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# Stable Phantom Cosmology based on (C)PT symmetry

**A.A. Andrianov**

*Universitat de Barcelona( ECM & ICC) & St.Petersburg State University*

*in collaboration with*

**F. Cannata, A.Y.Kamenshchik and D. Regoli**

**Highlights from:**

**AAA, F.Cannata and A.Y.Kamenshchik, Phys.Rev.D72:043531,2005; Int.J.Mod.Phys.D15:1299-1310,2006 J.Phys.A39:9975-9982,2006**

**ACK and D.Regoli, JCAP02:015,2008; JCAP10:019,2008; arXiv:0810.5076 [gr-qc]**

**Scalar field description of the Universe evolution**

↓

**Modern insights to the equation of state:**
**around cosmological constant but with *hints* on its evolution**

↓

**Phantom component of the large-scale Universe:**
**if not a myth then how to understand such a QFT?**

**Non-Hermitian Lagrangians!?**

# FRW cosmology for flat space

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a(t) & 0 & 0 \\ 0 & 0 & -a(t) & 0 \\ 0 & 0 & 0 & -a(t) \end{pmatrix}$$

**What is driving the Universe evolution?**
**Cosmological constant vs. matter+energy density?**

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**Consider our Universe filled by a barotropic fluid with an equation of state**

$$p = \mathrm{w}\,\rho$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

where $\rho$ and $p$ are energy density and pressure of the fluid,

## Einstein-Friedmann eqs. in units $G_{Newton} = \frac{3}{8\pi}$

$$h^2 = \rho,$$

$$h \equiv \frac{\dot{a}}{a}$$ Hubble variable

Energy conservation $$\dot{\rho} = -3h(\rho + p)$$

Pressure $$p = -\frac{2}{3}\dot{h} - h^2.$$

$$w = -1 - \frac{2}{3}\frac{\dot{h}}{h^2}$$

Dark energy is characterized by a negative pressure with $$w < -\frac{1}{3}$$

## Quintessence fields

**It is well accepted that for a given cosmological evolution $h(t)$ satisfying certain simple conditions one can employ a scalar field to build the cosmological model with the Lagrangian**

$$\mathcal{L} = \frac{1}{2}\,\dot{\phi}^2 - V(\phi)$$

**which contains the above mentioned evolution as a particular solution.**

# Einstein-Friedmann equations

**Gauge transformation:** $t \rightarrow \tau; \quad dt = N(\tau)\, d\tau$

$$h^2 \equiv \boxed{\frac{\dot{a}^2}{a^2} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \equiv \rho \qquad \text{density}$$

$$\delta a: \qquad \boxed{-h^2 - \frac{2}{3}\dot{h} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)} \equiv p \qquad \text{pressure}$$

$$\delta\varphi: \qquad \boxed{\ddot{\varphi} + 3\,h\,\varphi + V'(\varphi) = 0}$$

Eq. of state $\quad w = \frac{p}{\rho} \geq -1$ **!!!**

If quintessence fields are slowly rolling $\quad \dot{\varphi}^2 \ll V(\varphi)$
then
$w \searrow -1$

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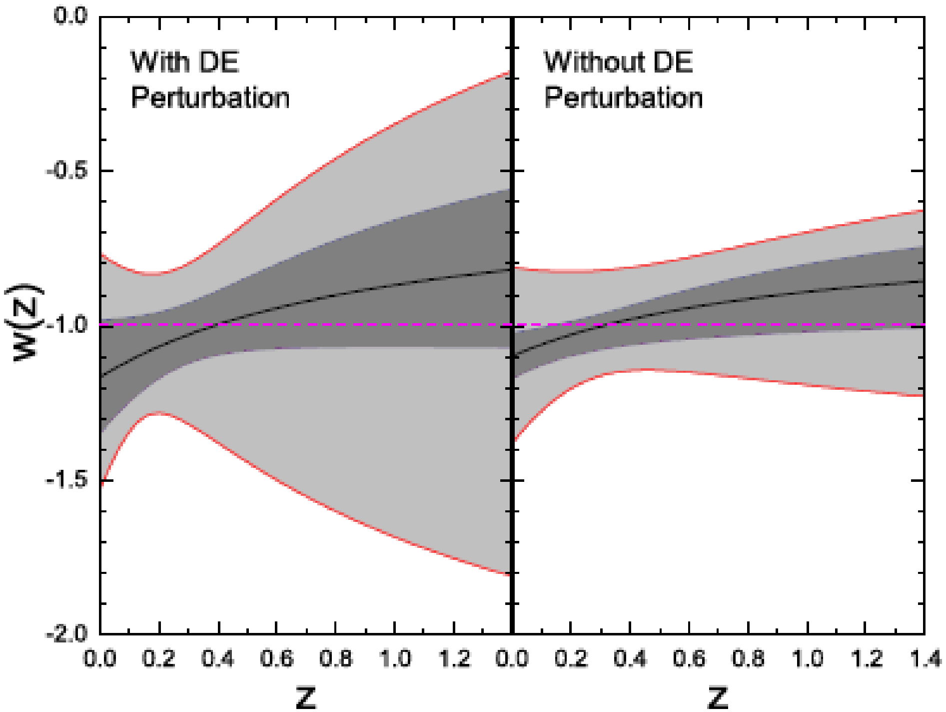
What about superaccelerated evolution $\quad w < -1$ ??

# Dark energy (DE) change (=evolution) with red shift z




(moderately optimistic!)

FIG. 4: Constrains on w(z) using WMAP + 157 "gold" SNIa data + SDSS with/without DE perturbation. Median(central line), 68%(inner, dark grey) and 95%(outer, light grey) intervals of w(z) using 2 parameter expansion of the EOS in (4).

$$w(z) = w_0 + w_1 \frac{z}{1+z}.$$

**Jun-Qing Xia et al, astro-ph/0511625**

**Wilkinson Microwave Anisotropy Probe (WMAP)**
**Type Ia supernova (SNIa)**
**Sloan Digital Sky Survey (SDSS)**

**Thus w = w(t) is a dynamical variable and one crosses the divide line w = -1 .**
**One needs a multicomponent scalar QFT to justify such a behavior .**
**Quintom, hessence, hybride → scalar QFT with two components.**

**Caldwell R. R., A Phantom menace ? , Phys. Lett. B 545, (2002) 23**

# *CHANDRA* CLUSTER COSMOLOGY PROJECT III: COSMOLOGICAL PARAMETER CONSTRAINTS

A. Vikhlinin[1,2], A. V. Kravtsov[3], R. A. Burenin[2], H. Ebeling[4], W. R. Forman[1], A. Hornstrup[5], C. Jones[1], S. S. Murray[1], D. Nagai[6], H. Quintana[7], A. Voevodkin[2,8]

arXiv:0812.2720v1 [astro-ph]

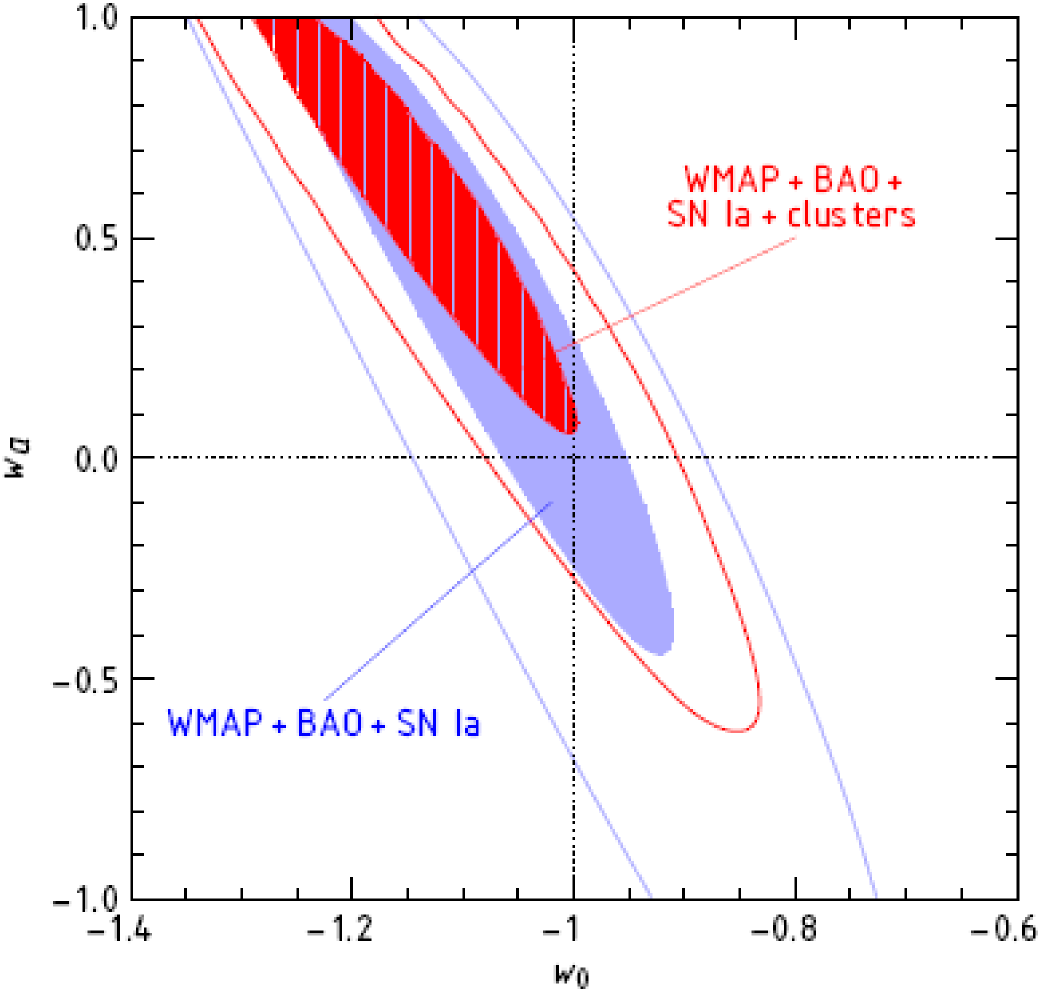



FIG. 12.— Constrains on evolving equation of state, $w(z) = w_0 + w_a z/(1+z)$, in flat universe.

## Superaccelerated evolution

**It can be designed with "phantom" fields** $L = -\frac{\dot{\phi}^2}{2} - V(\phi).$

**Evidently phantom fields have an energy unbounded from below**

$$\mathbf{L}_{\text{phantom}} \Rightarrow \mathbf{H}_{\text{phantom}} = -\pi^2 + V(\phi)$$

**Classically not! Because**

$$\rho \text{ is real} \Rightarrow -\dot{\varphi}^2 + V(\varphi) > 0 \text{ during classical evolution}$$

**BUT catastrophe against perturbations?**

**No ! Classical phantom fields may well be originated from a CPT invariant non-Hermitian but *crypto*-Hermitian QFT !**

Andrey Smilga

**Instructive exercise:**

**PT-symmetric QM oscillators**

$$H = p^2 + x^2 (ix)^{\varepsilon},\ 0 \le \varepsilon \le 2$$

**(rigorously proven to possess a real energy spectrum bounded from below)**

**Sample A:** $\varepsilon = 1$ Dorey,Dunning,Tateo, 2001

**Classics:** $H = p^2 + ix^3 \to \ddot{x}_{cl} = 3ix_{cl}^2 \xrightarrow{\text{a solution}} \mathrm{Re}\, x = 0,\ x_{cl} = i\, f(t)$

**Cosmology:** $\rho$ and $p$ are real $\Rightarrow$ $\dot{x}_{cl}^2$ and $V(x_{cl})$ are real !

$$S_{class} = \int dt \left( - (\dot{f}(t))^2 - f^3(t) \right)$$ real! but "phantom"

"phantom" oscillator

**Quasiclassics:** $x(t) = i\, f(t) + \delta x(t);\quad S^{(2)} = \int dt\, \delta x(t)\, (-\partial_t^2 + 6\, f(t))\, \delta x(t)$ real!

quantum fluctuations

**Sample B**: $\varepsilon = 2$ Andrianov, 1982

Classics: $H = p^2 - \lambda x^4$

$\xrightarrow{\text{a bad solution}}$ $\mathrm{Im}\, x = 0$ $\rightarrow$ $\ddot{x}_{cl} = 4\lambda x_{cl}^3$; $\dot{x}_{cl}^2 = C + \lambda x_{cl}^4$

infinite motion

$\xrightarrow{\text{a good solution}}$ $\mathrm{Re}\, x = 0;\quad x_{cl} = i\, f(t)$ $\rightarrow$ $\ddot{f} = -4\lambda f^3$; $\dot{f}^2 = C - \lambda f^4$

finite motion !

This classically “crazy” potential, on a curve in the complex coordinate plane (second solution), generates **exactly** the same energy spectrum as a two-dimensional O(2) symmetric quantum oscillator $V^{(4)}(\vec{q}) = \lambda(q_1^4 + q_2^4)/4$ with real coordinates in the sector of zero angular momentum.

Perturbations around an imaginary solution $q(t) = i\xi(t) + \delta q(t)$ can be performed along real axis and give a positive Hamiltonian

$$\mathcal{L}^{(2)} = p(t)\delta\dot{q}(t) - H, \qquad H = \frac{1}{4}p^2(t) + 12\lambda\xi^2(t)(\delta q(t))^2$$

# PT symmetric Quantum Mechanics ( $\Longrightarrow$ QFT?)

**Parity transformation** $\mathcal{P}\hat{x}\mathcal{P} = -\hat{x} \quad \text{and} \quad \mathcal{P}\hat{p}\mathcal{P} = -\hat{p}.$

**Time reversal** $\mathcal{T}\hat{x}\mathcal{T} = \hat{x} \quad \text{and} \quad \mathcal{T}\hat{p}\mathcal{T} = -\hat{p}. \qquad \mathcal{T}i\mathcal{T} = -i.$

**PT symmetry** $H(\mathcal{PT}) - (\mathcal{PT})H = 0.$

A $\mathcal{PT}$-symmetric Hamiltonian need not be Hermitian ! **!!** — **C.Bender et al 1998,…,2008**

**But if it has real spectrum it must be crypto-Hermitian !**

**Theorem:** $\exists C$, $H\,C = C\,H^{+}$; positive operator $C = C^{+} = \vartheta^{2}$; $\vartheta = \vartheta^{+}$

$\Rightarrow \vartheta^{-1} H \vartheta \equiv h\,; \quad h = h^{+}$ **(Mostafazadeh, 2003)**

**This operator is essentially non-local ! Thus one has two options:**
**either to work with a local but non-Hermitian QM**
**or with a non-local (non-Lagrangian!) one but Hermitian**

**What do you like?**

For $$H = \frac{p^2}{2m} + i\,\epsilon\, x^3,$$

a Hermitian counterpartner

$$\begin{aligned} h \;\; = \;\; & \frac{p^2}{2m} + \frac{3m}{16}\left(\{x^6, \frac{1}{p^2}\} + 22\hbar^2\{x^4, \frac{1}{p^4}\} + \alpha_2\,\hbar^4\{x^2, \frac{1}{p^6}\} + \right. \\ & \left. \frac{(14\alpha_2 + 1680)\hbar^6}{p^8} + \beta_2\,\hbar^3\{x^3, \frac{1}{p^5}\}\,\mathcal{P}\right)\epsilon^2 + \\ & \hbar^6\left(\alpha_3(\hbar\{x^2, \frac{1}{p^{11}}\} + \frac{44\hbar^3}{p^{13}}) + i\beta_3\,\{x^3, \frac{1}{p^{10}}\}\,\mathcal{P}\right)\epsilon^3 + \mathcal{O}(\epsilon^4), \end{aligned}$$

with a number of arbitrary parameters characterizing a non-uniqueness of C-operator

# Non-Hermitian (C)PT symmetric scalar QFT

(complex) Lagrangian of a scalar field

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi^* - V(\phi, \phi^*),$$

with the corresponding action,

**In units** $G_{Newton} = \frac{3}{8\pi}$

$$S(\phi, \phi^*, g) = \int d^4x \sqrt{-\|g\|}(L + \frac{1}{6}R(g)),$$

**Employ the potentials satisfying** $(V(\Phi, \Phi^*))^* = V(\Phi^*, \Phi),$

**However the Hermiticity is not required** $(V(\Phi, \Phi^*))^* \neq V(\Phi, \Phi^*)$

**This is a generalized condition of (C)PT symmetry**

## The model based on 1-dim non-Hermitian QM with real energy spectrum

**(proof of real spectrum in QM: Gasymov,1980;Curtright, Mezincescu, 2007)**

$$L = \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} - Ae^{\alpha\phi} + Be^{i\beta\chi}$$

**May be any function**

where $A$ and $B$ are real, positive constants

$$\ddot{\phi} + 3h\dot{\phi} + A\alpha e^{\alpha\phi} = 0,$$

$$\ddot{\chi} + 3h\dot{\chi} - iB\beta e^{i\beta\chi} = 0,$$

$$\chi = i\xi, \quad \xi \text{ real},$$

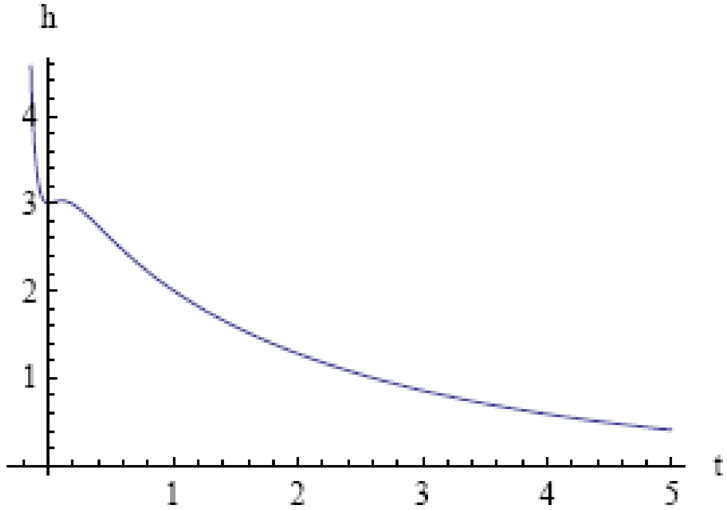
**$\rho$ and p are real $\Rightarrow$ $\dot{\varphi}^2 + \dot{\chi}^2$ and $V(\varphi, \chi)$ are real separately!**

$$h^2 = \frac{\dot{\phi}^2}{2} - \frac{\dot{\xi}^2}{2} + Ae^{\alpha\phi} - Be^{-\beta\xi}.$$

$$L_{eff} = \frac{1}{2}\dot{\delta\chi}^2 - B\beta^2 e^{i\beta\chi_0}(\delta\chi)^2,$$

where $\chi_0$ is a homogeneous purely imaginary solution of the dynamical system

# Cosmological evolution characteristic of the model



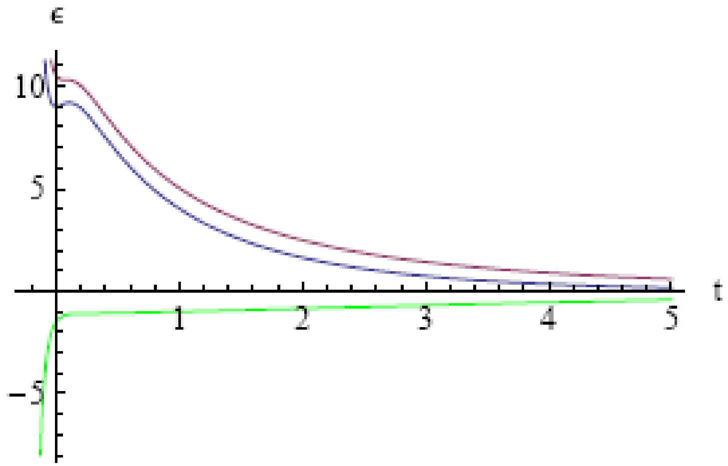




Fig. 1. (left) Plot of the Hubble parameter representing the cosmological evolution. The evolution starts from a Big Bang-type singularity and goes through a transient phase of superaccelerated expansion ("phantom era"), which lies between two crossings of PDL (when the derivative of $h$ crosses zero). Then the universe expands infinitely. (right) Plots of the total energy density (blue), and of the energy density of the normal field (purple) and of the phantom one (green).

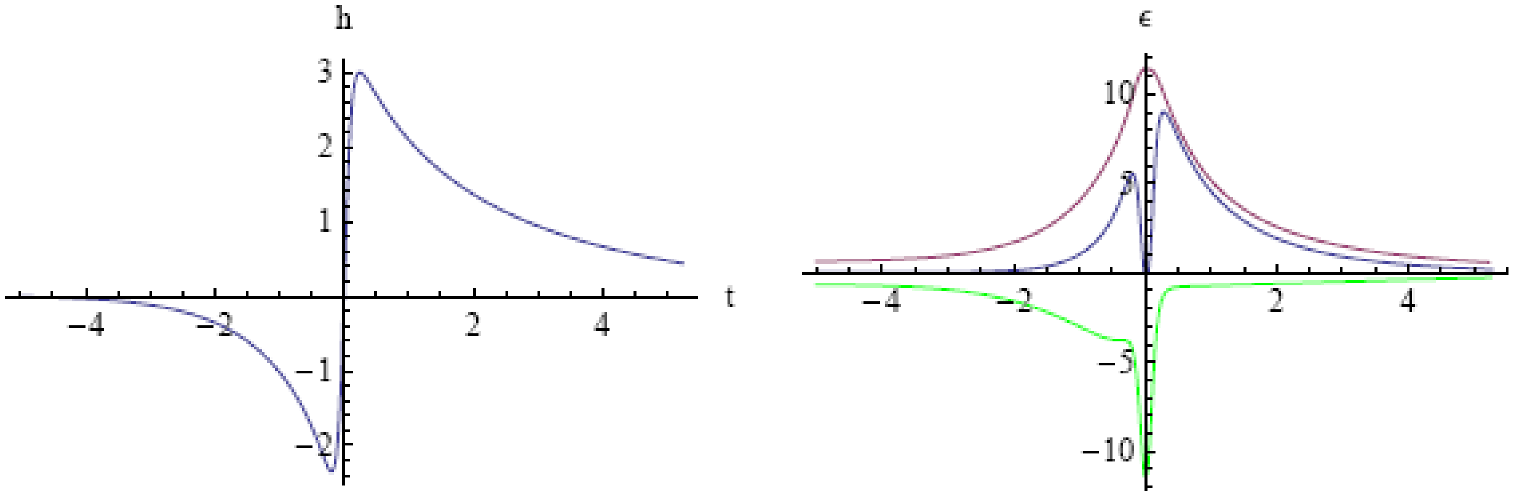



Fig. 2. (left) The evolution starts with a contraction in the infinitely remote past. At the point first PDL crossing the contraction becomes superdecelerated and turns in a superaccelerated expansion when $h$ crosses zero. The second PDL crossing ends the "phantom era"; the decelerated expansion continues till the universe begins contracting. In a finite time a Big Crunch-type singularity is reached. (right) Plots of the total energy density (blue), and of the energy density of the normal field (purple) and of the phantom one (green).

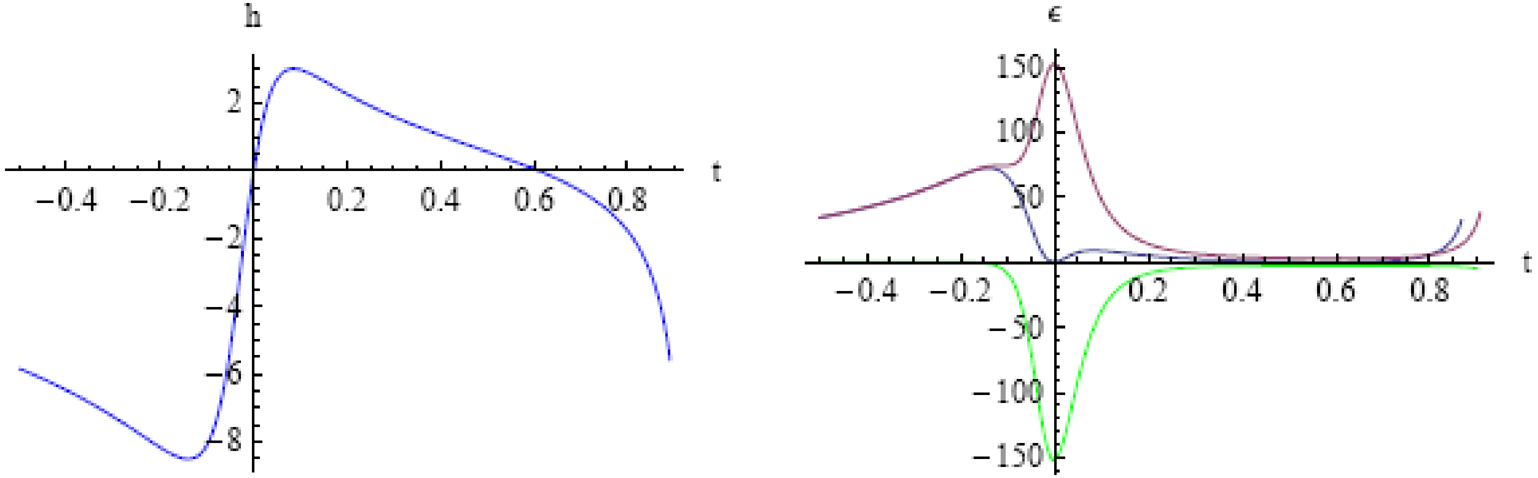



Fig. 3. (left) The cosmological evolution begins with a contraction in the infinitely remote past. with the first PDL crossing the contraction becomes superdecelerated until the universe stops ($h = 0$) and starts expanding. With the second crossing the "phantom era" ends and the expansion continues infinitely. (right) Plots of the total energy density (blue), and of the energy density of the normal field (purple) and of the phantom one (green).

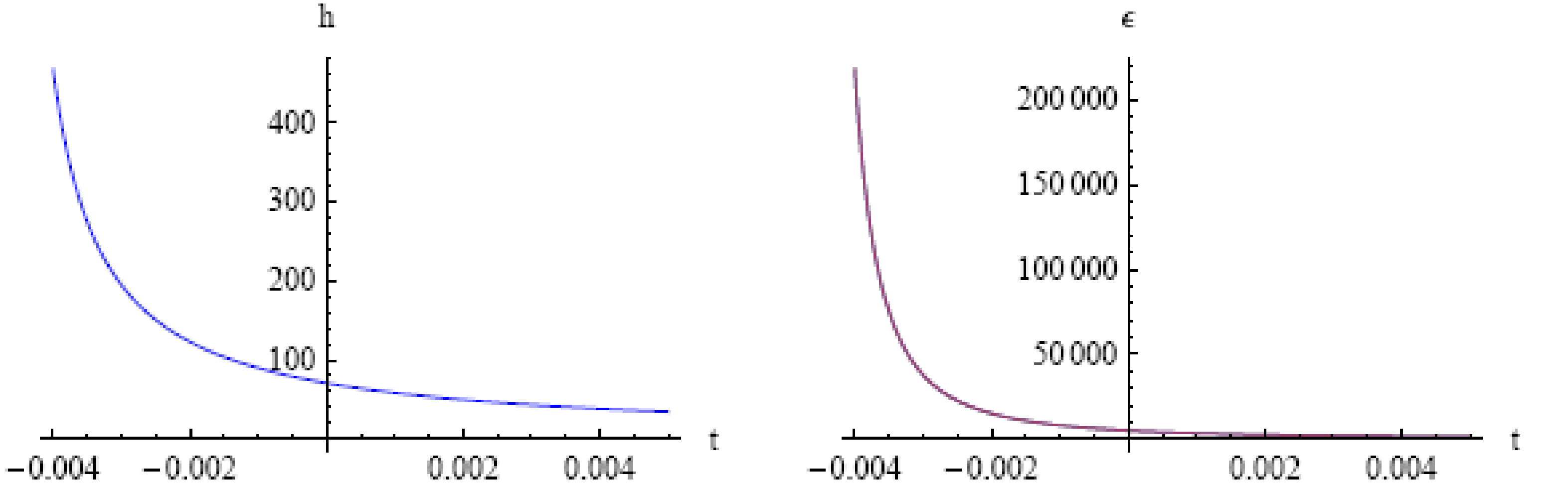



Fig. 4. (left) Evolution from a Big Bang-type singularity to an infinite expansion, without any crossing of PDL. This evolution is thus guided by the "normal" field $\phi$. (right) Plots of the total energy density (blue), and of the energy density of the normal field (purple) and of the phantom one (green). Notice that the energy density of the phantom field (green) is very close to zero, thus the total energy density is mainly due to the standard field.

the phantom dominance era is transient, the number of the phantom divide line crossings is even and the Big Rip singularity is excluded.

## For comparison a model for evolution with quintom = quintessence and phantom fields

the Hubble variable:

$$h(t) = \frac{A}{t\,(t_R - t)}.$$

The evolution begins at $t = 0$, which represents a standard initial big bang cosmological singularity, and comes to an end in the big rip type singularity at $t = t_R$.

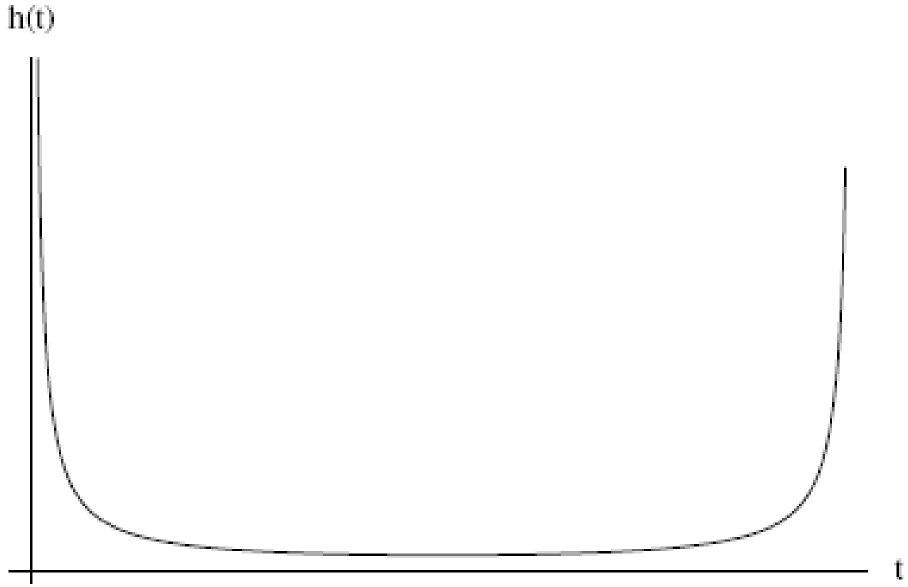



FIG. 2. $h(t)$ dependence in the model, describing the big rip.

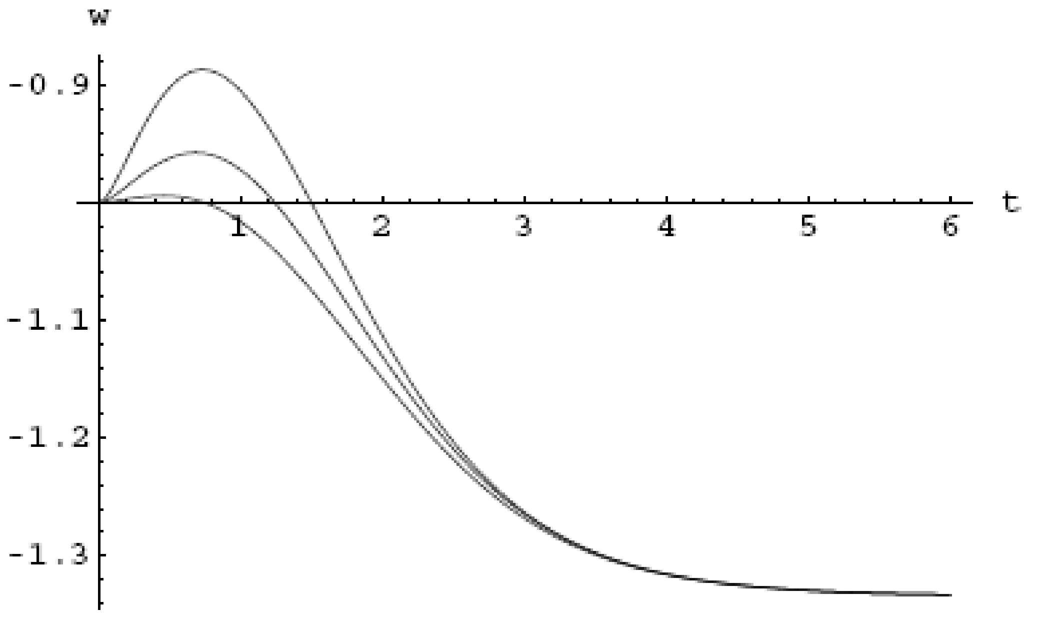



Figure 1: The evolution of the effective equation of state of the phantom and normal scalar fields with $V(\phi,\sigma) = V_{\phi 0}\, e^{-\lambda_\phi \kappa \phi} + V_{\sigma 0}\, e^{-\lambda_\sigma \kappa \sigma}$ for the case $\lambda_\phi = 1$.

**From: Guo Z. K. et al, Phys. Lett. B 608, (2005) 177**

# Perturbations around classical solution

## a) Scalar field perturbations

the quadratic part of the effective Lagrangian of perturbations:

$$L_{eff} = \frac{1}{2}\delta\dot{\chi}^2 - \frac{1}{2}(\vec{\nabla}(\delta\chi))^2 - B\beta^2 e^{i\beta\chi_0}(\delta\chi)^2,$$

$$H^{(2)}_{eff} = \frac{1}{2}\delta\pi^2 + \frac{1}{2}(\vec{\nabla}(\delta\chi))^2 + B\beta^2 e^{-\beta\xi_0}(\delta\chi)^2, \quad \delta\pi \Leftrightarrow \delta\dot{\chi}.$$

**It is positive, i.e. bounded from below**

## b) Metric perturbations $h_{\mu\nu} \equiv \delta g_{\mu\nu}$

The Hamiltonian of the metric perturbations should be naturally added to the above formulae. The mixed term, including both the metric and scalar field perturbations, appears superficially annoying because it contains the first derivative of the term $Be^{i\beta\chi}$, which is imaginary. However, one can show that by a proper choice of the gauge condition this imaginary term can be eliminated.

**The relevant terms**

$$\delta^2 S = \int d^4x \sqrt{-g}\delta\chi \left( h_{00}\chi^{(0)}_{,00} + \chi^{(0)}_{,0} \left( \nabla_\mu h^{\mu 0} - \frac{1}{2} h_{,0} + \frac{\dot{a}}{a}(h - h_{00}) \right) \right), \tag{18}$$

where $h = h^\mu_\mu$, commas stand for partial derivative and $\nabla_\mu$ is the covariant derivative operator. The expression (18) is purely imaginary because $\chi^{(0)}$ is purely imaginary

**This combination is not gauge invariant and the suitable gauge choice is evident: one has to constrain metric perturbations eliminating the linear combination in parentheses**

**In this gauge metric perturbations decouple from "phantom" ones and interplay only with perturbations of quintessence scalar field**

# Resume′

1) **The present-day knowledge of dark energy evolution leaves a room for eq. of state with $w < -1$ .**
2) **It poses the problem of existing of an unusual scalar matter with negative kinetic energy.**
3) **Such a classical FT can be derived from a (C)PT invariant non-Hermitian QFT, quantizable in quasiclassics and possessing a real energy spectrum.**
4) **From QM to QFT: are there crypto-Hermitian QFT? It has not yet been proven rigorously.**