30 Mpc voids

$$p \approx \frac{2}{3} + \frac{1}{12} \cdot \left(\frac{\log \epsilon}{\tau} \right)^2 > 0.7.$$

Cumulative!

For larger voids (100 Mpc ...) the situation is not clear.

Kolmogorov stochasticity parameter

(Kolmogorov 1933, Arnold 2008)

Let {X_1,X_2,…,X_n} be N independent values of the same real-valued random variable X ordered in increasing manner {X_1<X_2…} and the cumulative distribution function

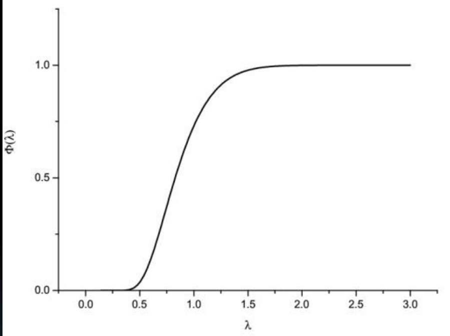
$$F(x) = P\{X \leq x\}$$

The empirical distribution function

$$F_n(x) = \begin{cases} 0, & x < X_1; \\ k/n, & X_k \leq x < X_{k+1}, \quad k = 1, 2, \ldots, n-1 \\ 1, & X_n \leq x. \end{cases}$$

Kolmogorov's stochasticity parameter

$$\lambda_n = \sqrt{n} \sup_x |F_n(x) - F(x)|.$$



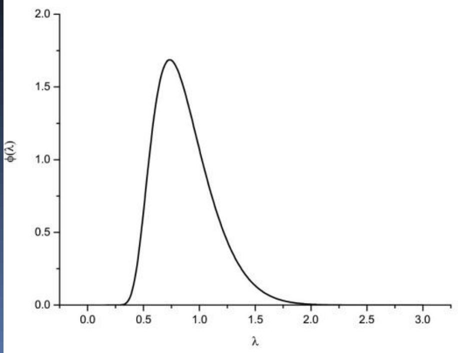
Kolmogorov (1933) proved that

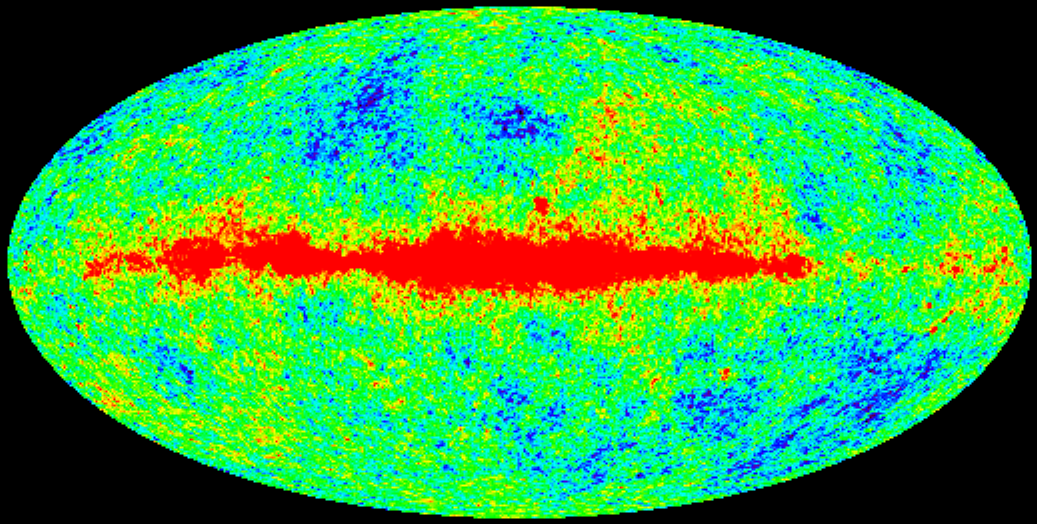
$$\lim_{n\to\infty} P\{\lambda_n \leq \lambda\} = \Phi(\lambda),$$

where $\Phi(0) = 0$,

$$\Phi(\lambda) = \sum_{k=-\infty}^{+\infty} (-1)^k \, e^{-2k^2\lambda^2}, \quad \lambda > 0,$$

independent on F.

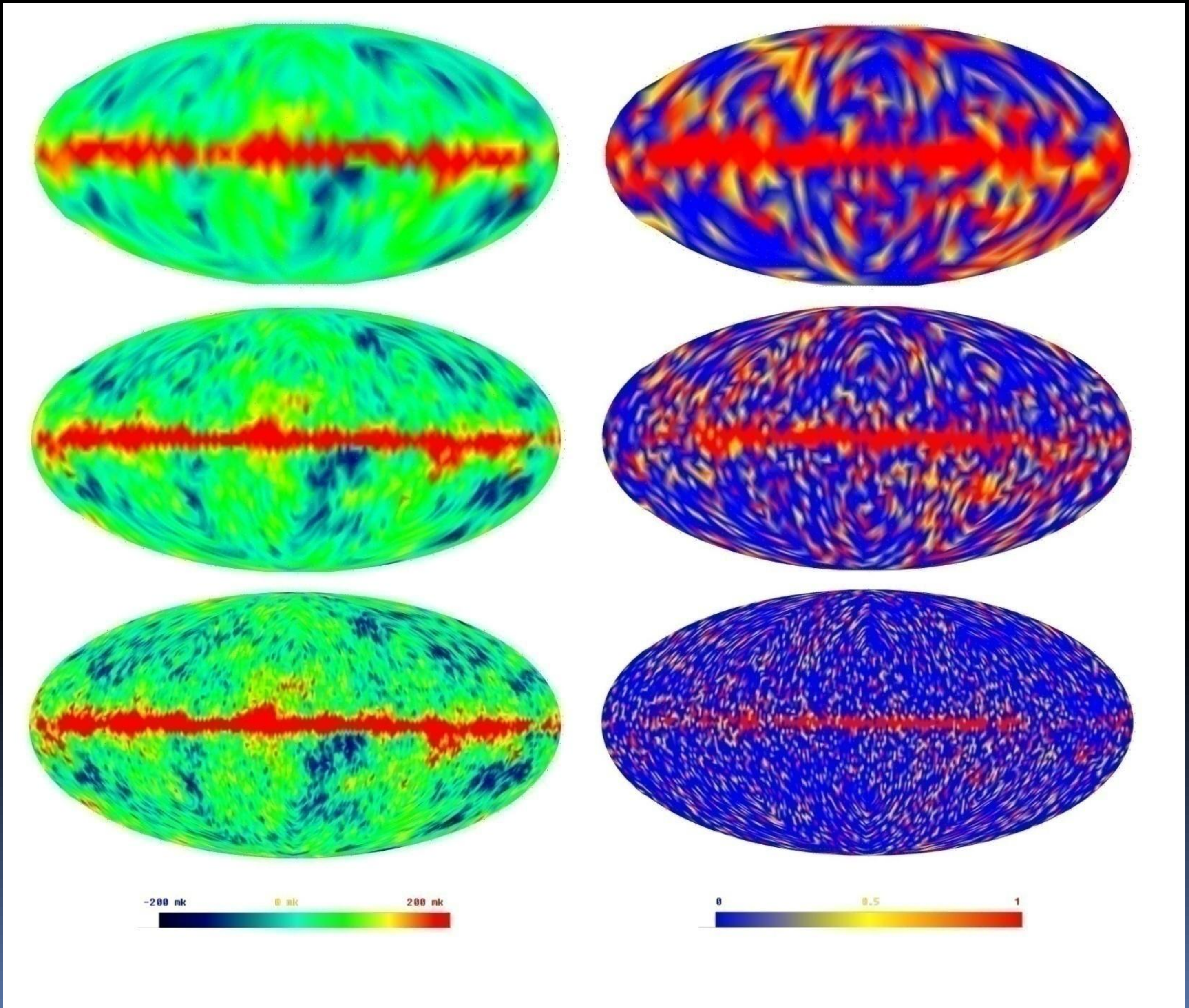




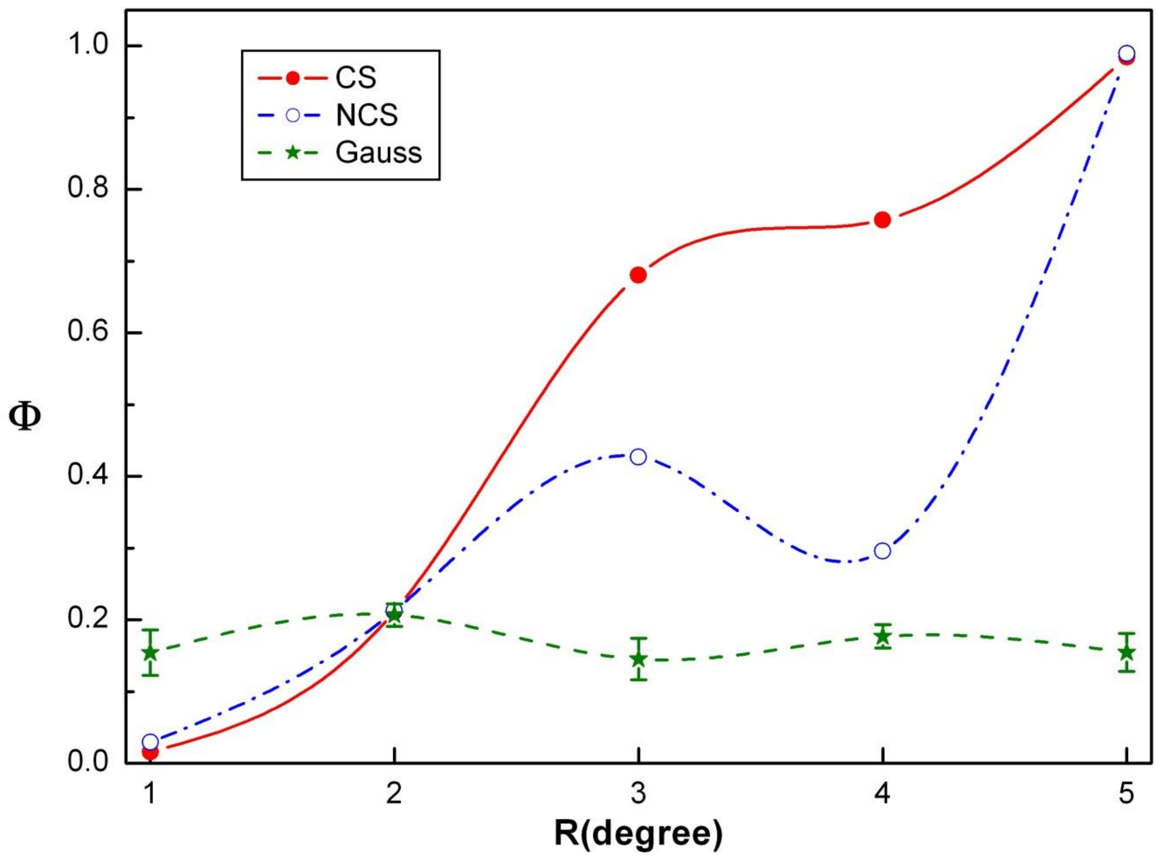
Source	Mean(Φ)	Var(Φ)
WMAP's data	0.353	0.07
Simulations	0.223	0.04
Cold Spot (W band)	0.749	0.00
Cold Spot (FR: W, Q, V bands)	0.859	0.01

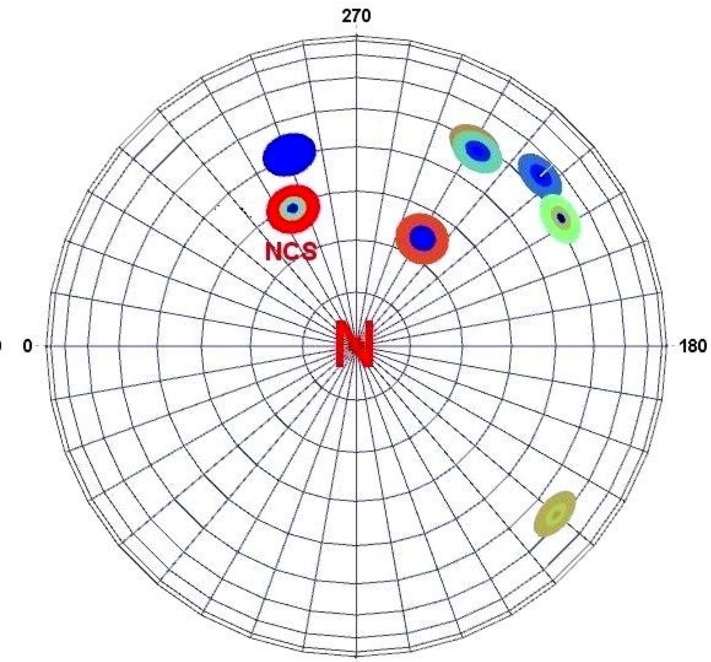
CMB sky is random with higher probability than the simulated one, i.e. there is an extra randomizing effect not included in the cosmological model.

Kolmogorov CMB map

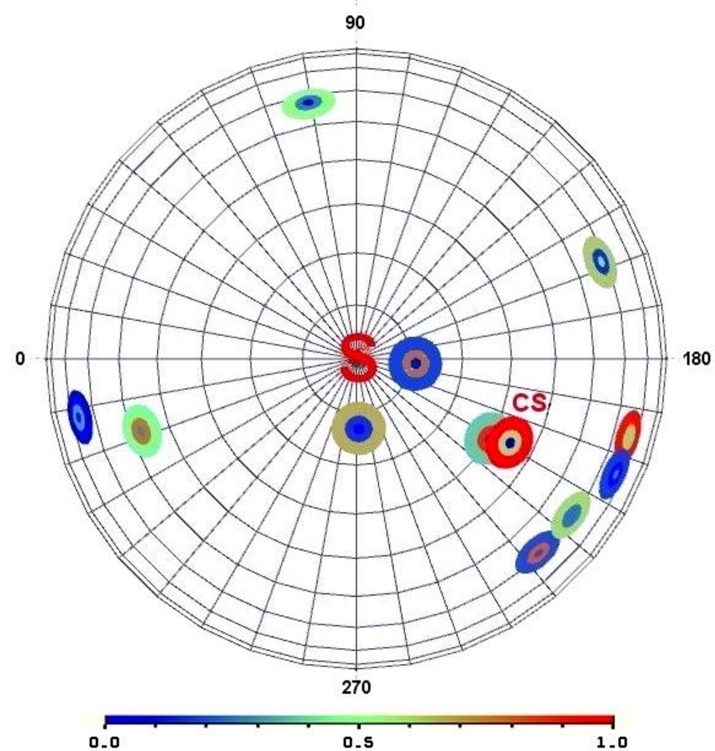


Hemisphere	Mean(Φ)	Var(Φ)
Northern	0.53	0.002
Southern	0.61	0.002

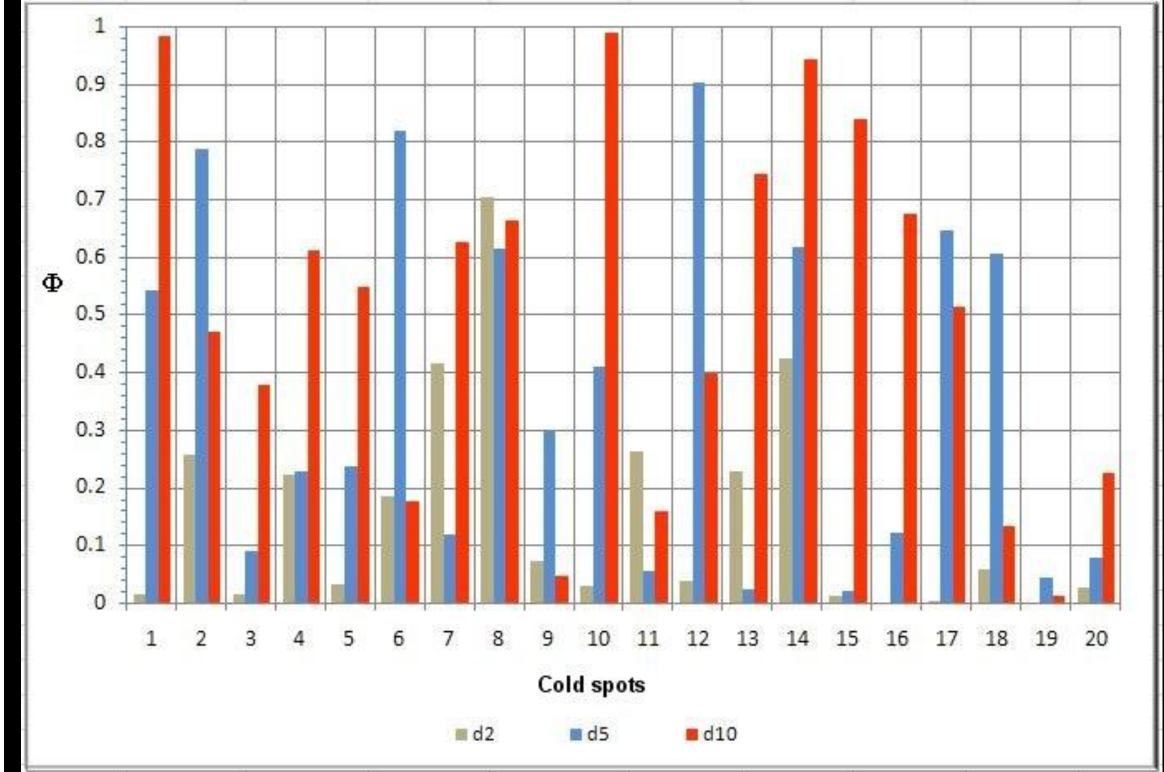




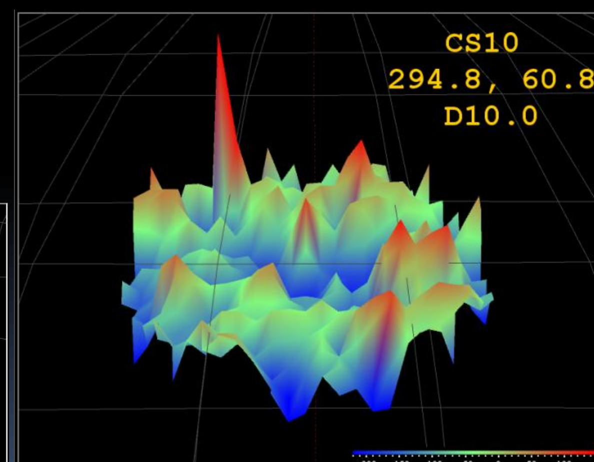
270
0
180
90
NCS
N



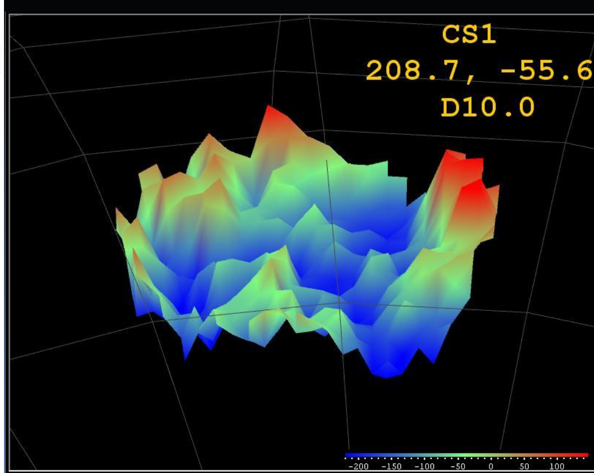
90
0
180
270
S
CS
0.0
0.5
1.0



Φ
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Cold spots
d2
d5
d10

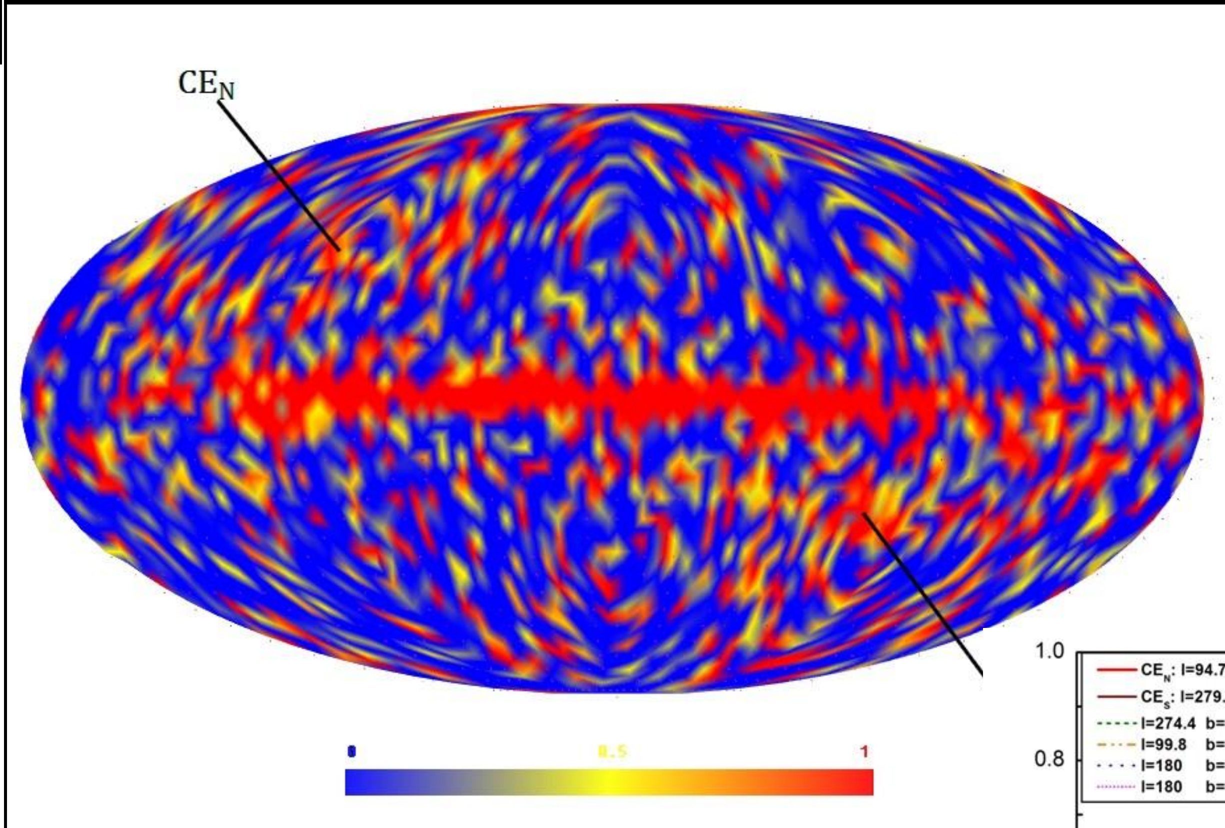


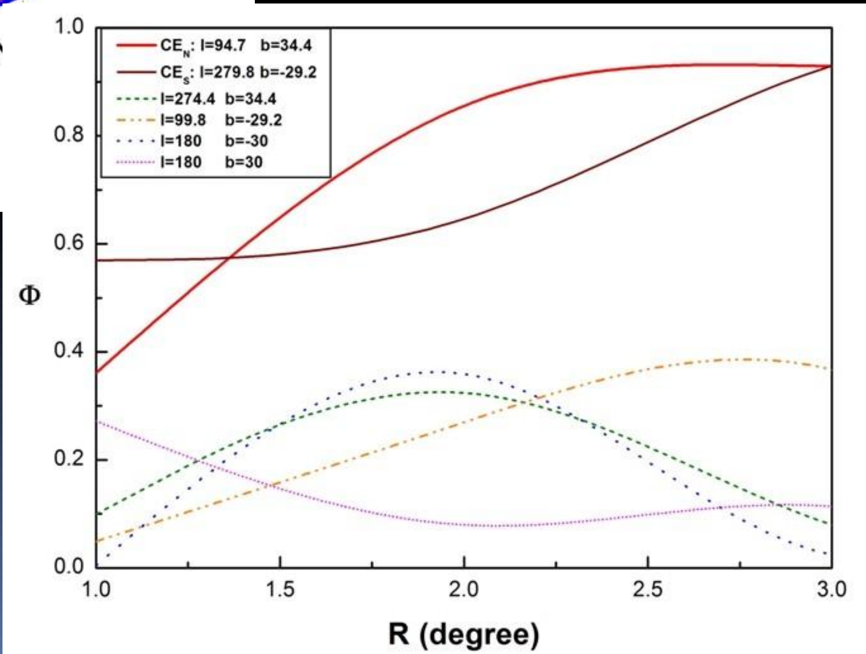
CS10
294.8, 60.8
D10.0



CS1
208.7, -55.6
D10.0

Plane mirror symmetry





V.G.Gurzadyan, A. A. Starobinsky, et al,
Astr. & Astrophys., 490 (2008) 929

Conclusions

CMB-filament link can be studied quantitatively using Kolmogorov statistic and the stochasticity parameter.

Kolmogorov maps reflect properties of variety of effects, cosmological and non-cosmological, open new possibilities in separating signals, by information power complementing the temperature and polarization maps.