# Cosmological Pistons

S. A. Fulling
Texas A&M University

K. Kirsten
Baylor University

SAF & KK, *Phys. Lett. B* **671** (2009) 179–180.

KK & SAF, *Phys. Rev. D* **79** (2009) 065019.

SAF & KK, *Phys. Lett. B* **671** (2009) 179–180.

KK & SAF, *Phys. Rev. D* **79** (2009) 065019.

Stimulated by

H. Cheng, *Phys. Lett. B* **668** (2008) 72.

Related observations by

A. Edery and V. Marachevsky, *JHEP* **12** (2008) 035;

L. P. Teo, *Phys. Lett. B* **672** (2009) 190–195.

# Real physics: The Casimir effect

Neutral metallic plates attract, because of the energy balance in the electromagnetic field's zero-point fluctuations

(or because of the van der Waals interaction at long range).

E.g.: Bressi et al., *Phys. Rev. Lett.* **88** (2002) 041804.

# Idealized physics

Scalar field $\phi$;

Dirichlet boundaries: $\phi = 0$ on the plates.

Extra Kaluza–Klein dimensions may exist.

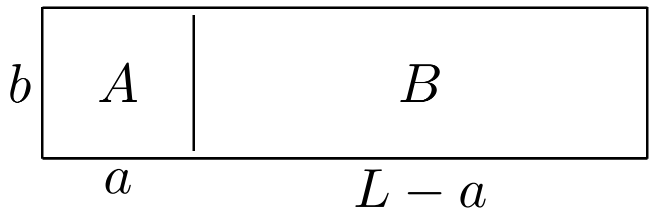
We do *not* discuss

- dark energy as vacuum energy of KK dimensions;
- vacuum force on dynamics of the KK geometry;
- attraction between Randall–Sundrum branes.

The plates are in the macroscopic dimensions.

# Classic derivation of the Casimir effect

(1960s: Fierz, Power, Boyer, . . . )




Enclosing the movable plate in a large, finite box makes all $a$-dependent energies finite. Take $b \to \infty$ and $L \to \infty$; the results are the same as in a naive calculation where transverse boundaries and third distant plate were never present. $F \propto -a^{-(d+1)}$.

Some people got into the habit of jumping straight to the infinite configuration without the precautionary cutoffs.

## Adding small dimensions

Cheng [*Phys. Lett. B* **643** (2006) 311] in this way calculated the Casimir force between 2D plates in 3 macroscopic dimensions and some microscopic Kaluza–Klein dimensions.

Cheng in 2006 found a *repulsive* force (asymptotically constant) at large separation, contrary to observation.

Two years later he redid the calculation as a "piston" problem. The repulsive force disappeared.

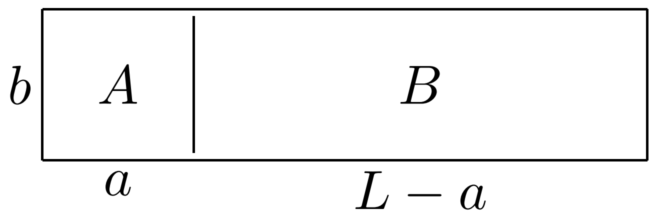
## Pistons

R. M. Cavalcanti, *Phys. Rev. D* **69** (2004) 065015.

M. P. Herzberg et al., *Phys. Rev. Lett.* **95** (2005) 250402; *Phys. Rev. D* **76** (2007) 045016.

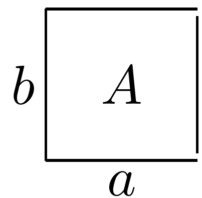
Motivation: Calculable model with no "renormalization".

Restore the transverse box and the distant third plate, and keep the transverse dimensions finite!




Energy inside the box $A$ includes that of Casimir attraction between top and bottom sides, $\propto a$ (constant outward force). This is *canceled* by its counterpart in the outer shaft $B$! (Reverses conclusion of Lukosz.)

One may question the relevance of this analysis to a Lukosz box without an attached shaft.

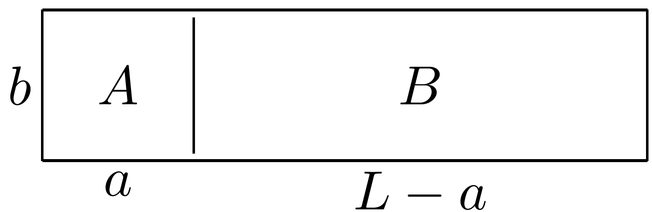



But the KK dimensions are unquestionably present outside the parallel plates if they are present inside! The naive calculation is wrong. KK dimensions contain negative vacuum energy and act as a piston of finite cross section. The canceling force from outside ("shaft") must not be ignored.

A piston model has two elements:

Finite transverse dimensions create the paradox of a spurious repulsive force. ($b =$ KK circumference now)

The third plate demarcates an outer chamber that removes that paradox.



## Other cosmological calculations

Hofmann et al., *Phys. Lett. B* **582** (2004) 1.

Frank et al., *Phys. Rev. D* **76** (2007) 015008;
*Phys. Rev. D* **78** (2008) 055014.

Pascoal et al., *Braz. J. Phys.* **38** (2008) 581.

They did *not* consider the outer chamber $B$, but they did *not* find Cheng's outward force. Why not?

They subtract the Casimir energy in $A$ (caused by the small compact dimensions) that would be there even if the plates were not present. The result is the same.

## Detailed calculations (summarized)

To compare with Cheng and others, we used zeta-function regularization:

$$E = \frac{1}{2} \sum_j {\omega_j}^{-2s} \quad \text{anal. cont. to } s = -\frac{1}{2}\,.$$

Ultraviolet cutoff (more physical) can be used instead.

"Because [analytic regularization] automatically subtracts all divergent contributions . . . , it also eliminates the possibility of understanding them in detail."

T. Konopka, *Phys. Rev. D* **79** (2009) 085012

# Energy between parallel plates alone

$M = [0, a] \times \mathbf{R}^2 \times N$. "Naive" calculation.

$\zeta_N(-2) = 0 \;\Rightarrow\;$ Lukosz repulsion (or attraction), asymptotically constant(!), can occur, depending on sign of $\zeta'_N(-2)$.

$\zeta_N(-2) \neq 0 \;\Rightarrow\;$ renormalization ambiguity (pole in zeta; logarithmic term in cutoff series).

## Parallel plates, with exterior contribution

Force is always well-defined and finite. (The energy is not, but the divergence is associated with the fixed surface area — that is the point of the piston model.)

The plate is always *attracted* to the nearest wall.

Force decays with distance, as it should:

$$F \propto a^{-4} + \text{exponential corrections.}$$

# Pistons with finite cross section

$M = ([0,a] \cup [a,\infty]) \times C \times N$. $(d \equiv \dim N)$

If both plates are Dirichlet (or both Neumann), the plate always goes to the nearest wall, regardless of geometry and topology of $C$ and $N$.

Large $a$ (compared to *all* other dimensions) $\Rightarrow$ $F \propto a^{-2}$.

Small $a$ $\Rightarrow$ $F \propto a^{-d-4}$, with higher-order corrections revealing geometry of $C \times N$ (through heat-kernel coefficients).

## Torus as KK manifold

$M = ([0,a] \cup [a,\infty]) \times \mathbf{R}^2 \times \mathbf{T}^d. \qquad \zeta_N(-2) = 0.$

Explicit formulas in terms of Epstein zeta functions and hence modified Bessel functions, etc.

## Finite temperature

It enhances the attraction to the nearest wall. (Teo)

## Conclusions

1. Like parallel plates always attract (if the transverse manifolds have no negative Laplacian eigenvalues).
2. Spurious repulsive forces arise from ignoring the effects of KK dimensions in the space *outside* the apparatus.
3. When $a$ is very small, the plates see the dimensions of all space, so the force is the Casimir force for the *full* dimension.
4. When $a$ is large, the plates do not see KK dimensions, so the force is the traditional Casimir force for

$([0,a]\cup[a,\infty])\times C$ (dimension $3+1$ or smaller, depending on size of $C$).

## Support

National Science Foundation Grants PHY-07857791 (Kirsten) and PHY-0554849 (Fulling)