Limiting Polarization – Missing Link in the Theory of the Pulsar Radio Emission

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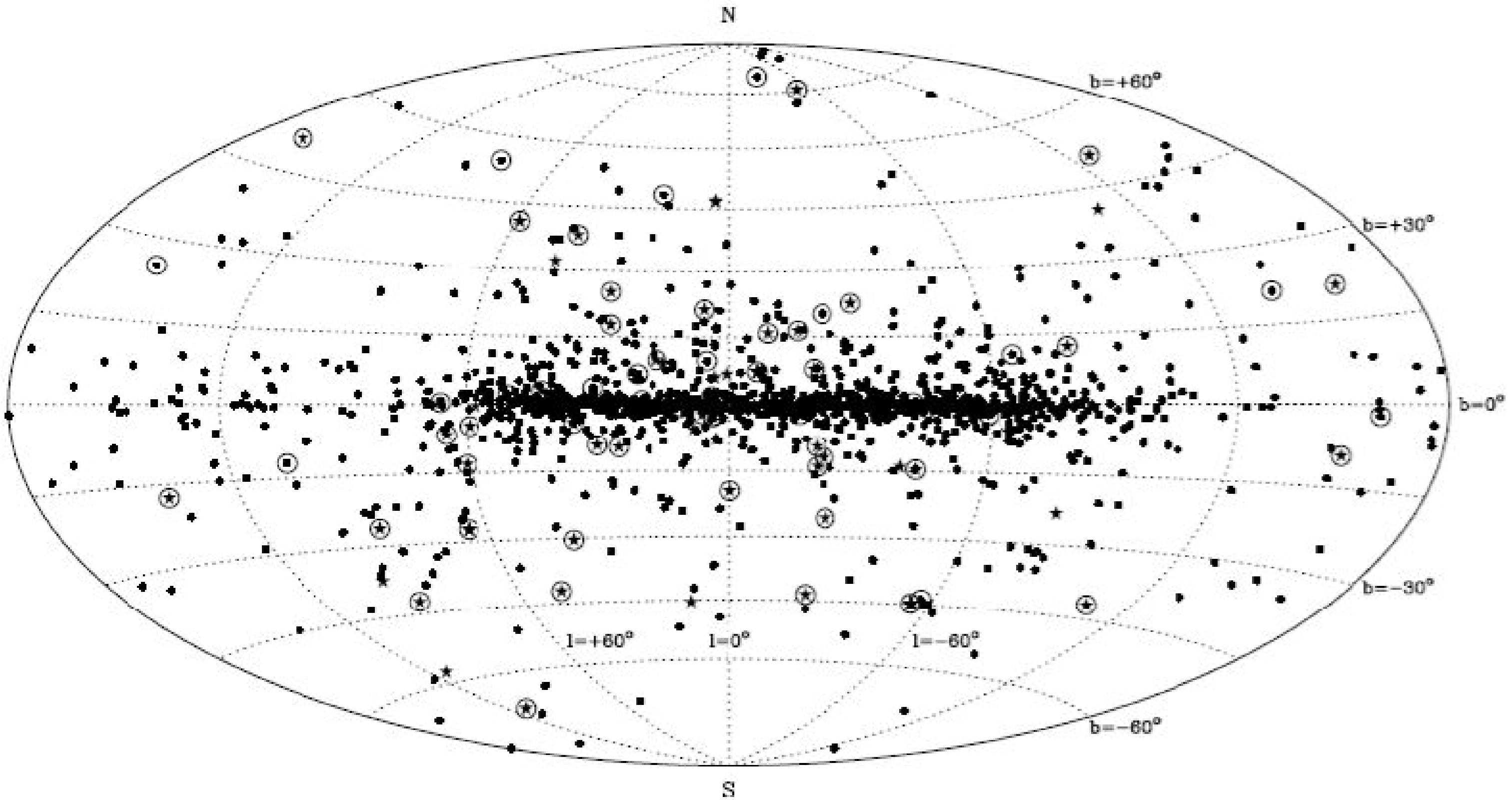
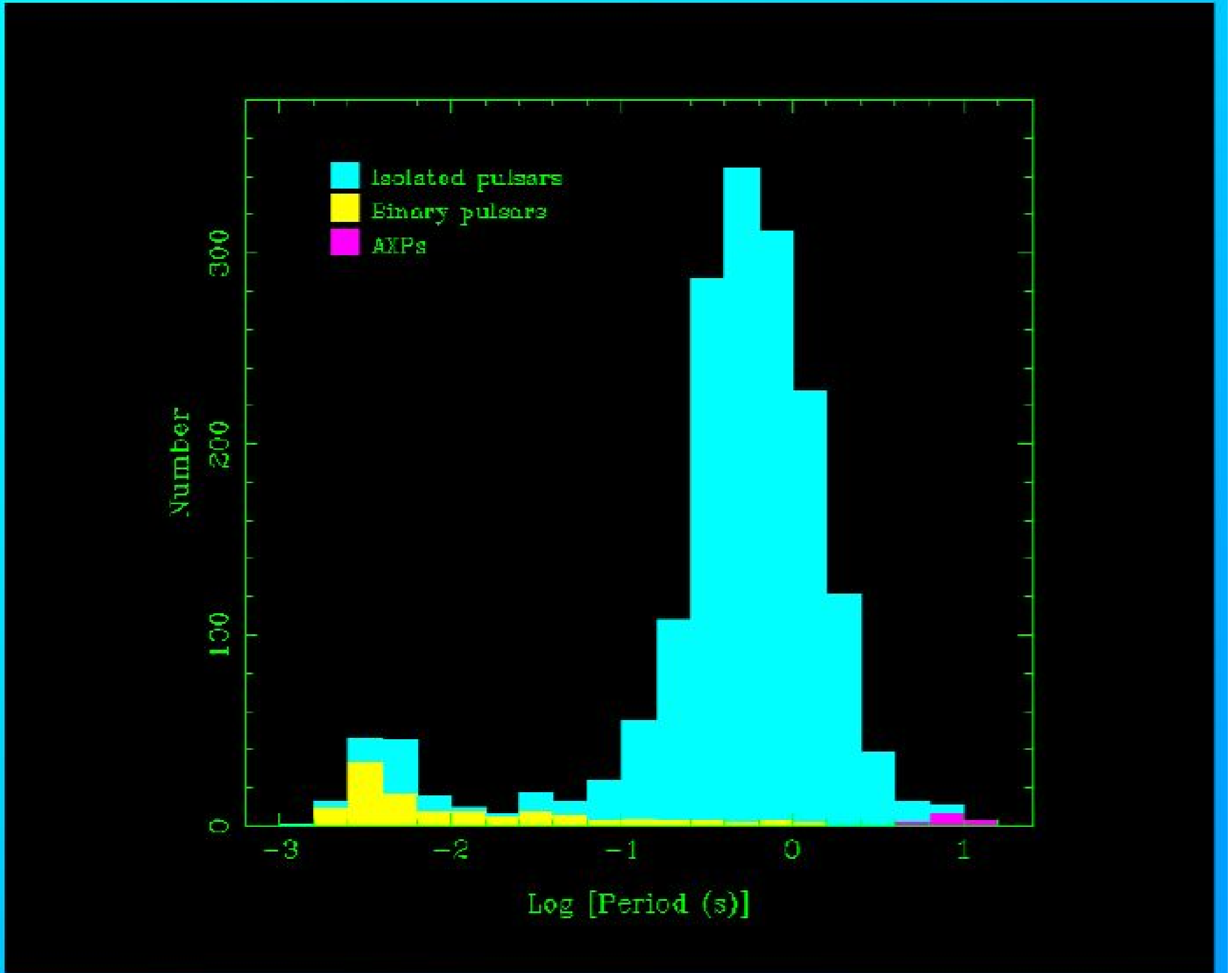
Radio pulsars – rotating solitary* neutron stars

- Mass $M \sim 1.4\ M_{\odot}$
- Radius $R \sim$ (10–15) km
- Rotating period $P \sim 1$ s
- Magnetic field $B_0 \sim 10^{12}$ G
- Radio luminosity $L_r \sim 10^{28}$ erg/s (~10^{-4} –10^{-6})
- Coherent mechanism: $T \sim 10^{28}$ K (~10^{40} ???)


N
b=+60°
b=+30°
b=0°
b=-30°
b=-60°
l=+60°
l=0°
l=-60°
S

Spin-Powered Pulsars: A Census

- Number of known pulsars: 1765
- Number of millisecond pulsars: 170
- Number of binary pulsars: 131
- Number of AXPs: 12
- Number of pulsars in globular clusters: 99*
- Number of extragalactic pulsars: 20



* Total known: 129 in 24 clusters
(Paulo Freire's web page)

Data from ATNF Pulsar Catalogue, V1.25
(www.atnf.csiro.au/research/pulsar/psrcat; Manchester et al. 2005)

The key electrodynamic idea

(Kardashev,1964; Pacini, 1967)

Magneto-dipole (vacuum) radiation

$$W_{\mathrm{tot}} = \frac{1}{6}\frac{B_0^2\Omega^4 R^6}{c^3}\sin^2\chi$$

$W_{\mathrm{tot}} \sim 10^{32}$ erg/s

In reality is it not so (magnetosphere is filled with plasma), but is enough for evaluation

The key electrodynamic idea

The moment of the true – Crab pulsar

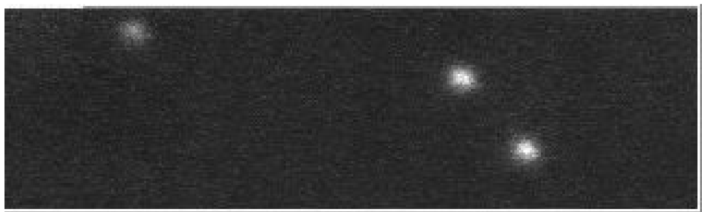
$$P = 0.033 \text{ c},$$

$$\mathrm{d}P/\mathrm{d}t = 4\ 10^{-13}$$

Total energy losses $W_{\mathrm{tot}} = -I\,\Omega\,\mathrm{d}\Omega/\mathrm{d}t \sim 5\ 10^{38}$ erg/s

Dynamical age $\tau_{\mathrm{D}} = P/(2\ \mathrm{d}P/\mathrm{d}t) \sim 1000$ years

Optical pulsations

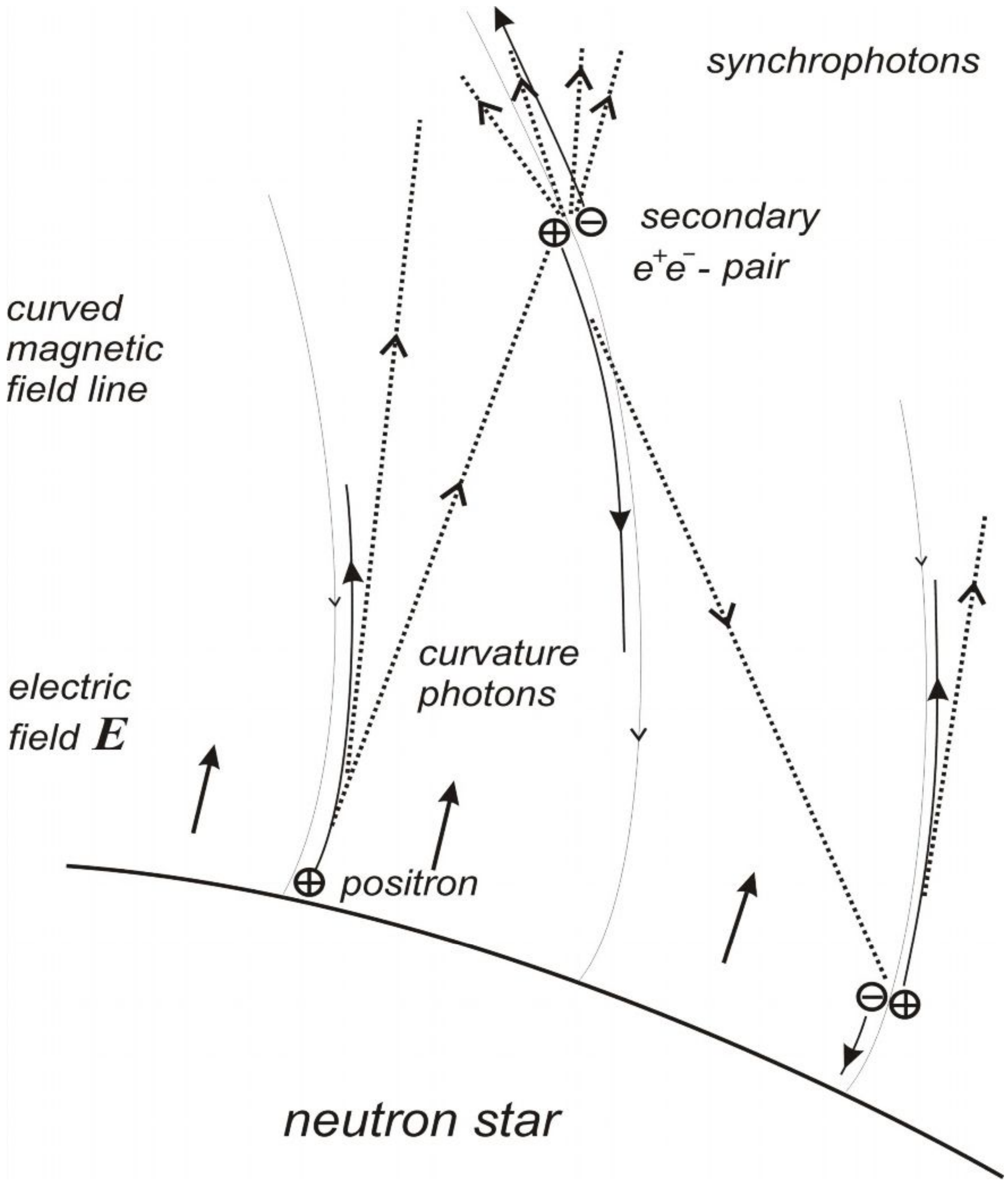


Strong magnetic field

$B \sim 10^{12}$ G $\sim B_{crit} = m_e^2 c^3/e\hbar = 4.4\ 10^{13}$ G

- $$w(\gamma \rightarrow e^+ e^-) \sim \exp\left(-\frac{8}{3}\frac{B_{crit}}{B_\perp}\frac{m_e c^2}{E_{ph}}\right)$$
- 1D motion $$\tau_s \approx \frac{1}{\omega_B}\left(\frac{c}{\omega_B r_e}\right) \sim 10^{-15}\ \text{s}$$
- Electric field $$E_{\parallel} \sim \frac{\Omega R}{c} B_0$$

Particle creation



To create pairs it's necessary

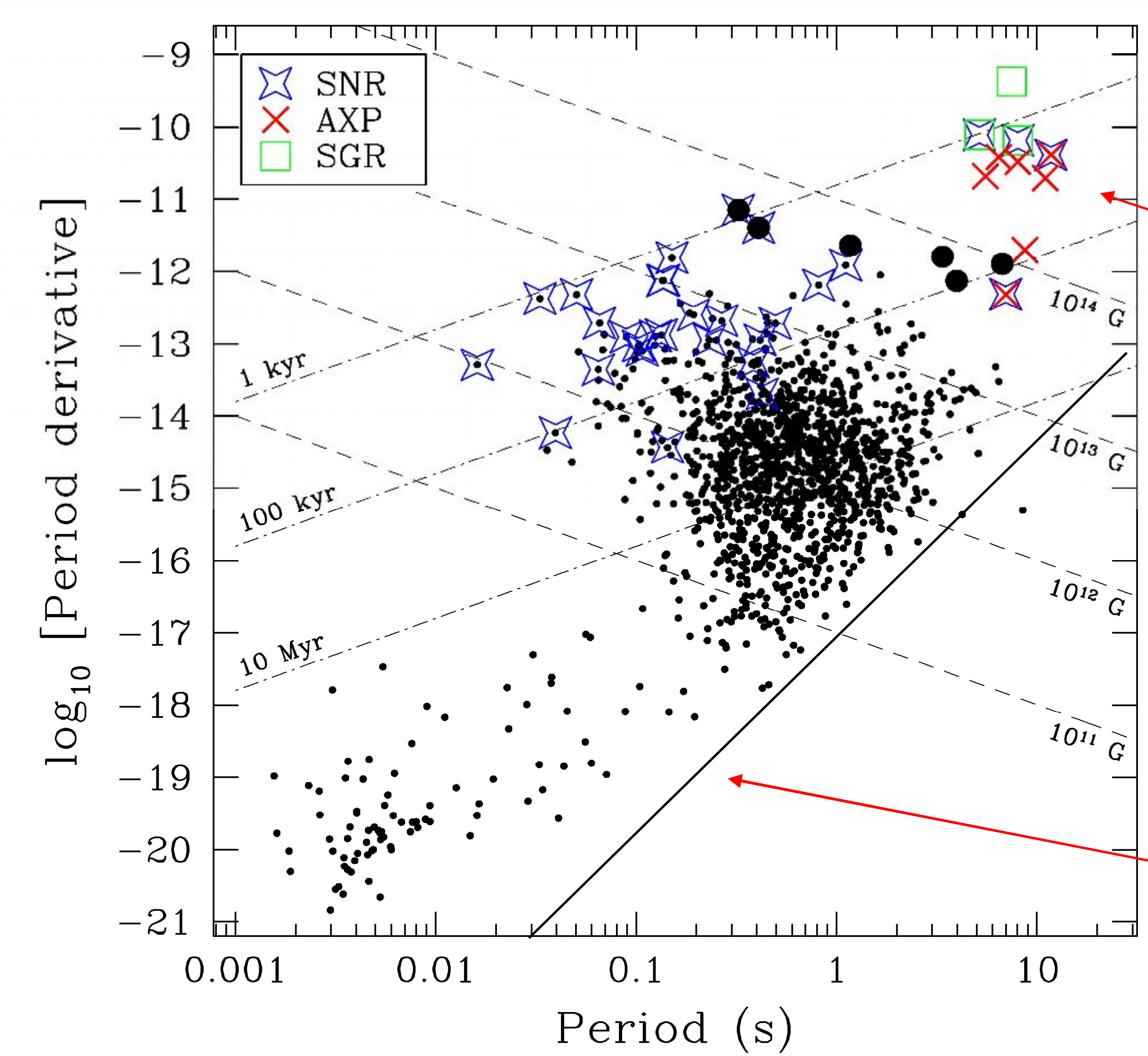
- Large enough electric field

  (hence, small enough rotation period P)
- Curvature of the magnetic field lines

  (impossible near the very magnetic pole)
- Critical charge density (which is necessary to screen longitudinal electric field)

$$\rho_{\mathrm{GJ}} = -\frac{\Omega B}{2\pi c}$$

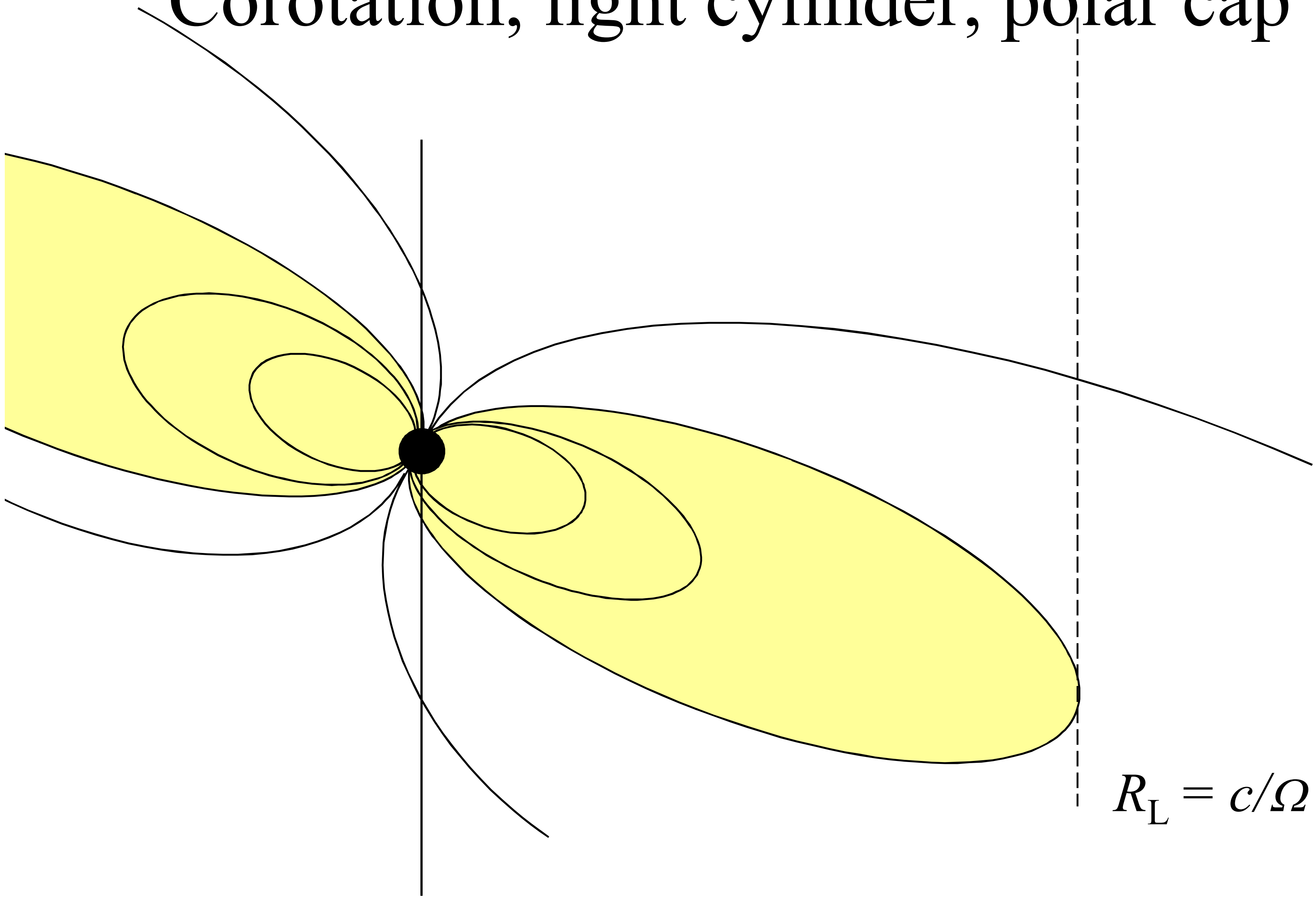
PPdot – diagram



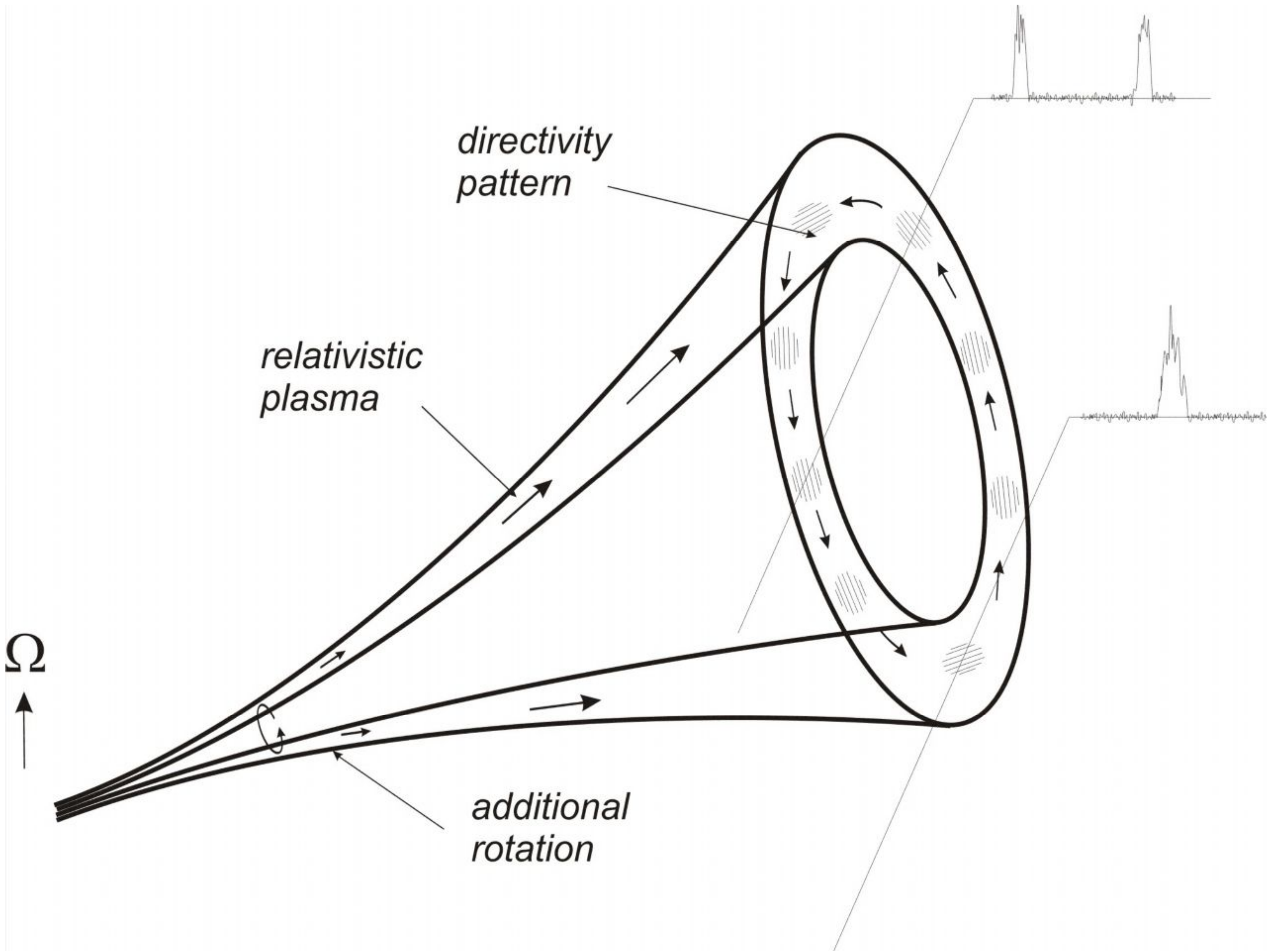
Magnetars
(X-ray sources)

Death line

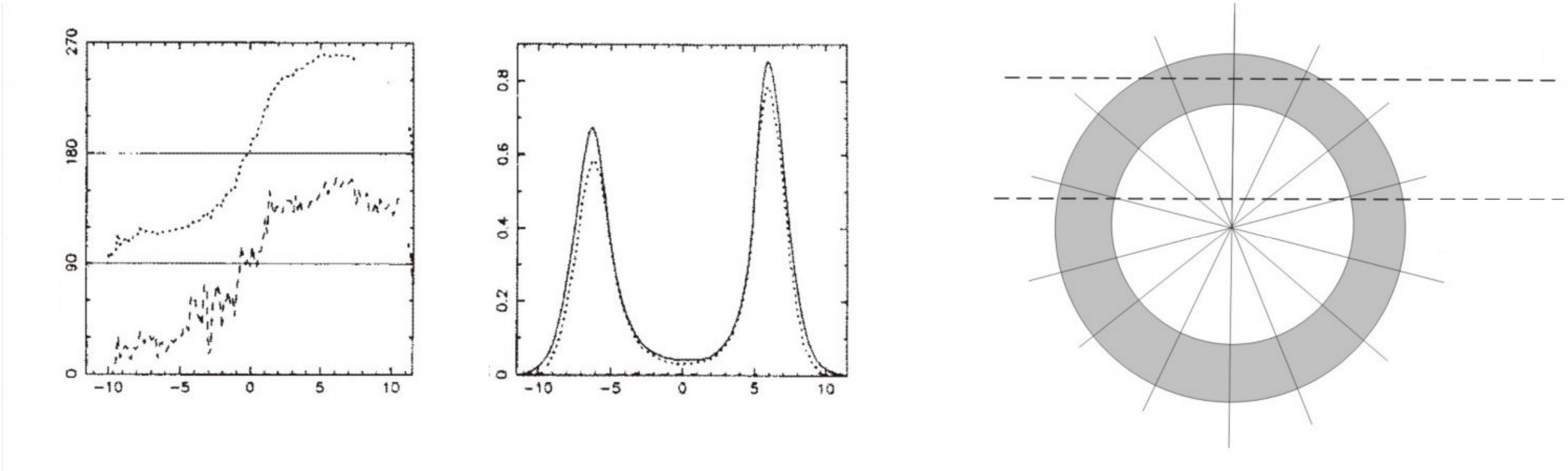
Corotation, light cylinder, polar cap



“Hollow cone” model



Correlation, orthogonal modes



Periphery passage

– single profiles, small change of the *p.a.*

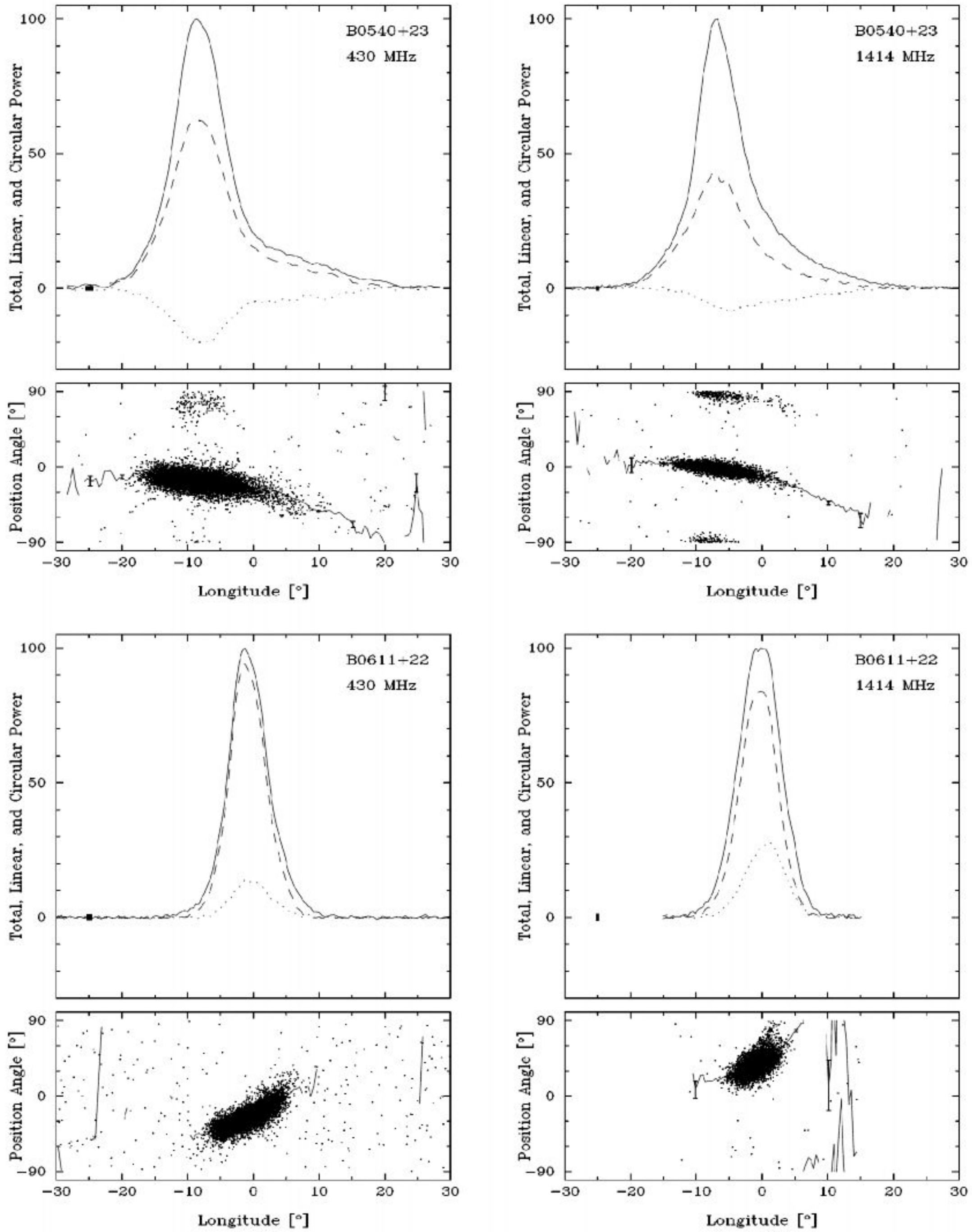
Central passage

– double profiles, *p.a.* changes up to 180°.

Position angle *p.a.*

$$p.a. = \arctan\left(\frac{\sin\chi\sin\varphi}{\sin\xi\sin\chi - \sin\xi\cos\chi\cos\varphi}\right)$$

T.Hankins,
J.Rankin, 2008



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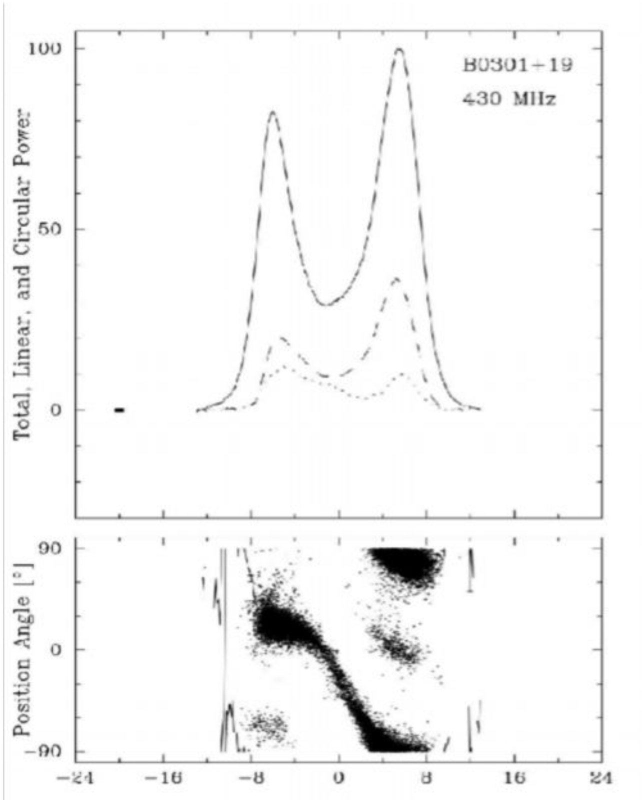

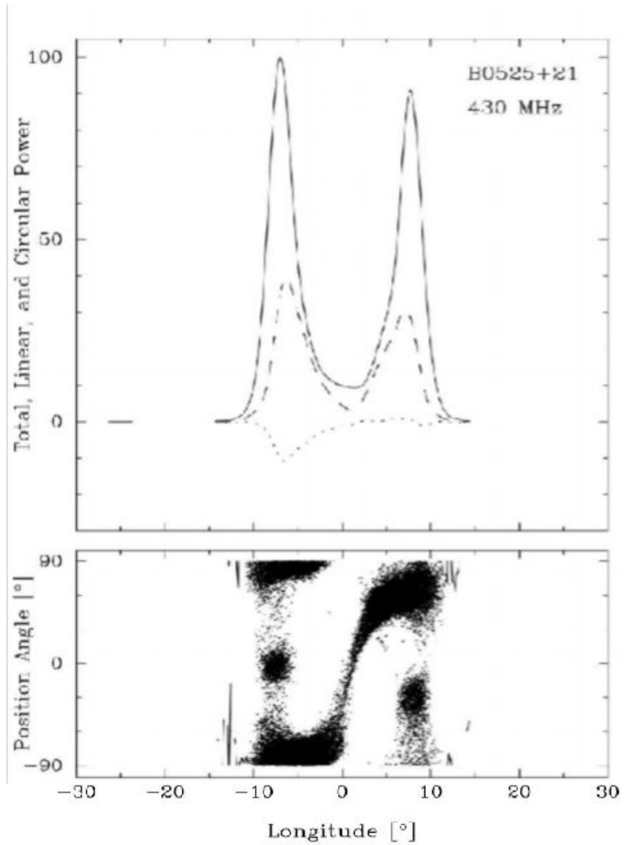

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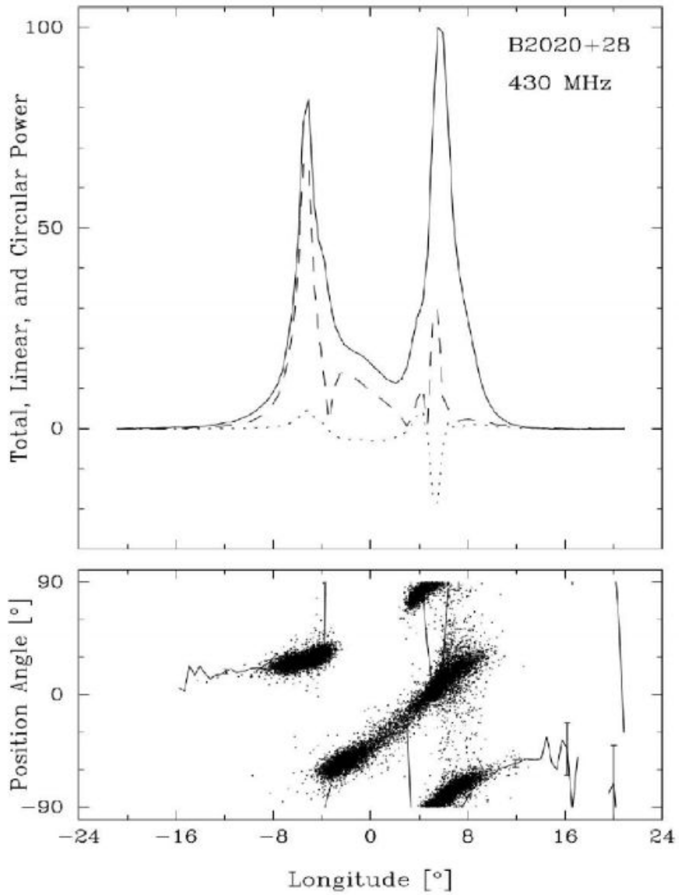
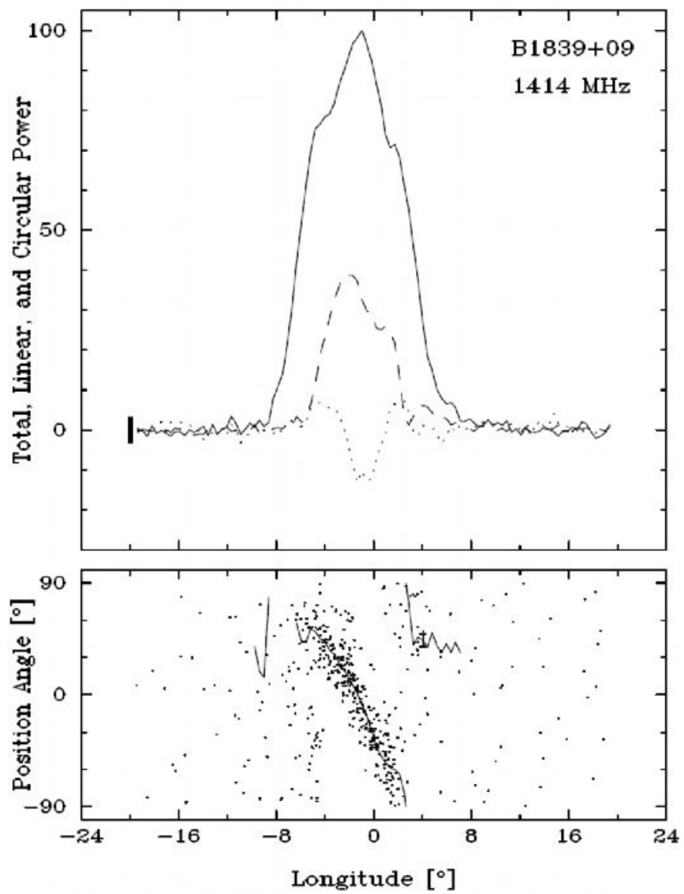
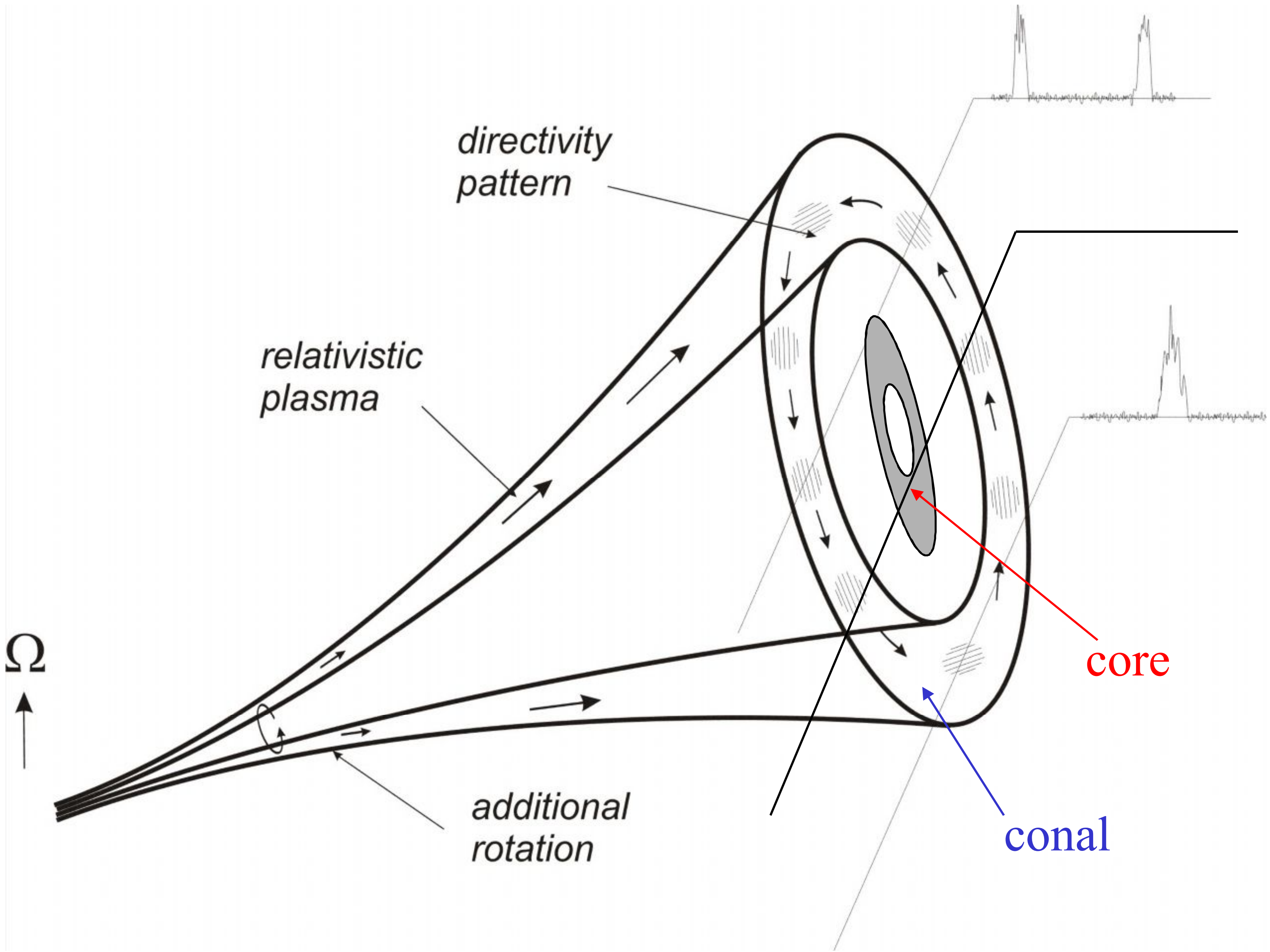




“Core-conal” model



P.Weltevrede, S.Johnston, MNRAS **391**, 1210 (2008)

$\dot{E}$ [erg s^{-1}]	Single	Double	Multiple	Total
$10^{35} - 10^{38}$	27 (53%)	17 (33%)	7 (14%)	51
$10^{33} - 10^{35}$	53 (47%)	43 (38%)	16 (14%)	112
$10^{28} - 10^{33}$	52 (46%)	46 (40%)	16 (14%)	114

Everything is clear

- Stability of pulsation – neutron star rotation
- Energy source – kinetic energy of rotation
- Mechanism of energy loss – electrodynamics
- Neutron star is a radio pulsar if there is secondary electron-positron generation near magnetic poles

Everything is clear?

- Stability of pulsation – neutron star rotation
- Energy source – kinetic energy of rotation
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- Neutron star is a radio pulsar if there is secondary electron-positron generation near magnetic poles
- Radio emission – ????

Theory of Radio Emission

- **Properties of the outgoing plasma**

(consensus)

- **Coherent mechanism**

Base instability

Saturation (nonlinear effects)

(no common point of view)

- **Propagation effects**

(there is the missing link)

Theory of Radio Emission

- **Properties of the outgoing plasma**

(consensus)

Concentration of the electon-positron plasma
$n = \lambda\, n_{\mathrm{GJ}}$ (primary beam $n \sim n_{\mathrm{GJ}}$)

Multiplicity parameter

$$\lambda \sim 10^4$$

Particle energy: beam – $\gamma \sim 10^7$, main flow

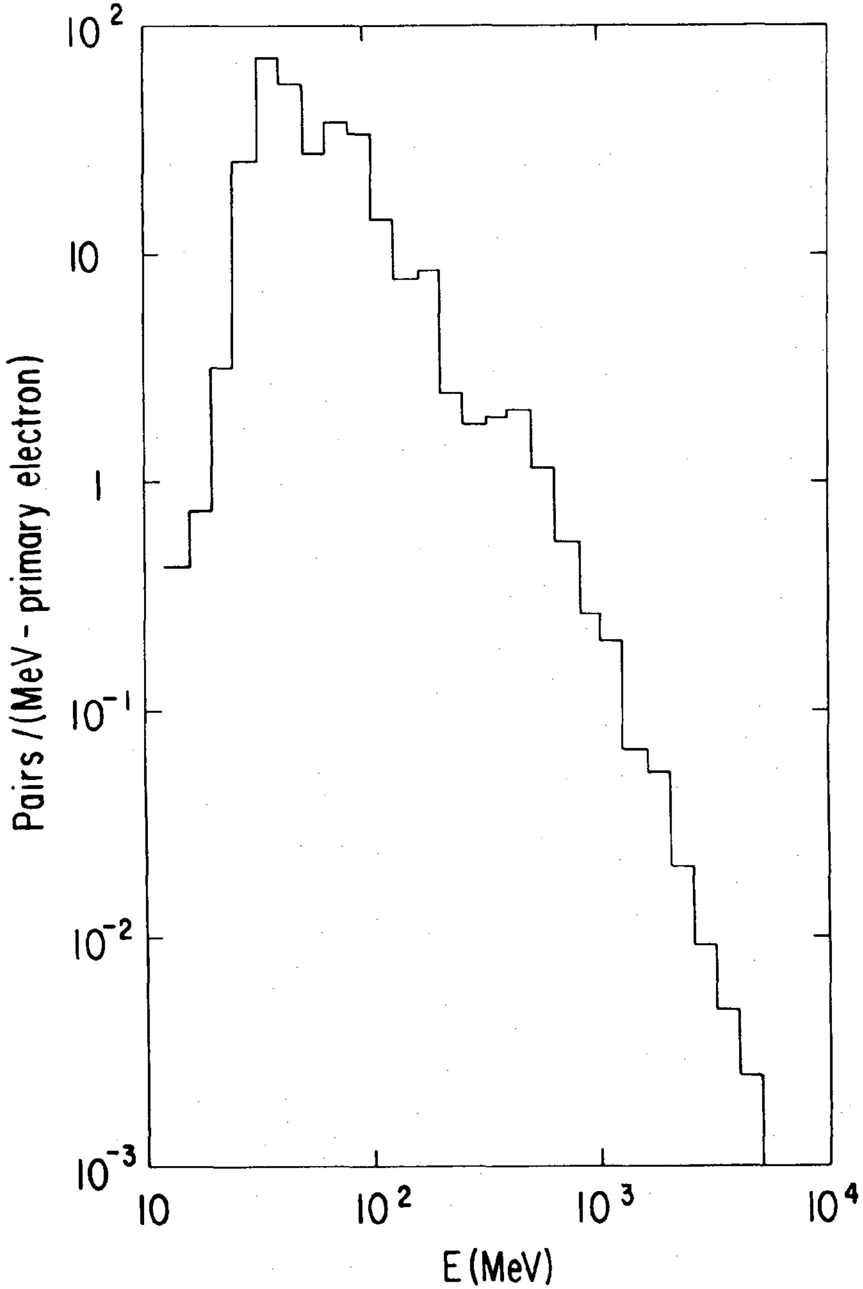
$$\gamma \sim 100$$

Ejection rate 10^{32} pairs/s (Crab – 10^{40} pairs/s)

Secondary electron-positron plasma

(J. K. Daugherty , A. K.Harding ApJ **252** 337 1982

A.V.Gurevich, Ya.N.Istomin, 1987)



Dielectric tensor

$$\varepsilon_{ij} = \begin{pmatrix} 1+ < \frac{\omega_p^2 \varpi^2 \gamma}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & i < \frac{\omega_p^2 \omega_B \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & < \frac{\omega_p^2 \gamma k_x v_\parallel \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \\ -i < \frac{\omega_p^2 \omega_B \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & 1+ < \frac{\omega_p^2 \varpi^2 \gamma}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & -i < \frac{\omega_p^2 \omega_B k_x v_\parallel}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \\ < \frac{\omega_p^2 \gamma k_x v_\parallel \varpi}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & i < \frac{\omega_p^2 \omega_B k_x v_\parallel}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > & 1- < \frac{\omega_p^2}{\varpi^2 \gamma^3} > + < \frac{\omega_p^2 \gamma k_x^2 v_\parallel^2}{\omega^2(\omega_B^2 - \gamma^2 \varpi^2)} > \end{pmatrix}$$

$$\varpi = \omega - k_x v_\parallel$$

Main parameters

$$\Delta n = -\frac{1}{2} < \frac{\omega_p^2 \omega_B^2}{\gamma^3 \varpi^2 (\omega_B^2 - \gamma^2 \varpi^2)} > \frac{\sqrt{q^2+1}}{q} \sin^2\theta$$

$$q = \frac{\omega_B}{2\omega} \frac{\sin^2\theta}{\cos\theta} \cdot \frac{\lambda}{\gamma^3 (1-\cos\theta)^3}$$

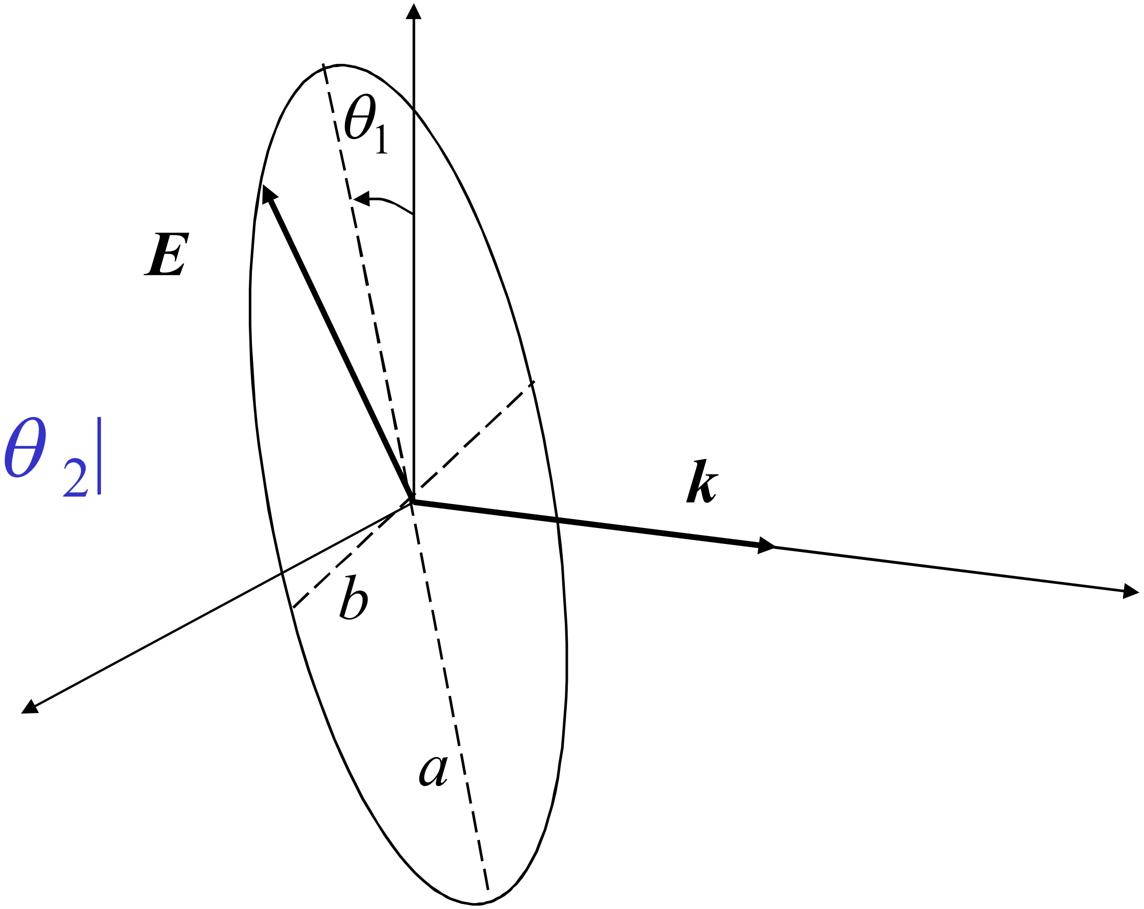
$$K_i^{-1} = i\frac{E_x}{E_y} = q \pm \sqrt{1+q^2}$$

$q >> 1$ ($K = 2q$, $1/2q$) – linear polarization

$q << 1$ ($K = +1, -1$) – circular polarization

Polarization

$K = a/b = |\mathrm{th}\theta_2|$



Beskin, Gurevich & Istomin (1988,1993)

In our theory we have included into consideration the curvature of magnetic field lines

$$\begin{pmatrix} \varepsilon_{xx} & \varepsilon_{xz} \\ \varepsilon_{zx} & \varepsilon_{zz} \end{pmatrix}$$

$$= \delta_{ij} - 2\pi\mathrm{i}\,\frac{R_{\mathrm{c}}^{2/3}}{k_{\parallel}^{1/3}}\int \mathrm{d}p_{\varphi}\,\frac{\omega_{\mathrm{p}}^{2}}{\omega}\,\frac{\partial f^{(0)}}{\partial p_{\varphi}}\begin{pmatrix} \dfrac{\mathcal{F}''(\zeta)}{(k_{\parallel}R_{\mathrm{c}})^{2/3}} & -\mathrm{i}\,\dfrac{\mathcal{F}'(\zeta)}{(k_{\parallel}R_{\mathrm{c}})^{1/3}} \\ \mathrm{i}\,\dfrac{\mathcal{F}'(\zeta)}{(k_{\parallel}R_{\mathrm{c}})^{1/3}} & \mathcal{F}(\zeta) \end{pmatrix}$$

$$\mathcal{F}(\zeta) = \mathrm{Ai}(\zeta) + \mathrm{iGi}(\zeta) = \frac{1}{\pi}\int_{0}^{\infty}\mathrm{d}\tau\,\exp\left(\mathrm{i}\tau\zeta + \mathrm{i}\,\frac{\tau^{3}}{3}\right)$$

$$\zeta = 2(\omega - k_{\parallel}v_{\varphi})\,\frac{R_{\mathrm{c}}^{2/3}}{k_{\parallel}^{1/3}v_{\varphi}}$$

‘Hollow cone’ – implicit assumptions

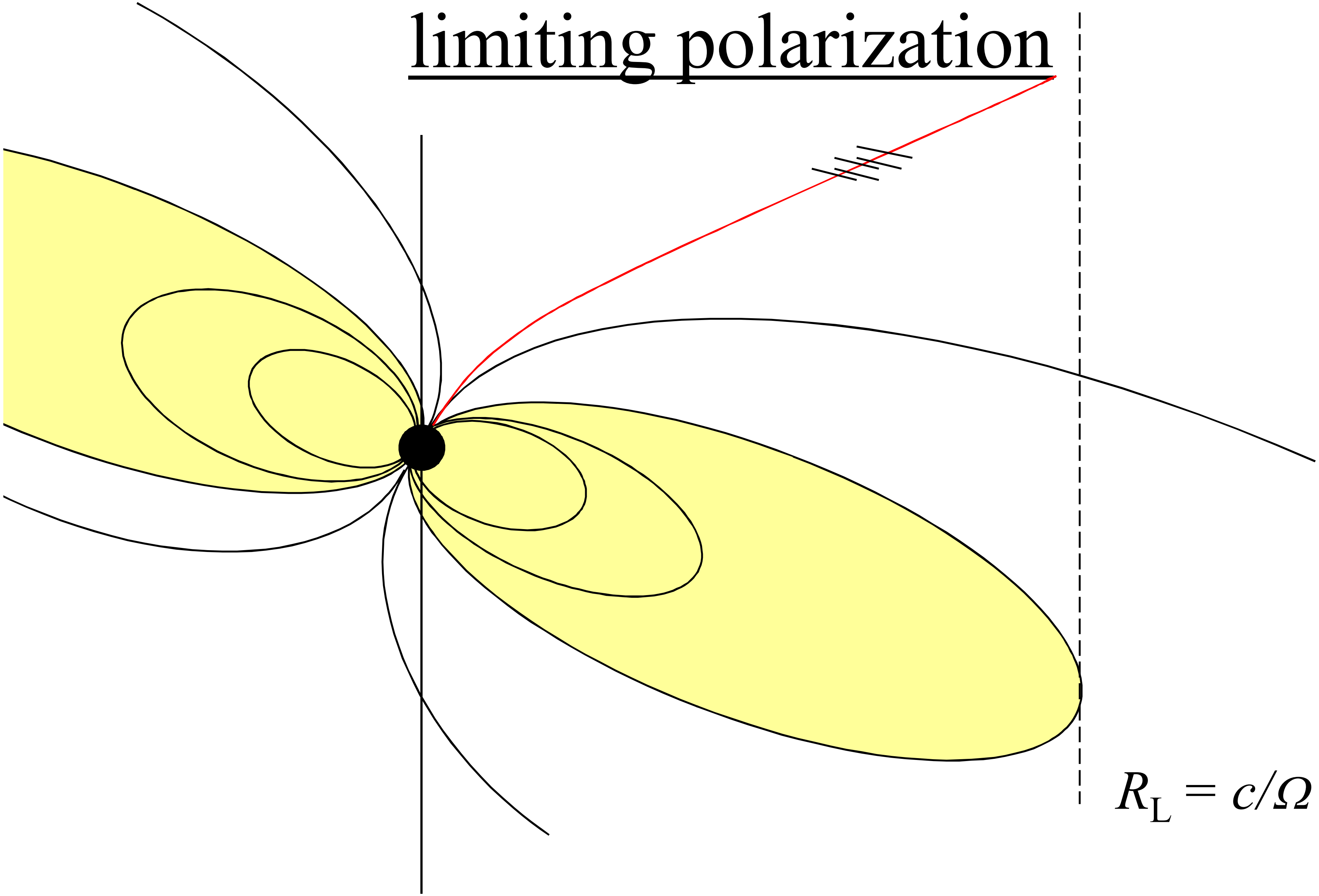
- Rectilinear propagation of radio waves
- Cyclotron absorption is not important
- Polarization is formed in the region of radiation

‘Hollow cone’ – implicit assumptions

- Rectilinear propagation of radio waves
- Cyclotron absorption is not important
- Polarization is formed in the region of radiation

All these points are incorrect

Refraction, cyclotron absorption, limiting polarization



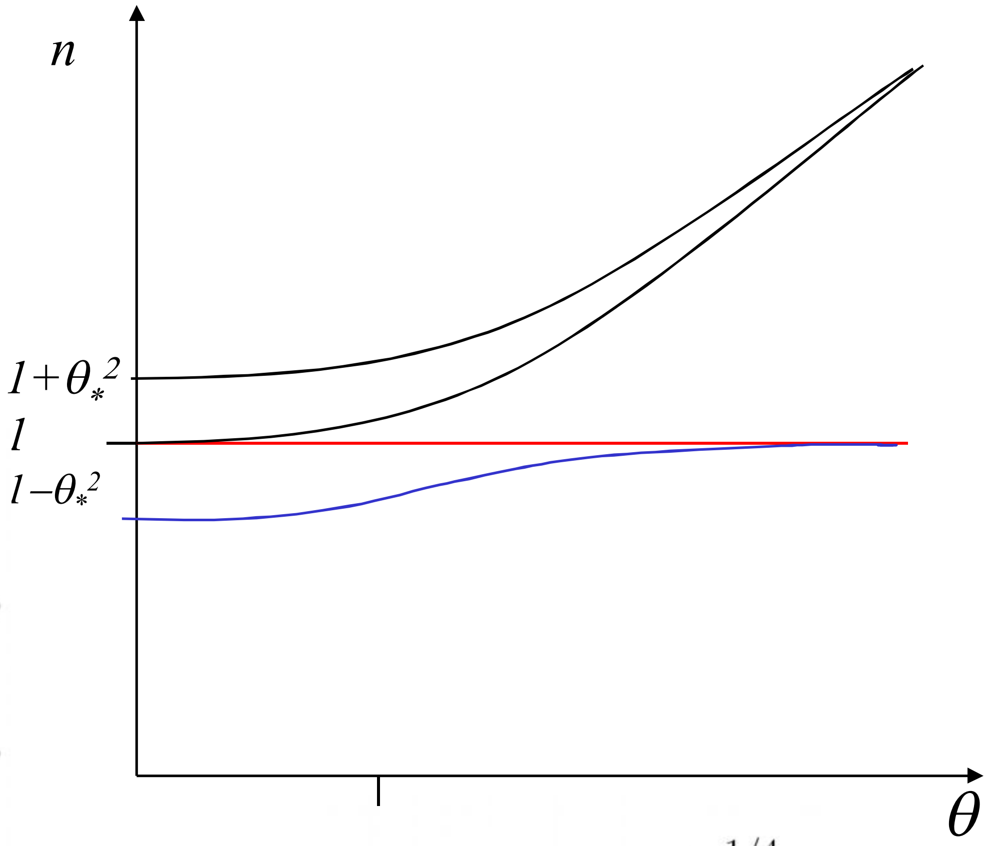
Refraction

J.Barnard, J.Arons, ApJ, **302**, 138 (1986)

$$\varepsilon_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1- < \frac{\omega_p^2}{\varpi^2\gamma^3} > \end{pmatrix}$$

If $$A_{\rm p} = \frac{\omega_p^2}{\omega^2} < \gamma > \gg 1$$

$$\begin{aligned} n_1 &= 1, \\ n_2 &= 1 + \frac{\theta^2}{4} - \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2}, \\ n_3 &= 1 + \frac{\theta^2}{4} + \left(\frac{\omega_p^2}{\omega^2} < \frac{1}{\gamma^3} > + \frac{\theta^4}{16} \right)^{1/2}, \\ n_4 &= \frac{1}{\cos\theta}. \end{aligned}$$



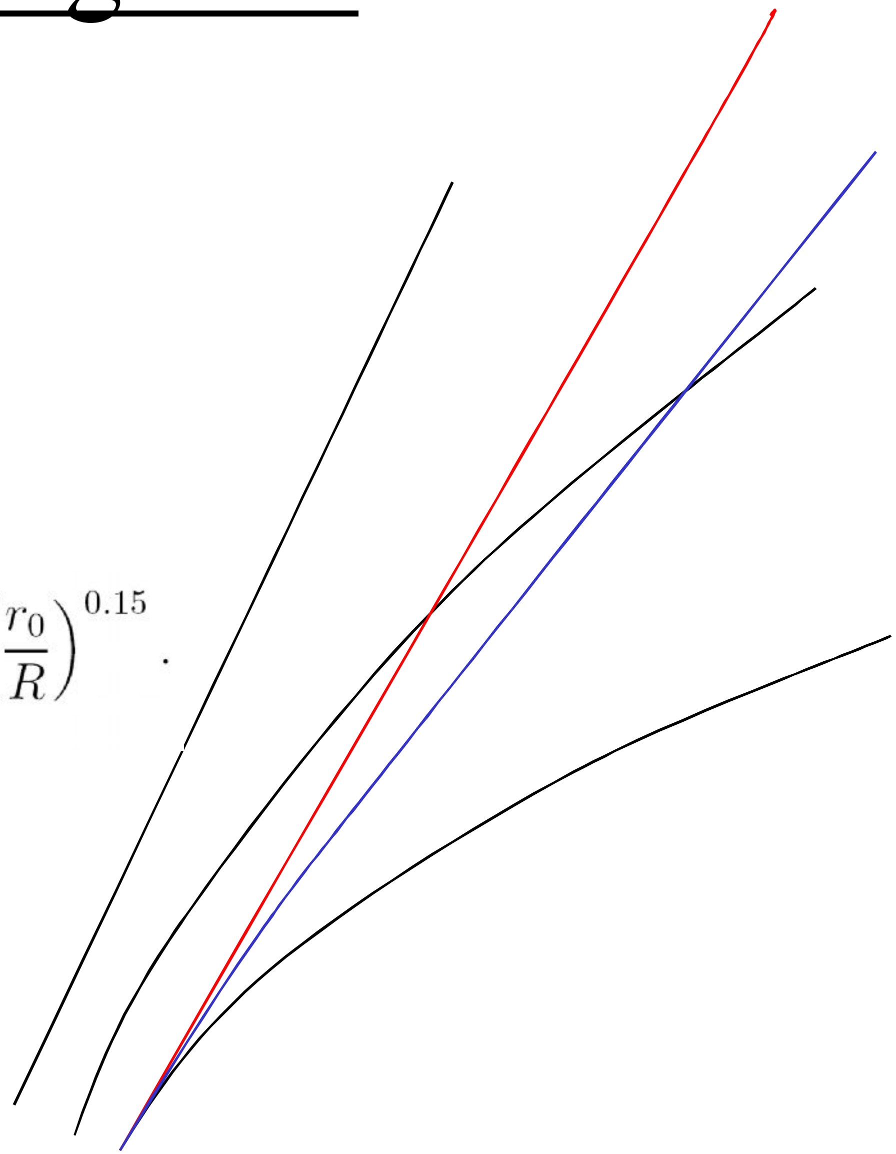
$$\theta_* = < \left(\frac{\omega_p^2}{\omega^2\gamma^3} \right)^{1/4} >$$

Propagation

$$\frac{\mathrm{d}r_\perp}{\mathrm{d}l} = \frac{\partial}{\partial k_\perp}\left(\frac{k}{n_j}\right),$$

$$\frac{\mathrm{d}k_\perp}{\mathrm{d}l} = -\frac{\partial}{\partial r_\perp}\left(\frac{k}{n_j}\right)$$

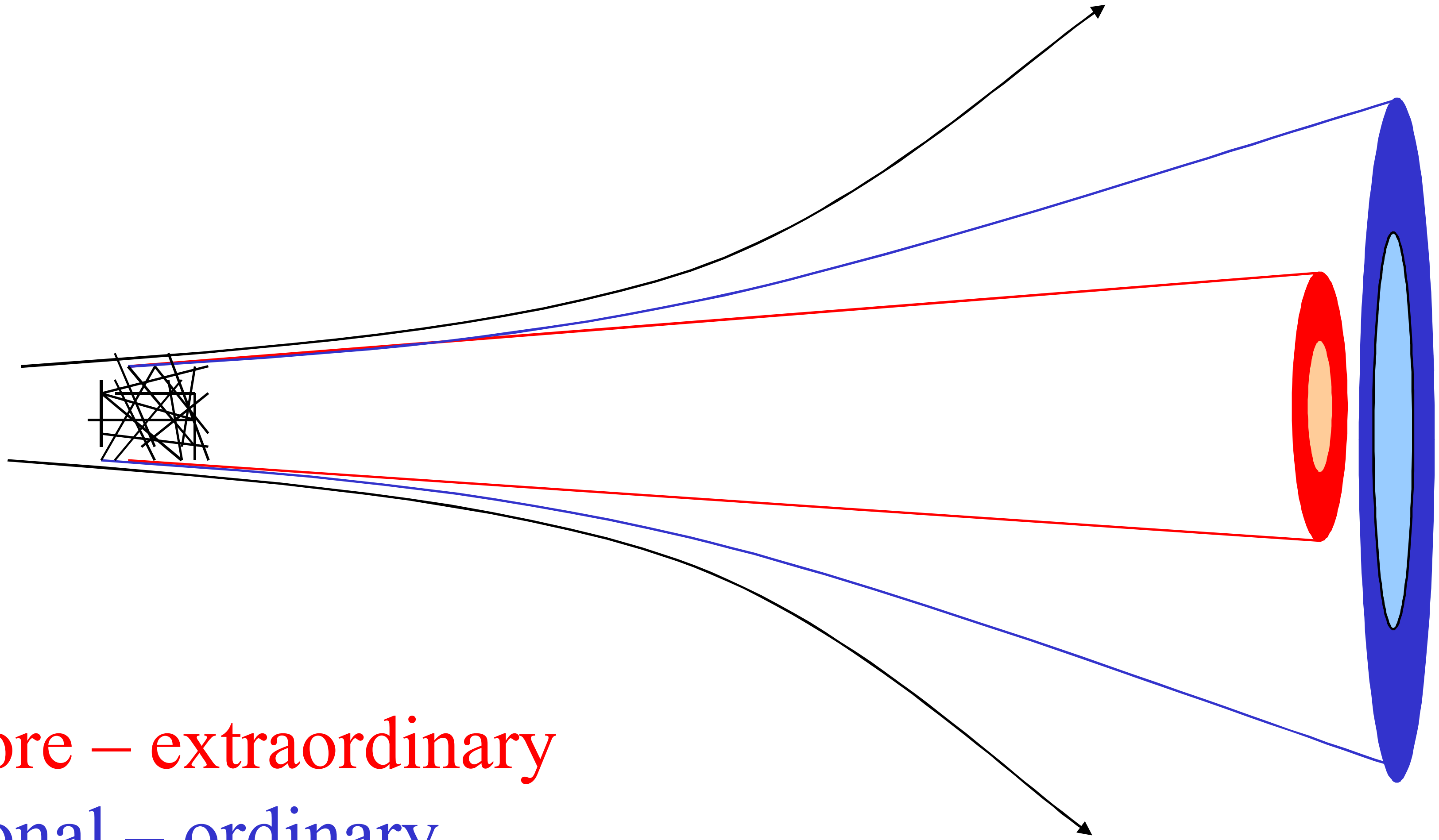
$$W \approx \left(\frac{\Omega R}{c}\right)^{0.36}\left(\frac{\omega_{p0}}{\omega}\right)^{0.14} < \gamma^{-3} >^{0.07}\left(\frac{r_0}{R}\right)^{0.15}.$$



On can know $r_0(\nu)$

Yu. Lyubarsky, S.Petrova

Core & Conal



Core – extraordinary

Conal – ordinary

Cyclotron absorption

(A.B.Mikhailovsky, O.G.Onishchenko, G.I.Suramlishvli, S.E.Sharapov, Sov. Astron. 1982)

$$\varepsilon \approx 1 + \frac{\omega_p^2}{\omega^2} < \frac{\varpi}{(\omega_B - \gamma\varpi)} >$$

$$\mathrm{Im}\, k \approx - \pi \frac{\omega_p^2}{2\omega c} < \varpi\delta(\omega_B - \gamma\varpi) >$$

$$\kappa \approx \lambda(1 - \cos\theta_{\mathrm{res}})\frac{r_{\mathrm{res}}}{R_{\mathrm{L}}}$$

Cyclotron absorption

(A.B.Mikhailovsky, O.G.Onishchenko, G.I.Suramlishvli, S.E.Sharapov, Sov. Astron. 1982)

If $\lambda \sim 10^4$, then cyclotron absorption is too large…

Limiting polarization

- V.V.Zheleznyakov (Budden)
- Yu. A.Kravtsov, Yu.I.Orlov

Escaping into vacuum, where $\Delta n = 0$, and, hence, the geometric optics approximation becomes invalid, polarizations of normal modes do not follow the orientation of the magnetic field in the picture plane.

Limiting polarization

- V.V.Zheleznyakov (Budden)

$$\frac{\mathrm{d}^2 V}{\mathrm{d}r^2} + \left\{ \frac{1}{4}\left[\frac{\mathrm{d}q/\mathrm{d}r}{1+q^2}\right]^2 + \frac{1}{4}\frac{\omega^2}{c^2}(\Delta n)^2 + \frac{i\omega}{2c}\frac{\mathrm{d}}{\mathrm{d}r}(\Delta n) \right\} V = 0$$

Δn is large, $\mathrm{d}q/\mathrm{d}r$ is small – geometric optics
Δn is small, $\mathrm{d}q/\mathrm{d}r$ is large, – vacuum propagation

Does not give the direct information about the polarization of outgoing radiation (here V isn't the Stokes parameter)

Limiting polarization

Location of the region where the polarization of the outgoing radiation is formed
(e.g., A.F.Cheng, M.A.Ruderman, **229**, 348, 1979)

$$r \sim 1000R \sim 0.1\ R_{\mathrm{L}}$$

$$q \sim 10\text{–}100$$

$$K \sim 1\text{–}10\ \%$$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov (1980)

$$\varepsilon_{ij} = \varepsilon\delta_{ij} + \chi_{ij}$$

$$\frac{d\Theta}{d\sigma} = \kappa + \frac{i\omega}{4c}[(\chi_{b\nu} - \chi_{\nu b}) + (\chi_{b\nu} + \chi_{\nu b})\cos 2\Theta - (\chi_{\nu\nu} - \chi_{bb})\sin 2\Theta]$$

$$\Theta = \theta_1 + i\theta_2$$

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Gives the direct information about the polarization of outgoing radiation

$$\theta_1 = \beta, \quad \beta + \pi/2,$$

$$\mathrm{sh}2\theta_2 = \pm\frac{1}{q}, \quad |\mathrm{th}\theta_2| = K.$$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

Ordinary wave – $\theta_1 = \beta(r)$
Extraordinary wave – $\theta_1 = \beta(r) + \pi/2$

Limiting polarization

- Yu. A.Kravtsov, Yu.I.Orlov

$$\frac{d\theta_1}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n}{\sqrt{q^2+1}} + \frac{1}{2}\frac{\omega}{c}\cos[2\theta_1 - 2\beta(r)]\frac{\Delta n q}{\sqrt{q^2+1}}\mathrm{sh}2\theta_2,$$

$$\frac{d\theta_2}{dr} = -\frac{1}{2}\frac{\omega}{c}\frac{\Delta n q}{\sqrt{q^2+1}}\sin[2\theta_1 - 2\beta(r)]\mathrm{ch}2\theta_2.$$

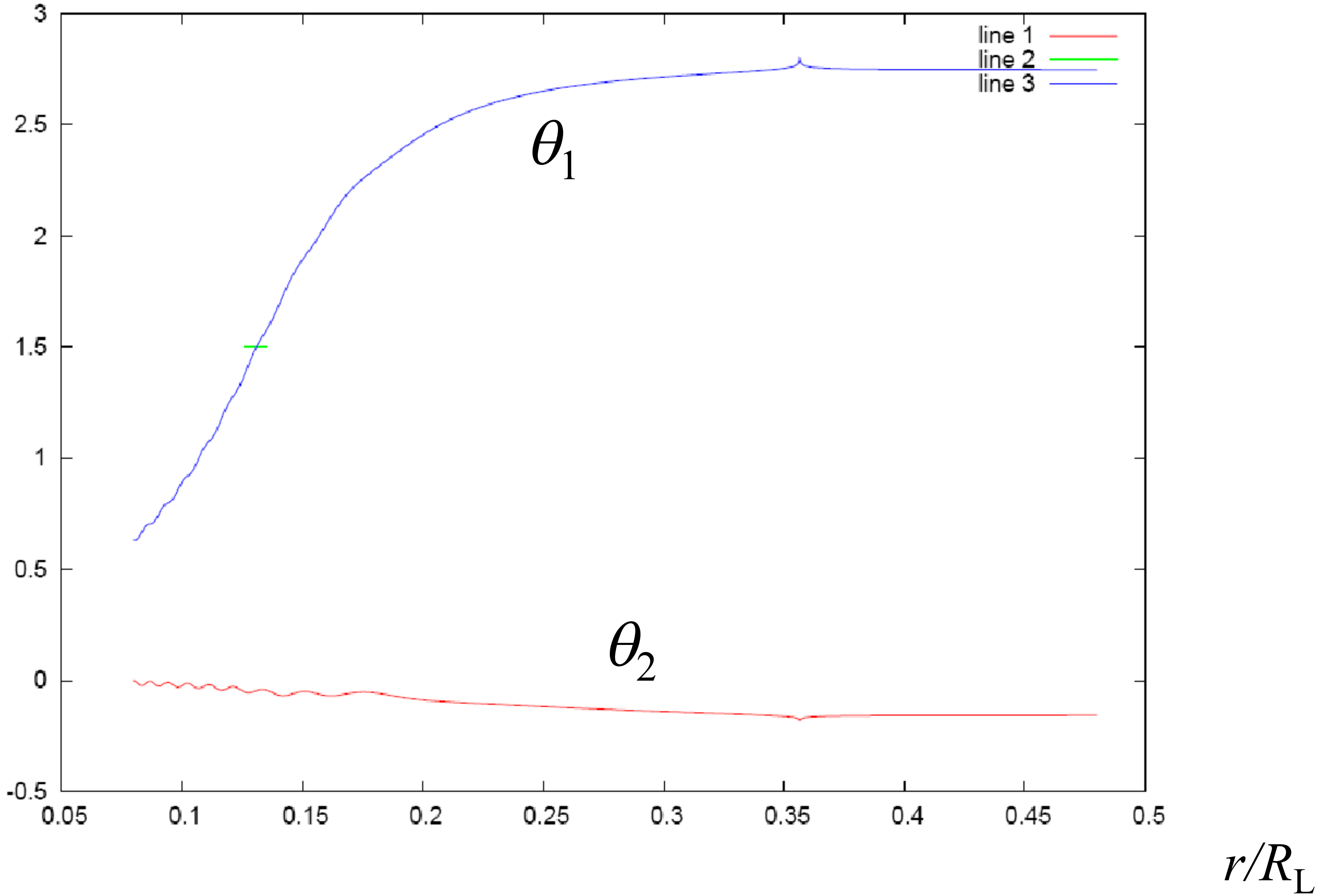
If the shear of the magnetic field is large, and $\omega_B > \gamma\varpi$

$$\theta_2 \approx -\frac{1}{2|q|}\cdot\frac{d\beta/dx}{|v_{||}/c - \cos\theta|}\cos[2\theta_1 - 2\beta(r)], \qquad x = \Omega r/c$$

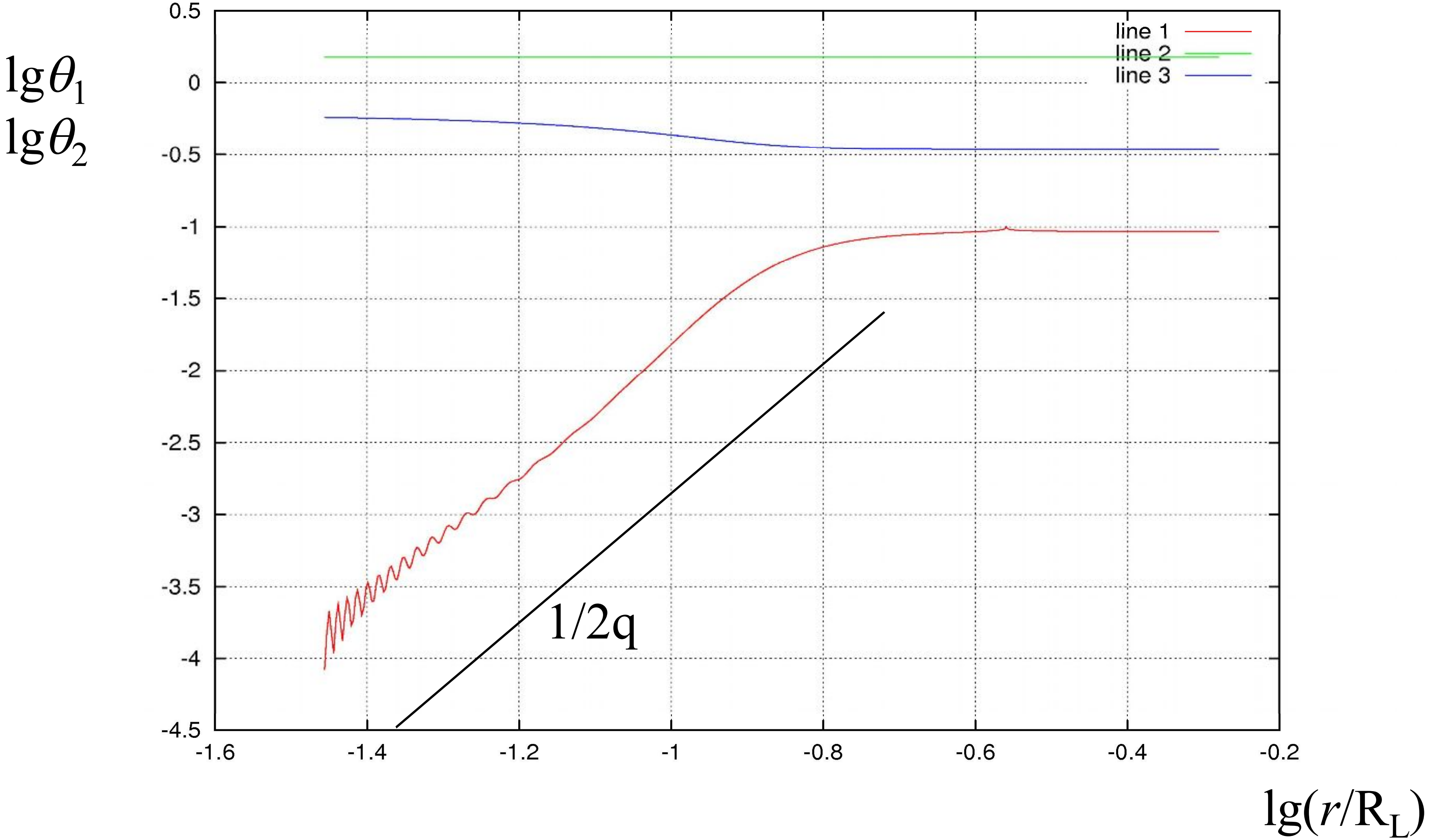
The sign of the circular polarization is determined by $d\beta / dx$

P = 1 s, B = 10^{12} G, ν = 1 GHz,

r_0 = 10R, γ = 100, λ = 10^4, χ =45°, α = 46°, r_{in}=0.1

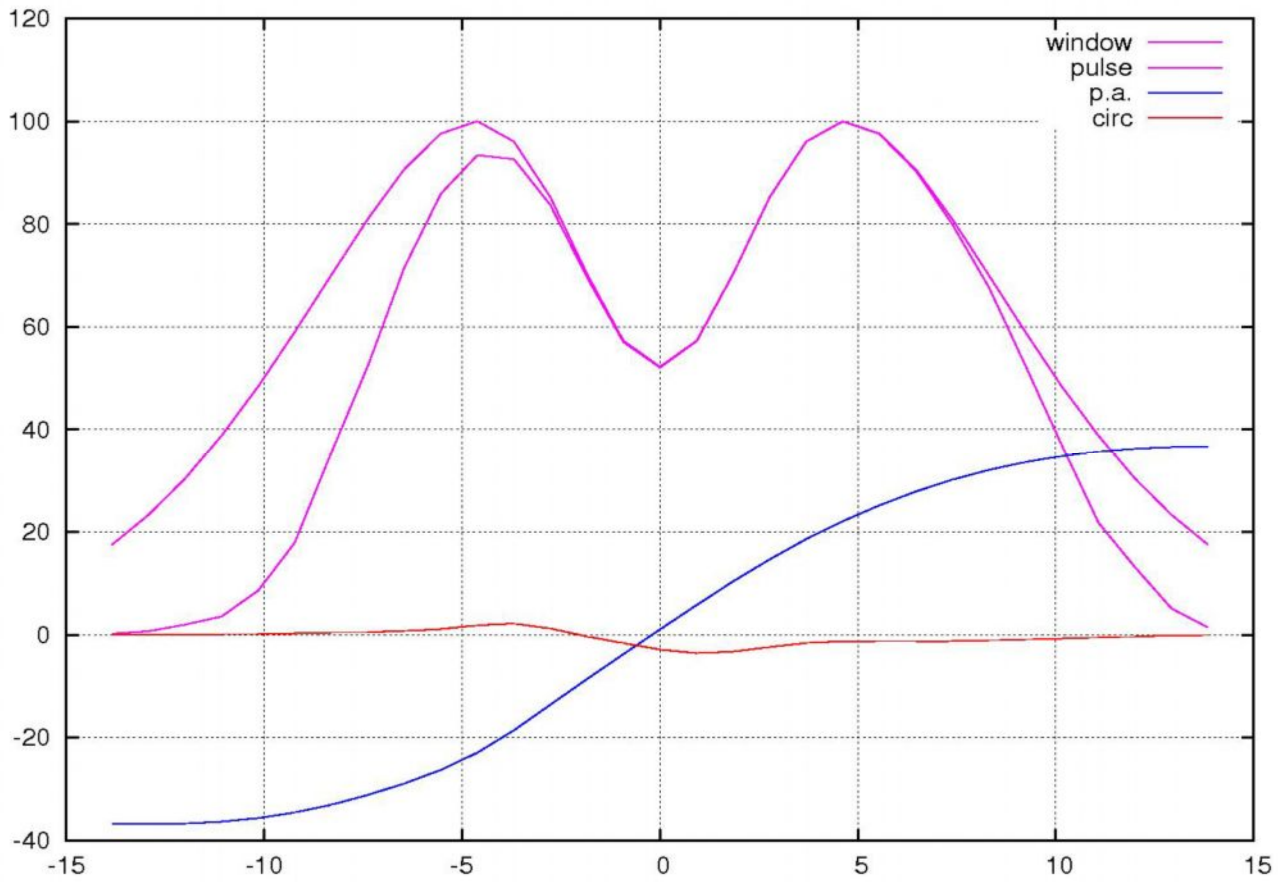


P = 1.5 с, B_0 = 0.6 10^{12} Гс, ν = 1 ГГц,
r_0 = 10R, γ = 100, λ = 5$10^5$, χ =45^o, α = 47^o, r_{in}=0.5



$P = 1.5$ s, $B_0 = 0.6\ 10^{12}$ G, $\nu = 1$ GHz, $r_0 = 10R$, $\gamma = 100$, $\lambda = 5\ 10^4$, $\chi = 45^o$, $\alpha = 47^o$, $r_{in} = 0.5$

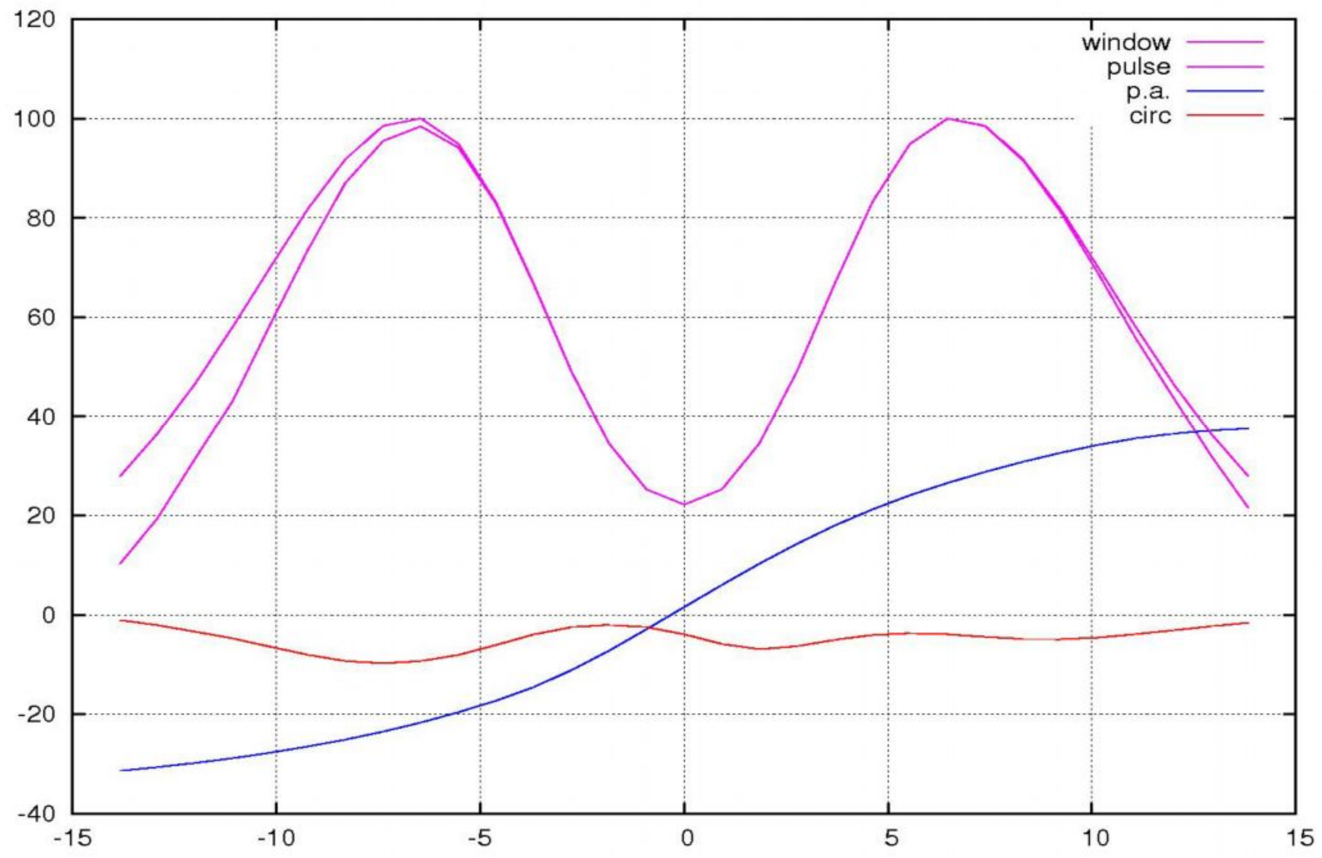
”nonrotating dipole”, ordinary wave



$P = 1.5$ s, $B_0 = 0.6\ 10^{12}$ G, **$\nu = 10$ GHz**,

$r_0 = 10R$, $\gamma = 100$, $\lambda = 5\ 10^4$, $\chi = 45^o$, $\alpha = 47^o$, $r_{in} = 0.5$

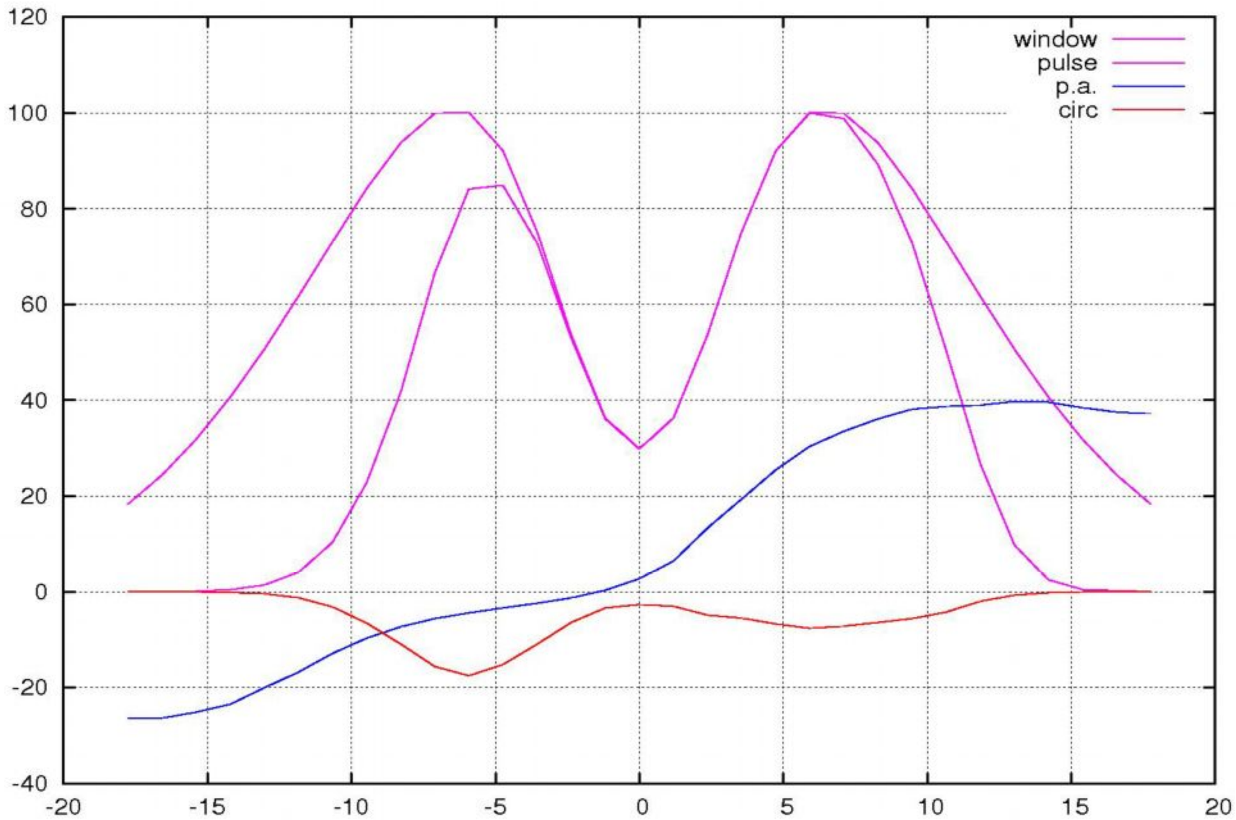
”rotating dipole”, ordinary wave



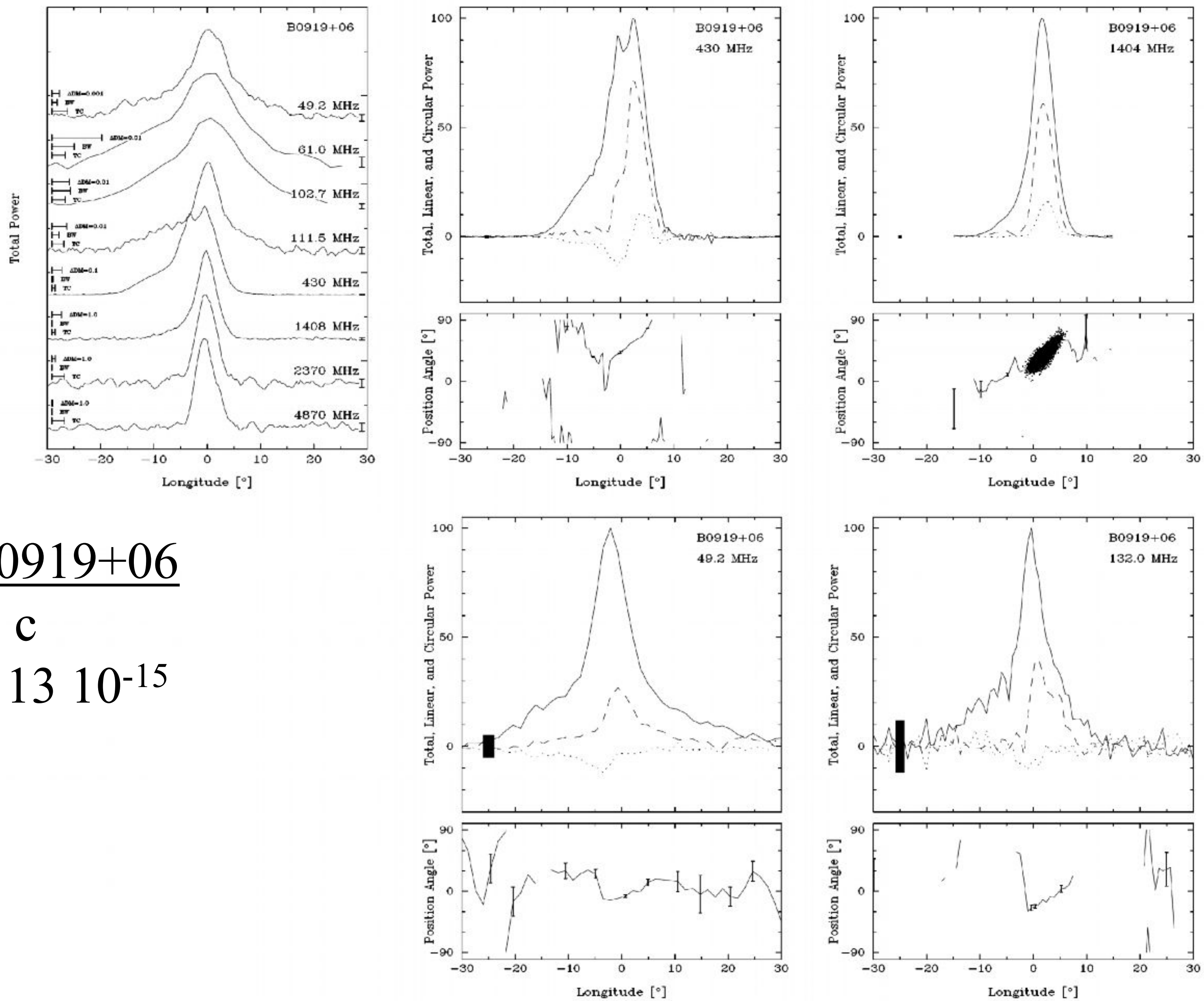
P = 1.5 c, B_0 = 0.6 10^{12} G, <u>**ν = 1 GHz**</u>,

r_0 = 10R, γ = 100, λ = 5 10^4, χ =45°, α = 47°, r_{in}=0.5

"rotating dipole", ordinary wave



T.Hankins, J.Rankin, 2008

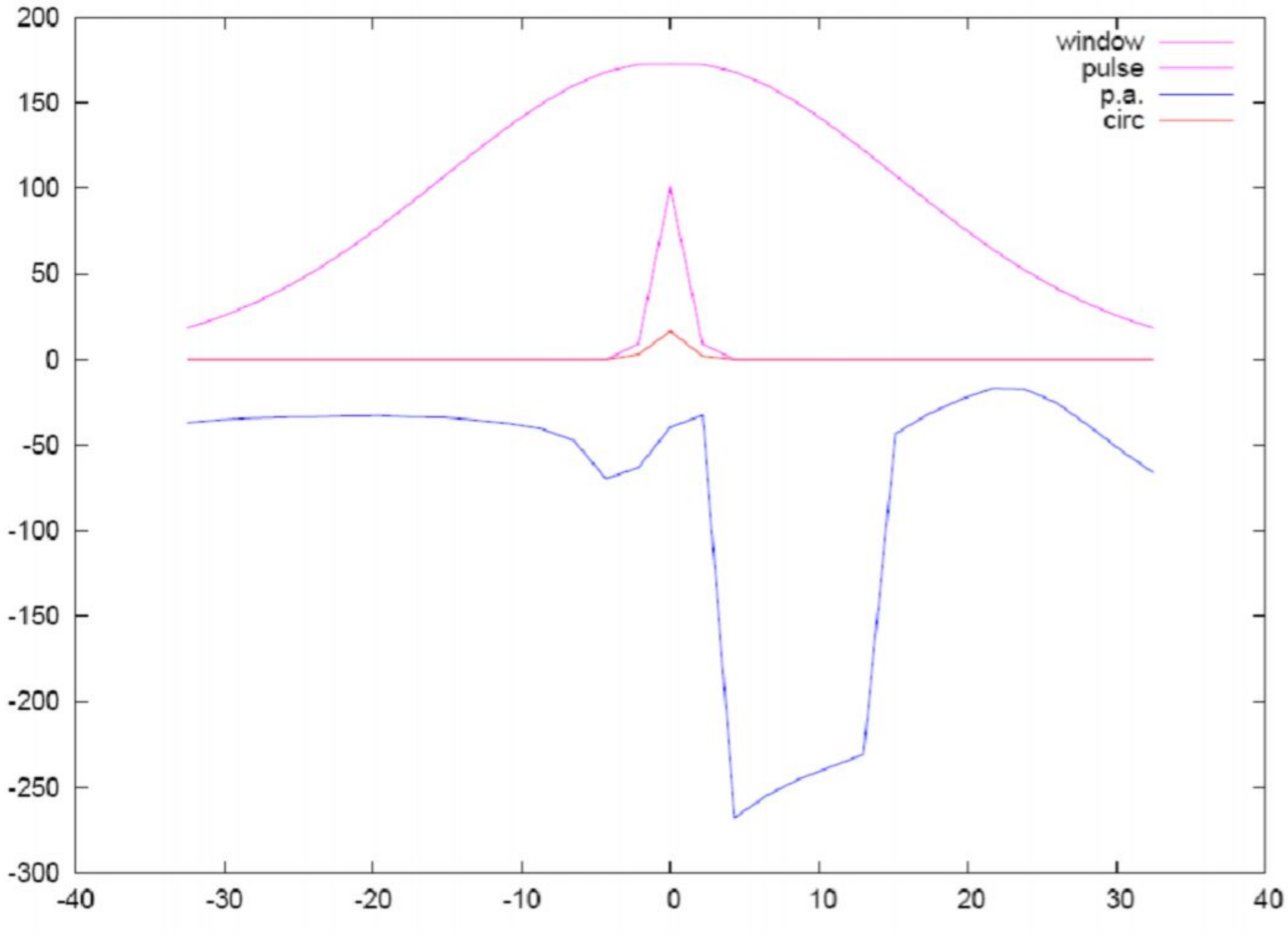


<u>PSR B0919+06</u>

P = 0.4 c

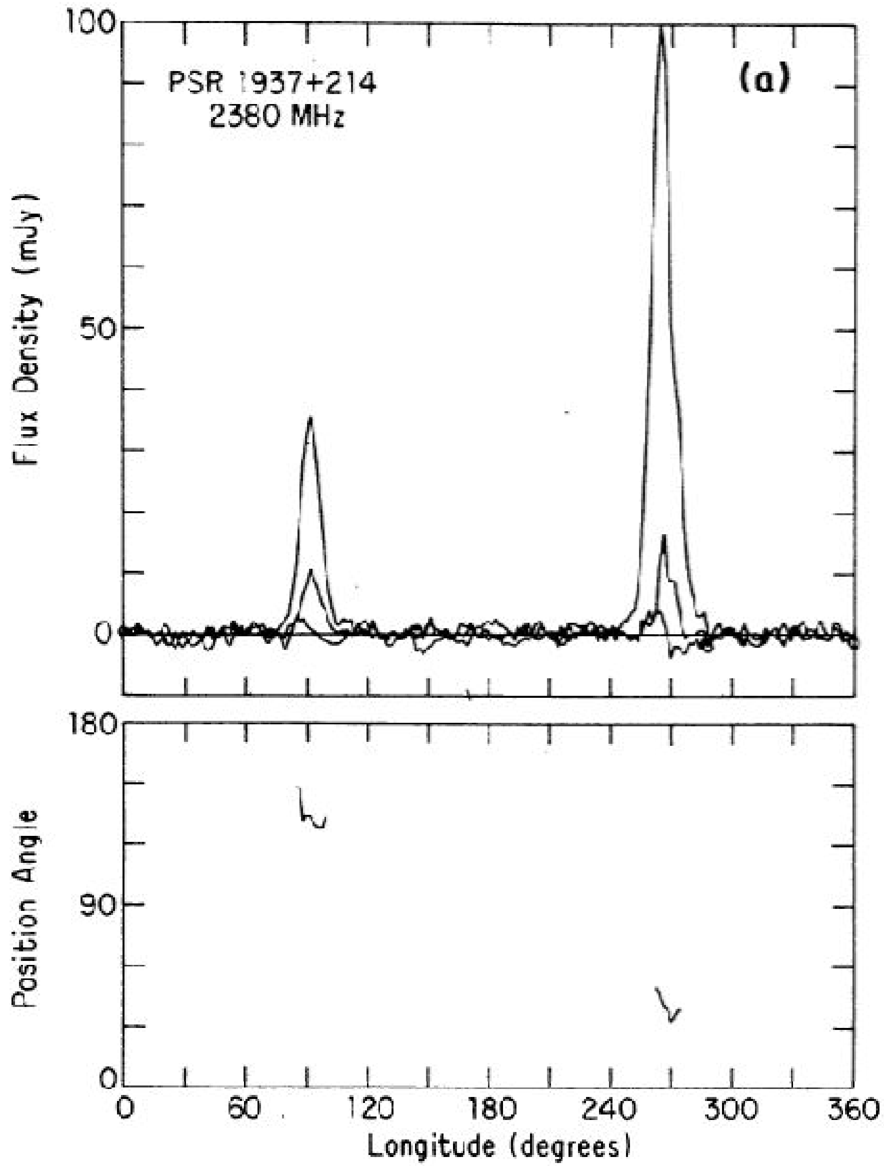
Pdot = 13 10^{-15}

$P = 0.01$ s, $B = 10^{12}$ G, $\nu = 1$ GHz, $r_0 = 2R$, $\gamma = 100$, $\lambda = 10^4$, $\chi = 45^o$ $\alpha = 46^o$ **$\underline{r_{in} = 0.03}$**



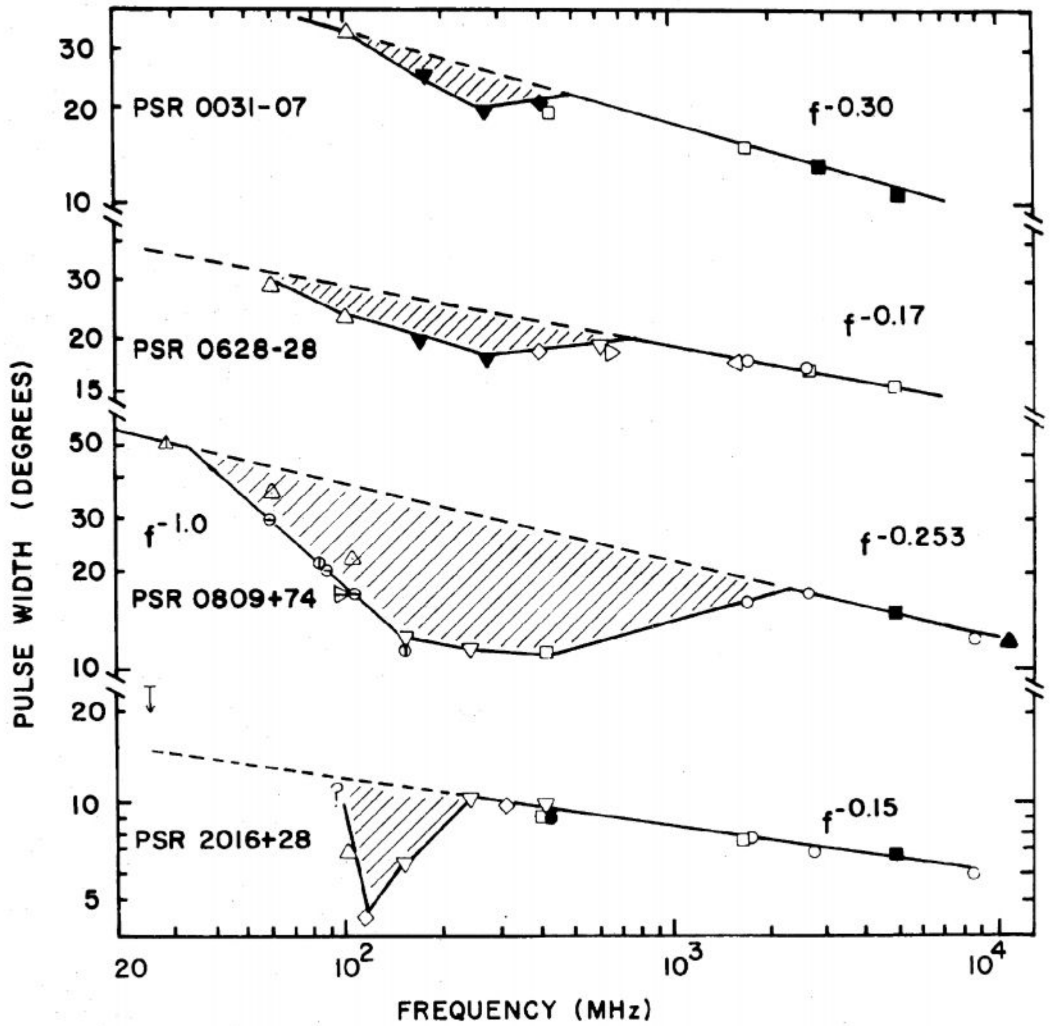
PSR B1937+21

$W = 14^{o}$



Absorption

(J.Rankin, 1983)



Main result

- It turns out that the sign of $\mathrm{d}\beta/\mathrm{d}x$ coincides with the sign of $\mathrm{d}p.a./\mathrm{d}\phi$.
- Hence, for ordinary wave (conal) the signs $\mathrm{d}p.a./\mathrm{d}\phi$ and V are to be opposite, and for the extraordinary wave (core) are to be the same.
- This property depends neither on the sign $\boldsymbol{\Omega m}$, nor on the pole of the neutron star.

Core & Conal

	O_S	O_D	e_S	e_D	$O_D + e_S$
N	5	20	41	6	5
$P^{1/2}W_{50}$	7.4	10.9	6.0	5.3	***

T.Hankins, J.Rankin, 2008
P.Weltevrede, S.Johnston, MNRAS **391**, 1210 (2008)

Conclusion

- Polarization is determined near the light cylinder, not in the radiation region.
- Polarization is determined by the magnetic structure near the light cylinder.
- Circular polarization – 5 - 20% (as observed).
- Small damping in the cyclotron resonance is possible only because the cone is holow.

Conclusion

Only now it’s possible to compare the theory with the observational data.