On Higher Spin Interactions with a Scalar Matter Field

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based on X. B. , E. Joung and J. Mourad, arXiv:0903.3338 [hep-th].

Perturbative & Power counting: UV behaviour

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But *not* always for an *infinite* set of fields with unbounded spin.

A famous example: string theory

String theory properties:

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Conclusion:
Adding an infinite number of problems with increasing difficulty can be a solution!

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$\exists$? another example

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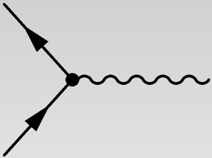
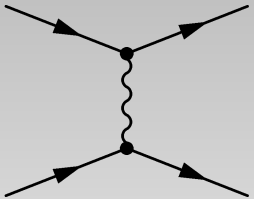
Strategy

Make use of the known propagators and cubic vertices including:

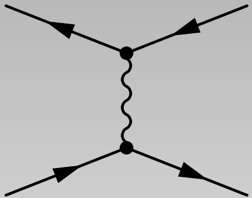
- scalar matter field (straight lines) and
- higher-spin gauge field (curly line).



Compute the tree-level exchange amplitude when the interaction is mediated by a massless higher-spin particle in the elastic scattering process $\phi\,\phi \;\rightarrow\; \phi\,\phi$



or in the elastic scattering process $\phi\,\bar{\phi} \rightarrow \phi\,\bar{\phi}$



Plan of the talk

Klein-Gordon action

$$\mathcal{S}_0^{\text{kin}}[\,\phi\,] = -\frac{1}{2}\int d^n x\,\big(\,\eta^{\mu\nu}\,\partial_\mu\phi^*(x)\,\partial_\nu\phi(x) + m^2\,\phi^*(x)\,\phi(x)\,\big)\,,$$

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$\Rightarrow$ *Scalar field propagator* $= \frac{1}{p^2+m^2}$.

Quadratic action

$$\mathcal{S}_2^{\text{kin}}[\,h\,] = -\sum_{S\geqslant 0} \frac{1}{2\,S!} \int d^n x \,\overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(x)\,\Box\,\overset{(S)}{h}{}^{\mu_1\ldots\mu_S}(x) \,+\, \ldots$$

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$\Rightarrow$ *Symmetric tensor gauge field propagators* $= \;\frac{1}{p^2}\,\mathit{Res}_{\mu_1\ldots\mu_S\,|\,\nu_1\ldots\nu_S}\,.$

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Unconstrained formalism (Francia, Mourad, Sagnotti; 2007)
No trace constraints $\Rightarrow$ easier to couple with currents

Minimal coupling

$$\mathcal{S}_1^{\text{min}}[\,\phi, h\,] = -\sum_{S\geqslant 0} \frac{c_S}{S!} \int d^n x \,\overset{(S)}{h}{}_{\mu_1\ldots\mu_S}(x)\, J^{\mu_1\ldots\mu_S}(x)$$

Arbitrary coupling constants $c_S \in \mathbb{R}$

Gauge invariance of the action

$$\mathcal{S}[\,\phi,h\,] = \mathcal{S}_0^{\text{kin}}[\,\phi\,] + \mathcal{S}_1^{\text{min}}[\,\phi,h\,] + \mathcal{S}_2^{\text{kin}}[\,h\,] + \text{higher}\,.$$

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At linear order in the gauge fields:

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Minimal coupling of gauge fields with
conserved currents for the matter field

Conserved current

Set of symmetric conserved currents of all ranks
(Berends, Burgers, van Dam; 1986)

$$J_{\mu_1 \ldots \mu_S}(x) = \left(\frac{i}{2}\right)^S \phi(x) \overleftrightarrow{\partial}_{\mu_1} \cdots \overleftrightarrow{\partial}_{\mu_S} \phi^*(x)$$

- Real
- Bilinear in the complex scalar field
- Number of derivatives = Rank

Cubic vertex

$$
\begin{aligned}
S_1[\,\phi, h^{(S)}\,] &= \frac{c_S}{S!}\int d^n x \;\overset{(S)}{h}{}_{\mu_1\dots\mu_S}(x)\, J^{\mu_1\dots\mu_S}(x) \\
&= -\int \frac{d^n\ell}{(2\pi)^n}\,\frac{d^n k}{(2\pi)^n}\,\phi^*(\ell)\;\phi(k)\;\overset{(S)}{h}{}_{\mu_1\dots\mu_S}(\ell-k)\times \\
&\qquad \times\,\frac{c_S}{S!}\left(\frac{k^{\mu_1}+\ell^{\mu_1}}{2}\right)\dots\left(\frac{k^{\mu_S}+\ell^{\mu_S}}{2}\right).
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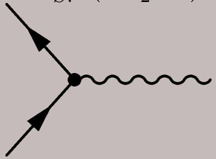
$\Rightarrow$ *Cubic vertex* $= -\frac{c_S}{S!}\left(\frac{k^{\mu_1}+\ell^{\mu_1}}{2}\right)\ldots\left(\frac{k^{\mu_r}+\ell^{\mu_S}}{2}\right).$



Mandelstam variables

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- θ = Scattering angle, determined by

$$\sin^2(\theta/2) = -t/(s - 4\,m^2)\,, \quad \cos^2(\theta/2) = -u/(s - 4\,m^2)$$

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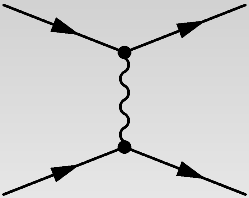
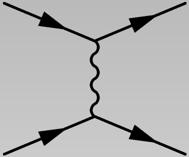
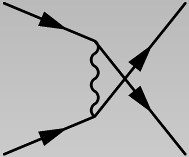


For bosons, the total amplitude for the scattering process $\phi(k_1)\,\phi(k_2) \,\rightarrow\, \phi(\ell_1)\,\phi(\ell_2)$ contains the sum of the t and u channel amplitudes:

$$\overset{(S)}{\mathcal{A}}_{\text{total}}(\phi\,\phi \rightarrow \phi\,\phi) \quad =$$



$$+$$



$$= \quad \overset{(S)}{\mathcal{A}}(s,t,u) \quad + \quad \overset{(S)}{\mathcal{A}}(s,u,t)\,.$$

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- In higher dimensions ($n \geqslant 6$), the amplitude can be expressed in terms of Gegenbauer polynomials $C_S^{\frac{n}{2}-2}$.

Crossing

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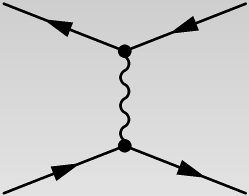
$$\overset{(S)}{\mathcal{A}}(u,t,s) = (-1)^S\,\overset{(S)}{\mathcal{A}}(s,t,u)$$



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- Fixed scattering-angle limit (s and t large, t/s fixed)

$$\overset{(S)}{\mathcal{A}}(s,t,u) \sim -\,\frac{1}{4}\,\frac{a_S}{S!}\,\Big(-\frac{\ell_P^2}{8}\,\sin^2(\theta/2)\,s\Big)^{S-1}\,T_S\left(\frac{1+\cos^2(\theta/2)}{\sin^2(\theta/2)}\right)\,.$$

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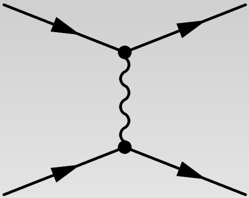

Summation of the amplitudes for all spins

The total amplitude including all possible gauge boson exchanges is the (possibly infinite) sum

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Let us denote by $a(z)$ the generating function of the coefficients $a_S \geqslant 0$, in the sense that

$$a(z) := \sum_{S\geqslant 0} \frac{a_S}{S!}\; z^S\,.$$

Summation of the amplitudes for all spins

Exact sum

$$\mathcal{A}(s,t,u) \;=\; -\frac{1}{\ell_P^2\, t}\,\Big[a\Big(-\frac{\ell_P^2}{8}\,\big(\sqrt{s}+\sqrt{-u}\,\big)^2\Big)+ \\ +a\Big(-\frac{\ell_P^2}{8}\,\big(\sqrt{s}-\sqrt{-u}\,\big)^2\Big)-a_0\Big]\,.$$

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Remark: $a(z)$ analytic around the origin $\Longrightarrow$ $\mathcal{A}(s,t,u)$ also is

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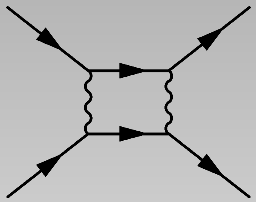
Example: Box diagram

Box diagram



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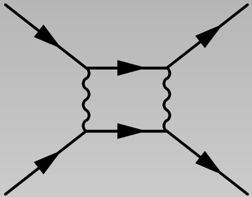

is proportional to

$$\int d^4p \, \frac{\mathcal{A}\Big(\phi(k_1)\phi(k_2) \to \phi(k_1+p)\phi(k_2-p)\Big)\, \mathcal{A}\Big(\phi(k_1+p)\phi(k_2-p) \to \phi(\ell_1)\phi(}{\Big((k_1+p)^2+m^2\Big)\Big((k_2-p)^2+m^2\Big)}$$

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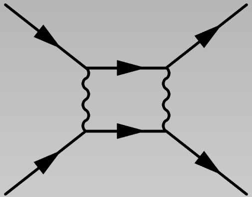

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and is UV finite if $a(z)$ goes to some constant when $z \to \pm\infty$.

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  Therefore, in the UV the higher-spin interactions may effectively behave as the spin-zero interaction (which is super-renormalizable).

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- Some Feynman diagrams can be seen to be UV finite if the generating function of coupling constants $a(z)$ goes to some constant when $z \to \pm\infty$.

Of course, this does not imply that the corresponding total one-loop amplitudes are finite because other diagrams should be taken into account, some of which might include higher-order vertices which are not considered in the present paper.
Nevertheless, it is already suggestive to observe that some Feynman diagrams may be UV finite if all contributions of the whole infinite tower of gauge fields are summed and if the coupling constants c_S behave nicely for large spin S .

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    - Test AdS_4/CFT_3 higher-spin/O(N)-model conjecture (Klebanov & Polyakov 2002, Petkou 2003, Sezgin & Sundell 2003, Leonhardt-Manvelyan-Rühl 2004, ...)