Dilatonic Black Holes in Gauss-Bonnet Gravity

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CMC, Gal'tsov, Orlov, Phys Rev D (2007) [hep-th/0701004]

CMC, Gal'tsov, Orlov, Phys Rev D (2008) arXiv:0809:1720 [hep-th]

CMC, Gal'tsov, Ohta, Orlov, to appear

Outline

- Motivation
  - Black Hole Thermodynamics
  - Charged BHs
  - Dilatonic BHs (small BHs)
- Gauss-Bonnet Gravity
  - 4D Electric Charged BHs
  - 4D Dyonic BHs
  - Higher Dimensional BHs
- Conclusion

BH Thermodynamics

- Area increasing law: (Hawking, 1971)

$$\delta A \geq 0$$

  which analogous to the 2nd law for entropy ($\delta S \geq 0$) in thermodynamics.

- 1st law: (Bekenstein, Smarr, 1972)

$$\delta M = \frac{\kappa}{2\pi}\,\delta\frac{A}{4} \qquad (\delta E = T\delta S)$$

- Surface gravity: for Schwarzschild $\kappa = \frac{1}{4M}$.
  It is a constant (0th law).

BH Thermodynamics

- Hawking Temperature:

$$T = \frac{\kappa}{2\pi} \quad \longrightarrow \quad T = \frac{\hbar c^3}{8\pi k G M},$$

  Entropy:

$$S = \frac{A}{4} \quad \longrightarrow \quad S = \frac{k c^3 A}{4 \hbar G}$$

- The non-vanishing temperature indicates that the black hole is unstable, emitting thermal radiation (quantum effect).

Charged Black Holes

- The Einstein-Maxwell theory

$$S = \int d^4x \sqrt{-g}\left(R - F_{[2]}^2\right).$$

- The Reissner-Nordström (RN) spacetime is

$$ds_{\mathrm{RN}}^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2\, d\Omega_2^2,$$

$$F_{[2]} = \frac{Q}{r^2}\, dt \wedge dr,$$

$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}$, $\quad M$: mass, Q: charge.

Charged Black Holes

- Essential Properties:
  - Coordinate singularities (event/Cauchy horizons) at $f(r) = 0$, $\left(r_\pm = M \pm \sqrt{M^2 - Q^2}\right)$.
  - Spacetime singularity at $r = 0$.
- There are three types:
  - $M > |Q|$: black hole with two horizons,
  - $M = |Q|$: extreme limit with degenerated horizon at $r = M$,
  - $M < |Q|$: naked singularity.

Charged Black Holes

- For the extremal limit ($M^2 = Q^2, r_+ = r_-$), the horizons degenerate.
- The Hawking temperature is vanishing ($\kappa = (r_+ - r_-)/2r_+^2$), but the entropy (area of horizon) is non-vanishing ($r_H = M$).
- The non-vanishing entropy indicates the quantum degrees of freedom (maybe consequence of stringy effect — microscopic interpretation).

New Ingredients

- Scalar fields: Brans-Dicke, dilaton
- Extra dimensions: Kaluza-Klein
- Higher-rank form fields: holes $\longrightarrow$ branes

- Those new ingredients are all essential in the low energy effective string theory.

- Extremal black holes are generally corresponding to the SUSY configurations and the symmetry of solution is enhanced.

Dilatonic black holes

- The 4-dim low-energy Lagrangian of string theory

$$S = \int d^4x \sqrt{-g}\,\big(R - 2(\partial\phi)^2 - e^{2a\phi}\,F_{[2]}^2\big)\,.$$

- Dilatonic black holes (for $a = -1$): Gibbons-Maeda ’88, Garfinkle-Horowitz-Strominger ’91

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + R^2(r)\,d\Omega_2^2,$$

$$f(r) = (1 - r_+/r), \quad R^2(r) = r^2(1 - r_-/r)$$

$$M = r_+/2, \quad Q^2 = r_+ r_-/2$$

Dilatonic black holes

- Outer horizon: $r = r_+, \quad (r_+ = 2M, r_- = Q^2/M)$,
  Extreme case: $r_+ = r_-, \qquad (Q^2 = 2M^2)$.
- The dilaton charge is a secondary type parameter, namely it is not free but determined by mass and charge ($D \simeq -Q^2/2M$).
- The Hawking temperature is $T \simeq 1/8\pi M$.
- $R(r_+ = r_-) = 0 \quad \Rightarrow \quad$ The entropy vanishes for the extreme case.

Higher curvature correction

- Puzzles:
  - Zero entropy: one degree of freedom, (with non-zero temperature for $a^2 = 1$).
  - Singularity and horizon are coincident.
- Expectation: higher curvature correction, in particular to include Gauss-Bonnet term.

Gauss-Bonnet Gravity

- The 4D Gauss-Bonnet gravity

$$\mathcal{L} = R - 2(\partial\phi)^2 - \mathrm{e}^{2a\phi}\left(F_{[2]}^2 - \alpha\mathcal{L}_{\mathrm{GB}}\right).$$

where $\mathcal{L}_{\mathrm{GB}} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$

- Field equations are second order in the metric, linear in second derivative.
- It does not contain new propagating degrees of freedom besides the graviton.
- It appears in the low-energy expansions of string theory.

Entropy function

- Entropy for higher curvature theories: Wald's Noether charge approach

$$S = 4\pi \int_{r=r_+} \frac{\partial \mathcal{L}}{\partial R_{rtrt}}$$

- For the BHs with near horizon geometry of $AdS_2 \times S^{D-2}$, the entropy can be calculated from the near horizon data via entropy function

$$I = \int f \, dtdr$$

Entropy Calculation

- The entropy (calculated by Sen's entropy function) is twice the Bekenstein-Hawking entropy:

$$S = 2\pi\rho_0^2 = 2\left(\frac{A}{4}\right).$$

- This S-A relation has also been observed in

  Dabholkar, Kallosh, Maloney, '04;
  Hubeny, Maloney, Rangamani, '05;
  Bak, Kim, Rey, '05

- How general is this relation?

  Cai, CMC, Maeda, Ohta, Pang, Phys Rev D '08 arXiv:0712.4212 [hep-th]

Extremal BH in 4D GB Gravity

- Ansatz: metric and Maxwell field

$$ds^2 = -w(r)dt^2 + \frac{dr^2}{w(r)} + \rho^2(r)d\Omega_2^2,$$

$$A = -f(r)\,dt - q_m \cos\theta\, d\varphi.$$

- The Maxwell field can be directly solved

$$f'(r) = q_e \rho^{-2} \mathrm{e}^{-2a\phi}.$$

- The GB term breaks the discrete S-duality (electric $\leftrightarrow$ magnetic).

Near Horizon Expansion

- Consider the series expansions around some point $r = r_H$ (supposed to be a horizon) in powers of $x = r - r_H$: $(P(r) := \mathrm{e}^{2a\phi(r)})$

$$w(r) = \sum_{k=2}^{\infty} w_k x^k,\ \rho(r) = \sum_{k=0}^{\infty} \rho_k x^k,\ P(r) = \sum_{k=0}^{\infty} P_k x^k.$$

- The function w starts from the quadratic term (vanishing of w_0 means that $r = r_H$ is a horizon, vanishing of w_1 means that the horizon is degenerate).

Near Horizon Expansion

- Electric charge:

$$q_e = \frac{\sqrt{4\alpha + q_m^2}}{2\alpha + q_m^2}\,\frac{{\rho_0}^2}{2}.$$

- Any solution with finite horizon radius ρ_0 must have non-zero electric charge.
- With fixed ρ_0, q_e decreases with increasing q_m and approaches zero in the limit $q_m \to \infty$.
- For simplicity, lets firstly focus on purely electric charged black holes.

CMC, Gal'tsov, Orlov, Phys Rev D '07 [hep-th/0701004]

Near Horizon Expansion

- Near horizon expansion for electric solution

$$
\begin{aligned}
w(r) &\simeq \frac{1}{\rho_0^2}\left[x^2 - \frac{2(5a^2-3)}{3}\left(\frac{\alpha P_1}{a^2\rho_0^2}\right)x^3\right] + O(x^4),\\
\rho(r) &\simeq \rho_0\left[1 + (a^2-1)\left(\frac{\alpha P_1}{a^2\rho_0^2}\right)x - \frac{2a^2(a^4-6)}{(5a^2-3)}\left(\frac{\alpha P_1}{a^2\rho_0^2}\right)^2 x^2\right] + O(x^3),\\
P(r) &\simeq \frac{\rho_0^2}{\alpha}\left[\frac{1}{4} + a^2\left(\frac{\alpha P_1}{a^2\rho_0^2}\right)x + \frac{a^2(a^4-5a^2-3)}{(5a^2-3)}\left(\frac{\alpha P_1}{a^2\rho_0^2}\right)^2 x^2\right] + O(x^3).
\end{aligned}
$$

- There are two parameters: ρ_0 and P_1.
- The near horizon geometry is $AdS_2 \times S^2$

$$
ds^2 = -\frac{x^2}{\rho_0^2}dt^2 + \frac{\rho_0^2}{x^2}dx^2 + \rho_0^2 d\Omega_2^2.
$$

Asymptotic Expansion

- Asymptotic expansion: (asymptotically flat)

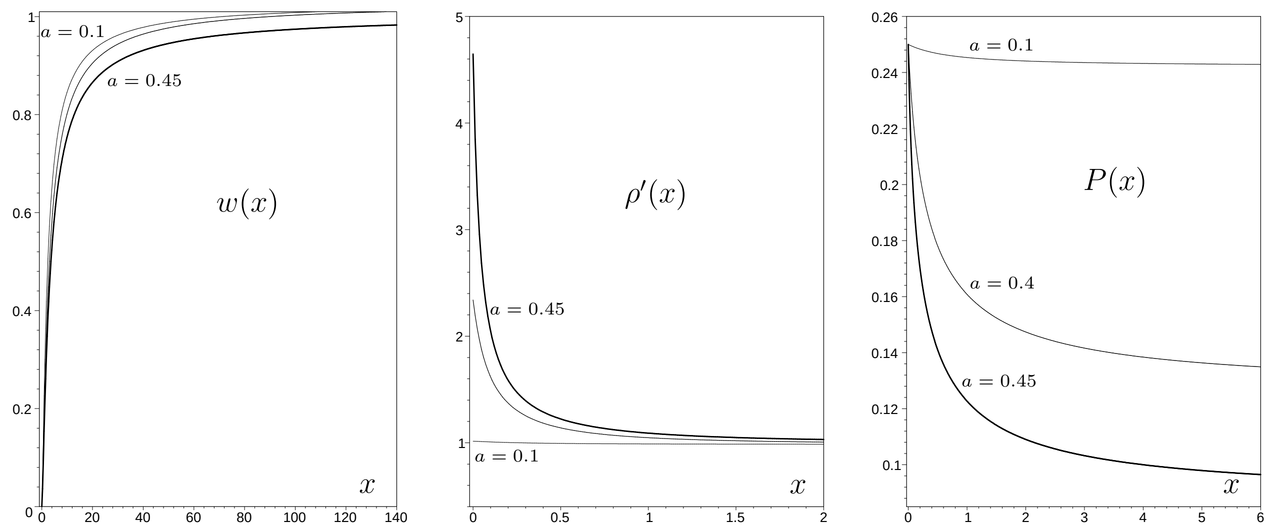
$$
\begin{aligned}
w(r) &= 1 - \frac{2M}{r} + \frac{\alpha Q_e^2}{r^2} + O(r^{-3}), \\
\rho(r) &= r - \frac{D^2}{2r} - \frac{D(2MD - \alpha a Q_e^2)}{3r^2} + O(r^{-3}), \\
\phi(r) &= \phi_\infty + \frac{D}{r} + \frac{2DM - \alpha a Q_e^2}{2r^2} + O(r^{-3}),
\end{aligned}
$$

where

$$
Q_e = q_e \mathrm{e}^{-a\phi_\infty}.
$$

Numerical Result

- The value of P_1 is fixed (depending on a, α) in order to get asymptotic flat solution.
- Mass M, dilaton charge D and asymptotic value of dilaton ϕ_∞ are determined by the value of only parameter ρ_0 (i.e. charge).

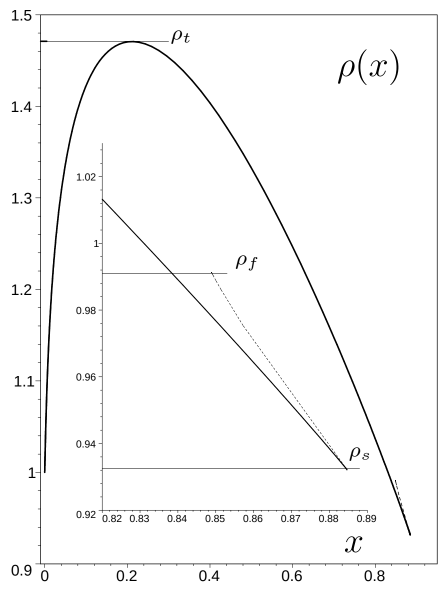


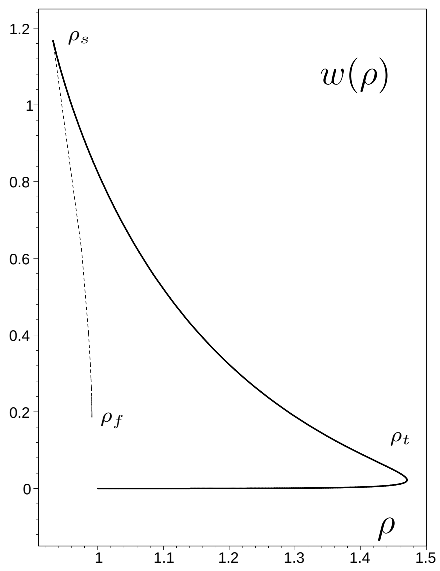
Critical Point

- The global solution exist only when a less than a critical value

$$a_{\rm cr} \simeq 0.488219703.$$

- If $a > a_{\rm cr}$ singularity appears outside horizon.





Extremal dyonic black holes

- Einstein-Maxwell-dilaton (EMD) theory
  - The horizon does not shrink to a point.
  - The extremal dyon solutions exist only for discrete values of the dilaton coupling constant a

$$a_i^2 = 1 + 2 + \cdots + i = \frac{i(i+1)}{2}.$$

Poletti, Twamley, Wiltshire, CQG ’95 [hep-th/9502054]

Extremal dyonic black holes

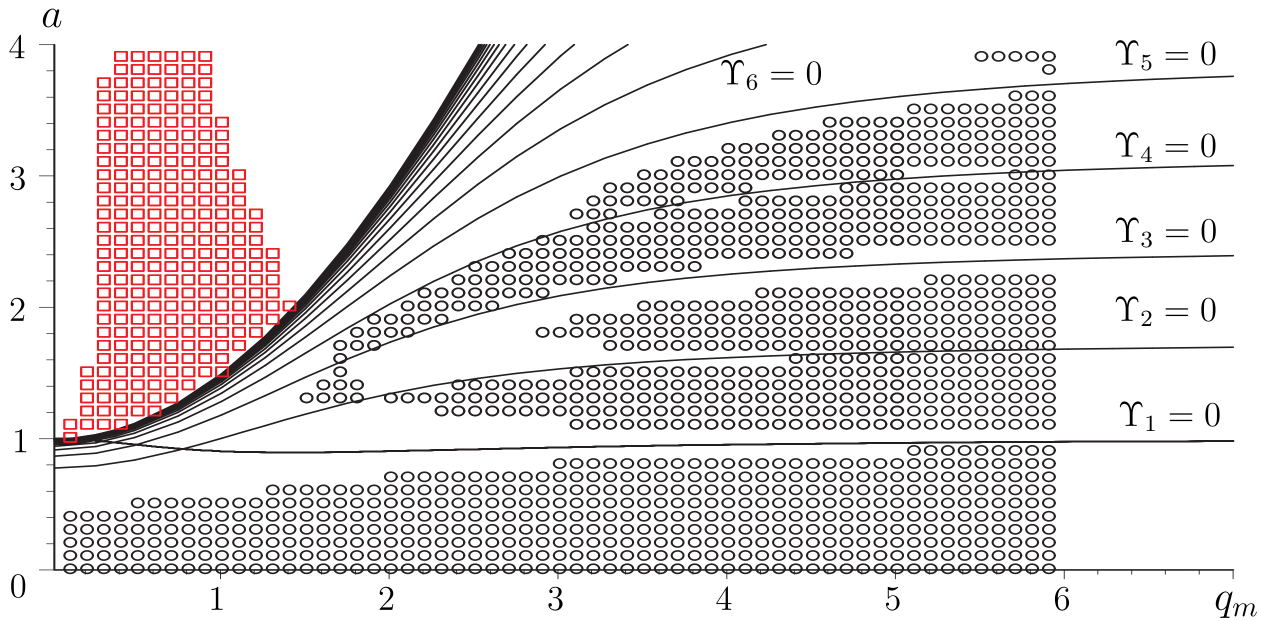
- EMD theory with Gauss-Bonnet correction (EMDGB)

CMC, Gal'tsov, Orlov, Phys Rev D '08 arXiv:0809:1720 [hep-th]

- There are two classes of solution:
  - Asymptotic Flat: The Gauss-Bonnet term acts as a dyon hair tonic enlarging the allowed values of a to continuous domains in the plane (a, q_m).
  - Asymptotic Linear Dilaton Background: black holes (magnetic) on the linear dilaton background (electric).

Extremal dyonic black holes

- The domains of existence of EDGB dyons



Black circles: asymptotical flat;

Red squares: black holes on the linear dilaton background

Extremal dyonic black holes

- The entropy-area relation

$$S = 2\pi q_e \sqrt{q_m^2 + 4\alpha} = \pi\rho_0^2 + \frac{2\pi\alpha\rho_0^2}{2\alpha + q_m^2}.$$

  - The Bekenstein-Hawking entropy-area relation, $S = A/4, \ A = 4\pi\rho_0^2$, is recovered when $\alpha = 0$ or $q_m \to \infty$.
  - The magnetic charge parameter q_m vanishing, the entropy has double Bekenstein-Hawking value.
  - For a generic extremal dyonic solution, the black hole entropy can not be completely expressed in terms of its horizon area.

Higher dimensional BHs

- The solutions in $D = 5, 6$ are similar with solutions in $D = 4$.
- However, for $D \geq 7$ the property of extremal black holes is drastically different.
  - The turning points appear in pair and the singularity does not appear after turning point.

Conclusion

- Black hole thermodynamics:
  - Extremal Charged BHs: $T = 0, S > 0$,
  - Extremal Dilatonic Charged BHs: $S = 0$.
- Higher curvature corrections are essential.
- 4D GB gravity admits black hole solutions with the horizons of $AdS_2 \times S^2$.
- The extremal dilaton BH (electric) with higher curvature correction consists: stretching its horizon and fixing the value of ϕ_∞.

Conclusion

- A purely local analysis is insufficient to fully understand the entropy of the curvature corrected black holes.
- The existence of the threshold value of the dilaton coupling constant under which the global solutions cease to exist is an interesting new phenomenon which may be related to the string-black hole transition.