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Spontaneous parity violation in hot and dense baryon matter in QCD motivated hadronic models

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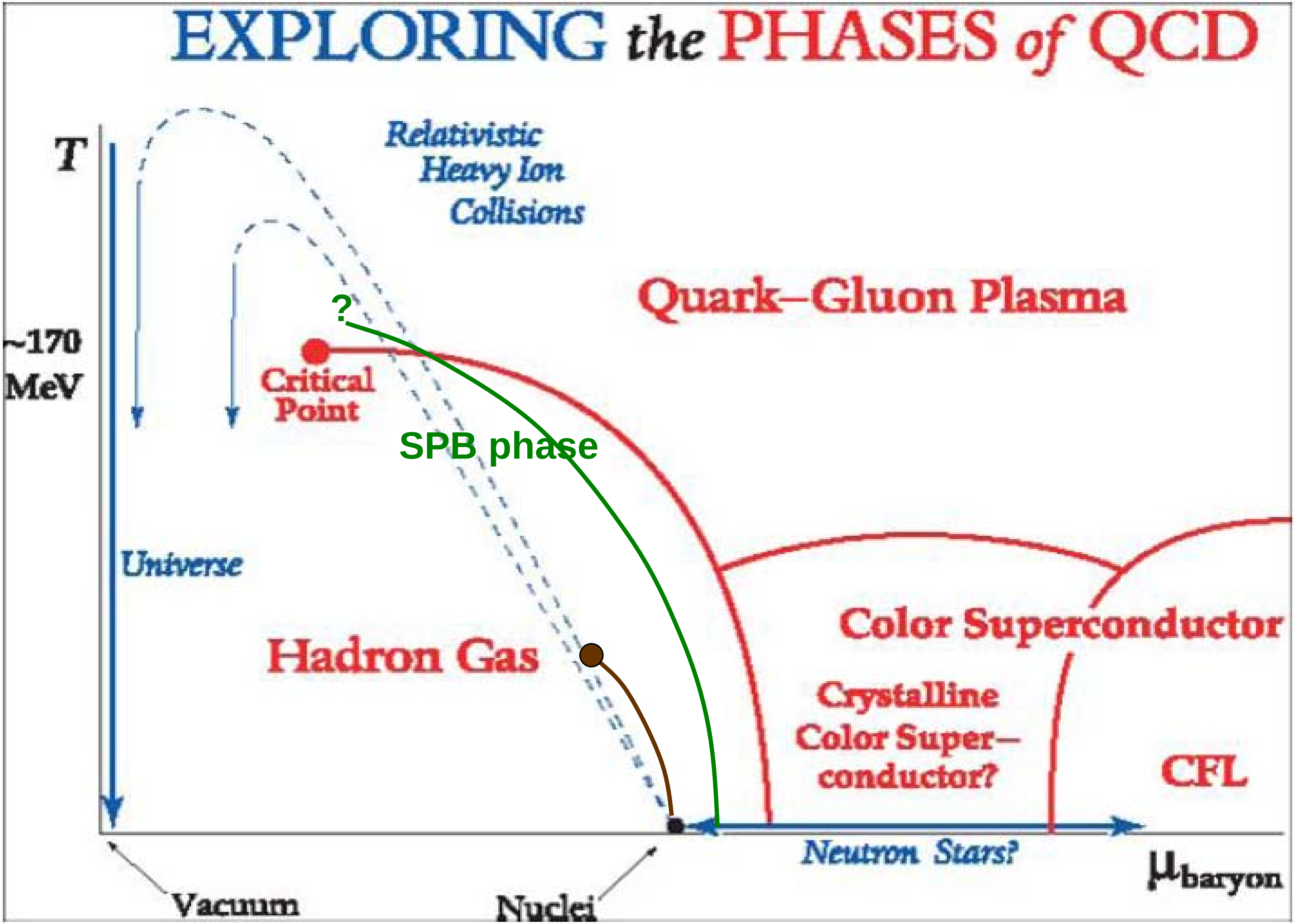
A.A. & D.Espriu, Phys.Lett.B, 663 (2008) 450;
A.A., V.Andrianov & D.Espriu, hep-ph/0904.0413.


EXPLORING the PHASES of QCD
T
~170
MeV
Relativistic
Heavy Ion
Collisions
?
Quark–Gluon Plasma
Critical
Point
SPB phase
Universe
Hadron Gas
Color Superconductor
Crystalline
Color Super–
conductor?
CFL
Neutron Stars?
μ_{baryon}
Vacuum
Nuclei

We guess P- violation to occur at nearly zero temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their heavy twins)

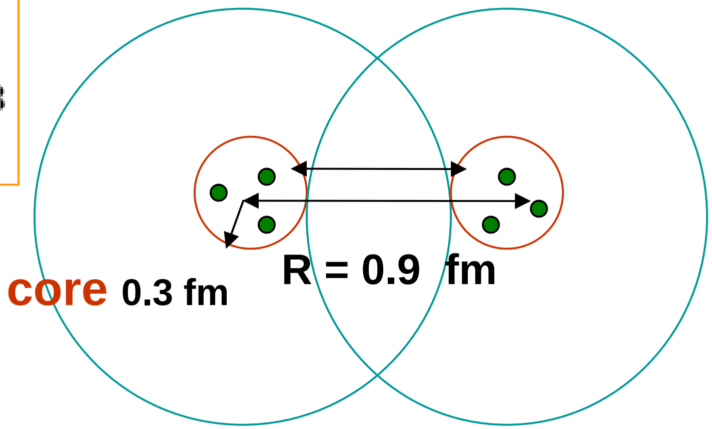
$$\varrho_B \gg \varrho_N \simeq 0.17 fm^{-3} = (1.8 fm)^{-3}$$

How large?

Beyond the range of validity of pion-nucleon effective Lagrangian but not large enough for quark percolation, $\varrho_B \sim (3 \div 8)\varrho_N$
i.e. **in the hadronic phase with heavy meson excitations playing an essential role in dense nuclear matter where quark-nuclear matter duality can be effectively used.**

Example

$$\varrho_B = 8\varrho_N \simeq (0.9 fm)^{-3}$$



Higher-mass mesons + dressed quarks

$$R_{\pi,\sigma,\omega,\rho,\pi',\sigma'...} = (0.3 - 0.4)\ \mathrm{fm} = (500\text{-}700\ \mathrm{MeV})^{-1}$$

Extended linear sigma model

with two multiplets of scalar and pseudoscalar mesons

(with matching to QCD and to nuclear matter as much as possible)

$$H_j = \sigma_j \mathbf{I} + i\hat{\pi}_j, \quad j = 1,2; \quad H_j H_j^\dagger = (\sigma_j^2 + (\pi_j^a)^2)\mathbf{I}, \qquad \hat{\pi}_j \equiv \pi_j^a \tau^a$$

Chiral limit ⟶ $SU_L(2) \times SU_R(2)$ **symmetry**

"Quark/nuclear matter free energy" after bosonization

$$\begin{aligned} V_{\rm eff} \ &= \ \frac{1}{2}\mathrm{tr}\left\{ -\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k + \lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \right. \\ &+ \frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \\ &\left. + \frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \right\} + \mathcal{O}(\frac{|H|^6}{\Lambda^2}) \end{aligned}$$

9 real constants (in fact 7 independent)

$$\Delta_{jk} \sim \lambda_A \sim N_c$$

Chiral expansion in $1/\Lambda$ **in hadron phase of QCD**

$$\Lambda \simeq 4\pi F_\pi \sim 4 M_{dyn}$$

Chirally symmetric parameterization

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \qquad \langle H_1 \rangle = \langle \sigma_1 \rangle > 0$$

$$H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Effective potential

$$\begin{aligned} V_{\mathrm{eff}} &= -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 \\ &+\lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3 + \lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5\sigma_1^3\sigma_2 + \lambda_6\sigma_1\sigma_2^3 \\ &+(\pi_2^a)^2\Big((\lambda_3 - \lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) + \lambda_2\Big((\pi_2^a)^2\Big)^2 \end{aligned}$$

Vacuum states for symmetric baryon matter

A pseudoscalar condensate breaking P-parity?

$$\pi_2^a = \delta^{a0}\rho$$

No! (Vafa-Witten theorem) $\rho = 0$ **in QCD at zero quark density**

Eqs. for vacuum states

$$\begin{aligned}
2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&\quad + \rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\
2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\
&\quad + \rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\
0 &= 2\pi_2^a\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)
\end{aligned}$$

Necessary and sufficient condition to avoid P-parity breaking in normal QCD vacuum (μ = 0)

$$(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22} \qquad \lambda_2 > 0$$

positive mass of $\pi(1300)$

Search for stable "nuclear matter"

4-parameters of linear transformations

$$\tilde{H}_j = \sum_{k=1,2} L_{jk} H_k.$$

are sufficient to provide

$$\Delta_{jk} = \Delta\delta_{jk} \text{ and } \lambda_5 = 0$$

One of the solution of the simplified equations is $\sigma_2 = 0$ and $\sigma_1^2 = \Delta/2\lambda_1$ provided that $\lambda_3 \pm \lambda_4 > 2\lambda_1 > 0$.

It is a minimum!

If $\sigma_2 \neq 0$, another set of solutions for σ_i given in term of the ratio $x \equiv \sigma_2/\sigma_1$

$$2\lambda_1 - (\lambda_3 + \lambda_4) - \frac{3}{2}\lambda_6 x + (\lambda_3 + \lambda_4 - 2\lambda_2)x^2 + \frac{1}{2}\lambda_6 x^3 = 0.$$

This equation may have one or three real roots.

We need three roots!

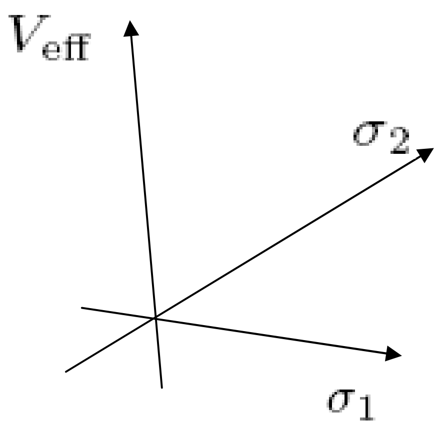
Conditions: $\lambda_6 < 0, \quad (\lambda_3 + \lambda_4)\Delta_{22} > 2\lambda_2\Delta_{11}, \quad \lambda_3^2 - \lambda_4^2 > 4\lambda_1\lambda_2.$

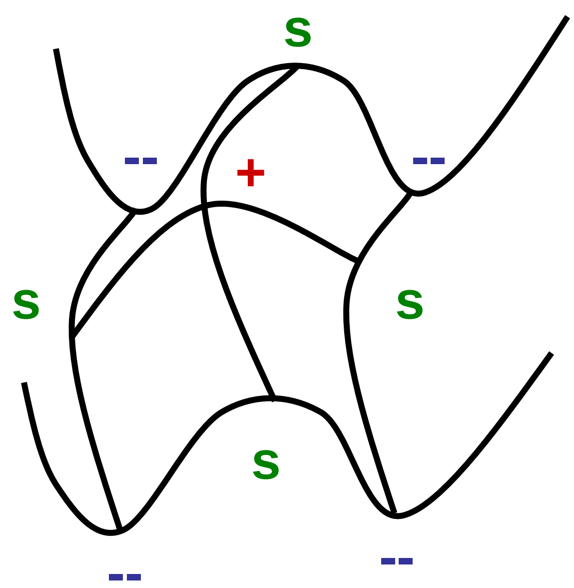
Landscape of solutions

One negative root – saddle point,
two positive roots – one minimum and one saddle point

effective potential is symmetric against the transformation $\sigma_j \to -\sigma_j$

$\longrightarrow$ **9 extremums = 4 minima + 4 saddle points + 1 maximum at the origin**





Fix the sign of σ_1 $\longrightarrow$ **two competitive minima** $\longrightarrow$ **phase transition of 1st order**

Embedding a chemical potential into "quark matter free energy"

$$\langle N^\dagger N \rangle \quad \Longleftrightarrow \quad \int d^3x \mu_B (\bar{N}\gamma_0 N(x) - \varrho_B)$$

Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states

From a quark model

$$\Delta V_{\text{eff}}(\mu) = \frac{\mathcal{N}}{2}\Theta(\mu - |H_1|)\left[\mu|H_1|^2\sqrt{\mu^2 - |H_1|^2} - \frac{2\mu}{3}(\mu^2 - |H_1|^2)^{3/2} - |H_1|^4 \ln\frac{\mu + \sqrt{\mu^2 - |H_1|^2}}{|H_1|}\right]\left(1 + O\left(\frac{\mu^2}{\Lambda^2}\right)\right)$$

$$\mathcal{N} \equiv \frac{N_c N_f}{4\pi^2}$$

Monotonous function

Only the first Eq. for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3 + \lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$

$$+\rho^2\Big(2(\lambda_3 - \lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu - \sigma_1)\left[\mu\sigma_1\sqrt{\mu^2 - \sigma_1^2} - \sigma_1^3 \ln\frac{\mu + \sqrt{\mu^2 - \sigma_1^2}}{\sigma_1}\right]$$

$$\equiv \sigma_1 \mathcal{A}(\sigma_1, \mu)$$

Repulsive forces are wanted for stabilization!

Chemical potential triggers condensation of omega mesons

$$\Delta\mathcal{L}_\omega = -\frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - g_{\omega\bar{q}q}\bar{q}\gamma_\mu\omega^\mu q,$$

Constant v.e.v. $g_{\omega\bar{q}q}\langle\omega_0\rangle \equiv \bar{\omega}.$ **Shift** $\mu \to \mu + \bar{\omega} \equiv \bar{\mu}.$

Modification of effective potential

$$\Delta V_\omega = -\frac{1}{2}m_\omega^2\langle\omega_0^2\rangle = -\frac{1}{2}\frac{(\bar{\mu}-\mu)^2}{G_\omega}, \quad G_\omega \equiv \frac{g_{\omega\bar{q}q}^2}{m_\omega^2} \simeq \mathcal{O}(\frac{1}{N_c}).$$

Stationary solution

$$\frac{\bar{\mu}-\mu}{G_\omega} = -N_c\varrho_B(\mu) = -\frac{N_cN_f}{3\pi^2}(\bar{\mu}^2 - \sigma_1(\bar{\mu})^2)^{3/2}.$$

Extension to Hot and Dense matter:

replace $\mathcal{A}(\sigma_1, \mu)$

$$\mathcal{A}(\sigma_1, \mu, \beta) = \int_{\sigma_1}^{\infty} dE \sqrt{E^2 - \sigma_1^2}\, \frac{\cosh(\beta\mu) + \exp(-\beta E)}{\cosh(\beta\mu) + \cosh(\beta E)} + \mathcal{O}\left(\mu^2/\Lambda^2, \sigma_1^2/\Lambda^2\right).$$

after summing up Matsubara frequences in one quark loop

On classical solutions

$$V_{\text{eff}}(\mu) = -\frac{1}{2}\sum_{j,k=1}^{2} \sigma_j(\mu)\Delta_{jk}\sigma_k(\mu) - \frac{1}{2}\Delta_{22}\rho^2(\mu) - \frac{\mathcal{N}}{3}\mu\left(\mu^2 - \sigma_1(\mu)^2\right)^{3/2}\theta\left(\mu - \sigma_1(\mu)\right)$$

$$+\Delta V_\omega(\bar{\mu}, \mu)$$

Thermodynamic properties at zero temperature

Pressure $$p(\sigma_j(\mu),\mu) \equiv V_{\rm eff}\left(\sigma_j^0\right) - V_{\rm eff}\left(\sigma_j(\mu),\rho(\mu),\mu\right),$$

$$\partial_\mu p = N_c \varrho_B.$$

Energy density $$\varepsilon = -p + N_c \mu \varrho_B.$$

relation between baryon density, Fermi momenta and the chemical potential is for quark matter

$$\varrho_B = -\frac{1}{N_c}\partial_\mu V_{\rm eff} = \frac{N_f}{3\pi^2} p_F^3 = \frac{N_f}{3\pi^2}(\mu^2 - \sigma_1(\mu)^2)^{3/2}.$$

Pressure vanishes in vacuum and at a minimum of energy per baryon

$$p = \varrho_B^2 \partial_{\varrho_B}\left(\frac{\varepsilon}{\varrho_B}\right),$$

This minimum realizes the stable baryon matter. Since pressure is increasing with density the phase diagram in the p, ϱ_B plane must necessarily exhibit discontinuity – 1st order phase transition "vapor-liquid".

It occurs at $$\bar\mu^* < \sigma_1^0, \sigma_j^* \equiv \sigma_j(\bar\mu^*)$$

To realize it one needs two competitive minima in the effective potential in order to interchange them at a phase transition point.

Saturation point (phase transition at p = 0)

$$\sum_{j,k=1}^{2}\left(\sigma_j^0\Delta_{jk}\sigma_k^0-\sigma_j^*\Delta_{jk}\sigma_k^*\right) = \frac{N_cN_f}{6\pi^2}\bar{\mu}^* p_F^3(\bar{\mu}^*)+G_\omega\frac{N_c^2N_f^2}{9\pi^4}p_F^6(\bar{\mu}^*)$$

$$= \frac{N_c}{2}\bar{\mu}^*\varrho_B(\mu^*)+G_\omega N_c^2\varrho_B^2(\mu^*),$$

$$\mu_* \simeq 303MeV$$

$$G_\omega \sim (10 \div 15)GeV^{-2}$$

In hot medium

$$\sum_{j,k=1}^{2}\left(\sigma_j^0(\mu^*,T)\Delta_{jk}\sigma_k^0(\mu^*,T)-\sigma_j^*(\mu^*,T)\Delta_{jk}\sigma_k^*(\mu^*,T)\right)=\frac{N_c}{2}\bar{\mu}^*\left(\varrho_B(\beta,\mu^*,\sigma_1^*)-\varrho_B(\beta,\mu^*,\sigma_1^0)\right)$$
$$+\frac{1}{2}T\left(S(\beta,\mu^*,\sigma_1^*)-S(\beta,\mu^*,\sigma_1^0)\right)+G_\omega N_c^2\left(\varrho_B^2(\beta,\mu^*,\sigma_1^*)-\varrho_B^2(\beta,\mu^*,\sigma_1^0)\right),$$

Gas-vapor tricritical point for nuclear matter arises when two minima and a saddle point merge
= triple root of mass-gap Eqs:
! Two constraints on coupling constants!

Necessary conditions to approach to P-violation phase

$$\partial_\mu\Big[(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\sigma_2^2\Big]<0$$

or

$$\Big(\lambda_6\sigma_1+4\lambda_2\sigma_2\Big)V^{(2)}_{\sigma_1\sigma_2}<\Big(2(\lambda_3-\lambda_4)\sigma_1+\lambda_6\sigma_2\Big)V^{(2)}_{\sigma_2\sigma_2}$$

One condition for 7 parameters:

P-violation is not exceptional but rather typical!

The type of phase transition: it is of a second order

$$\partial_\mu\sigma_1\Big|_{\mu_{crit}+i0}-\partial_\mu\sigma_1\Big|_{\mu_{crit}-i0}=-4\mathcal{N}\sigma_1\sqrt{\mu^2-\sigma_1^2}\,\frac{\big(\mathcal{V}_{10}\mathcal{V}_{22}-\mathcal{V}_{20}\mathcal{V}_{12}\big)^2}{\mathrm{Det}\mathcal{V}\;\mathrm{Det}V^{(2)}_{\sigma}}\Big|_{\rho\to0}<0,$$

Second variation matrices for effective potential before and after phase transition

P-parity violation phase

Critical points μ_c **when** $\rho(\mu_c) = 0$

$$(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})x^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})x + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0,$$

For $x = \frac{\sigma_2}{\sigma_1}.$

There are in general two solutions for two $\mu_c^- < \mu_c^+$

$$\boxed{\text{solution for } \mu > \mu_{crit}}$$

$$\sigma_2 = A\sigma_1 + \frac{B}{\sigma_1}, \qquad A \equiv \frac{2\lambda_5\lambda_2 + \lambda_6(\lambda_4 - \lambda_3)}{\lambda_6^2 - 8\lambda_2\lambda_4}, \qquad B \equiv \frac{\lambda_6\Delta_{22} - 4\lambda_2\Delta_{12}}{\lambda_6^2 - 8\lambda_2\lambda_4}.$$

P-parity violation strip

Boundary of the P-breaking phase,

$$\mathcal{N}\mathcal{A}(\sigma_1^{\pm}, \mu_c^{\pm}, \beta) = \Delta_{11} - 2\lambda_1(\sigma_1^{\pm})^2 - \lambda_5\sigma_1^{\pm}\sigma_2^{\pm} - (\lambda_3 - \lambda_4)(\sigma_2^{\pm})^2$$

It defines the P-breaking strip in the $T - \mu$ plane.

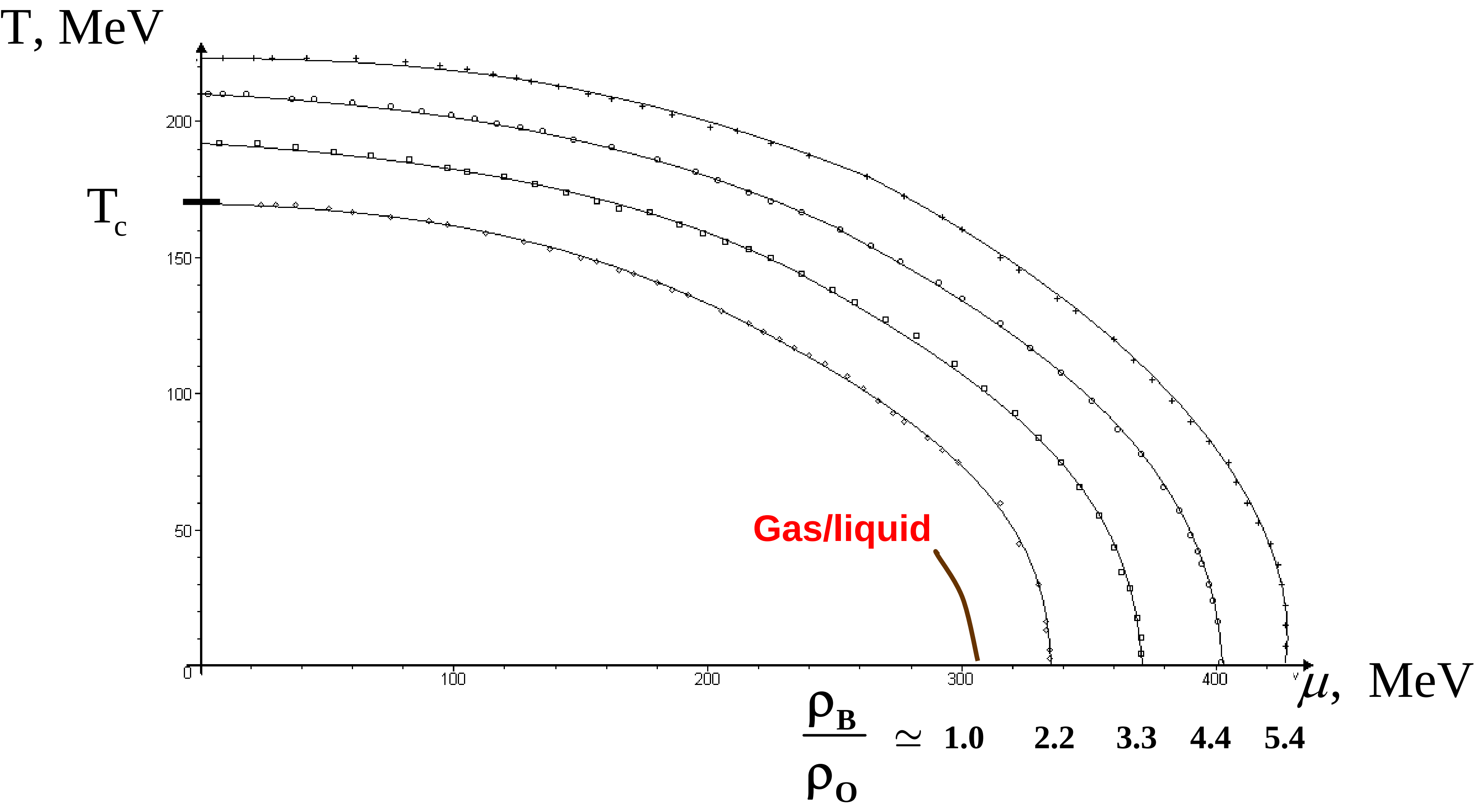
$\mathcal{A} > 0$ and $\mathcal{A} \to \infty$ when $T, \mu \to \infty$.

It means that for *any* nontrivial solution $r_{\pm}, \sigma_1^{\pm}, \sigma_2^{\pm}$ with $\mathcal{C}^{\pm}(\Delta_{jk}, \lambda_j) > 0$ the P-breaking phase boundary exists.

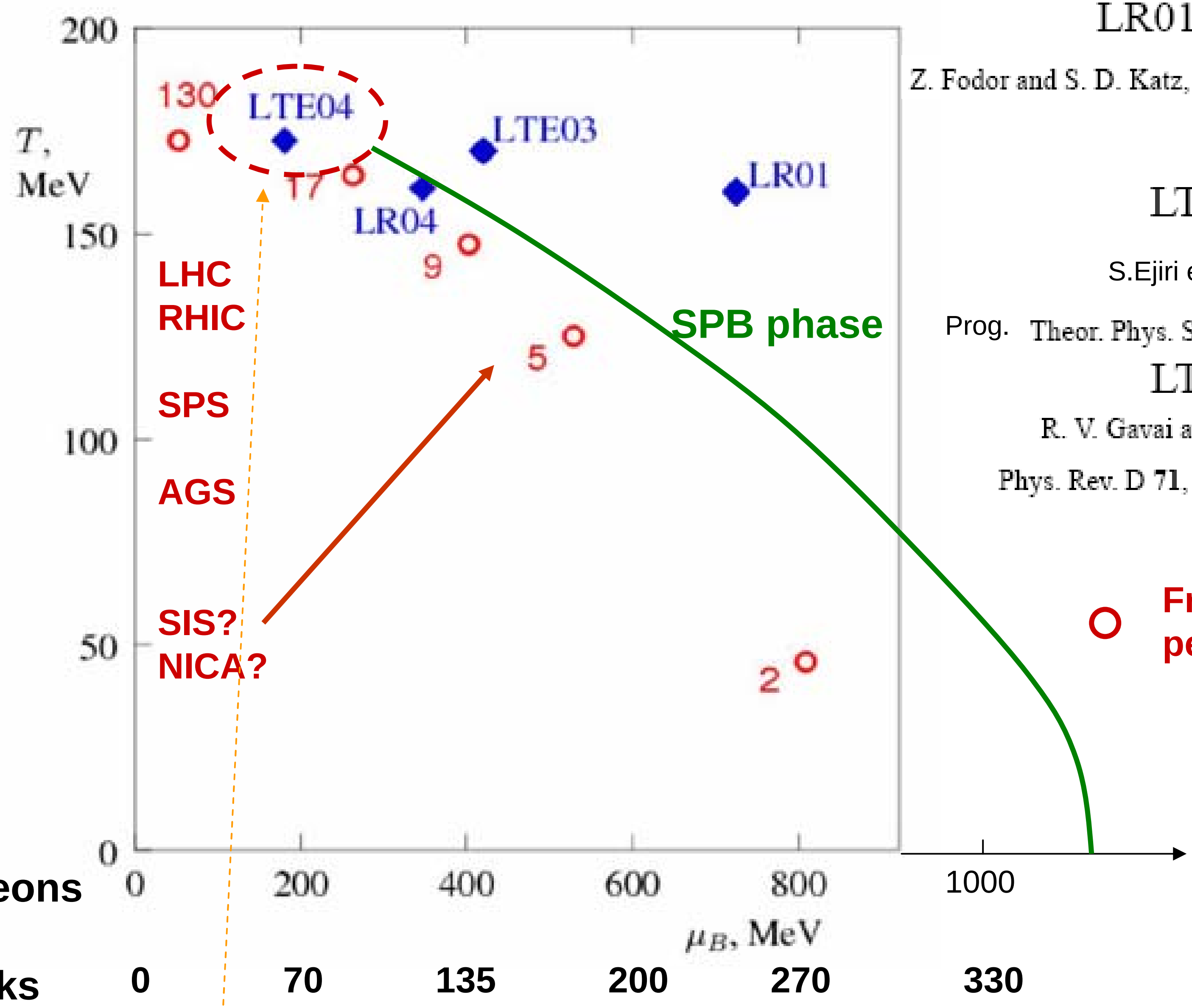
Thus if the phenomenon of P-breaking is realized for zero temperature it will take place in a strip including lower chemical potentials but higher temperatures.

However within the validity of chiral expansion the temperature cannot be too high!?

Parity breaking phase divide line



QCD tricritical point



Lattice estimations

LR01, LR04

Z. Fodor and S. D. Katz, JHEP **0203** (2002) 014

JHEP **0404**, 050 (2004)

LTE03

S.Ejiri et al.

Prog. Theor. Phys. Suppl. **153**, 118 (2004)

LTE04

R. V. Gavai and S. Gupta,

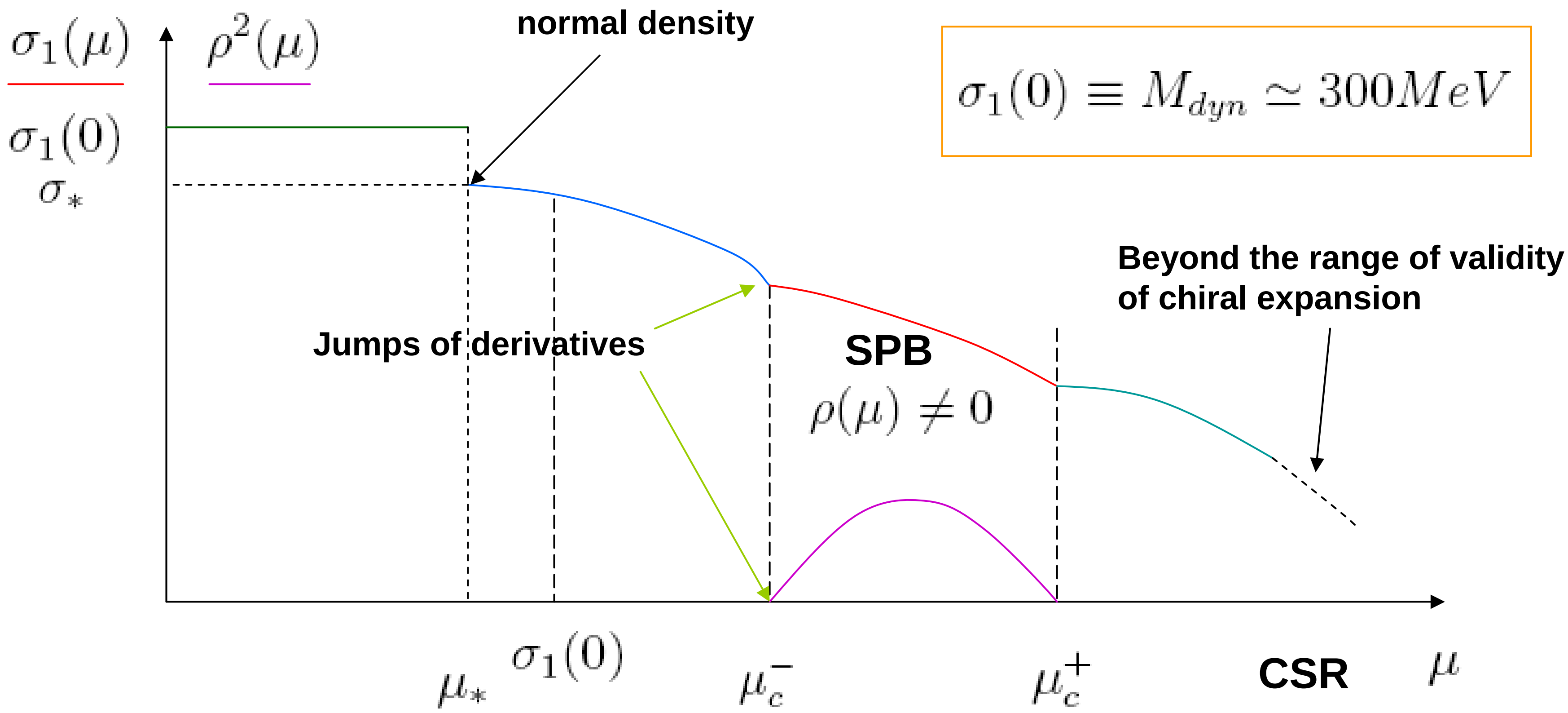
Phys. Rev. D **71**, 114014 (2005)

○ Freezout energy per baryon

!! $\mu_B^{\text{cep}} \sim 150 - 180$ MeV and $T_{\text{cep}} \sim 165 - 170$ MeV

R.A.Lacey et al 0708.3512, elliptic flow v2

Id order phase transition to saturation point (normal nuclear density) and spontaneous P-parity breaking (IId order phase transition)



With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL …)

P-breaking phase

Partially diagonalized kinetic term

$$
\begin{aligned}
\mathcal{L}^{(2)}_{kin} &= \partial_\mu\tilde{\pi}^\pm\partial^\mu\tilde{\pi}^\mp + (A_{22}-\zeta^2)\partial_\mu\Pi^\pm\partial^\mu\Pi^\mp + \frac{1}{2}\left(1+\frac{A_{22}\rho^2}{F_0^2}\right)\partial_\mu\tilde{\pi}^0\partial^\mu\tilde{\pi}^0 \\
&+\frac{1}{2}\left(A_{22}-\frac{F_0^2}{F_0^2+A_{22}\rho^2}\zeta^2\right)\partial_\mu\Pi^0\partial^\mu\Pi^0 + \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2+\rho^2\det A\,\delta_{1j}\delta_{1k}}{F_0^2+A_{22}\rho^2}\partial_\mu\Sigma_j\partial^\mu\Sigma_k \\
&-\frac{F_0\rho}{F_0^2+A_{22}\rho^2}\zeta\partial_\mu\Pi^0\sum_{j=1}^{2}A_{j2}\partial^\mu\Sigma_j
\end{aligned}
$$

Isospin breaking $SU_V(2)\rightarrow U(1)$

Meson spectrum in SPB phase

Neutral pi-prime condensate breaks vector SU(2) to U(1) and two charged pi-prime mesons become massless

$$
\begin{aligned}
\frac{1}{2}V_{11}^{(2)\sigma} &= 4\lambda_1\sigma_1^2 + 2\lambda_5\sigma_1\sigma_2 + 2\lambda_4\sigma_2^2 - 2\mathcal{N}\sigma_1^2 \ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1} \\
V_{12}^{(2)\sigma} &= 2\lambda_5\sigma_1^2 + 4\lambda_3\sigma_1\sigma_2 + 2\lambda_6\sigma_2^2 \\
\frac{1}{2}V_{22}^{(2)\sigma} &= 2\lambda_4\sigma_1^2 + 2\lambda_6\sigma_1\sigma_2 + 4\lambda_2\sigma_2^2 \\
V_{10}^{(2)\sigma\pi} &= \Big(4(\lambda_3-\lambda_4)\sigma_1 + 2\lambda_6\sigma_2\Big)\rho \\
V_{20}^{(2)\sigma\pi} &= \Big(2\lambda_6\sigma_1 + 8\lambda_2\sigma_2\Big)\rho \\
\frac{1}{2}V_{00}^{(2)\pi} &= 4\lambda_2\rho^2
\end{aligned}
$$

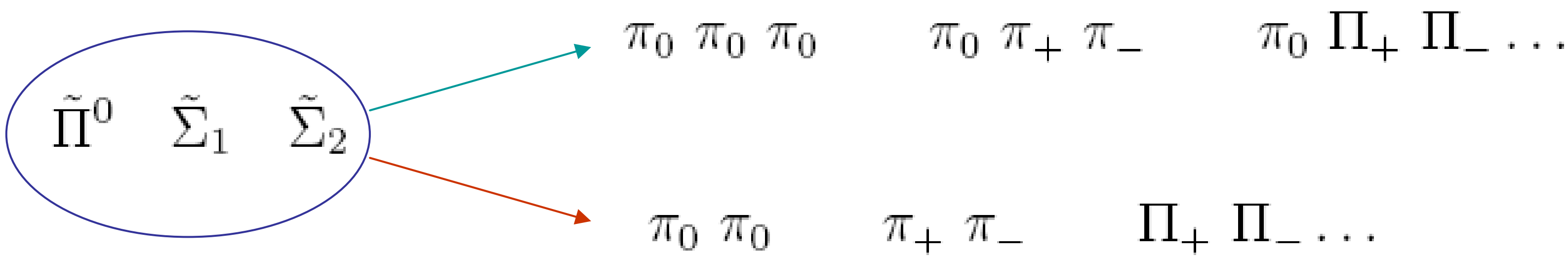
Mixture of massive scalar and neutral pseudoscalar states (for $V_{10}^{(2)\sigma\pi}$ and $V_{20}^{(2)\sigma\pi}$)

$$\frac{1}{2}V_{\pm\mp}^{(2)\pi} = 0$$

Mass matrix diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

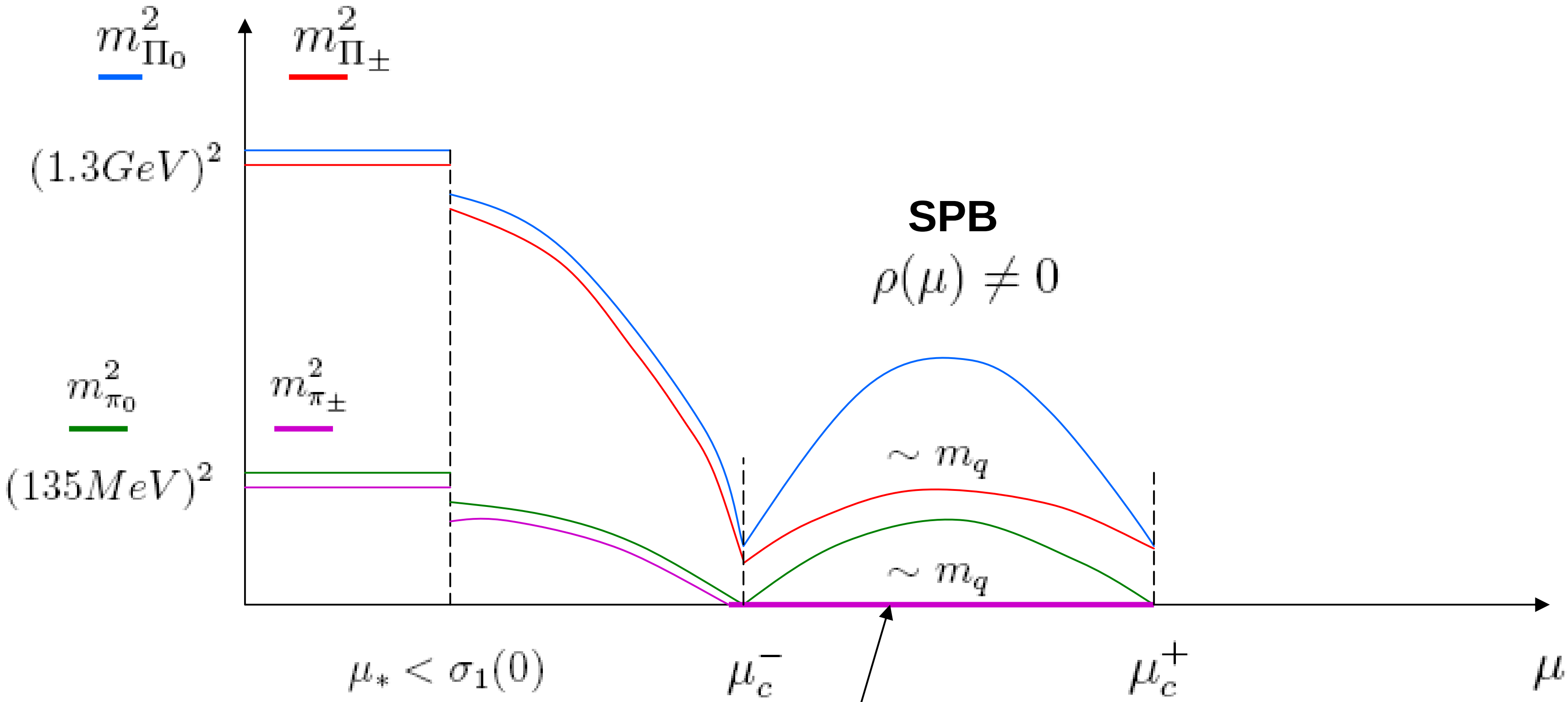
Therefore genuine mass states don't possess a definite parity in decays



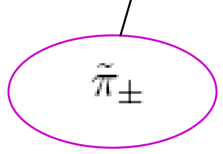
How to trigger a large volume P-breaking
= a large scale pseudoscalar condensate?
Strong electromagnetic fields! **Talk of D. Kharzeev**

Mass spectrum of "pseudoscalar" states

(massive quarks)



In P-breaking phase there are 2 massless pseudoscalar mesons



Where to see and how to check SPV ?

a) Approaching to SPV phase is indicated by a rapid decrease in heavy resonance masses . Below phase transition point one finds an abnormally *light and long-living pseudoscalar in-medium resonances*!

b) At the very point of the P-breaking phase transition one has three massless (very light) pion-like state.

c) After phase transition two massless (very light) charged pseudoscalars remain as Goldstone bosons enhancing charged pion production .

Hunting for new light pseudoscalars in medium!

d) F_{Π} and extended PCAC: it is modified for massless charged pions giving an enhancement of electroweak decays of heavy pions.

e) Additional isospin breaking: $f_{\pi_0} \neq f_{\pi_\pm}$

Perspectives

One can search for enhancement of long-range correlations in the pseudoscalar channel in lattice simulations? $T \neq 0;\ \mathrm{Im}\,\mu \neq 0$

Program for GSI SIS 300 ? For NICA, Dubna ?