Super Yang-Mills Theory in 10+2 dims. Another Step Toward M-theory

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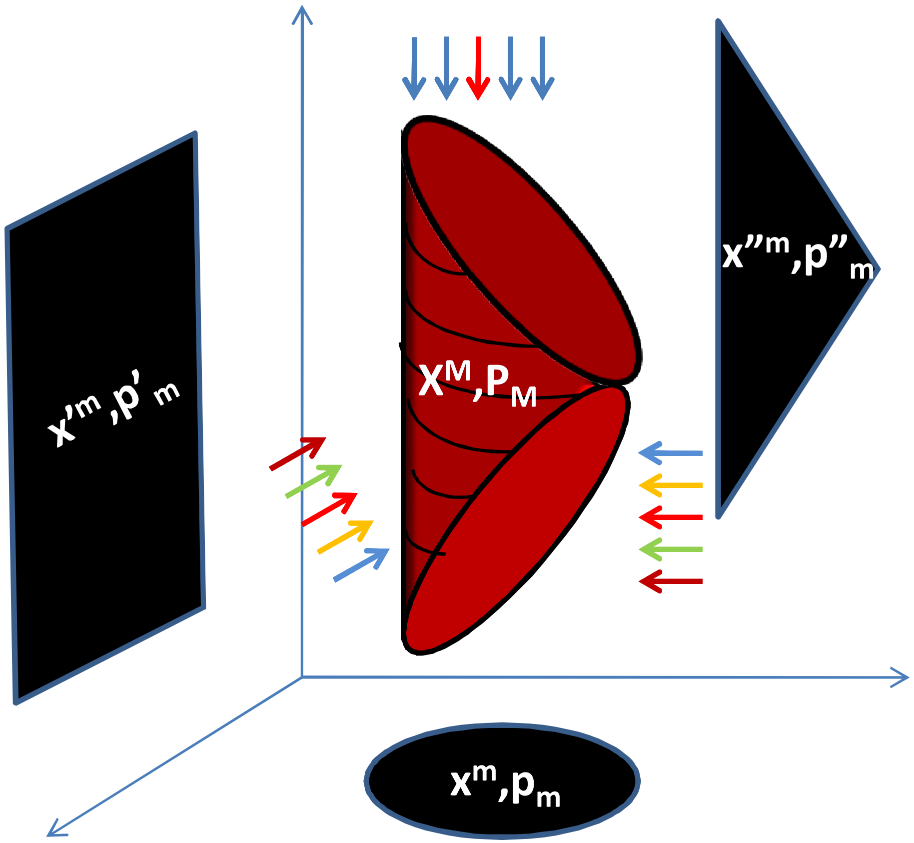
http://physics.usc.edu/~bars/homepage/moscow2009_bars.pdf

- SYM exists only in 2+1, 3+1, 5+1 and 9+1 dimensions. I will report on a new path which enlarges this horizon. I will show that the new theory is the mother of the N=4 SYM in 3+1 dims, the N=1 SYM in 9+1 dims, and M(atrix) theory, and others. The new theory is developed in the context of 2T-physics.
- Sakharov, who was one of the first to entertain the notion of two times, would have enjoyed what I now call 2T-physics.
- Strong hints for 2T-physics came from M-theory (IB -1995): Extended SUSY of M-theory is really a SUSY in 12 dimensions $\mathbf{\{Q_{32},Q_{32}\}=Z_{[2]}+Z_{[6]+}}$, Q_{32} **real** Weyl spinor →**(10+2) signature!** But if this implies 2 times, how does one remove the ghosts?
- 2T-physics developed by finding the fundamental solution to this ghost problem, and related causality problem. The answer is a gauge symmetry in phase space X^M,P_M. Phase space gauge symmetry is reminiscent of U-duality in M-theory (electric-magnetic).
- After a crash review on 2T-physics, I will explain the new SYM theory.

2T-Physics as a unifying framework for 1T-physics

•2T-physics is a ghost-free general framework that correctly describes all physics.
•2T-physics and usual 1T-physics are related, but 2T-physics unifies a larger set of phenomena that 1T-physics is unable to predict, but is only able to verify.



The relation between 2T-physics and 1T-physics can be described by an analogy : Object in the room (4+2 dim. phase space, X^M, P_M) and its shadows on walls (3+1 dim many phase spaces, x^m, p_m).

Observers like us are stuck on the "walls" (3+1 dims.), no privilege to be in the room (4+2). We interpret the shadows as different dynamical systems (1T formalism).

One (2T) to many (1T's). Predict many relations among the shadows (dualities, symmetries). This is systematically missed information in 1T-physics approach.

1) 1T-physics is incomplete !!!
2) Is 2T-physics more suitable for fundamentals?

2T-physics principles in a nutshell

Basic principle: Position-Momentum symmetry at every instant, for all motion for all physics (?)

Sp(2,R) gauge symmetry, local on worldline $X^M(\tau)$, $P_M(\tau)$

3 generators: $Q_{11}(X,P)$, $Q_{22}(X,P)$, $Q_{12}(X,P)=Q_{21}(X,P)$

Generalize worldline action for $x^\mu(\tau)$, $p^\mu(\tau)$

$$\partial_\tau x^\mu p_\mu - \frac{1}{2} e p_\mu p_\nu \eta^{\mu\nu}$$

Example: spinless particle

$A^{ij}(\tau)$ is Sp(2,R) gauge potential

$$\mathcal{L}_{2T} = \partial_\tau X^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X,P)$$

simplest example: $Q_{ij}(X,P) = (X\cdot X,\ P\cdot P,\ X\cdot P)$

first class constraints $Q_{ij}(X,P)=0$: requires Sp(2,R) singlets

Generalize? nontrivial soln. and no ghosts : ONLY Sp(2,R) !! Only 2T !!

Physical sector: only gauge invariant motion is allowed (shadows)

Nontrivial solutions exist only with 2 times! No less and no more!

The "shadows" are in 1 less space and 1 less time: [(d-1)+1] (gauge fixed)

In the simple example, spacetime η_{MN}: flat d+2 dims., SO(d,2) global symmetry

Shadows from 2T-physics ⟶ hidden info in 1T-physics

Hidden Symm.
SO(d,2), (d=4)
$C_2 = 1 - d^2/4 = -3$
singleton

Emergent
spacetimes
and emergent
parameters:
mass,
couplings,
curvature, etc.

•**Holography**:
These emergent
holographic
shadows are only
some examples of
much broader
phenomena.

Free or interacting
systems, with or
without mass, in
flat or curved
3+1 spacetime
Analogy: shadows
on walls

2T-physics predicts
hidden symmetries
and dualities (with
parameters) among
the "shadows".
1T-physics misses
these phenomena.

2T-physics
Sp(2,R) gauge symm.
generators $Q_{ij}(X,P)$ vanish
simplest example
$X^2=P^2=X\cdot P=0$ → **gauge inv.**
space: flat 4+2 dims
SO(4,2) symmetry

Massless
relativistic
particle
$(p_\mu)^2=0$
conformal sym
Dirac

Particle in Robertson-Walker expanding Universe

Massive
relativistic
$(p_\mu)^2+m^2=0$
Non-relativistic
$H=\mathbf{p}^2/2m$

Particle in any Maximally Symmetric Space, e.g. $AdS_{4-n}\times S^n$

H-atom
3 space dims
$H=\mathbf{p}^2/2m - a/r$
SO(4)xSO(2)
SO(3)xSO(1,2)

Particle in any Conformally flat Space singular OK, some Black Holes

Harmonic oscillator
2 space dims
mass = 3rd dim
SO(2,2)xSO(2)

Twistors
SU(2,2)=SO(4,2)
twistor
transform
for all
these

These emerge in 2T-field theory as well

Rules for 2T field theory, spins=0,½,1

Impose Sp(2,R) singlet condition !!

Use BRST approach for Sp(2,R). Like string field theory: **I.B.+Kuo, hep-th/0605267 I.B. hep-th/060645**

Flat space

$$S_{kin} = \int d^{d+2}X\,\delta(X^2)\left[\begin{array}{c}\frac{1}{2}\bar{\Phi}D^2\Phi + \frac{i}{2}\bar{\Psi}X\not{D}\Psi + h.c. \\ -\frac{1}{4}F_{MN}F^{MN}\,\Omega^{\frac{2(d-4)}{d-2}}\end{array}\right],$$

Ω is dilaton

There is explicit X^M, no translation invariance, only **SO(d,2) spacetime invariance**. This SO(d,2) becomes conformal symmetry in the "conformal shadow", but a hidden SO(d,2) symmetry in other shadows.

$$S_{yukawa} = \int d^{d+2}X\,\delta(X^2)\,\Omega^{-\frac{d-4}{d-2}}\left[y\left(\bar{\Psi}_L X \Psi_R\right)\Phi + h.c.\right],$$

$\Psi_{L,R}$ spinors of $SO(d,2)$

Double the size spinor as SO(d-1,1) +Fermionic gauge sym.

$$S_{scalars} = \int d^{d+2}X\,\delta(X^2)\,V(\Omega,\Phi), \qquad V(\Omega,H) = \Omega^{\frac{2d}{d-2}}V\left(1,\frac{\Phi}{\Omega}\right)$$

Homogeneous $V(\Omega,\Phi)$ Only dimensionless couplings among scalars

$$S_{anomalies} \sim \int d^{d+2}X\,\delta(X^2)\,\varepsilon^{M_1M_2M_3\cdots M_{d+2}}\left(X_{M_1}\partial_{M_2}\ln\Omega\right)\left(A_{M_3\cdots M_{d+2}}\right)$$

$$\delta S(\Phi) = 2\gamma\int d^{d+2}X\,\delta\Phi\left\{\begin{array}{c}\delta(X^2)\left[\partial^2\Phi - V'(\Phi)\right] \\ +2\delta'(X^2)\left[X\cdot\partial\Phi + \frac{d-2}{2}\Phi\right]\end{array}\right\}$$

dynamical eq. **P^2 + … = 0**

kinematic eqs. **X^2=0, X.P+P.X=0**

Minimizing the action gives two equations, so get **all 3 Sp(2,R) constraints <u>for each field ,</u> including interaction !!**

New gauge symmetries + kinematic equations (<=> Sp(2,R)), eliminate all ghosts!!

IB: 0804.1585
IB+S.H.Chen 0811.2510

Gravity in 2T-physics Field Theory

Gauge symmetry and consistency with Sp(2,R) lead to a unique action in d+2 dims, with **no parameters at all**.

$$a = \frac{(d-2)}{8(d-1)}.$$

$$S^0 = \gamma \int d^{d+2}X \sqrt{G} \left\{ \begin{array}{c} \delta(W)\left[\Omega^2 R(G) + \frac{1}{2a}\partial\Omega\cdot\partial\Omega - V(\Omega)\right] \\ +\delta'(W)\left[\Omega^2\left(4-\nabla^2 W\right) + \partial W\cdot\partial\Omega^2\right] \end{array} \right\}$$

Pure gravity has triplet of fields:
$G_{MN}(X)$, metric
$\Omega(X)$, dilaton
$W(X)$, replaces X^2

It has unique coupling to matter: scalars, spinors & vectors. Imposes **severe constraints on scalar fields** coupled to gravity.

$$S_{shadow}(g,\phi,s_i) = \int d^d x \sqrt{-g} \left(\begin{array}{c} \frac{1}{2a} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2} g^{\mu\nu}\partial_\mu s_i \partial_\nu s_i \\ +\left(\phi^2 - a s_i^2\right) R - V(\phi, s_i) \end{array} \right)$$

Local scale symm $\lambda(x)$ comes from general coordinate symm in d+2. Can choose dilaton $\phi(x)$ arbitrarily, e.g. a constant

$$\phi_0^2 = \frac{1}{16\pi G_d}$$

=> **Gravitational scale**.

$$g'_{\mu\nu} = e^{2\lambda} g_{\mu\nu},\ \ \phi' = e^{-\frac{d-2}{2}\lambda}\phi,\ \ s'_i = e^{-\frac{d-2}{2}\lambda}\phi$$

Prediction from 2T-physics: The <u>gravitational constant</u> is determined by the vacuum values of all scalar fields. It **<u>increases</u>** after every cosmic phase transition at the scales of inflation, GUT, SUSY, electroweak. **Effect on cosmology** !!

Super Yang-Mills in 10+2 dimensions

General SUSY Field Theory, for N=1,2,4, in 4+2 dimensions
done: IB + Y-C.Kuo hep-th/ 0702089, 0703002, 0808.0537

Usual N=4 SYM in d=4 is the conformal shadow from 4+2

12D theory

$$S = 2\gamma \int (d^{10+2}X)\,\sqrt{G}\,\delta(W(X)) \quad L(A_m^a(X), \lambda_\alpha^a(X))$$

vector 10+2, spinor 32 (Weyl)

Note G,W,Ω general gravity background

$$L(A,\lambda)_{W,\Omega} = -\frac{1}{4}\Omega^{3/2} F^a_{mn} F^a_{m'n'} G^{mm'} G^{nn'} + \frac{i}{2}\left[\bar{\lambda}^a V \bar{D} \lambda^a + \bar{\lambda}^a \overleftarrow{\bar{D}} \bar{V} \lambda^a\right]$$

$$V \equiv \gamma^m V_m,\ \bar{D} \equiv \bar{\gamma}^m D_m = \bar{\gamma}^m \left(\partial_m + \frac{1}{4}\omega_m^{ij}\gamma_{ij} + A_m^a t^a\right)$$

forms of V_m, G_{mn} follow from Sp$(2,R)$

non-dynamical background W, Ω

$$V_m = \tfrac{1}{2}\partial_m W,\ \ V \cdot V = W$$

$$G_{mn} = \tfrac{1}{2}\nabla_m \nabla_n W,\ \ (G^{mn} V_m \partial_n + 4)\Omega = 0$$

Homothety: Lie derivative $\pounds_V G^{MN} = -2G^{MN}$

SUSY is possible due to special gamma matrix identity in 10+2 dims

$$(\gamma^{ik})_{(\alpha\beta}\left(\gamma_i{}^j\right)_{\gamma)\delta} + (\gamma^{ij})_{(\alpha\beta}\left(\gamma_i{}^k\right)_{\gamma)\delta} = \frac{1}{6}\eta^{kj}(\gamma^{il})_{(\alpha\beta}(\gamma_{il})_{\gamma)\delta}$$

$$2f_{abc}(V_n\bar{\varepsilon}\gamma^{qn}\lambda^a)\,(V^p\bar{\lambda}^b\gamma_{qp}\lambda^c)\delta(W)$$
$$= \frac{1}{6}f_{abc}(\bar{\lambda}^b\gamma_{il}\lambda^a)(\bar{\varepsilon}\gamma^{il}\lambda^c)W\delta(W) = 0$$

similar identity also in $(3+2)$, $(4+2)$, $(6+2)$

conserved current $\partial_m(\bar{\varepsilon}J^m) = 0$ ⇔ SUSY $\delta_\varepsilon A_m,\ \delta_\varepsilon \lambda_\alpha$

$\varepsilon_\alpha = 32$ spinor

$$\bar{\varepsilon}^\alpha J_\alpha^m = \delta(W)\sqrt{G}\,\Phi^{3/4} F_{pq}^a V_n (\bar{\varepsilon}\gamma^{pqn}\bar{\gamma}^m\lambda)$$

SUSY possible only iff ε_α satisfies SUSY condition

$$\left[-(\bar{\varepsilon}\gamma^m\bar{\gamma}^{pqn}\lambda)V_n\partial_m \ln\Phi^{\frac{d-4}{d-2}} + V_n(D_m\bar{\varepsilon})\gamma^{pqn}\bar{\gamma}^m\lambda\right]_{W=0}$$

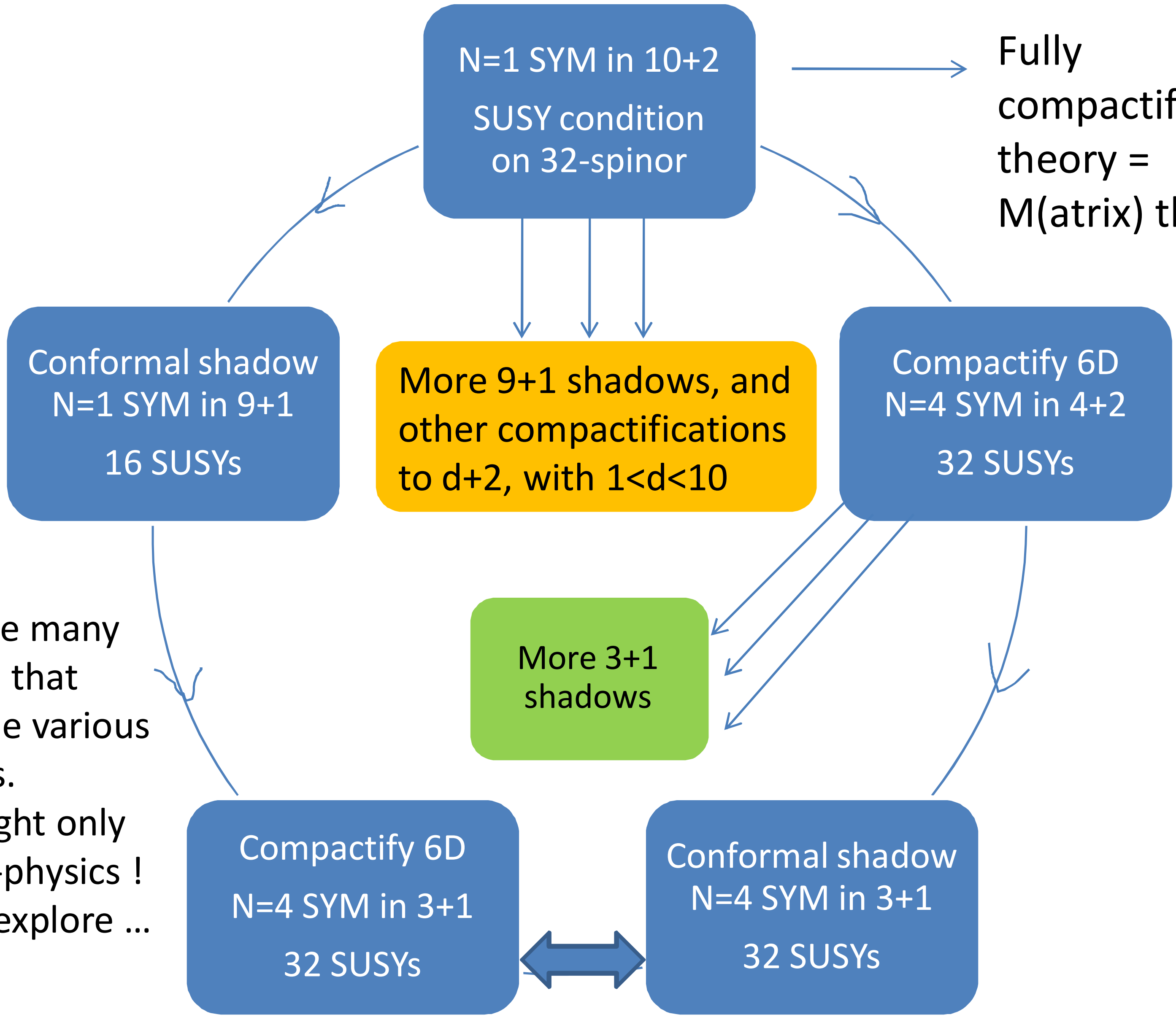
$$= (V^p U^q - V^q U^p), \quad \text{any } U^q(X)$$

$$D_m\varepsilon_\alpha = 0 \quad \text{and } \bar{\varepsilon}\gamma^m\partial_m \ln\Phi^{\frac{d-4}{d-2}} = 0$$

For most background geometries such $\varepsilon(X)$ can be found with only 16 independent components.

But there are special cases with 32 components. For example, dimensionally reduce 10+2 to (4+2)+(6+0), then we obtain 32 component ε which corresponds to N=4 SYM in 4+2 dimensions, which in turn has N=4 SYM in 3+1 dimensions in the conformal shadow.

10+2 SYM as parent of N=4 SYM in 3+1, and a web of dualities



Fully compactified theory = M(atrix) theory.

There are many dualities that relate the various shadows.
This insight only from 2T-physics !
A lot to explore …

Status of 2T-physics

- Local Sp(2,R) → 2T-physics, a principle in CM & QM:
  Seems to work generally to produce 1T Hamiltonians for particle dynamics, including spin, supersymmetry, backgrounds of all types, including gravity, E&M, etc.. **A new unification of 1T systems into classes that belong to the same 2T system**, and brings out hidden symmetries related to extra dims.

- Field Theory, The Standard Model & Gravity in 4+2 dimensions,
  In the "conformal shadow" in 3+1 dims. agree structurally with usual SM and GR, but include some new constraints that provide new phenomenological guidance for physics at the LHC and in Cosmology (e.g. $-\frac{1}{12} s^2 R$ is required!!)

- Beyond the Standard Model
  GUTS, SUSY, higher dims; all have been elevated to 2T-physics in d+2 dimensions.
  Strings, branes; tensionless, and twistor superstring, 2T OK. Tensionful incomplete.
  M-theory; expect 11+2 dimensions → OSp(1|64) global SUSY, S-theory.

IB+Chen+Quelin, **0705.2834 0802.1947,**

- New non-perturbative technical tools – a lot more to do here !!
  Emergent spacetimes and dynamics; unification; holography; duality; hidden sym. Expect to be useful for non-perturbative analysis of field theory, including QCD. (analogs of AdS-CFT, others …). Path integral approach for quantum field theory directly in d+2 dimensions will be useful. (still under development).

Do you need 2T? YES!

Hidden information in 1T-physics is revealed by 2T-physics (shadows)

1T-physics on its own is not equipped to capture these hidden symmetries and dualities, which actually **exist.**

1T-physics needs the additional guidance, so 1T-physics is definitely incomplete.

"Of course the elements are earth, water, fire and air. But what about chromium? Surely you can't ignore chromium."

2T-physics seems to be a promising idea on a new direction of higher dimensional unification.

extra 1+1 are LARGE, also not Kaluza-Klein, **not hidden.**
Different shadows are different perspectives, so you can "see" extra dims. indirectly by proper interpretation.

A lot more remains to be done with 2T-physics. Predictions at every scale of physics are expected from hidden dualities and symmetries by using the more powerful tools in future research ...

2T-physics works
in the known world so far

… and through work in progress
we hope to the extend
its domain of validity
to solve the remaining mysteries!!

The End