Superfluidity

in

ultracold Fermi gases

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- **Introduction**
- **Overview of theory and experiment**
- **Polarized ultracold Fermi gases**
- **1 spin ↓ + N spins ↑**

Review paper : S. Giorgini, L. P. Pitaevskii and S. Stringari, Rev. Mod. Phys. **80**, 1215 (2008)

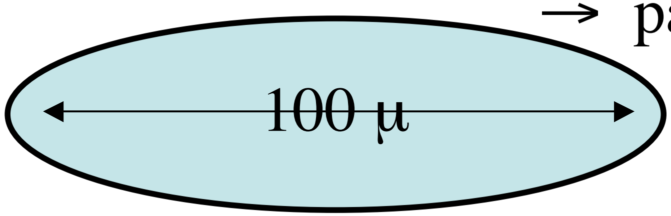
• Ultracold Fermi gases, some basics

- Interatomic distance ~ 10^2 - 10^3 nm

- In practice alkali with even number of nucleons: **^{6}Li** or **^{40}K**

-**Trapping** (**magnetic** or **optical**)

→ parabolic potential (**harmonic oscillator**)



→ **inhomogeneous system**

N ~ 10^6 - 10^7

- **Ultracold** : **optical** + **evaporative cooling** (temperature ~ 1nK - 1μK)

→ **degenerate** Fermi gas : **quantum** regime

- **Very low T** → very low energy → **s-wave** scattering only

- But prohibited for **fermions** by **Pauli exclusion** → **no interaction** !

(very good for atomic cloks)

⇒ **Physics usually with two fermionic species : " ↑ and ↓ "**

usually two lowest energy hyperfine states

of same element Ex: ^{6}Li

• **Low energy s-wave** scattering $\Rightarrow$ **single** parameter

$$\boxed{\textbf{scattering length } \mathbf{a}}$$

• **Short-range** interaction compared to interatomic distance

$$\boxed{V(\mathbf{r}) = (4\pi h^2 a/m)\ \delta(\mathbf{r})}$$

Very convenient ! $\Rightarrow$

Remarkable model system

for **strongly interacting** (and strongly correlated) **Fermi** systems
normal and **superfluid**

• **BCS Superfluidity**

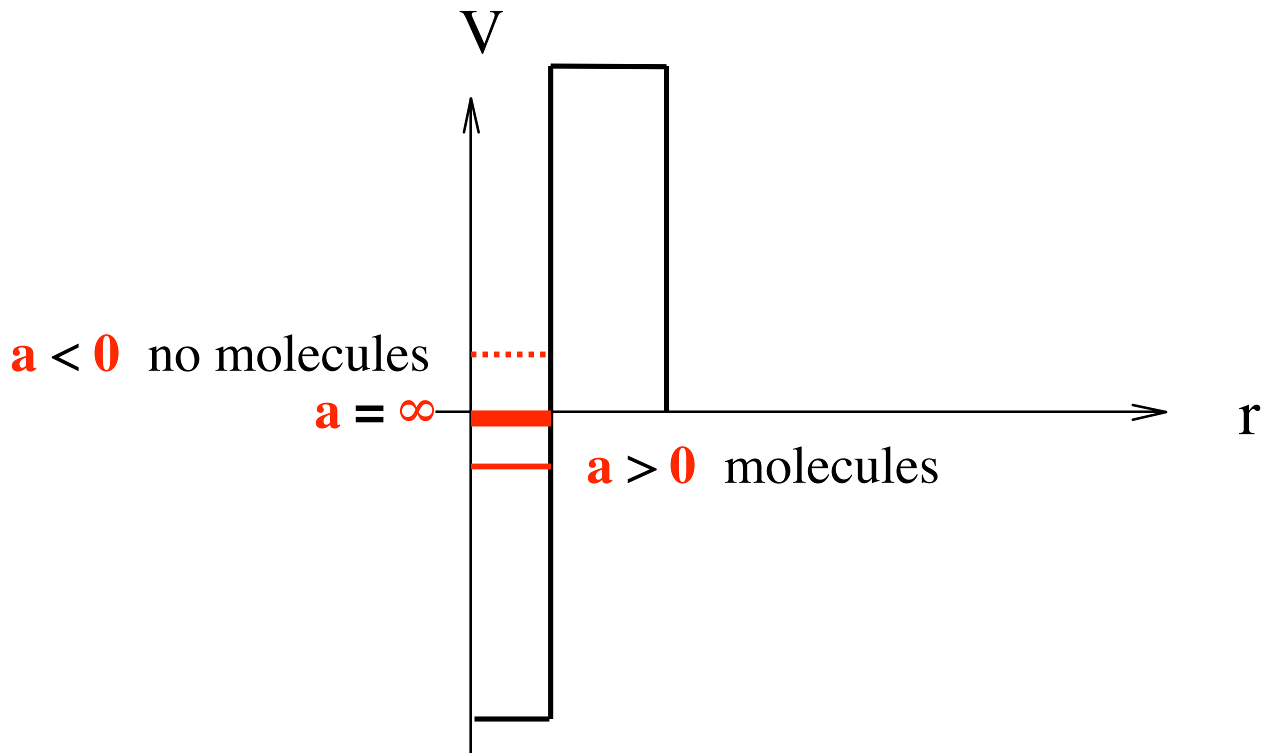
- **Formation of Cooper pairs** between $\uparrow$ and $\downarrow$ atoms
 requires **attractive interaction** + **degenerate** Fermi gas

- Attraction $\rightarrow$ $\mathbf{a} < \mathbf{0}$

- **Feshbach resonance**

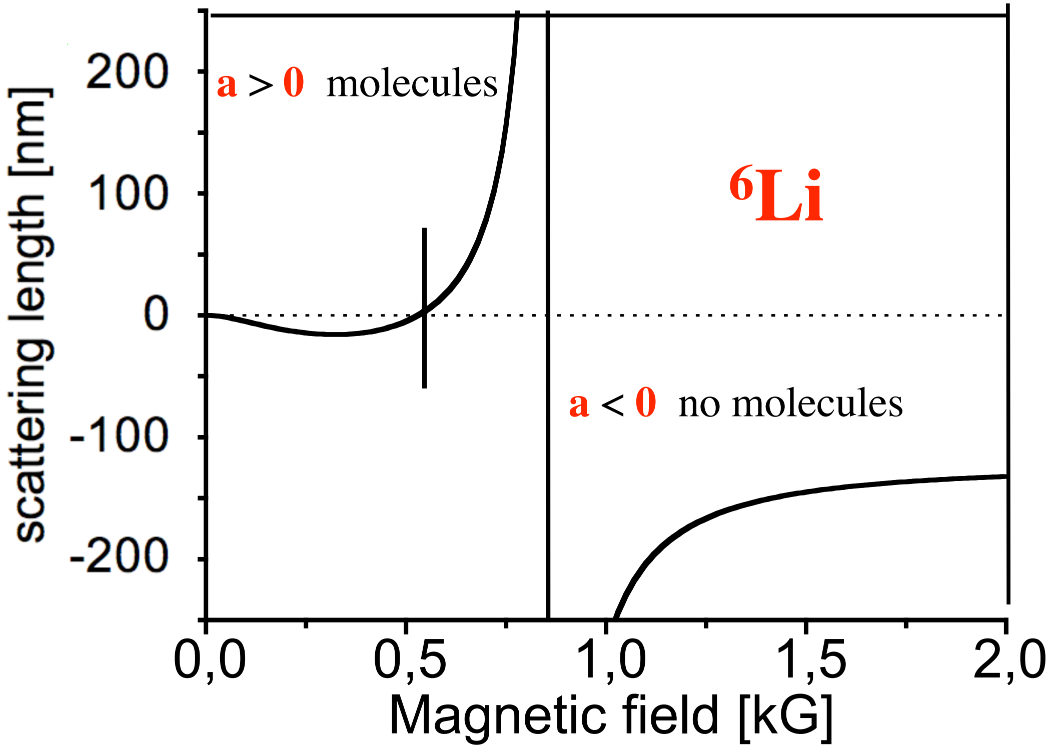
  - Allows to **control effective interaction** via **magnetic field** by changing **scattering length a**

  - **Scattering length** **a = ∞** if **bound state** with **energy = 0** exists



- Actually atoms very **near each other** are not in the same spin configuration as when they are very **far from each other** ("**closed channel**" and "**open channel**")
  → **sensitivity** of **bound state energy** to **magnetic field**

• **Feshbach resonance for two ^{6}Li particles in vacuum**



• Interaction **tunable** at will by **magnetic field** !

Dream !

$\Rightarrow$ Allows a **physical realization** of the **BEC-BCS crossover**

"**New**" **superfluid** !

• **BEC - BCS crossover**

- **BCS Ansatz** for **ground state** wavefunction:

$$\Psi(\mathbf{r}_1, \mathbf{r}_2, \ldots) = \mathrm{A}\,\{\Phi(\mathbf{r}_1 - \mathbf{r}_2)\,\Phi(\mathbf{r}_3 - \mathbf{r}_4)\ldots.\}$$

describes as well **dilute gas** of **molecules**, made of 2 fermions

- Known since Popov (66), Keldysh and Kozlov(68), Eagles(69)....
Leggett (80), Nozières and Schmitt-Rink (85)

- Accurate in **weak-coupling**(BCS) limit and **strong-coupling**(BEC) limit
In between : physically quite reasonable "**interpolation scheme**"

$\Rightarrow$ in between need **experiment to tell** what happens!

No exact theory

- **Theoretical interest**

- General interest : **strongly interacting superfluid** systems (**unitarity**)

- Interesting for **high T_c superconductivity** :
  very tight pairs, **pseudogap** ~ **preformed pairs** ? ⇒ **nearly BEC** ?

- **Ultracold gases** results **not in favor** of this model:
  BEC physics pushed **beyond unitarity** by **Fermi sea**

- Need to control **normal state** for good understanding of **superfluid**

$\epsilon_k = k^2/2m \qquad \xi_k = \epsilon_k - \mu$

- **Theoretical treatments**

  - **BCS** (mean field) or **equivalent**
  Use scattering length a known experimentally
  instead of BCS potential V (cf. Galitski and Belaiev)
  - single parameter **$1/k_F a$**

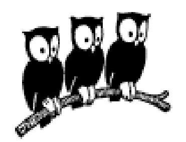
  - **fairly reasonable** !

$$E_k = \sqrt{\xi_k^2 + \Delta^2}$$

$$\frac{m}{4\pi a} = \sum_k \left(\frac{1}{2\epsilon_k} - \frac{1}{2E_k} \right)$$

$$\boxed{n \equiv \frac{k_F^3}{6\pi^2}} = \sum_k \frac{1}{2} \left(1 - \frac{\xi_k}{E_k} \right)$$

$$E_F \equiv \frac{k_F^2}{2m}$$

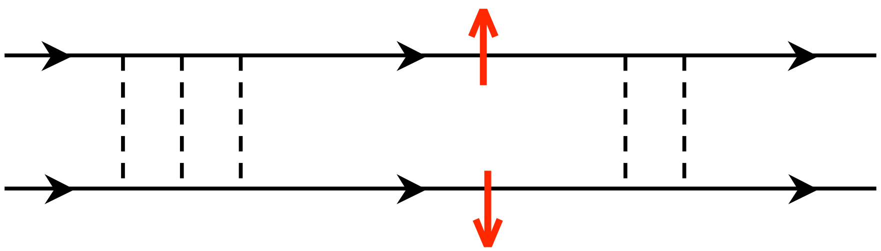


- **Quantum Monte-Carlo** **quite reliable**, **but numbers**

- **T-matrix**

Quite often used (**High T_c**)

Quite satisfactory for ultracold gases



Perali, Pieri and Strinati, PRL **93** (2004)

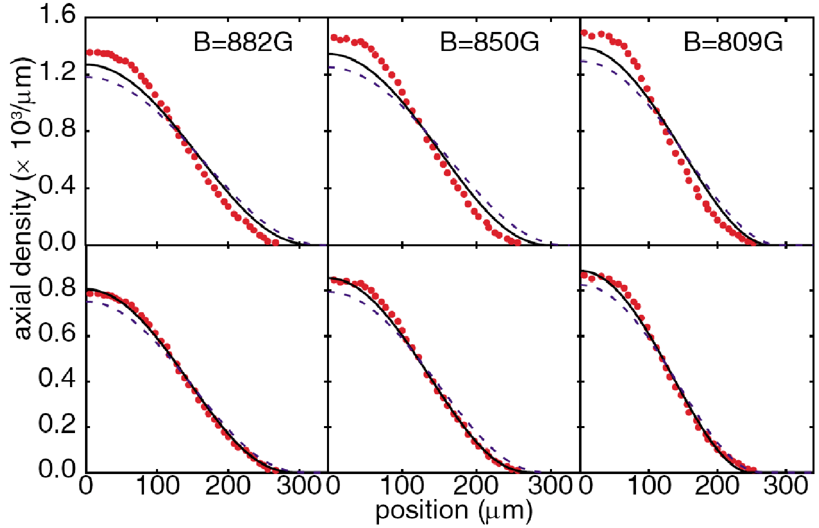


FIG. 1 (color online). Comparison between experimental and theoretical axial density profiles. Experimental data from Ref. [17] (dots) are shown for three different values of the magnetic field B tuning the FF resonance. Theoretical results at $T = 0$ obtained by our theory (solid lines) and by BCS mean field (dashed lines) are shown for the corresponding couplings $(k_F a_F)^{-1}$ given in the text. The upper (lower) panel refers to the estimated number of atoms $N = 4 \times 10^5$ ($N = 2.3 \times 10^5$).

- **Unitarity**

- **Quite interesting case:** **very strongly interacting**

- **parameter :** $\boxed{1/k_F a = 0}$

$\Rightarrow$ **single parameter k_F (or E_F) left** $\Rightarrow$ **dimensional analysis**

Example :

$$\boxed{\mu = \xi E_F}$$

BCS	$\xi = 0.59$
QMC	$\xi = 0.42\text{-}0.44$
T-matrix	$\xi = 0.455$
Exp	$\xi = 0.27\text{-}0.51(\pm 10)$

- **Away from unitarity: chemical potential**

Pieri, Pisani and Strinati, PRB **72** (2005)

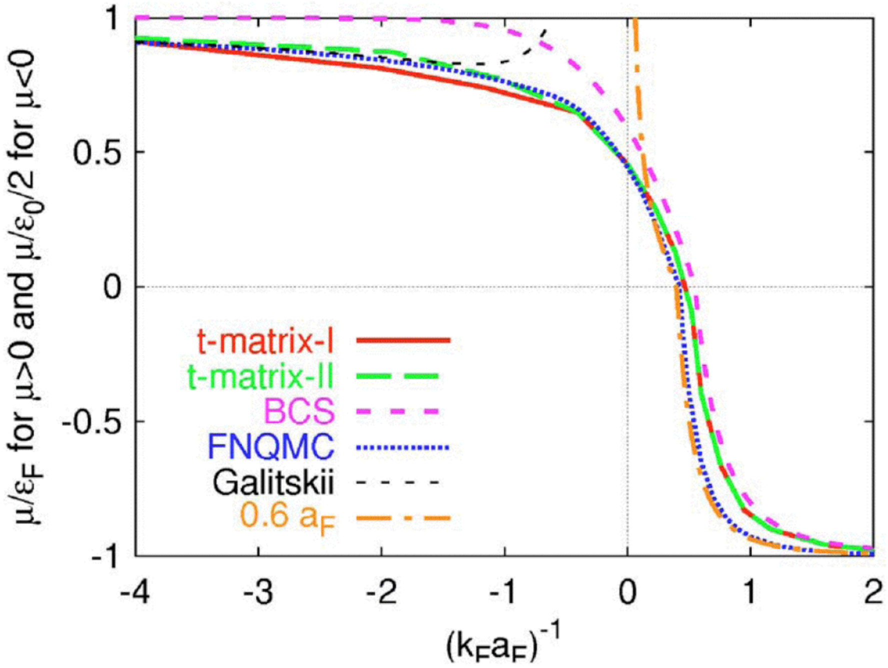


FIG. 1. (Color online) Chemical potential at zero temperature vs the coupling parameter $(k_F a_F)^{-1}$. The results of the present theory (t-matrix-I) and of its version without the inclusion of the self-energy shift Σ_0 (t-matrix-II) are compared with the BCS mean field (BCS), the fixed-node QMC data from Ref. 11 (FNQMC), the Galitskii's expression for the dilute Fermi gas (Galitskii), and the asymptotic expression for strong coupling using the result $a_B = 0.6 a_F$.

Gap

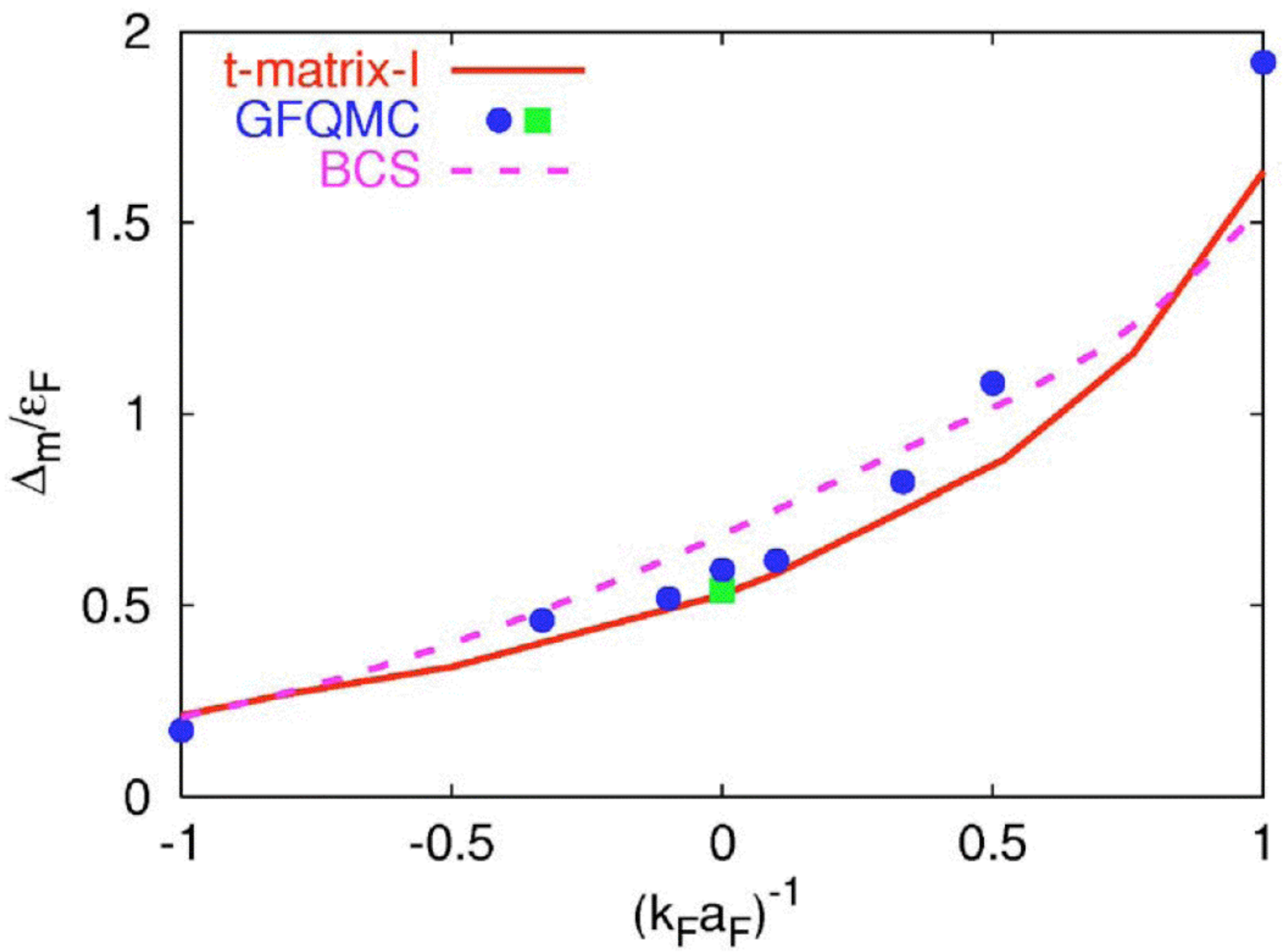
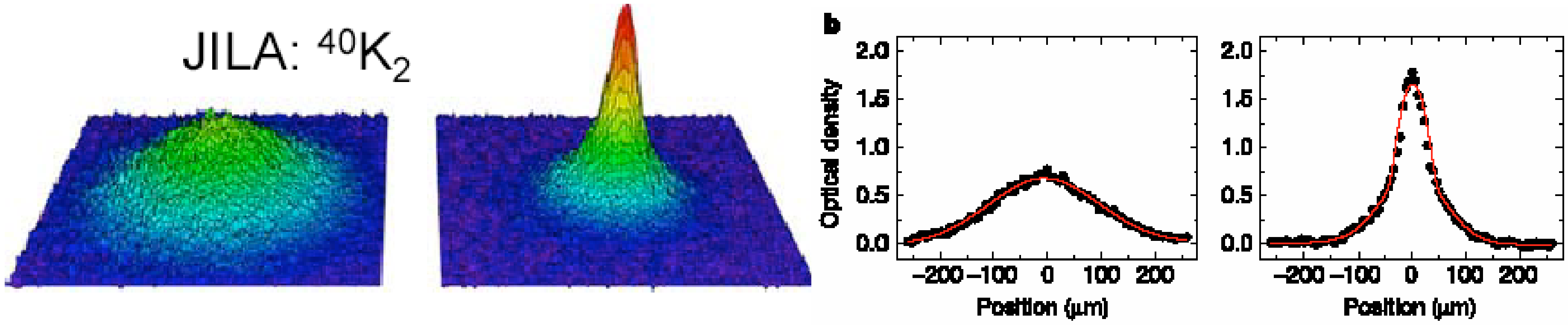
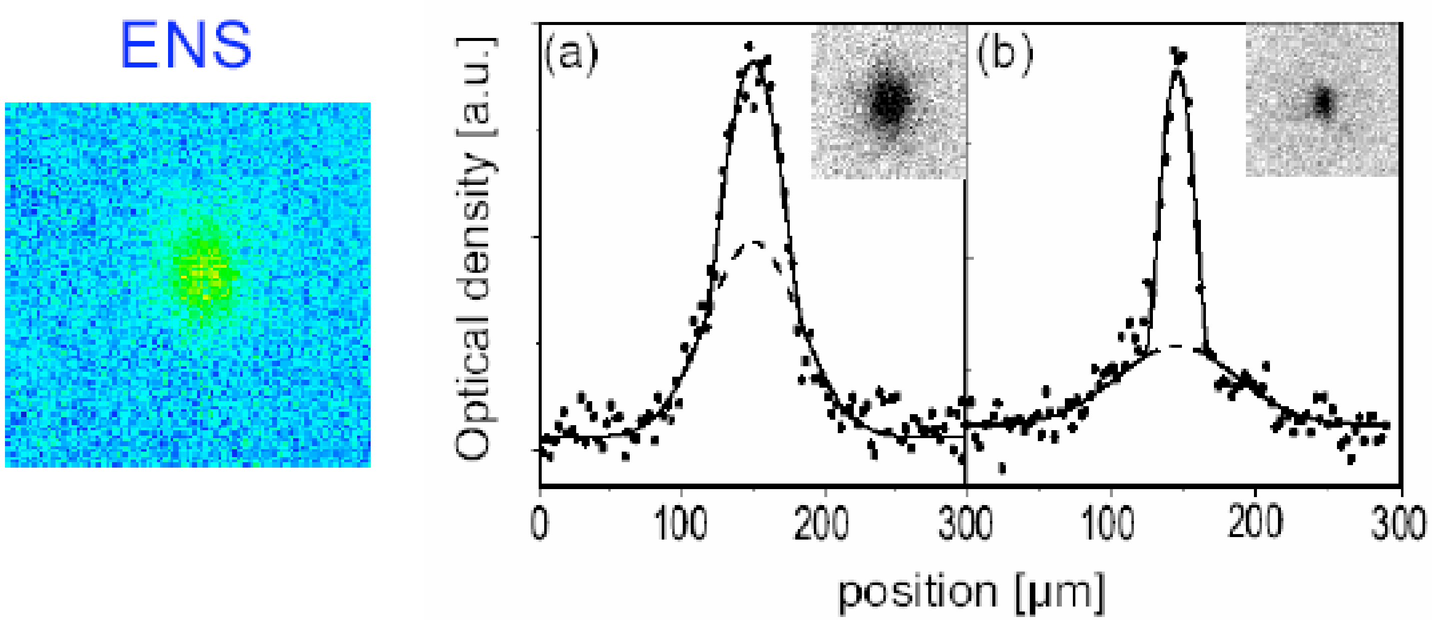


FIG. 2. (Color online) Excitation gap Δ_{m} at zero temperature vs the coupling parameter $(k_F a_F)^{-1}$. The results of the present theory (t-matrix-I) are compared with the Green's function QMC data of Refs. 8 and 10 (GFQMC) as well as with the BCS mean field (BCS).

• **Bose-Einstein condensates of molecules** (2003-2004)





• Also **MIT** and **Innsbrück**

• First Bose-Einstein **condensates** of **molecules** made of **fermions** !

• **Vortices as evidence for superfluidity** (2005)

- **Before** : **Anisotropic expansion** ~ **BEC**

Collective mode damping (much more convincing)

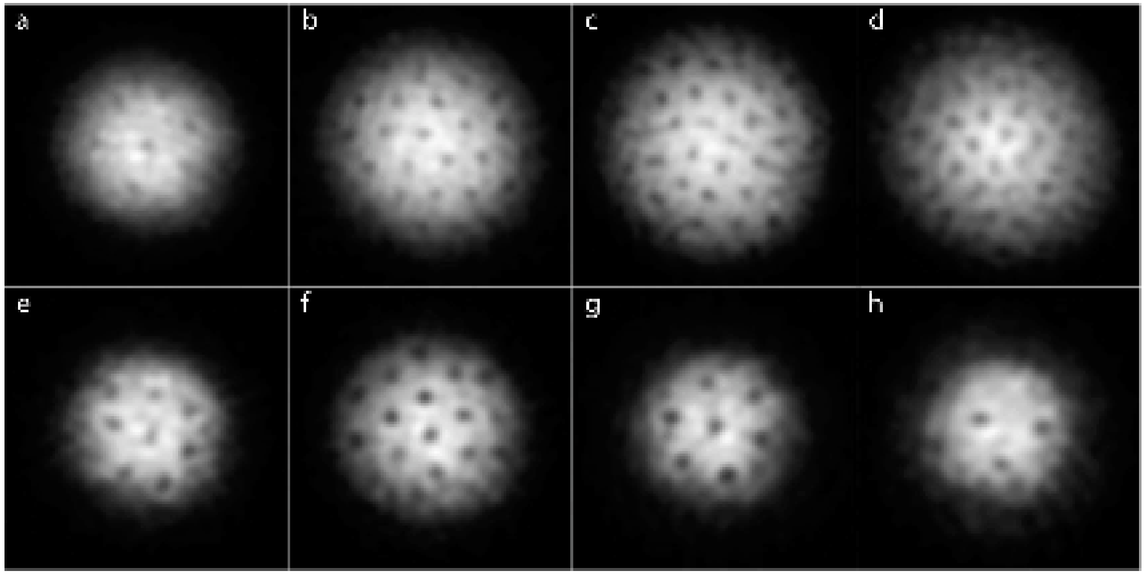
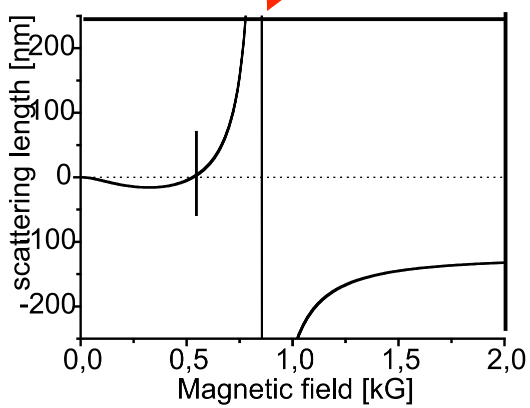


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is 880 m × 880 m .

MIT

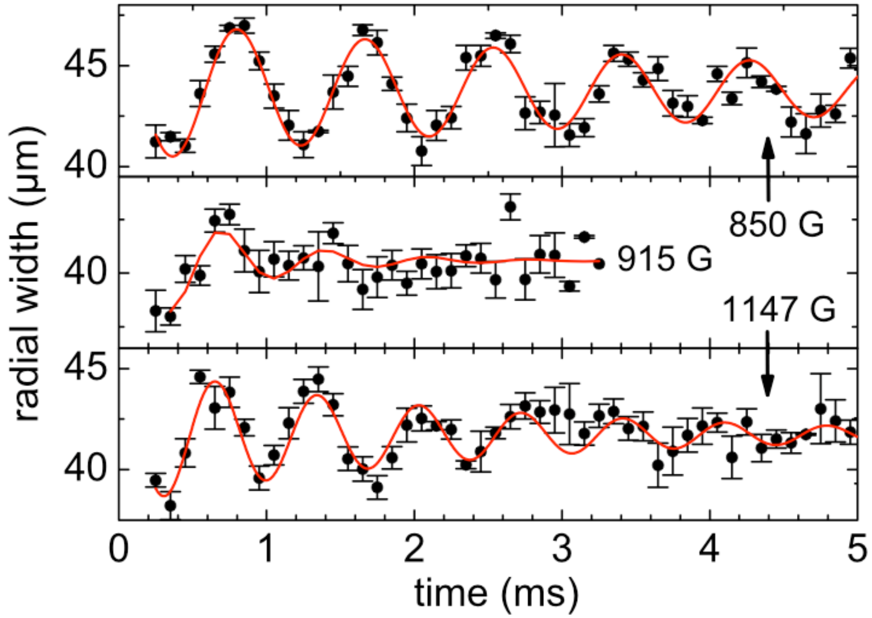
834 G



• Collective oscillations in harmonic trap (superfluid state)

Cigar geometry $\omega_z << \omega_{x,y}$

- **In situ** experiments
  (no need for interpretation)
- **High** experimental **precision** possible



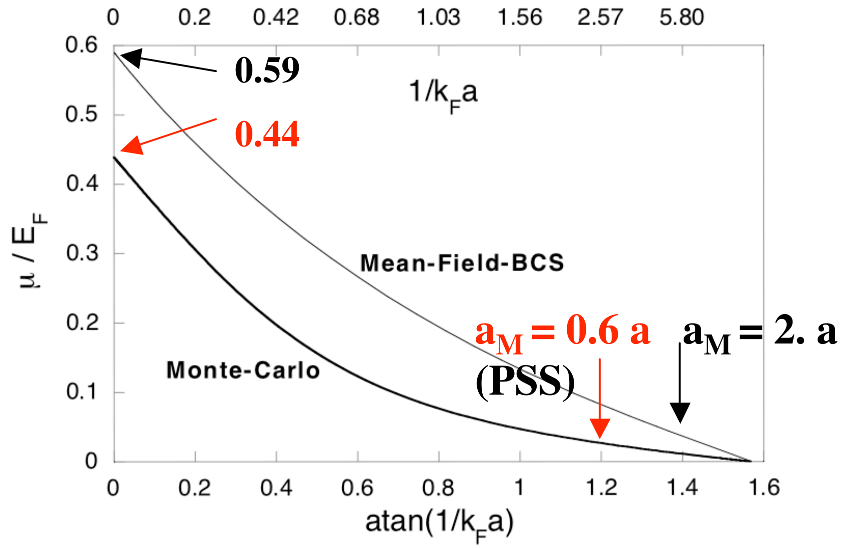
Most experiments require **expansion**

- Direct access to **equation of state $\mu(n)$**

- **Equations of state**

- **Monte-Carlo** : should be reasonably **accurate**

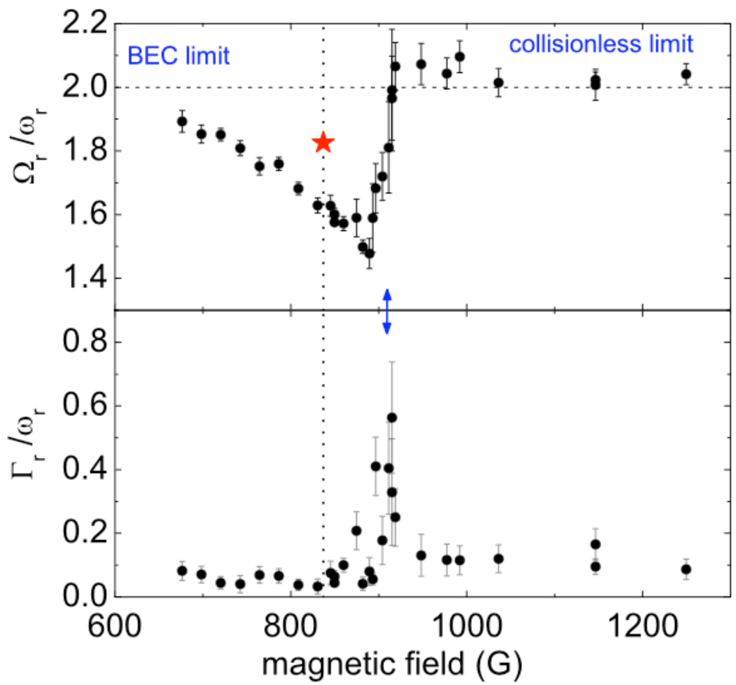
- **BCS** equation of state



- **Hydrodynamics**

- **T → 0** **Superfluid** satisfies hydrodynamics, but $\omega \ll E_b$ (**pair binding** energy, i.e. $\omega \ll \Delta$ on **BCS side**)

- **Radial geometry**

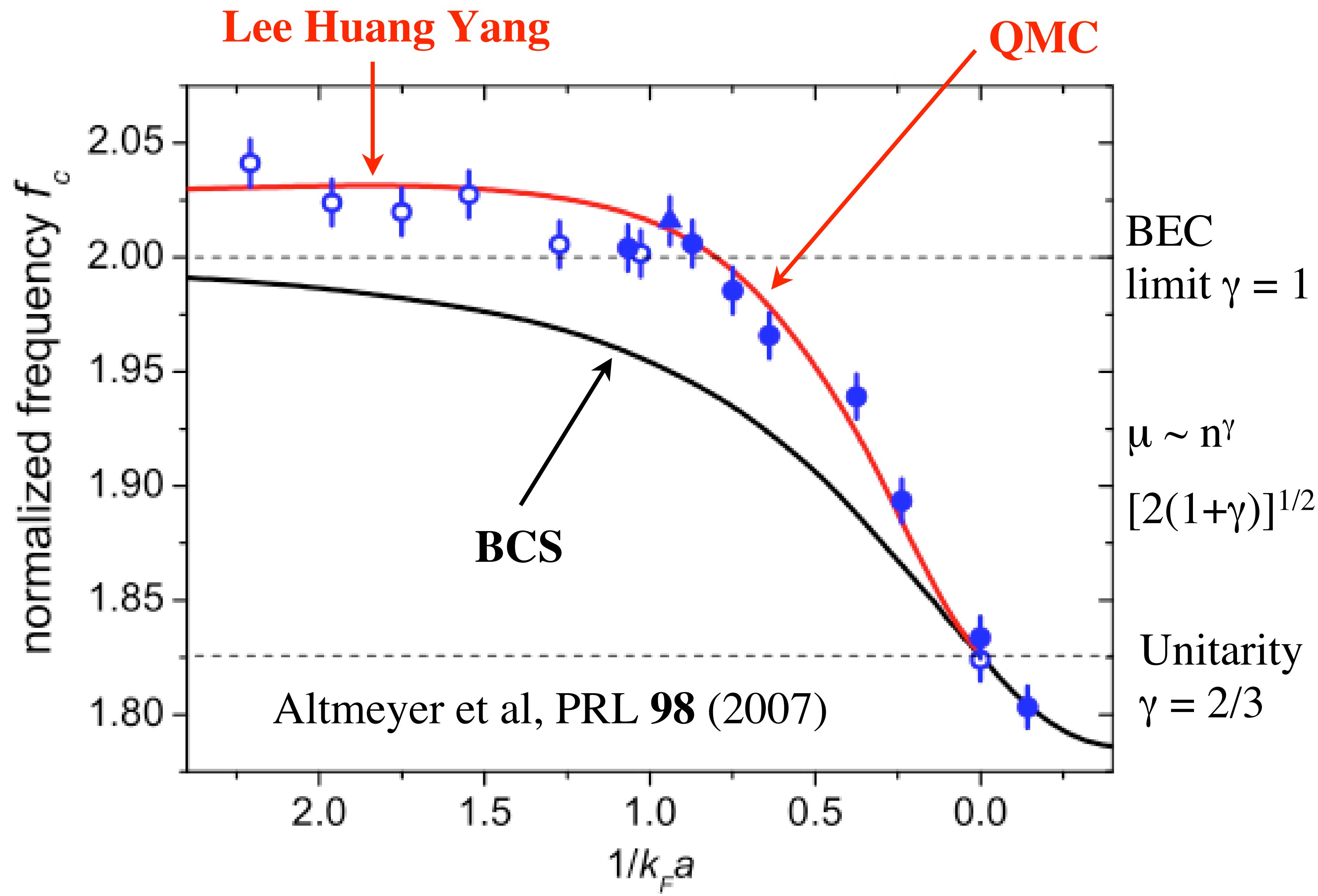


- **Strong attenuation** at 910 G

→ **pair-breaking** peak $\Omega_r = 2\,\Delta$ (T,B)

→ **superfluid !**

Astrakharchik, RC, Leyronas and Stringari PRL **95** (2005)



- **Equation of state for composite bosons**

  \- Purely fermionic exact theory

  Leyronas and RC, PRL **99** (2007)

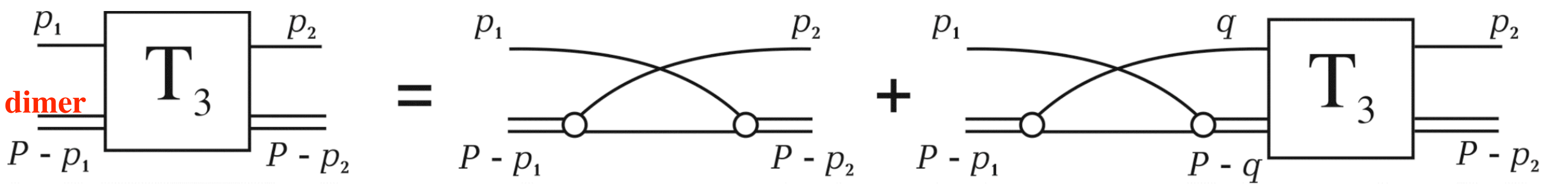
  same spirit as **Keldysh and Kozlov,** Sov. Phys. JETP **23** (1968)

- **Diagrammatic formulation of 4-body problem**

  Brodsky, Klaptsov, Kagan, RC and Leyronas, JETP Letters **82** (2005)

  PRA **73** (2006)

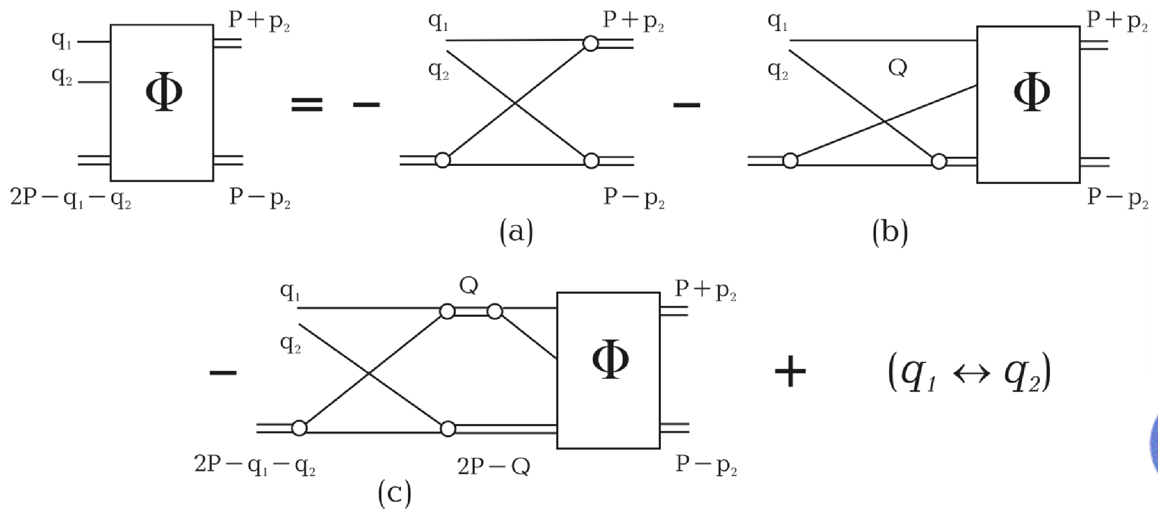
  \- **3-body**

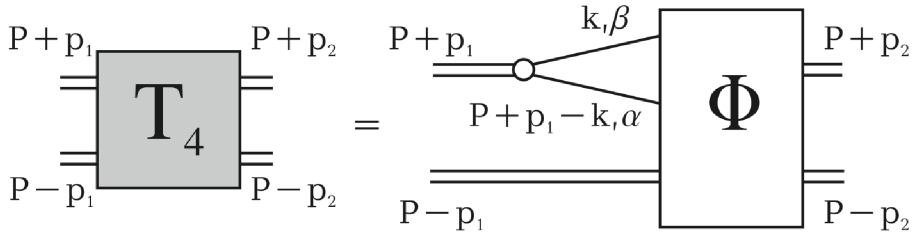


leads to $a_3 = 1.18\ a$

Skorniakov and Ter-Martirosian, Sov. Phys. JETP **4** (1957)

  \- **4-body**



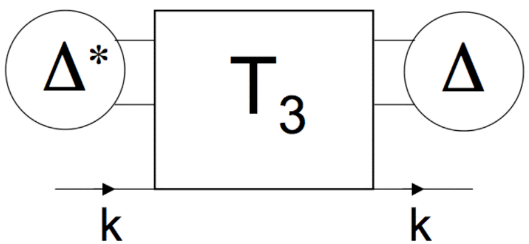


leads to $a_M = 0.60\ a$

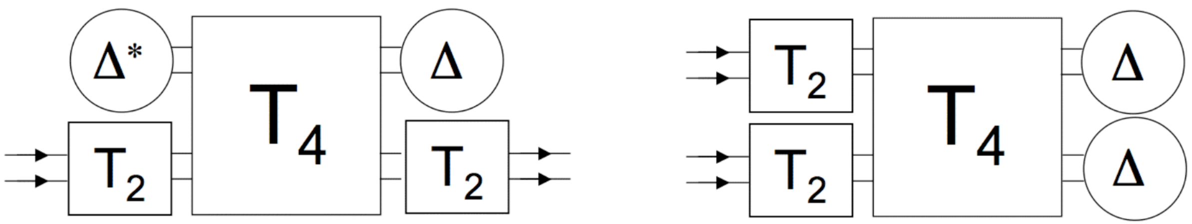
Petrov, Salomon and Shlyapnikov, PRL **93** (2004) (solving Schrödinger equation)

- **Many-body (superfluid T=0)**

- Systematic **expansion** in **powers** of **anomalous self-energy Δ(*k*)**
- **Collective mode** contributions



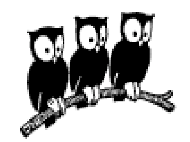
Lowest order normal self-energy



Irreducible vertices for collective mode propagator

$$\cdots\cdots\cdots \Rightarrow \boxed{\mu = -\frac{E_b}{2} + \frac{\pi a_M}{m} n\left[1 + \frac{32}{3\sqrt{\pi}}(n a_M^3)^{1/2}\right]}$$

LHY



- **New physical situations**

  - **Unequal masses :** $m_\uparrow \neq m_\downarrow$ experiments just beginning
  - **Bose - Fermi mixtures**

- **" Polarized " gases**

  - **Strong imbalance** $n_\uparrow \neq n_\downarrow$ easily achieved (stable) $\Rightarrow \mu_\uparrow \neq \mu_\downarrow$

  - **Weak imbalance** in **superconductors** in magnetic field

by coupling to electronic **spin** $E = -\mathbf{M}.\mathbf{B}$

(with **orbital** currents **suppressed** in planar geometry)

  - Very interesting for high critical field (**High** T_c !)

  - Very interesting for **quark matter** :

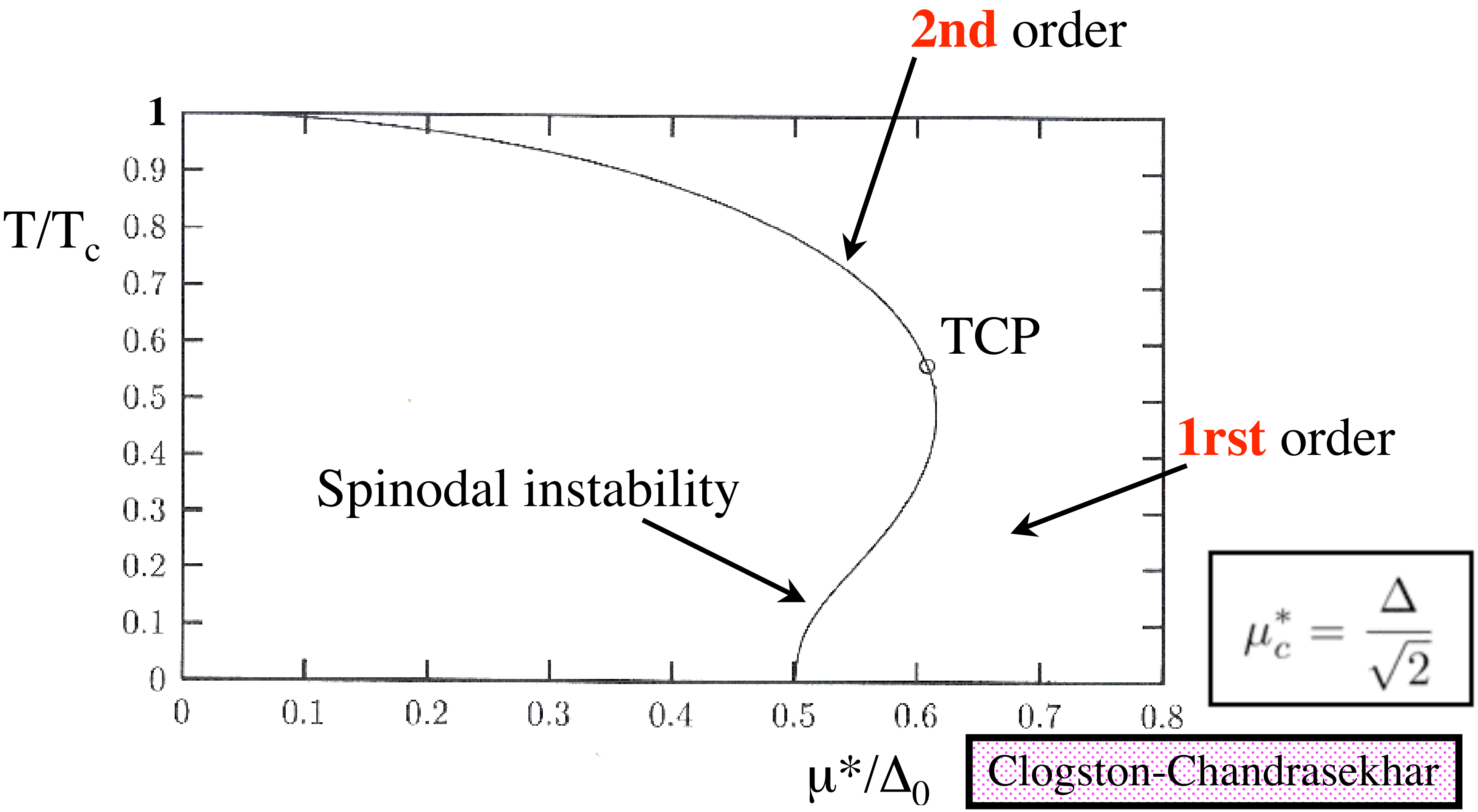
    superfluid core of **neutron stars**, **heavy ions** collisions

Casalbuoni and Nardulli Rev.Mod.Phys. **76** (2004)

- **Imbalance breaks pairs** since **pairing** $\Rightarrow$ $n_\uparrow = n_\downarrow$

$\Rightarrow$ **critical "field"** μ_c^* $\mu^* = (\mu_\uparrow - \mu_\downarrow)/2$

- **Weak coupling BCS**



$$\mu_c^* = \frac{\Delta}{\sqrt{2}}$$

- Fulde - Ferrell and Larkin - Ovchinnikov (1964)

  - FFLO or LOFF phases

- Pairs with $\boxed{\text{nonzero total momentum} \quad \mathbf{q} \neq 0}$

better in high effective field $\Longrightarrow$

extension of superfluid stability domain

- spontaneous symmetry breaking ! (~ vorticity)
- No very clear observation in standard superconductors
- Larkin - Ovchinnikov :
- T = 0 Ginzburg-Landau investigation : 2nd order transition
- Best LO solution : $\boxed{\Delta(\mathbf{r}) \sim \cos(\mathbf{q}.\mathbf{r})}$

- Reinvestigated for 1rst order transition

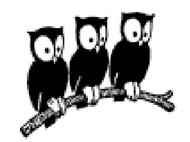
RC and C. Mora, Europhys. Lett. **68** (2004)
RC and C. Mora. PRB **71** (2005)

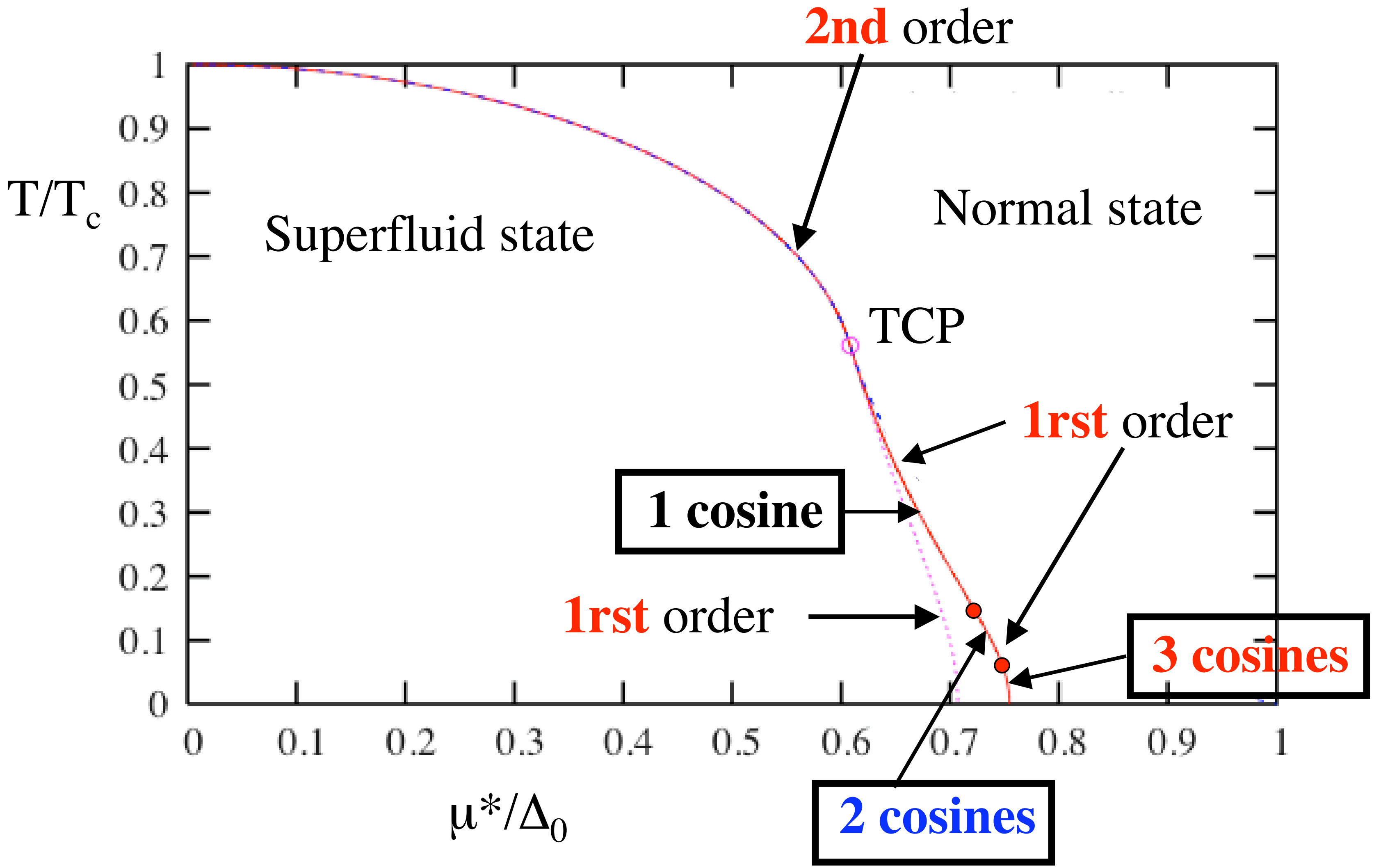
better !

but transition line very near LO

$$\boxed{\Delta(\mathbf{r}) = \sum_{i=x,y,z} \cos(\mathbf{q}_i.\mathbf{r})}$$

T=0





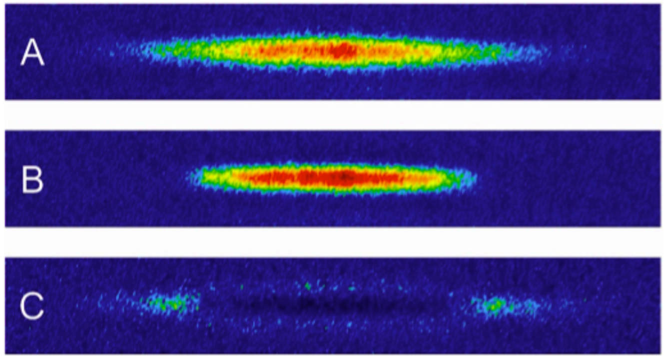
• No LOFF state seen (**yet**?) in **ultracold gases**:

only for **weakly interacting** systems ??

- Difficult: inhomogeneous systems
- Mostly ignored by theories
- 2D or 1D better

• Experiments on polarized ultracold Fermi gases

- Exp. systems **inhomogeneous** !



total

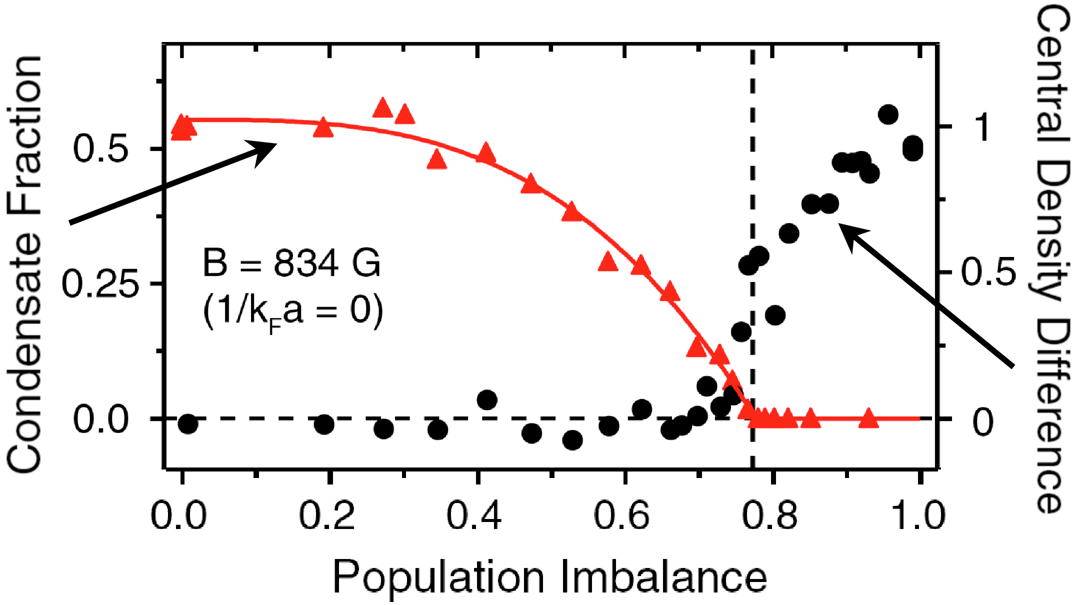
BCS

normal

- Partridge et al. (Rice) Science **311** (2006)
  Zwierlein et al. (MIT) Science **311** (2006)
  Shin et al. (MIT) PRL **97** (2006)

- Phase separation **seen** between
- BCS phase $\mathbf{n_\uparrow = n_\downarrow}$
  and (strongly polarized) normal phase $\mathbf{n_\uparrow \neq n_\downarrow}$

- Disagreement between Rice and MIT : Rice **smaller** and more **elongated** ?



Unitarity

- Many theoretical papers !
- C. Lobo, A. Recati, S. Giorgini and S. Stringari, PRL **97** (2006)

- Generalizes **Clogston-Chandrasekhar** for **strongly interacting** system
- **Strongly polarized normal state** at **unitarity** (**for simplicity**)

- single ↓ spin with (non interacting) Fermi sea of ↑ spins

$$e_\downarrow(k) = \mu_\downarrow + \frac{k^2}{2m^*}$$

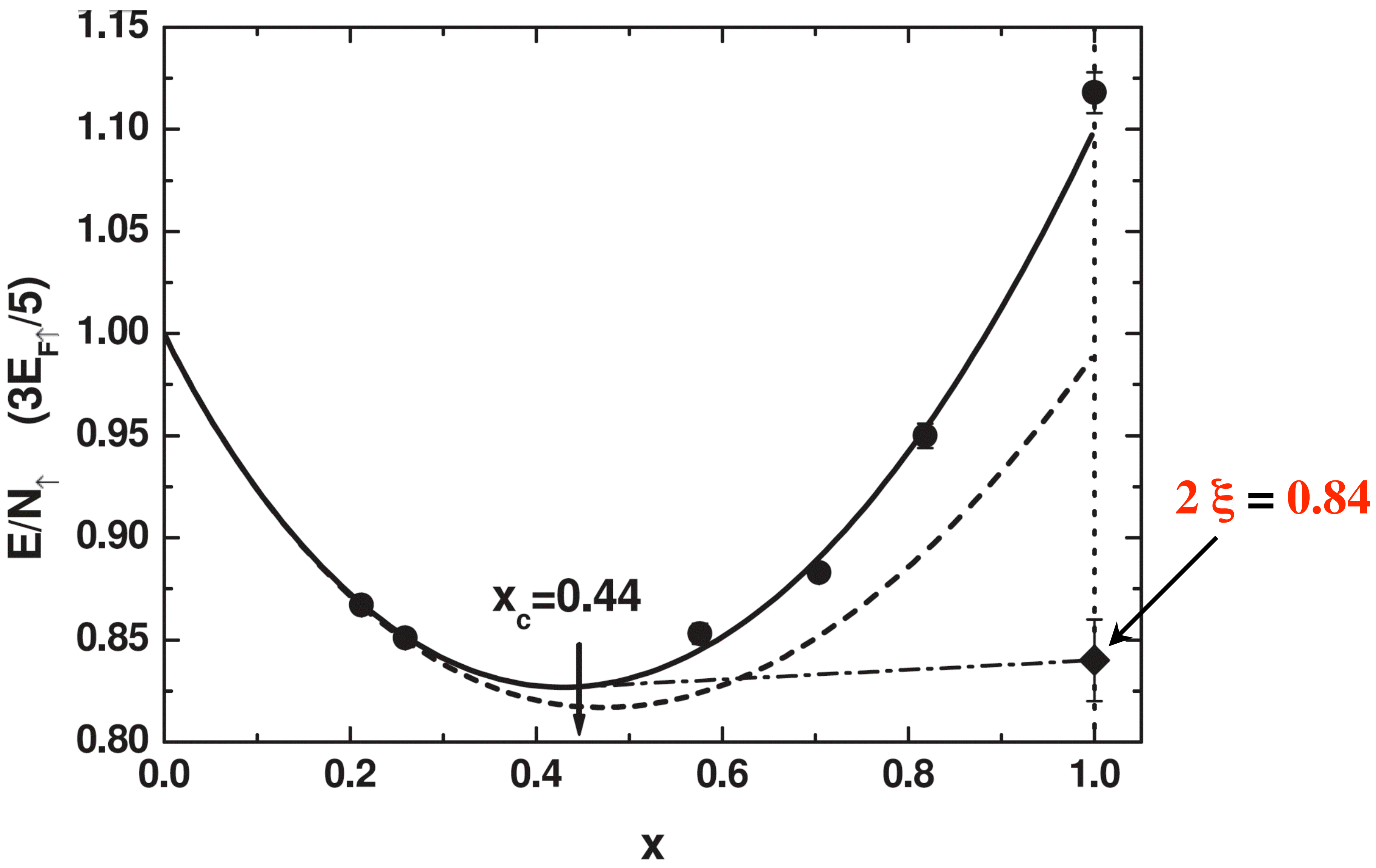
- small Fermi sea of ↓ spin with (non interacting) Fermi sea of ↑ spins

$$E_\downarrow = \mu_\downarrow n_\downarrow + \frac{3}{5}\frac{k_{F\downarrow}^2}{2m^*} n_\downarrow \qquad x = \frac{n_\downarrow}{n_\uparrow}$$

$$\frac{E(x)}{n_\uparrow} = \frac{3}{5}E_F\left(1 - \frac{5}{3}\frac{|\mu_\downarrow|}{E_F}x + \frac{m}{m^*}x^{5/3}\right)$$

- QMC : $\frac{|\mu_\downarrow|}{E_F}$ = **0.58** $\qquad \frac{m}{m^*}$ = **1/1.04**

- **Superfluid** : $x = 1 \qquad \frac{E}{n_\uparrow} = 2\,\xi\,\frac{3}{5}E_F$



First order transition for $x_c = 0.44$

• **Trapped gas** with **Local Density Approximation** (LDA)

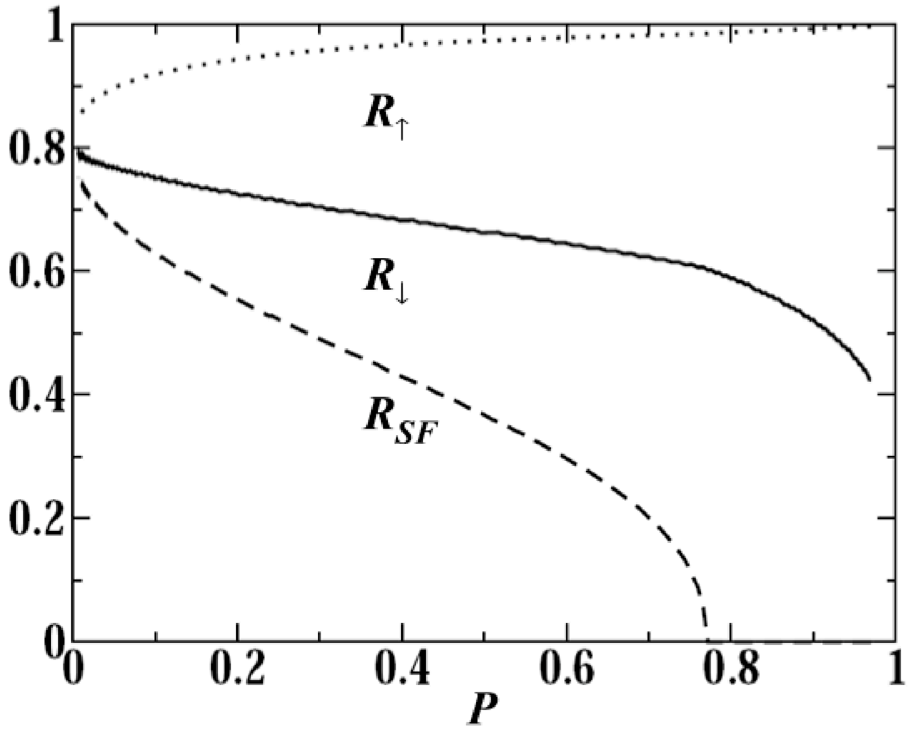
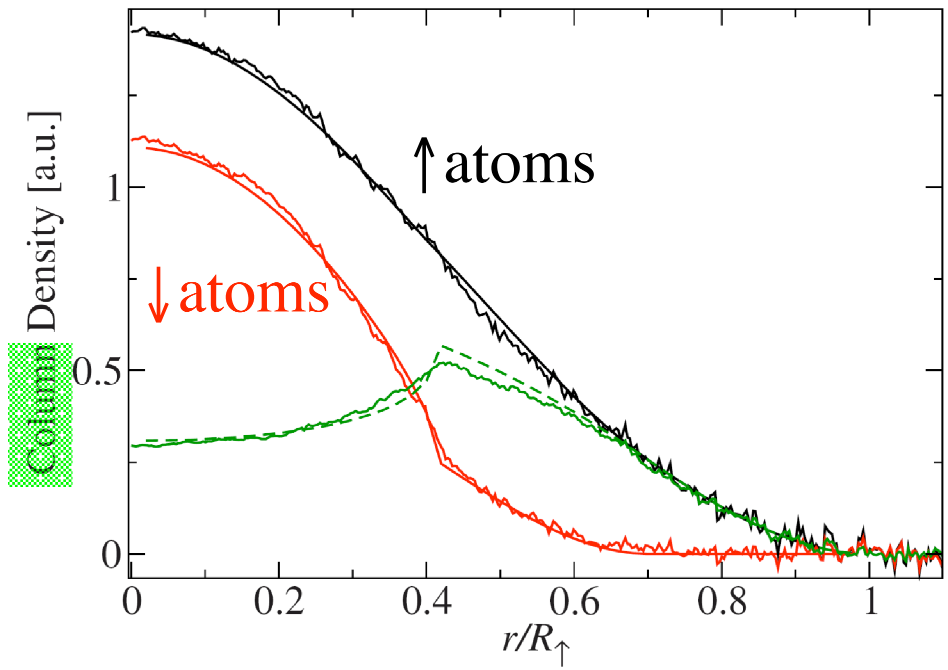
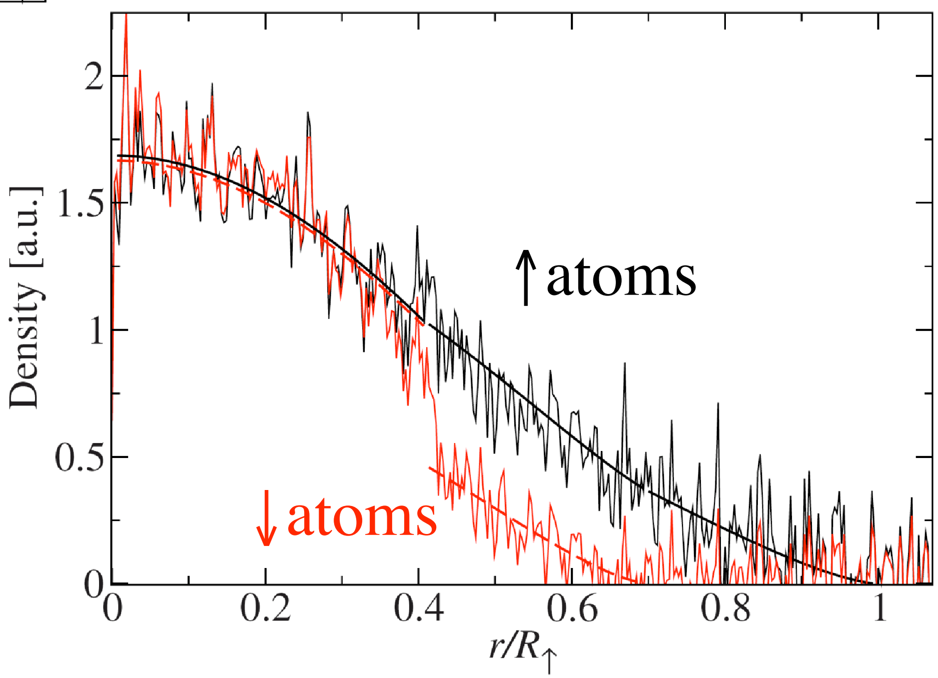


FIG. 3: Radii of the three phases in the trap in units of the radius $R^0_\uparrow = a_{\rm ho}(48N_\uparrow)^{1/6}$ of a noninteracting fully polarized gas, where $a_{\rm ho}$ is the harmonic oscillator length.

• Critical polarization $\mathbf{P_c}$ = **0.77** in good agreement with experiments

• **Very good agreement** for **density** of **trapped gas**



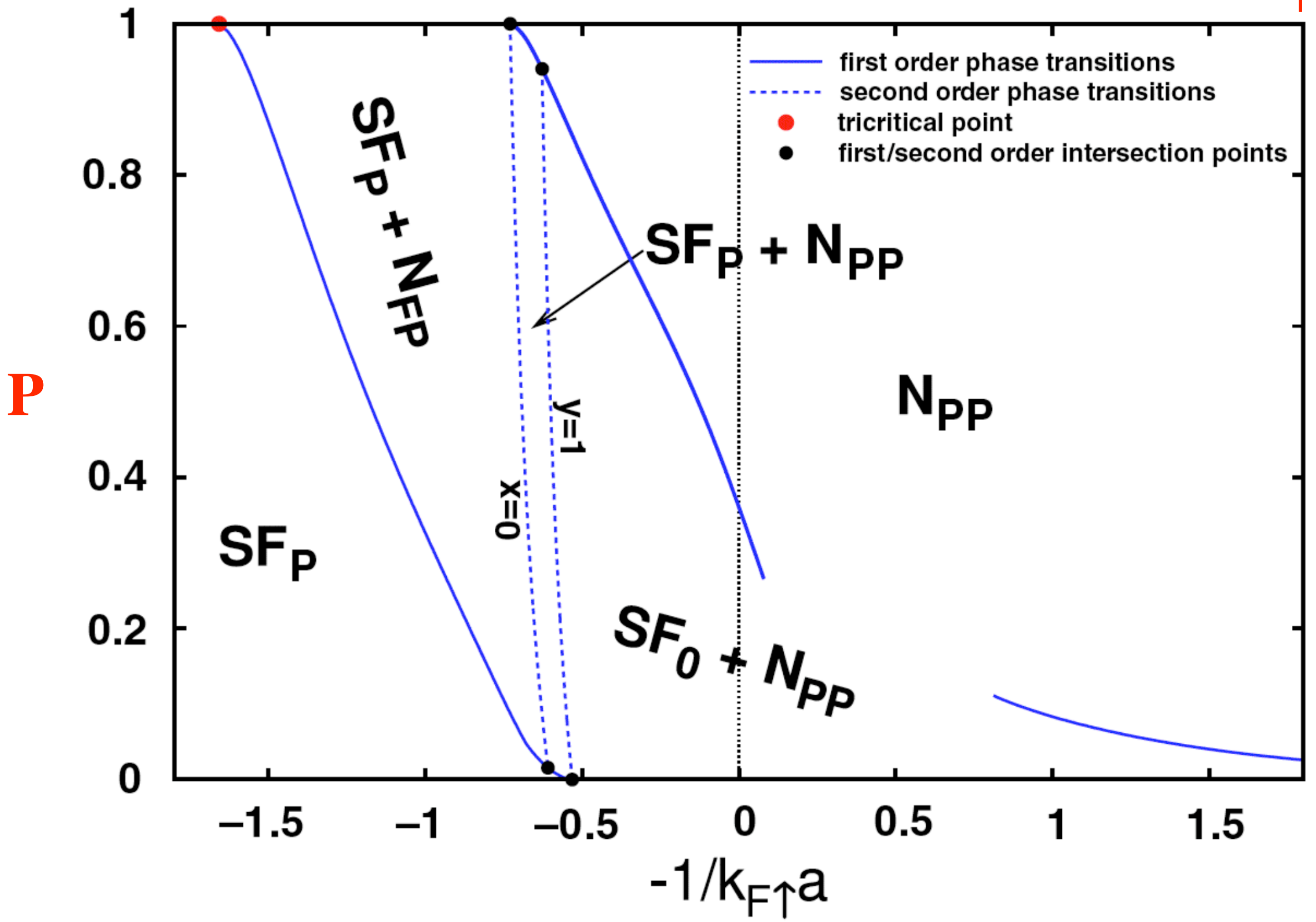


- **Generalization out of unitarity**

Pilati and Giorgini, PRL **100** (2008)

Polarization:

$\mathbf{P} = (\mathbf{n}_\uparrow - \mathbf{n}_\downarrow)/(\mathbf{n}_\uparrow + \mathbf{n}_\downarrow)$



- QMC for N_{PP} (partially polarized normal state), N_{FP} (fully polarized)
  for SF_0 (unpolarized superfluid)
  for SF_P (polarized superfluid)

• 1 spin ↓ + N spins ↑

- For full control of "**Clogston-Chandrasekhar**", solution **needed** for this very interesting "**many body**" (**normal state**) problem

- "**Simple**" since spins ↑ non interacting

- First step: **T**- **matrix** approach (quite often used)

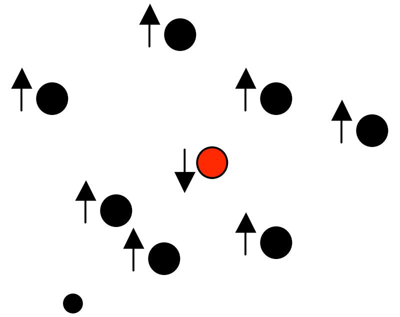
- Unitarity

$$\frac{\mu_\downarrow}{E_F} = -0.6066 \qquad\qquad \frac{m*}{m} = 1.17$$

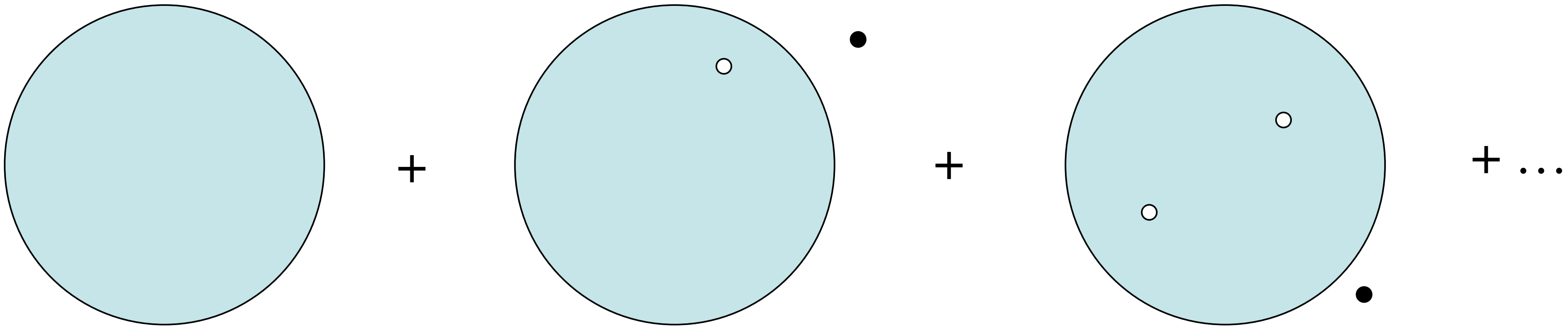
- Very near **Monte Carlo** !

- Very surprising → **coincidental** ?

- **Hamiltonian** RC and Giraud, PRL **101** (2008)



- **Wave function**



- $\alpha_{\{\mathbf{k}_i\}\{\mathbf{q}_j\}}$ antisymetric in particle and hole variables

- $$\boxed{H|\psi\rangle = E|\psi\rangle}$$

-If weak dependence of kinetic energies on hole variables $\mathbf{q}_i$ neglected

$\rightarrow$ **Exact decoupling** of higher order terms by **destructive interference**

- In practice **very fast convergence** $\longrightarrow$ **n = 2 quite enough**

- **Check** $\boxed{m_\downarrow \to \infty}$

  - **Fermi sea + impurity : exactly solvable (one-body)**

  - **Unitarity** ρ **= E_b/E_F = 0.5**

  1rst order ρ **= 0.465** **2nd order** ρ **= 0.498**

  - **Convergence essentially complete !**

- **1D** Excellent agreement with exact results for energy + effective mass

- **Results for** $\boxed{m_\uparrow = m_\downarrow}$

  1rst order ρ **= 0.6066** **2nd order** ρ **= 0.6156**

  - **Extremely fast convergence**

  - **Monte Carlo**

    Pilati-Giorgini FN-DMC ρ **= (3/5) (0.99 ±0.01) = 0.59 - 0.60**

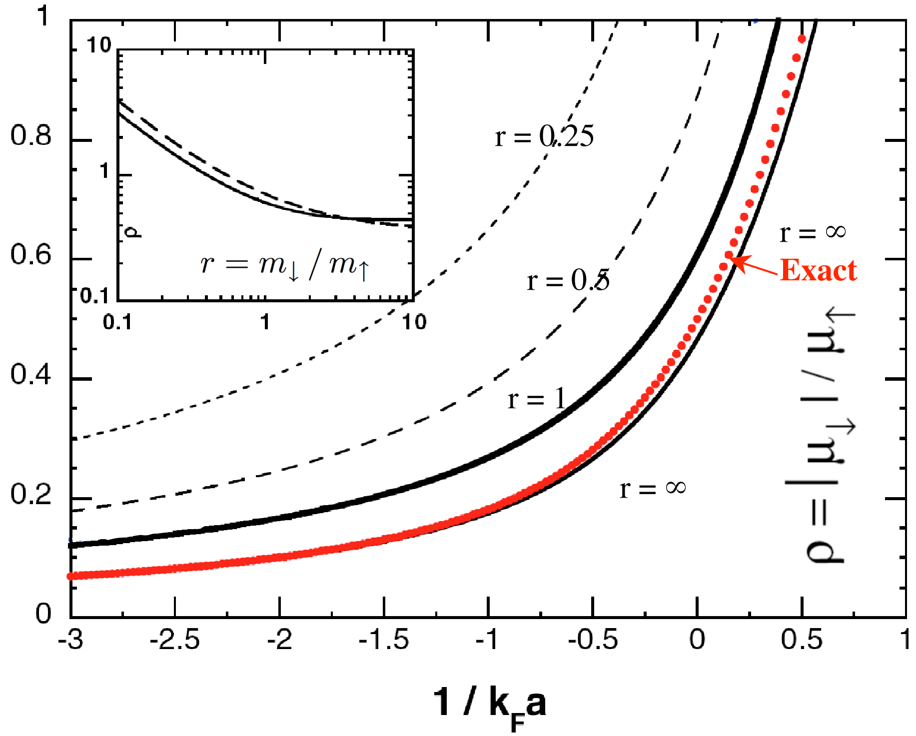
    Prokof'ev-Svistunov Diag.MC ρ **= 0.618** ρ **= 0.615**

  - **Excellent agreement not coincidental !**

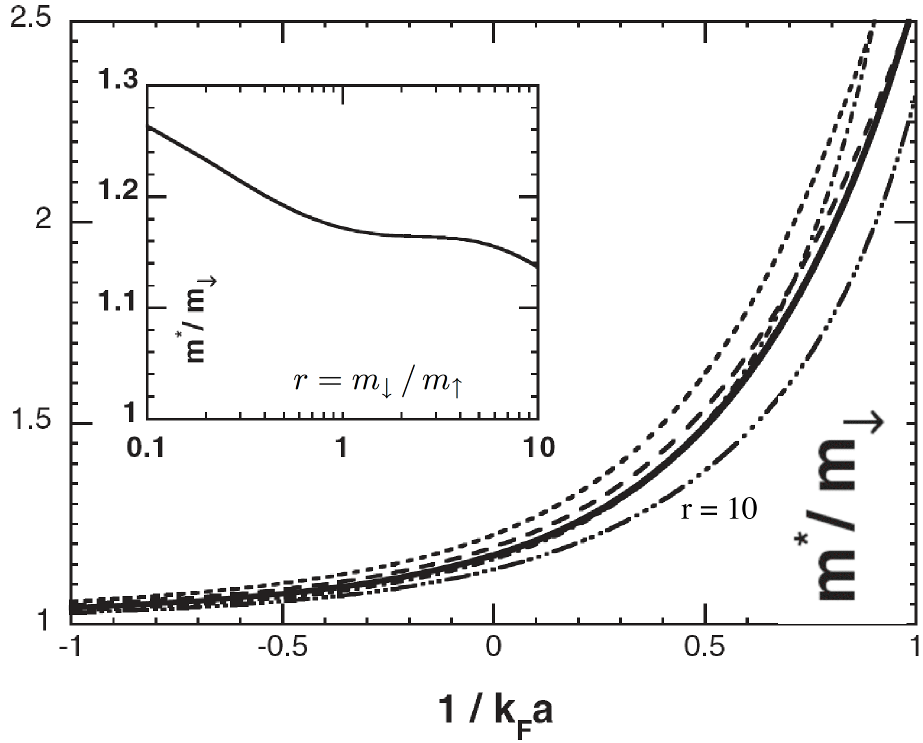
• Effective mass

1^{rst} order **$m^* / m = 1.17$** **2^{nd} order** **$m^* / m = 1.20$**

RC, Recati, Lobo and Chevy, PRL **98** (2007)



1^{rst} order



- **Nothing special** happens at **unitarity** :

unitarity physically not different from **BCS side**

Conclusion

Very interesting field !

Thank you for your attention !