Vacuum Fluctuations & Cosmology

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ICE/CSIC & IEEC, UAB, Barcelona

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Outline of this presentation

- Einstein's Cosmological Constant

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- On the Casimir Effect & the ζ Function Method

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- Fulling-Davies Theory (Dynamical CE)

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- Gravity Eqs as Eqs of State

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- Gravity Eqs as Eqs of State
- With THANKS to:
  G. Cognola, J. Haro, S.D. Odintsov, P.J. Silva, S. Zerbini, ...

Einstein's Cosmological Constant

Our universe seems to be spatially flat and to possess a non-vanishing cosmological constant

- For cosmologists and general relativists: a great mistake (Einstein)

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -(8\pi G/c^4) T_{\mu\nu} + \lambda g_{\mu\nu}$$

- For elementary particle physicists: a great embarrassment

  no way to get rid off (Coleman, Weinberg, Polchinski)

- The cc Λ is indeed a peculiar quantity
  - has to do with cosmology Einstein's eqs., FRW universe
  - has to do with the local structure of elementary particle physics stress-energy density μ of the vacuum

$$L_{cc} = \int d^4x \, \sqrt{-g} \, \mu^4 = \frac{1}{8\pi G} \int d^4x \, \sqrt{-g} \, \lambda$$

In other words: two contributions on the same footing (Zel'dovich, 68)

$$\frac{\Lambda \, c^2}{8\pi G} + \frac{1}{\text{Vol}} \frac{\hbar \, c}{2} \sum_i \omega_i$$

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QFT vacuum to vacuum transition: $\langle 0|H|0\rangle$

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Even then: Has the final value real sense ?

Existence of ζ_A for A a ΨDO

1. A a positive-definite elliptic ΨDO of positive order $m \in \mathbb{R}^+$
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3. E, n-dim vector bundle over
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$$\zeta_A(s) = \text{tr}\, A^{-s} = \sum_j \lambda_j^{-s}, \quad \text{Re}\, s > \frac{n}{m} := s_0$$

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(b) $\zeta_A(s)$ has a meromorphic continuation to the whole complex plane $\mathbb{C}$ (regular at $s = 0$), provided the principal symbol of A, $a_m(x, \xi)$, admits a spectral cut: $L_\theta = \{\lambda \in \mathbb{C}; \mathrm{Arg}\, \lambda = \theta, \theta_1 < \theta < \theta_2\}$, $\mathrm{Spec}\, A \cap L_\theta = \emptyset$ (the Agmon-Nirenberg condition)

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(c) The definition of $\zeta_A(s)$ depends on the position of the cut L_θ

(d) The only possible singularities of $\zeta_A(s)$ are poles at

$$s_j = (n - j)/m, \qquad j = 0, 1, 2, \ldots, n-1, n+1, \ldots$$

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As Mellin transform: $\zeta_H(s) = \frac{1}{\Gamma(s)} \int_0^{\infty} dt\, t^{s-1}\, \mathrm{tr}\, e^{-tH}$, Re$s > s_0$

Derivative: $\quad \zeta'_H(0) = -\sum_{i \in I} \ln \lambda_i$

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C. Soulé et al, Lectures on Arakelov Geometry, CUP 1992; A. Voros,...

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- Asymptotic expansion for the heat kernel:

$$\mathrm{tr}\, e^{-tA} = \sum\nolimits'_{\lambda \in \mathrm{Spec}\, A} e^{-t\lambda}$$

$$\sim \alpha_n(A) + \sum\nolimits_{n \neq j \geq 0} \alpha_j(A) t^{-s_j} + \sum\nolimits_{k \geq 1} \beta_k(A) t^k \ln t, \quad t \downarrow 0$$

$$\alpha_n(A) = \zeta_A(0), \quad \alpha_j(A) = \Gamma(s_j)\, \mathrm{Res}_{s=s_j}\, \zeta_A(s), \; s_j \notin -\mathbb{N}$$

$$\alpha_j(A) = \frac{(-1)^k}{k!} \left[\mathrm{PP}\, \zeta_A(-k) + \psi(k+1)\, \mathrm{Res}_{s=-k}\, \zeta_A(s)\right], \qquad s_j = -k, \; k \in \mathbb{N}$$

$$\beta_k(A) = \frac{(-1)^{k+1}}{k!} \mathrm{Res}_{s=-k}\, \zeta_A(s), \quad k \in \mathbb{N} \backslash \{0\}$$

$$\mathrm{PP}\, \phi := \lim_{s \to p} \left[\phi(s) - \frac{\mathrm{Res}_{s=p}\, \phi(s)}{s-p}\right]$$

The Chowla-Selberg Expansion Formula: Basics

- Jacobi's identity for the $\theta-$function

$$\theta_3(z,\tau) := 1 + 2\sum_{n=1}^\infty q^{n^2}\cos(2nz), \qquad q := e^{i\pi\tau},\ \tau\in\mathbb{C}$$

$$\theta_3(z,\tau) = \frac{1}{\sqrt{-i\tau}}\, e^{z^2/i\pi\tau}\,\theta_3\left(\frac{z}{\tau}\Big|\frac{-1}{\tau}\right)$$

equivalently:

$$\sum_{n=-\infty}^{\infty} e^{-(n+z)^2 t} = \sqrt{\frac{\pi}{t}}\sum_{n=0}^{\infty} e^{-\frac{\pi^2 n^2}{t}}\cos(2\pi n z), \quad z,t\in\mathbb{C},\ \text{Re}\, t>0$$

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- Higher dimensions: Poisson summ formula (Riemann)

$$\sum_{\vec{n}\in\mathbb{Z}^p} f(\vec{n}) = \sum_{\vec{m}\in\mathbb{Z}^p}\widetilde{f}(\vec{m})$$

$\widetilde{f}$ Fourier transform

[Gelbart + Miller, BAMS '03, Iwaniec, Morgan, ICM '06]

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- Truncated sums $\longrightarrow$ asymptotic series

Extended CS Formulas (ECS)

- Consider the zeta function (Re$s > p/2, A > 0,$ Re$q > 0$)

$$\zeta_{A,\vec{c},q}(s) = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[\frac{1}{2}\left(\vec{n}+\vec{c}\right)^T A\left(\vec{n}+\vec{c}\right)+q\right]^{-s} = \sum_{\vec{n}\in\mathbb{Z}^p}{}' \left[Q\left(\vec{n}+\vec{c}\right)+q\right]^{-s}$$

prime: point $\vec{n}=\vec{0}$ to be excluded from the sum
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$$\times \sum_{\vec{m}\in\mathbb{Z}^p_{1/2}}{}' \cos(2\pi\vec{m}\cdot\vec{c}) \left(\vec{m}^T A^{-1}\vec{m}\right)^{s/2-p/4} K_{p/2-s}\left(2\pi\sqrt{2q\,\vec{m}^T A^{-1}\vec{m}}\right)$$

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[ECS1]

- Pole: $s=p/2$ Residue:

$$\text{Res}_{s=p/2}\,\zeta_{A,\vec{c},q}(s) = \frac{(2\pi)^{p/2}}{\Gamma(p/2)}\left(\det A\right)^{-1/2}$$

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- K_ν modified Bessel function of the second kind and the subindex 1/2 in $\mathbb{Z}^p_{1/2}$ means that only half of the vectors $\vec{m} \in \mathbb{Z}^p$ participate in the sum. E.g., if we take an $\vec{m} \in \mathbb{Z}^p$ we must then exclude $-\vec{m}$ [simple criterion: one may select those vectors in $\mathbb{Z}^p \backslash \{\vec{0}\}$ whose first non-zero component is positive]

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- Case $c_1 = \cdots = c_p = q = 0$ [true extens of CS, diag subcase]

$$\zeta_{A_p}(s) = \frac{2^{1+s}}{\Gamma(s)} \sum_{j=0}^{p-1} (\det A_j)^{-1/2} \left[\pi^{j/2} a_{p-j}^{j/2-s}\, \Gamma\left(s - \frac{j}{2}\right) \zeta_R(2s-j) + \right.$$

$$\left. 4\pi^s a_{p-j}^{\frac{j}{4}-\frac{s}{2}} \sum_{n=1}^{\infty} \sum_{\vec{m}_j \in \mathbb{Z}^j}{}' n^{j/2-s} \left(\vec{m}_j^t A_j^{-1} \vec{m}_j\right)^{s/2-j/4} K_{j/2-s}\left(2\pi n \sqrt{a_{p-j} \vec{m}_j^t A_j^{-1} \vec{m}_j}\right) \right]$$

[ECS3d]

The Casimir Effect

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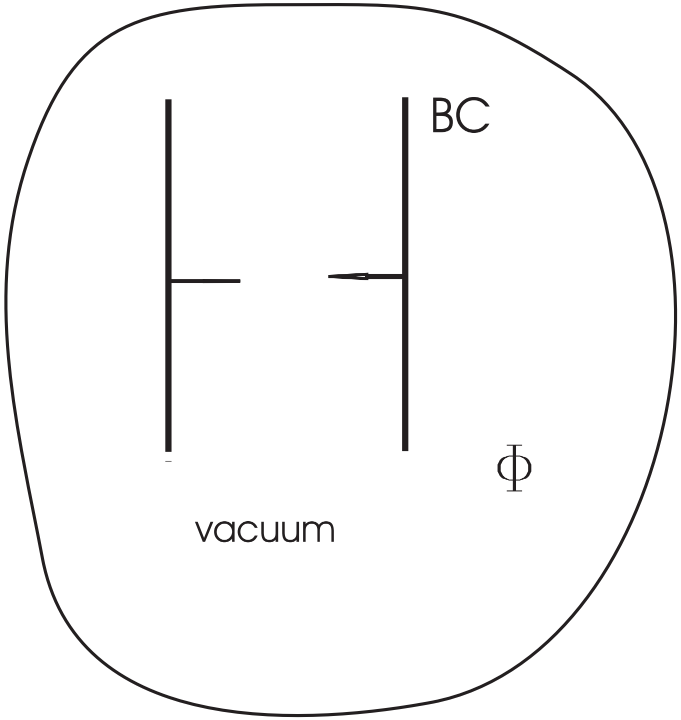
BC e.g. periodic



Casimir Effect

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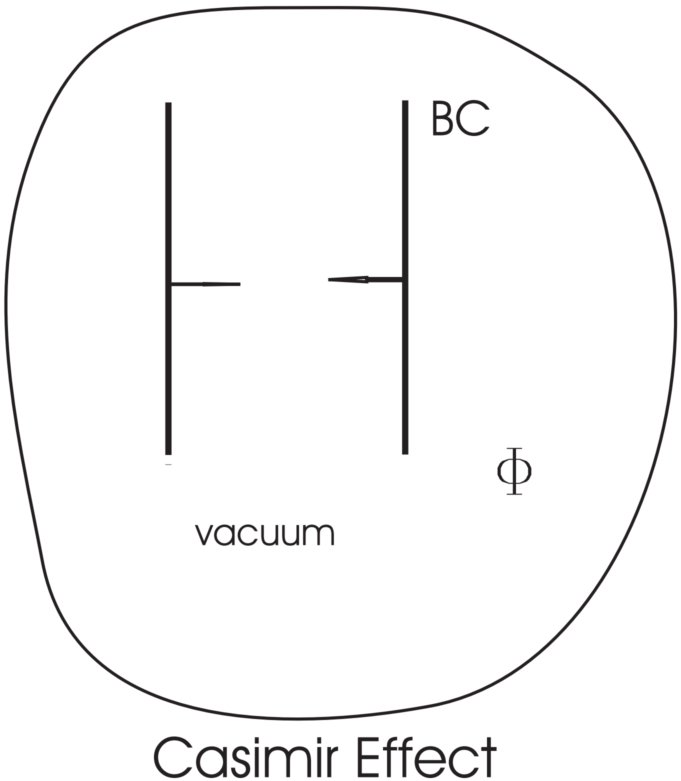

BC e.g. periodic

$\Longrightarrow$ all kind of fields

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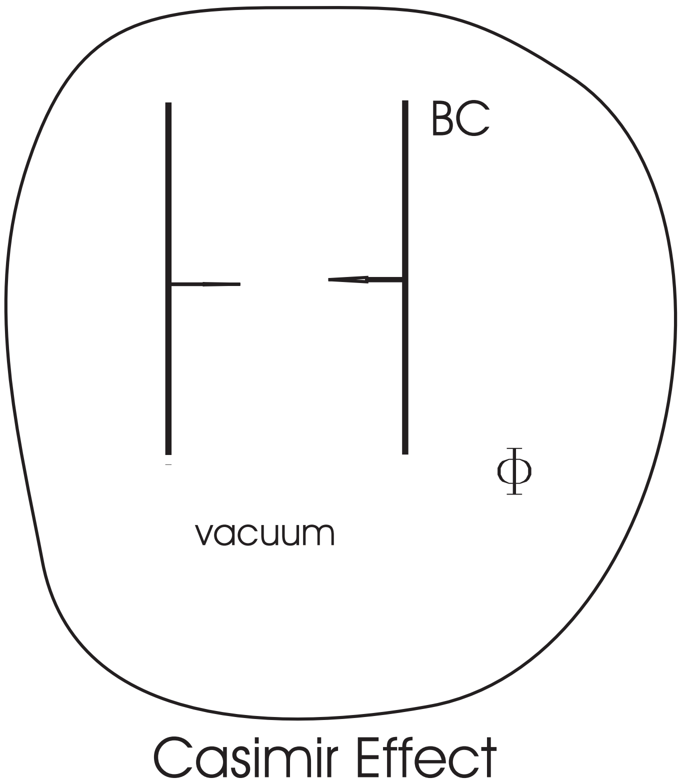

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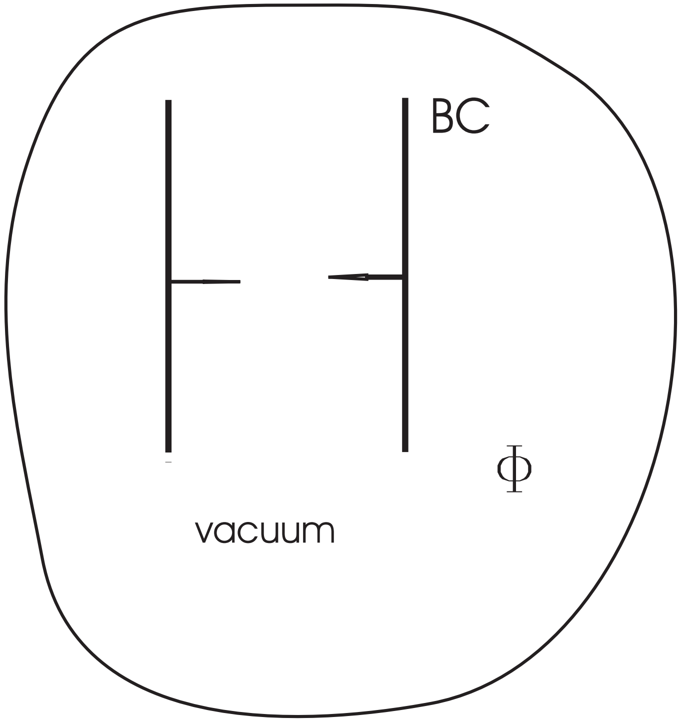

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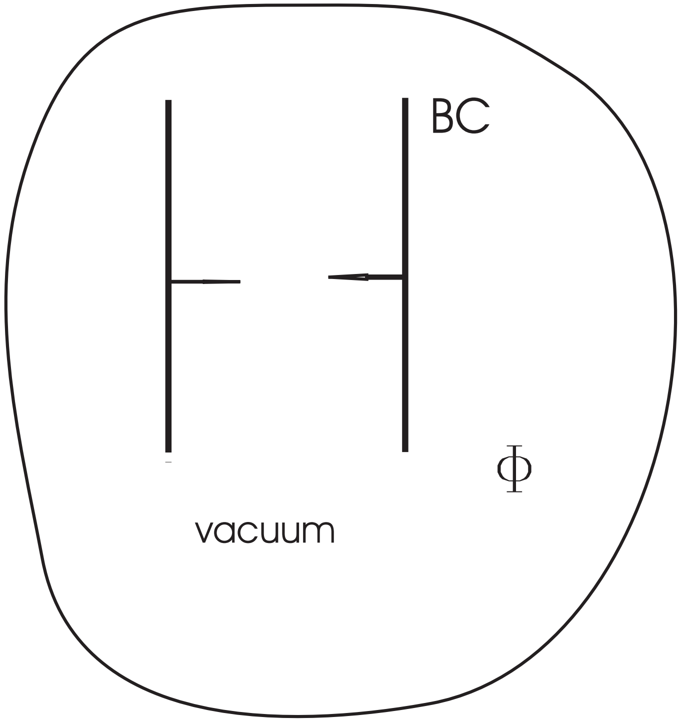
Universal process:

The Casimir Effect



Casimir Effect

BC e.g. periodic
$\Longrightarrow$ all kind of fields
$\Longrightarrow$ curvature or topology

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- Cond. matter (wetting ^{3}He alc.)
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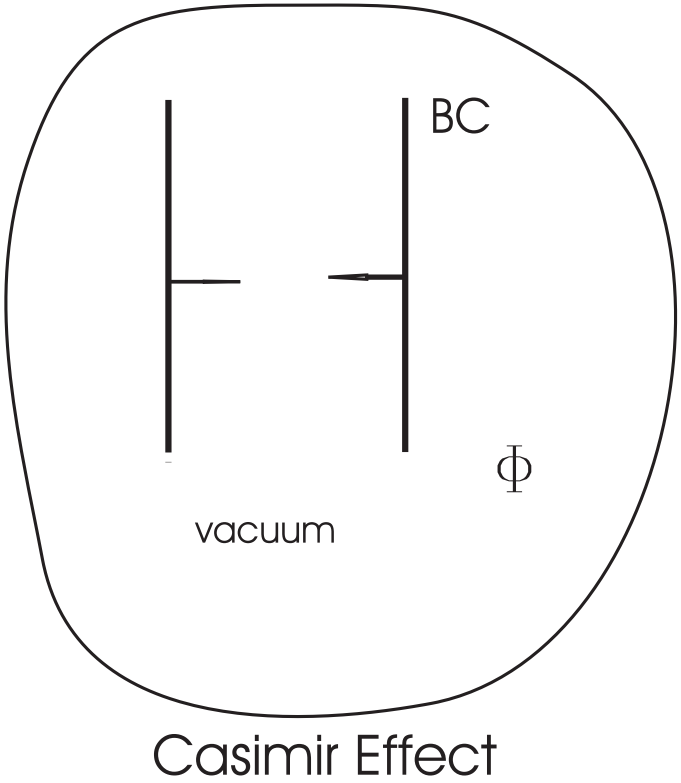

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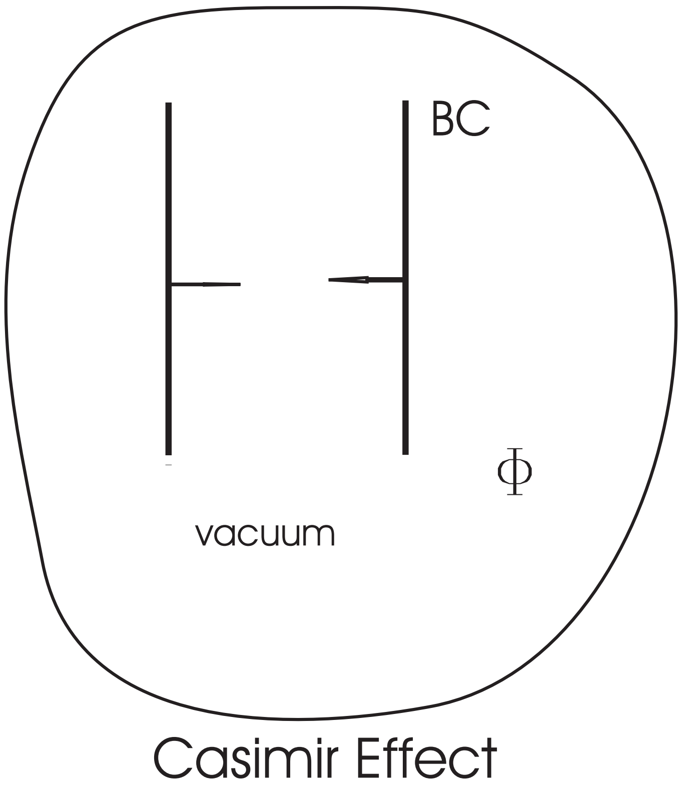

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- Dynamical CE $\Leftarrow$
- Lateral CE
- Extract energy from vacuum
- CE and the cosmological constant $\Leftarrow$

The standard approach

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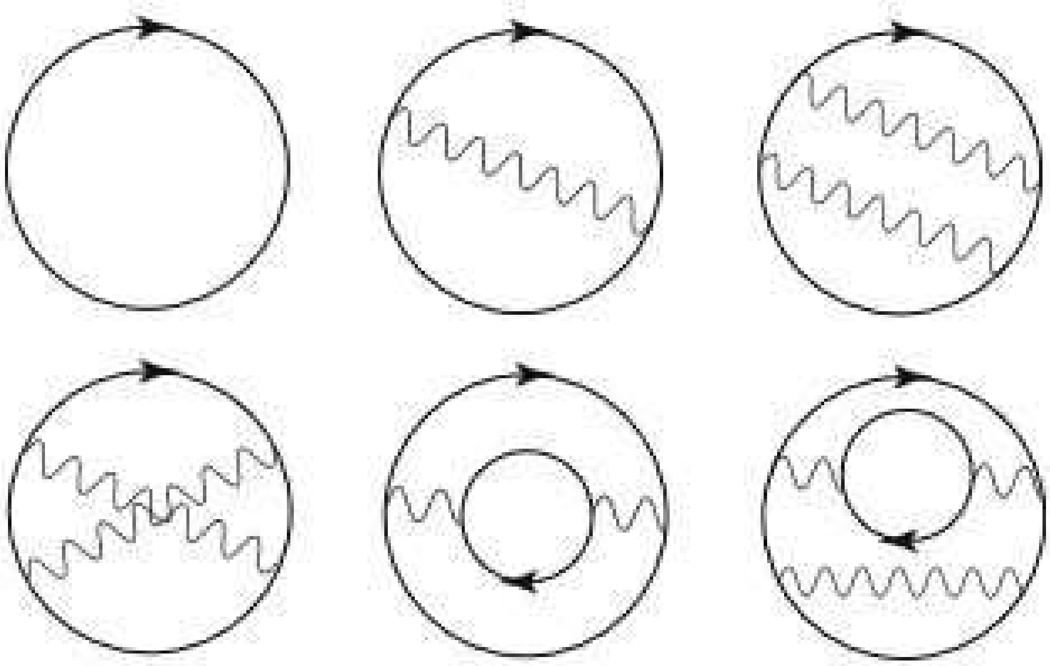

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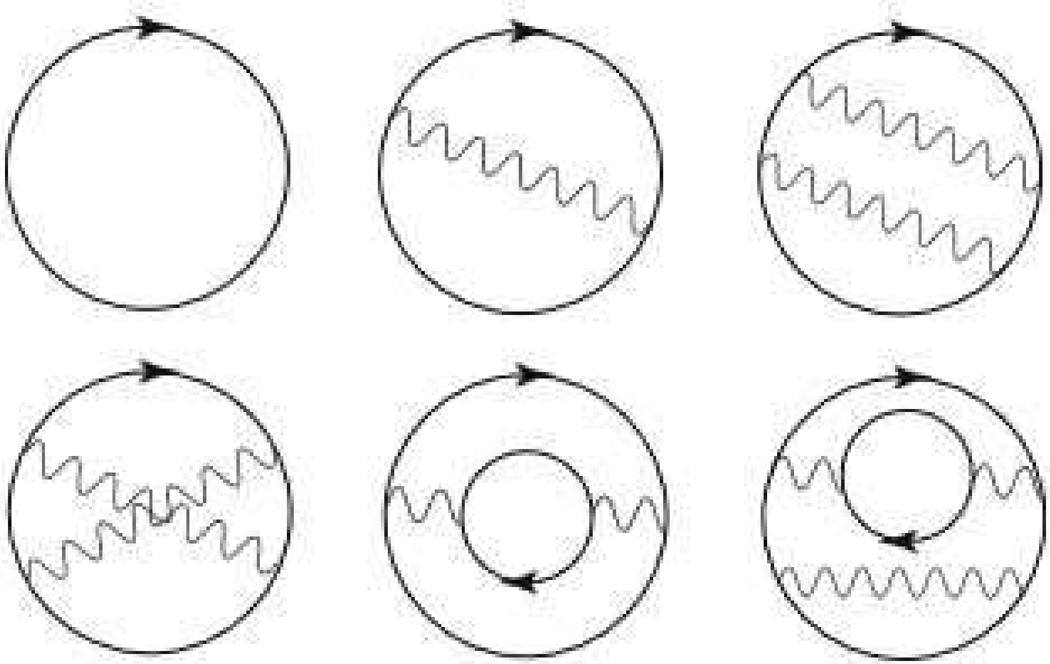

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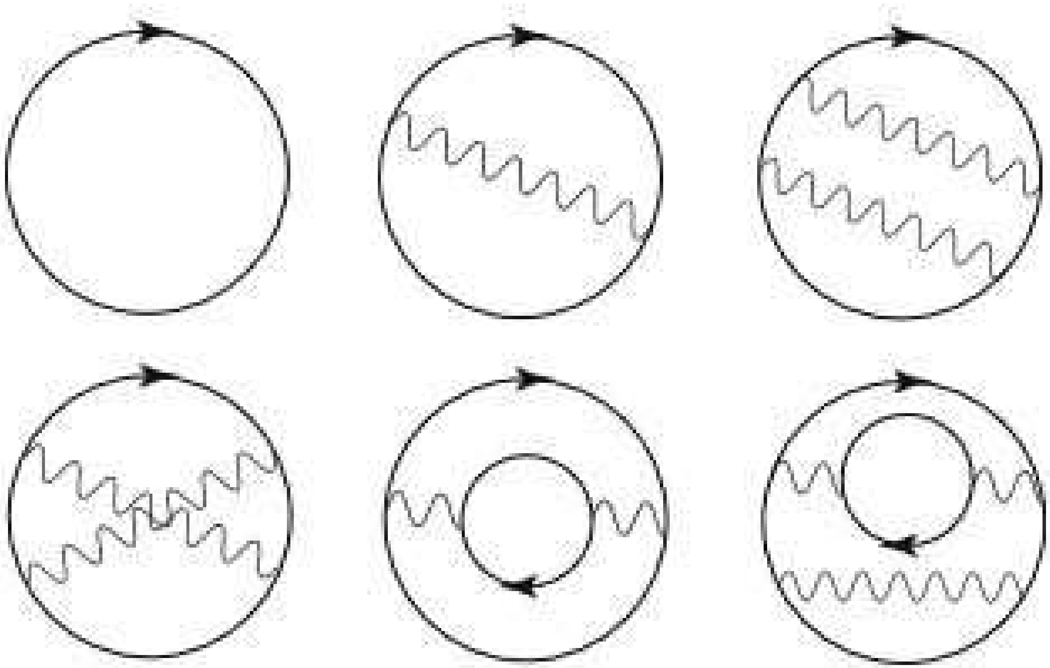

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In modern language the Casimir energy can be expressed in terms of the trace of the Greens function for the fluctuating field in the background of interest (conducting plates)

$$\mathcal{E} = \frac{\hbar}{2\pi}\mathrm{Im}\int d\omega\omega\ \mathrm{Tr}\int d^3x\,[\mathcal{G}(x,x,\omega+i\epsilon) - \mathcal{G}_0(x,x,\omega+i\epsilon)]$$

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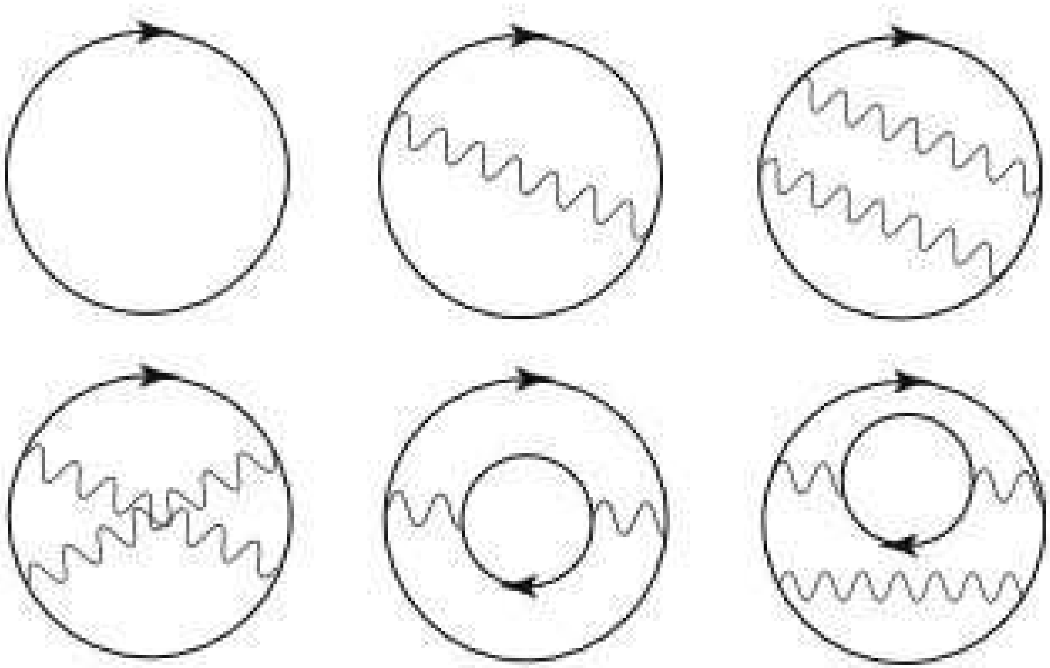

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$\mathcal{G}$ full Greens function for the fluctuating field

$\mathcal{G}_0$ free Greens function Trace is over spin

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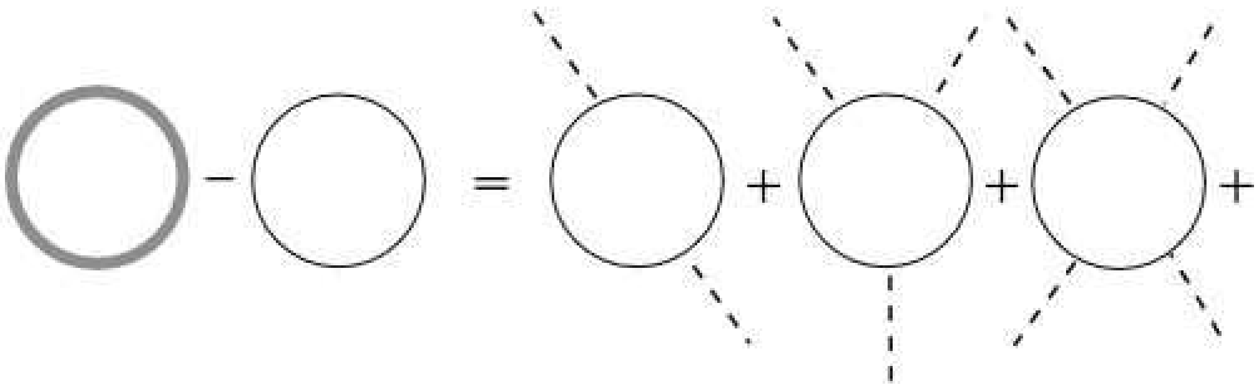

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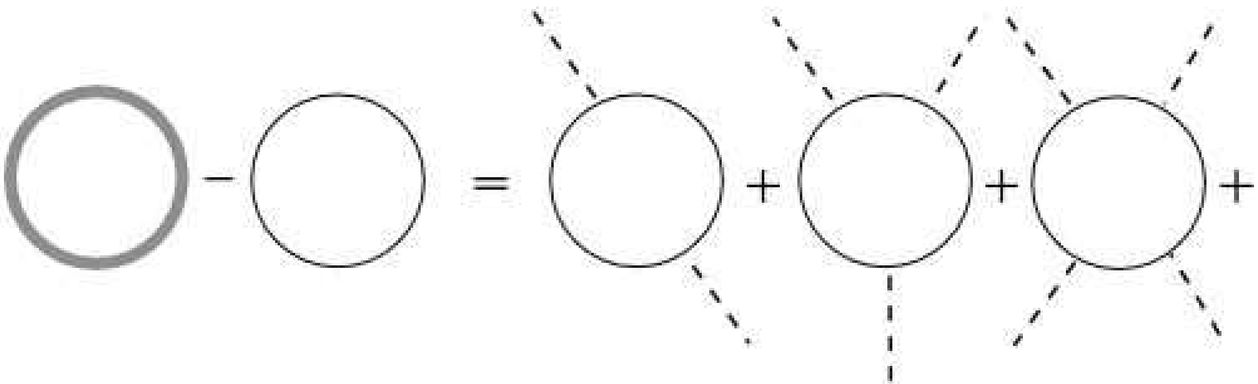

$\Longrightarrow$ "Experimental confirmation of the Casimir effect does not establish the reality of zero point fluctuations" [R. Jaffe et. al.]

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- The dissipative part we obtain agrees with the other methods. But those have problems with the reactive part, which in general yields a non-positive energy $\Longrightarrow$ EXPERIMENT

SOME DETAILS OF THE METHOD

- Hamiltonian method for neutral Klein-Gordon field in a cavity Ω_t, with boundaries moving at a certain speed $v << c$, $\epsilon = v/c$

(of order 10^{-8} in Kim, Brownell, Onofrio, PRL 96 (2006) 200402)

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- Hamiltonian. Transform moving boundary into fixed one by (non-conformal) change of coordinates

$$\mathcal{R} : (\bar{t}, \mathbf{y}) \to (t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$$

transform Ω_t into a fixed domain $\widetilde{\Omega}$

$\widetilde{\Omega}$: $(t(\bar{t}, \mathbf{y}), \mathbf{x}(\bar{t}, \mathbf{y})) = \mathcal{R}(\bar{t}, \mathbf{y}) = (\bar{t}, \mathbf{R}(\bar{t}, \mathbf{y}))$

(with $\bar{t}$ the new time)

CASE OF A SINGLE, PARTIALLY TRANSMITTING MIRROR

Case of a single, partially transmitting mirror

Seminal Davis-Fulling model [PRSL A348 (1976) 393]

renormalized energy negative while the mirror moves:

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$\rightarrow$ Real in the temporal domain: $S(-\omega) = S^*(\omega)$

$\rightarrow$ Causal: $S(\omega)$ is analytic for Im $(\omega) > 0$

$\rightarrow$ Unitary: $S(\omega)S^\dagger(\omega) =$ Id

$\rightarrow$ The identity at high frequencies: $S(\omega) \rightarrow$ Id, when $|\omega| \rightarrow \infty$

$s(\omega)$ and $r(\omega)$ meromorphic (cut-off) functions

(material's permitivity and resistivity)

Results are rewarding:

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$$\langle \hat{F}_{Ha}(t)\rangle = -\frac{\epsilon}{2\pi^2}\int_0^\infty\int_0^\infty \frac{d\omega d\omega'\omega\omega'}{\omega+\omega'}\,\text{Re}\left[e^{-i(\omega+\omega')t}\,\widehat{\dot{g}\theta_t}(\omega+\omega')\right]$$
$$\times\left[|r(\omega)+r^*(\omega')|^2+|s(\omega)-s^*(\omega')|^2\right]+\mathcal{O}(\epsilon^2)$$

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$\Longrightarrow$ Two mirrors; higher dimensions; fields of any kind

Quantum Vacuum Fluct's & the CC

- The main issue: S.A. Fulling et. al., hep-th/070209v2

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- Idea: zero point fluctuations can contribute to the cosmological constant Ya.B. Zeldovich '68

CC PROBLEM

- Relativistic field: collection of harmonic oscill's (scalar field)

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- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

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- Very difficult to solve and we do not address this question directly [Baum, Hawking, Coleman, Polchinsky, Weinberg,...]

- What we do consider —with relative success in some different approaches— is the additional contribution to the cc coming from the non-trivial topology of space or from specific boundary conditions imposed on braneworld models:

$\Longrightarrow$ kind of cosmological Casimir effect

Cosmolog Imprint of the Casimir Eff't?

- Assuming one will be able to prove (in the future) that the ground value of the cc is zero (as many had suspected until recently), we will be left with this incremental value coming from the topology or BCs

  * L. Parker & A. Raval, VCDM, vacuum energy density
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  - (b) dS & AdS worldbranes
  - (c) supergraviton theories (discret dims, deconstr)

The Braneworld Case

1. Braneworld may help to solve:
   - the hierarchy problem
   - the cosmological constant problem
2. Presumably, the bulk Casimir effect will play a role in the construction (radion stabilization) of braneworlds

- Bulk Casimir effect (effective potential) for a conformal or massive scalar field
- Bulk is a 5-dim AdS or dS space with 2/1 4-dim dS brane (our universe)
- Consistent with observational data even for relatively large extra dimension

Previous work: $\longrightarrow$ flat space brane
$\longrightarrow$ bulk conformal scalar field
$\longrightarrow$ conclusion: no CE

We used zeta regularization at full power, with positive results!

EE, S Nojiri, SD Odintsov, S Ogushi, Phys Rev D67 (2003) 063515 *Casimir effect in de Sitter and Anti-de Sitter braneworlds* EE, SD Odintsov, AA Saharian 0902.0717 *Repulsive Casimir effect from extra dimensions and Robin BC: from branes to pistons*

Casimir eff in brworl's w large extra dim

- Casimir energy for massive scalar field with an arbitrary curvature coupling, obeying Robin BCs on two codim-1 parallel plates embedded in background spacetime $R^{(D_1-1,1)} \times \Sigma$, Σ compact internal space

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  $\longrightarrow$ needed for conformally invariant theories with boundaries, to preserve this invariance
- Quantum scalar field with Robin BCs on boundary of cavity violates Bekenstein's entropy-to-energy bound near certain points in the space of the parameter defining the boundary condition [Solodukhin 01]

- Robin BCs can model the finite penetration of the field through the boundary: the ‘skin-depth’ param related to Robin coefficient [Mostep ea 85,Lebedev 01] Casimir forces between the boundary planes of films [Schmidt ea 08]

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For arbitrary internal space, interaction part of the Casimir energy given by

$$\Delta E_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2}}{\Gamma(D_1/2)} \sum_\beta \int_{m_\beta}^\infty dx\, x(x^2 - m_\beta^2)^{D_1/2-1}$$

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For Dirichlet and Neumann BCs on both plates this leads to

$$\Delta E_{[a_1,a_2]}^{(\mathrm{J},\mathrm{J})} = -\frac{2a^{-D_1}}{(8\pi)^{(D_1+1)/2}} \sum_\beta \sum_{n=1}^\infty \frac{f_{(D_1+1)/2}(2nam_\beta)}{n^{D_1+1}}$$

with $f_\nu(z) = z^\nu K_\nu(z)$ $\longrightarrow$ energy always negative

For Dirichlet BC on one plate and Neumann on the other, the interaction component of the vacuum energy is

$$\Delta E^{(\mathrm{D,N})}_{[a_1,a_2]} = \frac{(4\pi)^{-D_1/2} a}{\Gamma(D_1/2+1)} \sum_{\beta} \int_{m_\beta}^{\infty} dx \, \frac{(x^2 - m_\beta^2)^{D_1/2}}{e^{2ax}+1}$$

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In the case of a conformally coupled massless field on the background of a spacetime conformally related to the one described by the line element

$$ds^2 = g_{MN} dx^M dx^N = \eta_{\mu\nu} dx^\mu dx^\nu - \gamma_{il} dX^i dX^l$$

$\eta_{\mu\nu} = \mathrm{diag}(1, -1, \ldots, -1)$ metric of (D_1+1)-dim Minkowski st and X^i coordinates of Σ, with the conformal factor $\Omega^2(x^{D_1})$. Interaction part of Casimir energy is given (*), with coeffs β_j related to coeffs of the Robin BCs

$$(1 + \overline{\beta}_j n^M \nabla_M)\overline{\varphi}(x) = [1 + (-1)^{j-1}\Omega_j^{-1}\overline{\beta}_j \partial_{D_1}]\overline{\varphi}(x) = 0, \; \Omega_j = \Omega(x_j^{D_1})$$

& conformal factor $\beta_j = \left[\Omega_j + (-1)^j \frac{D-1}{2\Omega_j}\overline{\beta}_j \Omega_j'\right]^{-1} \overline{\beta}_j, \; \Omega_j' = \Omega_j'(x_j^{D_1})$

In Randall-Sundrum 2-brane model with compact internal space, the Robin coefficients are $\overline{\beta}_j^{-1} = (-1)^j c_j/2 - 2D\zeta/r_D$, c_1, c_2 mass parameters in the surface action of the scalar field for the left and right branes, respectively

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We have considered a piston-like geometry, introducing a third plate (then this plate is sent to infinity) Casimir force

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With independence of the geometry of the internal space, the force is attractive for Dirichlet or Neumann boundary conditions on both plates

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$$= \frac{2a^{-D_1-1}}{(8\pi)^{(D_1+1)/2} V_\Sigma} \sum_\beta \sum_{n=1}^{\infty} \frac{1}{n^{D_1+1}} \left[f_{(D_1+1)/2}(2nam_\beta) - f_{(D_1+3)/2}(2nam_\beta) \right]$$

$\mathrm{J} = \mathrm{D}, \mathrm{N}$, and repulsive for Dirichlet BC on one plate and Neumann on the other, a monotonic function of the distance

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Interesting remark: this property could be used in the proposal of a Casimir experiment with the purpose to carry out an explicit detailed observation of 'large' extra dimensions as allowed by some models of particle physics

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- The cosmological constant as an “integration constant”
  T. Padmanabhan; D. Blas, J. Garriga, E. Alvarez ...
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- C. Eling, R. Guedens, T. Jacobson [PRL2006]: extension to polynomial $f(R)$ gravity but as non-equilibrium thermodyn.
  Also Erik Verlinde (private discussions)

- Jacobson's argument: basic thermodynamic relation

$$\delta Q = T \delta S$$

  – entropy proport to variation of the horizon area: $\delta S = \eta \, \delta \mathcal{A}$
  – local temperature T defined as Unruh temp: $T = \hbar k / 2\pi$
  – functional dependence of S wrt energy and size of system

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- Key point in our generalization: the definition of the local entropy (Iyer+Wald 93: local boost inv, Noether charge)

$$S = -2\pi \int_\Sigma E_R^{pqrs} \epsilon_{pq} \epsilon_{rs}, \qquad \delta S = \delta \left(\eta_e A \right)$$

η_e is a function of the metric and its deriv's to a given order

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- Case of $\mathbf{f}(R)$ gravities: $\mathbf{L} = \mathbf{f}(R, \nabla^n R)$

- Also the concept of an effective Newton constant for graviton exchange (effective propagator)

$$\frac{1}{8\pi G_{eff}} = E_R^{pqrs}\epsilon_{pq}\epsilon_{rs} = \frac{\partial \mathbf{f}}{\partial R}(g^{pr}g^{qs} - g^{qr}g^{ps})\epsilon_{pq}\epsilon_{rs}$$

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- Final result, for $\mathbf{f}(R)$ gravities:
  the local field equations can be thought of as an equation of state of equilibrium thermodynamics (as in the GR case)

- Jacobson's argum non-trivially extended to $f(R)$ gravity field eqs as EoS of local space-time thermodynamics
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- S-F Wu, G-H Yang, P-M Zhang, arXiv:0805.4044, direct extension of our results to Brans-Dicke and scalar-tensor gravities
  T Zhu, Ji-R Ren and S-F Mo, arXiv:0805.1162 [gr-qc];
  C Eling, arXiv:0806.3165 [hep-th]; R-G Cai, L-M Cao and Y-P Hu, arXiv:0807.1232 [hep-th] & arXiv:0809.1554 [hep-th]

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Thanks for your attention