Transport in an array of Josephson junctions in the insulating state.

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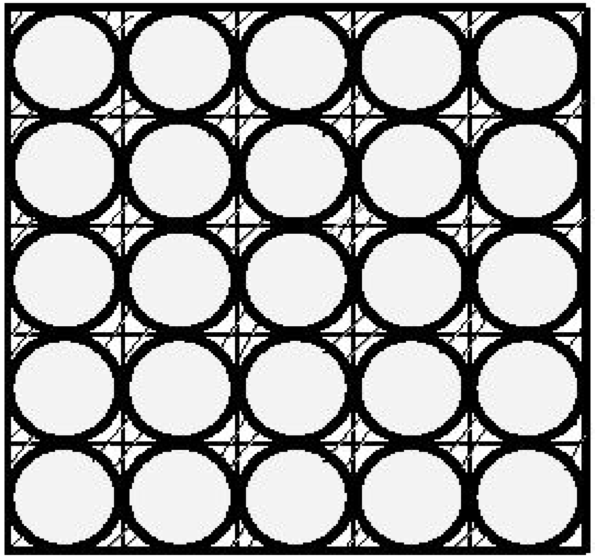
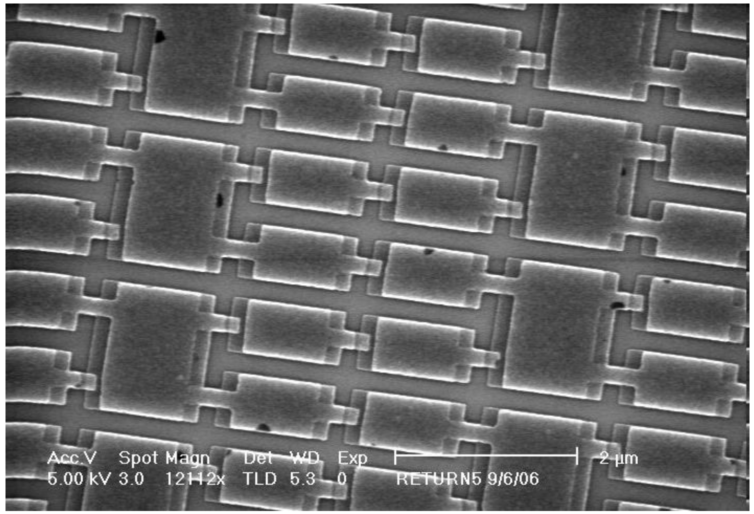
Typical structure of a granular (superconducting) metal: d=50-200A

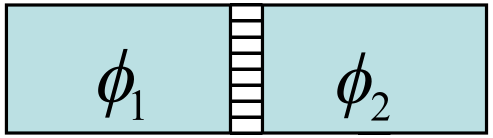


Arrays of artificial Josephson junctions



Also dirty superconductors (Sambandamurthy et al (2005), Baturina et al (2007))

Arrays of Josephson junctions:
Competition between quantum coherence and Coulomb blockade.



Josephson energy:

$$E_J = J\left(1-\cos\left(\phi_1-\phi_2\right)\right)$$

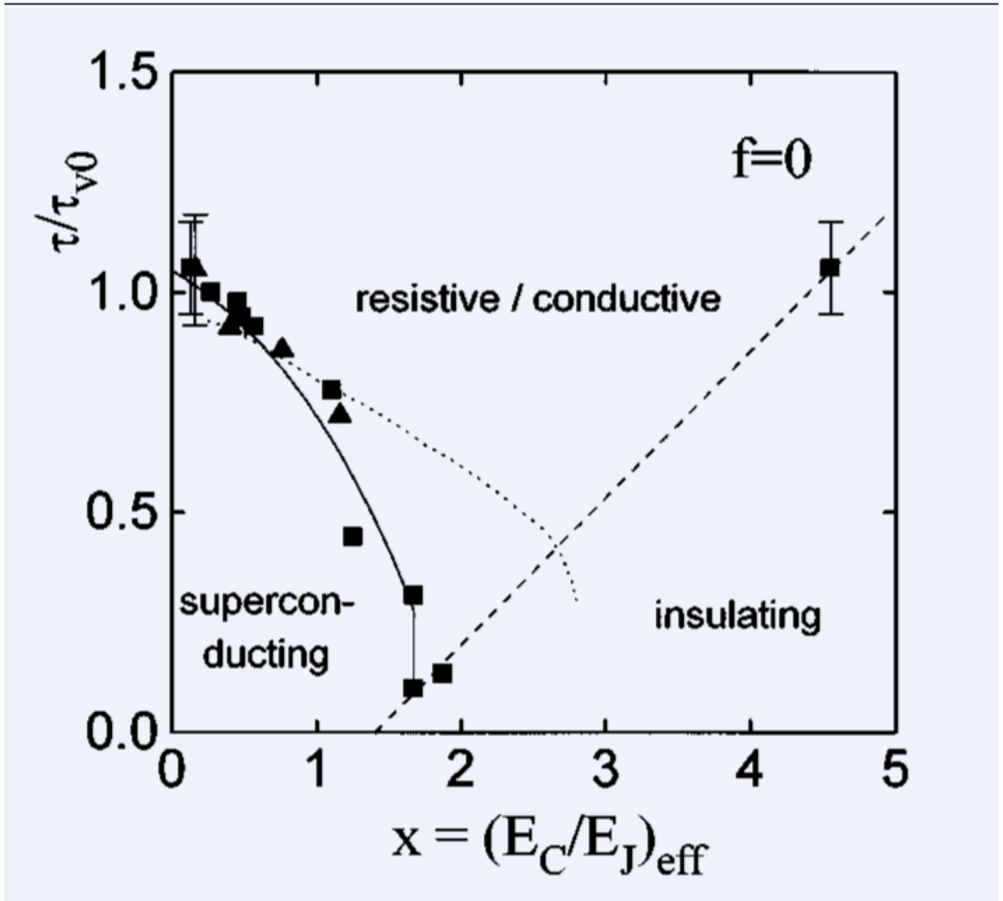
Minimum of the energy: $\boxed{\phi_1 = \phi_2}$ ⇒ Quantum coherence

However, finite size! $\boxed{\left[\hat{n}_i, \phi_i\right] = 1}$

$\hat{n}_i$-Operator of the number of particles in the island i

⇓

Fluctuations of ϕ , destruction of the coherence (superconductivity)!
Insulating state.



Van der Zant et al, (1996)

Microscopic description (phase Lagrangian L and Hamiltonian H).

$$L=\sum_{i,j}\int_0^{1/T}\left[\frac{1}{4e^2}C_{ij}\dot{\phi}_i(\tau)\dot{\phi}_j(\tau)-J_{ij}\cos\left(\phi_i(\tau)-\phi_j(\tau)\right)\right]d\tau$$

Efetov (1980)

$$H=\frac{1}{2}\sum_{i,j}B_{ij}\hat{n}_i\hat{n}_j-\sum_{i,j}J_{ij}\cos\left(\phi_i-\phi_j\right)$$

$$B_{ij}=(2e)^2\left(C^{-1}\right)_{ij}$$

$$\hat{n}_j=-i\partial/\partial\phi_j$$

C_{ij} -capacitance matrix

Existence of a quantum superconductor-insulator transition at $J\propto B_{ii}$

The description is well justified when the binding energy of the Cooper pairs is the largest energy scale, otherwise one should account for screening the Coulomb interaction by single particle excitations.

Response to an electric field $E(\omega)$

Current $\boxed{j = -2eJ_{1,2}\sin(\phi_2 - \phi_1 - 2eAa)}$

A-vector potential,
a-size of the grain

Linear response theory $\boxed{j_\omega = Q(\omega)A_\omega}$

$$Q(\omega) = \int_0^{1/T} \langle j(\tau) j(0)\rangle \exp(i\omega_n\tau)\,d\tau \Big|_{i\omega_n \to \omega + i\delta}$$

$$Q_\omega = 2e^2 aJ^2 (K(\omega) - K(0))$$

$$J = J_{1,2}$$

$$K(\omega) = \int_0^{1/T} \Pi^2(\tau)\exp(i\omega_n\tau)\,d\tau \Big|_{i\omega_n \to \omega + i\delta}$$

$$\Pi(\tau) = \langle \exp(i\phi(\tau) - i\phi(0))\rangle_H$$

Calculation in the limit J<<B

$(B \equiv B_{ii})$

eigenfunctions

$$\Psi_{n_1,n_2,n_3,\dots n_N}(\phi_1,\phi_2,\dots\phi_N) = \exp(i\phi_1 n_1 + i\phi_2 n_2 + \dots + i\phi_N n_N)$$

$$n_i = 0,\pm1,\pm2,\dots\dots$$

The response function K

$$K(\omega) = Z^{-1} \sum_{n_1,n_2,n_3\dots\dots} \frac{(B_{11}+B_{22}-2B_{12})\exp\left(-\sum_{ij} B_{ij} n_i n_j / 2T\right)}{1/4(B_{11}+B_{22}-2B_{12})^2 - \left(\sum_i (B_{2i}-B_{1i}) n_i + \omega + i\delta\right)^2}$$

$$Z = \sum_{n_1,n_2,n_3\dots\dots} \exp\left(-\sum_{ij} n_i n_j / 2T\right)$$

The ground state: $\boxed{n_i = 0}$ for all i ⇨ simple results at T=0

$$\boxed{Q(\omega) = (2e)^2 aJ^2 \frac{\omega^2}{E_d\left(E_d^2 - (\omega + i\delta)^2\right)}} \qquad \boxed{\sigma(\omega) = \frac{2e^2 aJ}{E_d}\delta(\omega - E_d)}$$

$$\boxed{\sigma(\omega) = \frac{\operatorname{Im} Q(\omega)}{\omega}}$$ -*ac* conductivity

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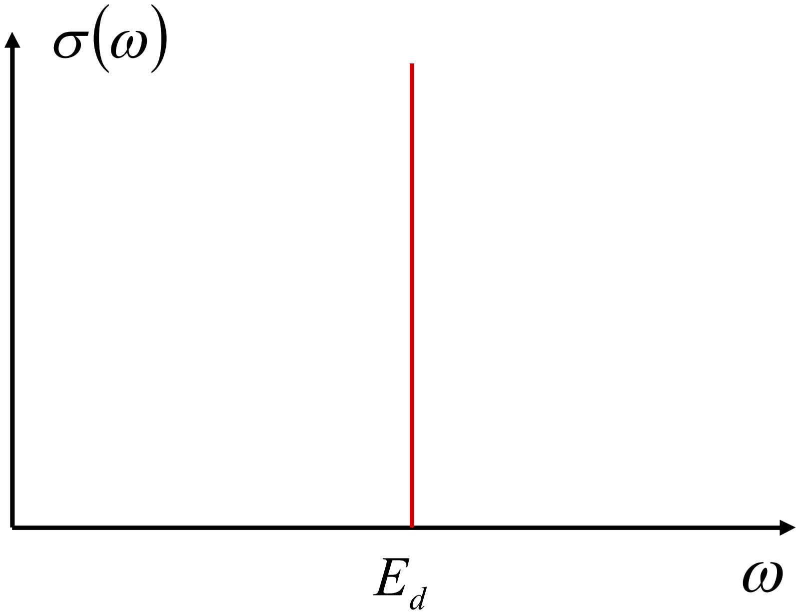
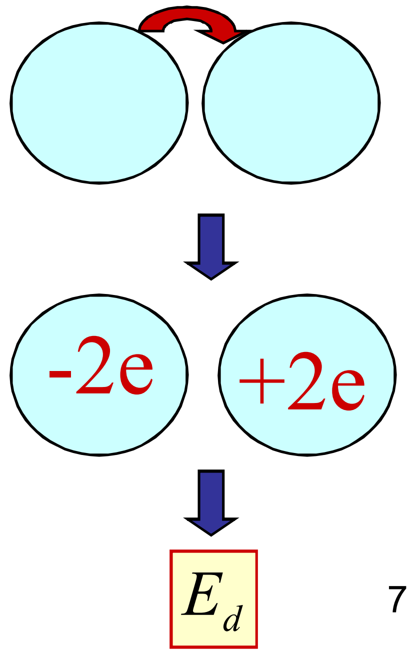
$$\boxed{\sigma(0) = 0}$$

$\boxed{E_d = B_{11} - B_{12}}$ -dipole energy



No chance for a finite conductivity at T=0!



What about finite temperatures?

$$\sigma(0)=\frac{e^2aJ^2}{T}Z^{-1}\sum_{n_1,n_2,\ldots,n_N}\delta\left(E_1-\sum_i\left(B_{2j}n_j-B_{1j}n_j\right)\right)\exp\left(-\sum_{i,j}\frac{B_{ij}n_in_j}{2T}\right)$$

The δ -function can be satisfied for $n_1=-1,\quad n_i\neq 0, i\neq 1$
and $\quad n_2=1,\quad n_i\neq 0, i\neq 2$

In both cases $\quad \sigma\propto\exp(-B/2T),\quad B=B_{ii}$

⇓

Conductivity might become finite due to contribution of freely moving (anti-) Cooper pairs (bosons or antibosons).

However, the energy levels are still discrete in this approximation and *dc* transport is still not possible.

The main idea:

a) transport is due the bosons

b) formation of bands of the bosons

c) density of the bosons $\propto \exp(-B/2T)$ and their interaction is weak

Reduced Hamiltonian for pseudospins S

$$H_{red} = \frac{1}{2}\sum_{ij} B_{ij} S_i^z S_j^z - \frac{1}{2}\sum_{ij} J_{ij} S_i^+ S_j^-$$

$$\left[S_i^+, S_j^-\right] = 2\delta_{ij} S_i^z$$

$$\left[\hat{S}_i^z, \hat{S}_j^\pm\right] = \pm\delta_{ij} S_i^\pm$$

eigenstates

$$S_i^\pm |0\rangle_i = \pm\sqrt{2}|\pm 1\rangle_i$$

$$S_i^\pm |\mp 1\rangle_i = \pm\sqrt{2}|0\rangle_i$$

$$S_i^\pm |\pm 1\rangle_i = 0$$

$$S_i^z |0\rangle_i = 0$$

$$S_i^z |\pm 1\rangle_i = \pm 1|\pm 1\rangle_i$$

The ground state:

$S_i^z = 0$ for all i

Relevant (for conductivity) excited states:

$S_i^z = \pm 1$ for i=j,

$S_i^z = 0$ otherwise

The eigenenergies $E_{J=0}$ in the limit J=0

$E_{J=0}^{(0)} = 0$

$E_{J=0}^{(\pm 1)} = B/2$

The main idea for $J \neq 0$: formation of bands for $S^z = \pm 1$

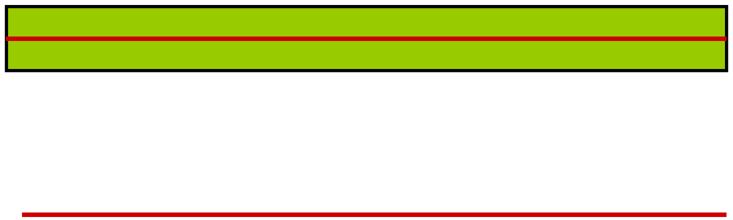
Eigenfunctions in the limit J<<B:

$$\boxed{|k\rangle = N^{-1/2}\sum_r \exp(ikr)|r\rangle}$$

(N is the number of sites)

Eigenenergies:

$$\boxed{E(k) = B/2 - 2J\cos(k_x x) - 2J\cos(k_y y)}$$



The gap $\boxed{E_0 = B/2 - 4J}$ is the energy of adding (subtracting) one Cooper pair.

Existence of the band $\Rightarrow$ possibility of *dc* transport!

The current:

$$\boxed{\hat{I}_{ij} = ieJ_{ij}\left(\hat{S}_i^+\hat{S}_j^- - \hat{S}_i^-\hat{S}_j^+\right)}$$

Standard linear response formalism for non-interacting systems

$\Downarrow$

infinite conductivity without disorder

However, a finite conductance G of a finite system

$$\boxed{G = (2/\pi)(2e)^2 \sinh(2J/T) I_0(2J/T)\exp(-E_0/T)}$$

I_0 is the Bessel function

Disorder, inelastic scattering, etc.: standard consideration of non-interacting particles with the density $\propto \exp(-E_0/T)$

Disorder correlations:

$$\langle \delta B_{ii} \delta B_{jj} \rangle = f_1 \delta_{ij}, \quad \langle \delta J_{ij} \delta J_{kl} \rangle = f_2 \left(\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} \right)$$

Elastic scattering time:

$$\tau^{-1} = \left(f_1/8 + 4 f_2 \right) / J$$

$$\tau^{-1} << \min\{J, T\}$$

Array conductivity

$$\sigma \sim e^2 T^{-1} \tau \left(\min\{J, T\} \right)^2 \exp(-E_0/T)$$

Weak localization correction:

$$\boxed{\delta\sigma_{WL} \sim -e^2 T^{-1} \min\{J, T\} \exp(-E_0 / T) \ln(L_\varphi / l)}$$

Possibility of Anderson (many-body (?)) localization of the Cooper pairs for $\tau^{-1} \ln(L_\varphi / l) \geq \min\{J, T\}$!

An interesting possibility (Mooji et al, 1990) for 2 D arrays:

Capacitance matrix: $$C_{ij} = \begin{cases} C_0, & i = j \\ C, & |i-j| = 1 \end{cases}$$ $$C >> C_0$$

$$\Downarrow$$

$$B = \frac{(2e)^2}{C} \ln(C / C_0)$$

A (smeared) Berezinskii, Kosterlitz, Thouless transition at

$$T_K \approx E_d = B_{11} - B_{12} \propto e^2 / C$$

No Coulomb blockade at $T > T_K$

Conclusions:

1) Transport of free bosons in the regime when the superconductivity is destroyed by the Coulomb blockade.

2) Infinite conductivity in the absence of a macroscopic disorder.

3) Drude formula and Anderson localization in the presence of disorder with exponentially low density of carriers.