Unexpected physical contents of the fluxbrane throat-like solutions in Type IIA and Type IIB supergravities

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Abstract

In Parts I, II of the Report the influence of background fluxes on spectra of masses of Fermions and KK gauge fields is studied for the AdS(5)xS(5) solution of the Type IIB supergravity. It is shown that presence of the flux-generated Pauli-type term in D10 Dirac equation permits to receive the light neutrino mass scale in the Fermion's "twisted" spectrum without any reference to the standard seesaw mechanism and heavy right neutrino. The difference of profiles in extra space of the wavefunctions of right and left chiral components of the "twisted" Dirac spinor results in their essentially different interactions with "ordinary" matter in 4 dimensions. In Part II it is shown that background magnetic flux plays the role of vacuum condensate of the charged Higgs scalar in generating the mass gap for Kaluza-Klein gauge field. Part III presents simple analytical expression for the radion effective potential which is conventionally calculated for the background magnetic fluxbrane throat-like solution of the supergravity theory and which meets the early inflation observational demands.

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1 Part I. Peculiarities of mass spectrum of Dirac Eq. with the Pauli-type flux term

This Section sums up the results of preprint [1]. To receive the observed spectra of Fermions from the higher-dimensional theories is a long-standing problem. Introduction of Higgs scalar is a conventional Standard Model approach to generate masses of Fermi fields. However mass-like terms in Dirac equations in higher-dimensional theories may appear also because of interaction of Fermion with gauge fields (see e.g. Review [2]) or with n-form fields ([3] and references therein). Thus the interesting task is to study the influence of the extra-dimensional Pauli-type terms in the bulk Fermi field Lagrangian on the properties of mass spectra of Fermi excitations on different supergravity backgrounds.

Here spectrum of D10 Dirac equation with the flux-generated bulk "mass term" in the Type IIB supergravity [4]

$$\left(\Gamma^M D_M - \frac{i}{2\cdot 5!}\Gamma^{M_1\dots M_5}F_{M_1\dots M_5}\right)\hat{\lambda} = 0 \tag{1}$$

is explored on the $AdS_5 \times S^5$ (+ self-dual 5-form) background:

$$ds_{10}^2 = e^{-2z/L}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 + L^2 d\Omega_5^2, \tag{2}$$

$$F_{0123z} = e^{-4z/L}\bar{Q}/L, \qquad F_{56789} = L^4\bar{Q}, \qquad \bar{Q} = 1. \tag{3}$$

The value of the 5-form charge $\bar{Q} = 1$ follows from the Einstein equations in 10 dimensions for the choice of normalization of the 5-form taken in the Type IIB supergravity action in [4]:

$$S = \frac{1}{2l_s^8}\int d^{10}x\,\sqrt{-g}\left(R - \frac{4}{5!}F_{M_1\dots M_5}F^{M_1\dots M_5} + \dots\right), \tag{4}$$

l_s is fundamental string length. We follow here the notations of [4]: $M, N = 0, 1\dots 9$, $x^M = (x^a, y^\alpha)$, $x^a = (x^\mu, z)$ ($\mu = 0, 1, 2, 3$) and y^α are five angles of S^5 ($\alpha = 5, 6, 7, 8, 9$). $\eta_{\mu\nu}$ in (2) is metric of Minkowski space-time with signature $(-, +, +, +)$.

D10 space-time is as ordinary orbifolded at the UV and IR boundaries given by the corresponding values of proper coordinate z:

$$z_{UV} = 0 < z < \pi R = z_{IR}, \tag{5}$$

$AdS_5 \times S^5$ space-time consists of two pasted copies with Z_2 symmetry imposed at its UV and IR ends. It is supposed that there are no additional surface terms of the Action which may influence the dynamics of Fermions.

The low-energy effective Action (4) makes sense if scale of curvature of space-time (2) is essentially below the fundamental scale, i.e. for $L \gg l_s$. Standard dimensional reduction of Einstein term in (4) with use of background metric (2) gives the following expression for Planck Mass in 4 dimensions through length parameters L, l_s (cf. e.g. [5]):

$$M_{Pl} = \sqrt{\frac{\pi^3}{2}} \cdot \left(\frac{L}{l_s}\right)^4 \frac{1}{L}, \tag{6}$$

here the exponentially small contribution from z_{IR} limit of integration over z in (4) is omitted and value of volume of unit 5-sphere $\Omega_5 = \pi^3$ is used.

As it was shown in [4] only one of two chiral components of 32-component $D10$ Dirac spinor $\hat{\lambda}$ in (1) "feels" the flux; we shall identify these components by the two-values number $Q = (\bar{Q}, 0) = (1, 0)$.

Expanding these 16-component $D10$ chiral spinors in two set (every numbered by $n = 0, 1, 2 \ldots$) of spherical harmonics of S^5, see in [4], taking the set where non-zero flux ($Q = 1$) effectively decreases by 1 spectral number n, and separating variables of the 4-component spinor λ in 5 dimensions:

$$\lambda(x^\mu, z) = (\lambda_L, \lambda_R) = (\psi_L(x^\mu)\, f_L(z),\ \psi_R(x^\mu)\, f_R(z)) \tag{7}$$

(here ψ_L, ψ_R are the left and right components of Dirac spinor $\psi(x^\mu) = (\psi_L,\ \psi_R)$ of mass m governed by the ordinary Dirac equation in 4 dimensions $(\gamma^\mu \partial_\mu - m)\psi = 0$; all indices of λ, $\psi_{L,R}$, $f_{L,R}$, m are omitted), the following system for profiles $f_{L,R}(z)$ is received:

$$\begin{aligned} &\left[\frac{d}{dz} - \frac{2}{L} - \frac{1}{L}\,(\nu + 1/2)\right] f_L + m\, e^{z/L} f_R = 0, \\ &\left[\frac{d}{dz} - \frac{2}{L} + \frac{1}{L}\,(\nu + 1/2)\right] f_R - m\, e^{z/L} f_L = 0. \end{aligned} \tag{8}$$

Parameter $\nu = n + 2 - Q$ in (8) essentially determines the looked for spectra of m; for $Q = 1$ (i.e. for Fermions which "feel" the flux) $\nu = 1, 2 \ldots$, and in case $Q = 0$ we have $\nu = 2, 3 \ldots$ We'll see that it is influence of flux which permits to receive the "seesaw" value of mass of Fermion (cf. expressions (13) and (14) below).

Equations (8) are typical in the Randall-Sundrum type models when bulk Dirac mass term is included in the Fermi field Lagrangian [6], [7], [8], [9]. However, contrary to these papers were value of bulk Dirac mass which determines the physically important parameter ν in (8) was taken "by hand", here we rely upon well grounded supergravity approach which gives definite values of parameter ν.

Solution of system (8) is a linear combination of Bessel and Neumann functions [7]-[9]:

$$
\begin{aligned}
f_L(z) &= e^{5z/2L} \left[A J_\nu(\tau) + B N_\nu(\tau) \right], \\
f_R(z) &= e^{5z/2L} \left[A J_{\nu+1}(\tau) + B N_{\nu+1}(\tau) \right],
\end{aligned}
\tag{9}
$$

where $\tau = mLe^{z/L}$; A, B are integration constants.

Spectra of m are determined from the boundary conditions at the orbifold points $z = 0$, $z = \pi R$ which are easily received from the transformation rule of spinor $\hat{\lambda}$ under reflection of coordinate z [10]:

$$
P_z \hat{\lambda}(z) = \Gamma^{\hat{z}} \hat{\lambda}(-z), \tag{10}
$$

$\Gamma^{\hat{z}}$ is corresponding $32 \otimes 32$ gamma-matrix in 10 dimensions [4], [1].

The crucial point is to consider (following [7], [8], [11]) two types of boundary conditions: the usual "untwisted" one (when e.g. $f_L(0) = f_L(\pi R) = 0$) and "twisted" one (when $f_L(0) = f_R(\pi R) = 0$). These conditions determine two essentially different towers of the eigenvalues of Dirac equation (1). The "twisted" boundary condition corresponds to breaking supersymmetry by the Scherk-Schwarz mechanism [12], [2] and its application for receiving small gravitino mass in the warped models was first proposed, as to our knowledge, in [8].

The "untwisted" spectrum is given by simple formula [7]:

$$m^{untw}_{q,n} \simeq \left(q + \frac{\nu}{2} - \frac{3}{4}\right) \frac{\pi}{L}\, e^{-\pi R/L} =$$

$$\left(q + \frac{n}{2} + \frac{1}{4} - \frac{Q}{2}\right) \sqrt{\frac{2}{\pi}} M_{Pl} \left(\frac{l_s}{L}\right)^4 e^{-\pi R/L} \cong M_{EW}, \tag{11}$$

where $q = 1, 2, 3 \ldots$, $n = 0, 1, 2 \ldots$, $Q = 1, 0$; formula (6) was used to express L^{-1} through M_{Pl}.

Physically mass scale in the RHS of (11) must be of order of the electroweak scale M_{EW}; its relation to the Planck scale ("first mass hierarchy") is basically given by the small Randall-Sundrum exponent $e^{-\pi R/L}$ in (11), although in the model under consideration it also depends on relation of fundamental string length l_s to the scale L of the Type IIB supergravity solution (2).

Since $\nu = n + 2 - Q > 0$ the profiles of eigenfunctions (9) of "untwisted" modes are concentrated in vicinity of the IR end of the slice (5) of $AdS_5 \times S^5$ space-time.

The spectrum of the "twisted" solution also possesses the "first hierarchy" massive modes with eigenvalues of type (11), but it also has the "inverse tower" of extremely small values of $m^{tw}_{Q,n}$ exponentially decreasing with growth of the 5-sphere spectral number n:

$$m^{tw}_{Q,n} = \frac{2\sqrt{n+2-Q}}{L}\, e^{-(n+3-Q)\pi R/L}, \tag{12}$$

which is received with account that in this case argument in (9) is small, in particular at the boundaries of slice (5) we have: $\tau_{UV} = mL \ll 1$ and $\tau_{IR} = mLe^{\pi R/L} \ll 1$. We also inserted $\nu = n + 2 - Q$ in (12).

The highest value of $m^{tw}_{Q,n}$ is achieved when Fermion interacts with flux, i.e. at $Q = 1$, $n = 0$ in (12):

$$m^{tw}_{1,0} = \frac{2}{L}\, e^{-2\pi R/L} = \left(\frac{L}{l_s}\right)^4 \frac{M^2_{EW}}{M_{Pl}}. \tag{13}$$

In deriving the RHS of (13) we expressed L^{-1} and $e^{-\pi R/L}$ through M_{Pl} and M_{EW} from (6), (11) and omitted coefficient of order one. For the choice $M_{EW} = 1TeV$, $(L/l_s)^4 = 10^3$ (13) gives the mass scale of order of mass of electron neutrino.

Theory surely must be more elaborated. The goal of the paper is to demonstrate interesting potential possibilities of the supergravity models, and to demonstrate the importance of presence of Pauli type terms in the bulk Dirac equations. In fact, let us calculate the first spectral value of tower (12) when there is no flux, i.e. for $Q = 0$, $n = 0$ (or this is the second spectral value in the presence of flux, i.e. for $Q = 1$, $n = 1$):

$$m^{tw}_{0,0} = m^{tw}_{1,1} = \frac{\sqrt{8}}{L}\, e^{-3\pi R/L} = \left(\frac{L}{l_s}\right)^8 \frac{M^3_{EW}}{M^2_{Pl}}. \tag{14}$$

In the absence of the 5-form term in (1) this would be the highest value of spectrum (12) and physically promising "seesaw" combination M^2_{EW}/M_{Pl} like in the RHS of (13) would not appear in the spectrum of Dirac equation. It must be noted that mass scale (13) M^2_{EW}/M_{Pl} is received here without any reference to large right neutrino mass and standard seesaw mechanism (cf. [6]).

Now let us look at the profiles of eigenfunctions (9) of "twisted" modes. With use of boundary conditions $f_L(0) = f_R(\pi R) = 0$, inserting expression for m given in (12), and taking into account that in this case argument of cylinder functions in (9) is small ($\tau \ll 1$) everywhere inside the slice (5), it is easy to receive the simple approximate expressions for "twisted" eigenfunctions (9):

$$\begin{aligned} f^{tw}_L(z) &= N_\nu\, e^{5z/2L} \sinh\left(\frac{\nu z}{L}\right), \\ f^{tw}_R(z) &= -N_\nu \sqrt{\nu}\, e^{5z/2L} \sinh\left[(\nu+1)\frac{(\pi R - z)}{L}\right], \end{aligned} \tag{15}$$

where N_ν is the normalization factor, $\nu = n + 2 - Q$.

From (15) it is immediately seen that "twisted" profile $f^{tw}_L(z)$ of the left component of 4-spinor is concentrated near the IR end of the warped space-time (2), whereas profile $f^{tw}_R(z)$ of the right component is located near the UV end. This must result in essential difference in interactions of the left and right chiral components of "twisted" Dirac spinor with massive modes of other fields which profiles in extra space are concentrated near IR end of the bulk.

In the extra-dimensional theories the strength of interaction of modes of different fields depends on overlapping of their wave functions in extra space.

That is why universality of electric charge is achieved in these theories only if zero-mode of electro-magnetic field is constant in extra space. The same is true of course for the interaction of matter with gravitational field, the constancy of its zero-mode was supposed in deduction of expression (6) for Planck mass in 4 dimensions.

If "twisted" 4D Dirac Fermions considered above are neutral, then their left chiral components ("LH neutrino") will be observed since their profiles (15) in extra space overlap with profiles of modes of other fields (trapped on the IR brane or "living" in the bulk in vicinity of its IR end). Whereas right chiral components ("RH neutrino") will not be observed in experiments because of the exponential suppression of the overlapping in extra space of their wave functions (15) with wave functions of the "ordinary" matter modes.

Thus in this approach there is no need to suppose the extra large mass of right neutrino as an explanation of its non-observability in experiments. On the other hand interpretation of the right and left chiral components of Dirac spinor considered above as corresponding neutrinos is hardly compatible with the different group nature of right and left neutrinos in Standard Model. It would be interesting to study the possibility to receive the "seesaw" scale of Majorana mass in frames of approach described above.

2 Part II. Higgs mechanism from fluxes for Kaluza-Klein gauge fields

In [13] the effective Actions in five-dimensional space-time (formulae (18), (19) in [13]) of the Kaluza-Klein gauge fields associated with isometries of subspaces S^4 and S^1 of the throat-like model in the Type IIA supergravity were received by reduction from 10 to 5 dimensions. The interesting feature of these Actions was the appearance of the mass terms of KK gauge potentials resulting from the 4-form and 2-form terms of the IIA supergravity Action when there are non-zero magnetic fluxes in the background solution.

Here we demonstrate this "flux instead of Higgs" phenomenon for the theory given by Action (4), i.e. in the Type IIB supergravity for its most familiar throat-like solution of the $AdS_5 \otimes S^5$ (plus self-dual 5-form) background (2), (3).

Following conventional Kaluza-Klein approach let us introduce non-diagonal components B_a^α of D10 metric which "mix" AdS_5 and S^5 subspaces and which are associated with isometries of S^5: $B_a^\alpha = B_a^q(x^\nu, z)\,\xi_q^\alpha(y)$, where ξ_q^α are the corresponding Killing vectors on S^5, q is group index (summing over q is supposed); indices $a = (\mu, z)$, $\alpha = 1\ldots 5$; y^α are angles of S^5.

Then standard reduction of D10 Action (4) to 5 dimensions with use of background (2), (3) with account of limits (5) gives following Action in 5 dimensions for KK gauge potentials $B_\mu^q(x^\nu, z)$ (we put $B_z^q = 0$ which is always possible with use of the gauge freedom in 5 dimensions; we also take Minkowski 4D space-time):

$$S^{(5)}(B_\mu^q) = \frac{\pi^3 L^7}{2 l_s^8} \int d^4x \int_0^{\pi R} \left[-\frac{1}{4} F_{\mu\nu}^q F^{q\mu\nu} - \right.$$
$$\left. -\frac{1}{2} e^{-2z/L} B_{\mu,z}^q B_{,z}^{q\mu} - \frac{2}{L^2} e^{-2z/L} B_\mu^q B^{q\mu} \right] dz, \tag{16}$$

where $F_{\mu\nu}^q$ is Yang-Mills field strength of potentials B_μ^q, comma means derivative over z, and contraction over μ, ν is performed with Minkowski metric. We omitted higher-order terms in B_μ^q in (16).

Mass term $B_\mu^q B^{q\mu}$ in (16) results from the 5-form term in (4) in direct analogy with Higgs mechanism - because of existence of non-zero background 5-form (3) which here substitutes the condensate of charged scalar field in the conventional Higgs mechanism. This is seen from direct calculation of the $F_{(5)}^2$ term in (4) when there are non-zero non-diagonal KK components of the 10-dimensional metric. With account of background (2), (3) and after integrating out 5-sphere angles y^α it is received:

$$\int F_{M_1\ldots M_5} F^{M_1\ldots M_5}\, dy = \frac{1}{L^2} \left[1 + L^2 e^{2z/L} B_\mu^q B^{q\mu} + \ldots \right]. \tag{17}$$

Dots mean higher order terms in B_μ^q - up to B^{10}. Background electric 5-form component F_{0123z} in (3) gives B^5 term in RHS of (17). Thus quadratic B^2 term which we are interested in is formed only by the background magnetic flux.

Evident gauge non-invariance of (17) is not a mistake at all since in calculating RHS of (17) we took the lower indices components of 5-form like in (3). This is the analogy of taking zero phase of the charged Higgs scalar which gives manifestly non-gauge-invariant scalar field Action in the unitary gauge. The LHS of (17) is surely gauge invariant (i.e. invariant of the corresponding KK general coordinate transformations $y^\alpha \to y^\alpha + \xi_q^\alpha(y) f^q(x^\mu)$) if in parallel with the gauge transformation of potentials, $B^q_\mu \to B^q_\mu + f^q_{,\mu}$, all non-zero components of the 5-form $F_{M_1...M_5}$ appearing because of this transformation are taken into account in calculation in (17).

Standard procedure gives spectrum of linear excitations of KK gauge field on the background (2), (3). Variation of Action (16) over B^q_μ and separation of variables: $B^q_\mu(x^\nu, z) = \Sigma_n B^q_{\mu n}(x^\nu) F_n(z)$, where $B^q_{\mu n}(x^\nu)$ are 4D gauge fields of mass m_n, give finally the following equation for the wave functions $F_n(z)$:

$$\left[\frac{d^2}{dz^2} - \frac{2}{L}\frac{d}{dz} + m^2 e^{2z/L} - \frac{4}{L^2}\right] F(z) = 0, \tag{18}$$

index n of F, m is omitted here. It is seen immediately that the flux-generated bulk mass term $4/L^2$ in (18) excludes zero-mass gauge field from the spectrum of excitations.

Solution of (18) is well known (see e.g. general analysis in [7]). Like in (9) it is expressed through the Bessell and Neumann functions of the argument $\tau = mLe^{z/L}$

$$F(z) = e^{z/L}[AJ_{\sqrt{5}}(\tau) + BN_{\sqrt{5}}(\tau)]. \tag{19}$$

Imposing the standard boundary conditions at the UV and IR ends (5) of the bulk gives mass spectrum m_n which linearly depends on spectral number n. Mass gap is given by formula (cf. (11)):

$$m_{min} \cong L^{-1} e^{-\pi R/L} \approx 1TeV. \tag{20}$$

Thus background magnetic flux of solution (2), (3) "fulfills the job" of Higgs field in making KK gauge fields massive.

This mechanism of generating masses of KK gauge fields is not associated only with the throat-like fluxbrane solutions but is of general nature. Presence in the background solution of the fluxes which components "live" in some extra subspace will inevitably make massive KK gauge fields corresponding to the isometries of this subspace.

Sometimes it may give curious results. E.g. in the model of the Type IIA supergravity considered in [13], where the bulk mass term in equation of type (18) (Eq. (24) of [13]) generated by the 2-form background flux quickly decreases up the throat ("up" means from IR end to UV end), the "seesaw" scale, M^2_{EW}/M_{Pl}, in spectrum of the Abelian KK gauge field associated with isometry of subspace S^1 of the background solution appears. Although this mathematical result looks interesting its physical interpretation is vague.

3 Part III. Radion as inflaton. Simple calculation of potential

Here another curious feature of the fluxbrane throat-like solutions of IIA and IIB supergravities will be demonstrated. This is the possibility of receiving the exact analytical expression for the radion effective potential which meets the strict early inflation demands. This Section covers in short the results of [14], [15].

Radion field is defined as the position $\rho(x)$ of the UV boundary of the 10-dimensional space-time "moved" from the point r_0 fixed by the Israel junction conditions and slowly depending on coordinates of D4 space-time x^μ, see (23) below. Radion's potential is, as ordinary, calculated for constant scalar field $\rho = const$ and is conventionally defined as a value of D10 supergravity Action calculated on the throat-like background where 6 extra dimensions are integrated out. Arbitrary "upper" (UV) limit ρ of integration over the isotropic coordinate, i.e. over length of the throat, is an argument of radion potential. This integration must be fulfilled over two pasted copies of the bulk with Z_2-symmetry imposed at the UV boundary. To get the ordinary 4D Einstein gravity with self-interacting scalar field (see effective Action (29)) metric in 4 dimensions must be rescaled to the Einstein frame and radion must be transformed to scalar field $\psi(\rho)$ ("inflaton") possessing canonical kinetic term (see (25)-(28) below).

Surely the possibility to stabilize the modulus of the overall volume of extra space and to receive the potential appears only because the introduction of a co-dimension one local source (heavy isotropic brane which terminates the UV end of the throat) breaks the no-scale structure of supergravity ([16] and references therein) and evades the no-go theorem [16], [17]. Indeed it is easy to show that combination $\widetilde{T}$ (defined by expression (34) of paper [17]) of the components of the energy-momentum tensor does not meet demands of the no-go theorem in case the positive tension co-dimension one local source is introduced in the action; this is not true however for the positive tension local sources of lower dimensions.

Radion's potential calculated in a way specified above is always non-negative and possesses stable extremum at the top of the throat where anisotropic Israel junction conditions imposed at the UV brane are fulfilled. It is nontrivial which is seen in particular from the fact that physically meaningful radion effective potential (including the mentioned features) may be received here only in case the bulk magnetic monopole fluxbrane solution is considered as a background, not the dual electric one. The nonequivalence of two solutions is immediately seen when the higher-dimensional consistency condition of [18] is applied, see Appendix in [15].

Minimum of potential where Universe supposedly resides after reheating is protected by the barrier from the spontaneous decompactification [19] to 10 flat dimensions. The value of potential in this extremum is zero for the elementary throat-like solution. However in the Type IIA supergravity the deformation of the elementary solution in a Reissner-Nordstrom way (it is done with use of the additional 2-form flux) violates UV Israel junction conditions in case 4D space-time remains flat; junction conditions are restored with introduction of the extremely small curvature of 4D space-time. This in turn means that there is shift of the radion potential in its extremum from zero to the positive value which may be adjusted to the value of observed Dark Energy density [14]. And again like in the end of the previous Section it must be repeated that although this result looks interesting its physical meaning is unclear.

The form of potential depends on the choice of the theory. For the considered below throat-like solution in the Type IIA supergravity potential decreases exponentially (exponent is equal to 0.21 in Planck units, see (32)) when position of the UV boundary is moved from the region deep inside the throat up to its top; then potential steeply falls down to its stable extremum (see curves in [15] for different theories).

It is most interesting that all features of radion potential described above with words are contained in the simple analytical expression. This I'll demonstrate now on the example of the elementary throat-like solution of the Type IIA supergravity equations. The primary Action is the truncated low-energy Action of Bose sector of the Type IIA supergravity in Einstein frame in 10 dimensions where in addition to the conventional bulk terms the Action of the co-dimension one heavy isotropic brane is included:

$$S^{(10)} = M^8 \Big\{ \int \Big[R^{(10)} - \frac{1}{2}(\nabla\varphi)^2 - \frac{1}{2\cdot 4!} e^{\varphi/2} F^2_{(4)} - \frac{1}{2\cdot 2!} e^{3\varphi/2} F^2_{(2)} - \\ - \sigma\, e^{-\varphi/12} \delta^{(1)} \frac{\sqrt{-h^{(9)}}}{\sqrt{-g^{(10)}}} \Big] \sqrt{-g^{(10)}}\, d^{10}x + \mathrm{GH} \Big\}. \tag{21}$$

Action (21) is easily received by compactification of the corresponding Action of D11 M-theory supplied with additional surface brane's Action. In (21) M is Planck mass in 10 dimensions, σ is the UV brane's tension parameter, $\delta^{(1)}$ is the Dirac delta function fixing the position of the UV brane, $h^{(9)}$ is the induced metric on the brane.

The background bulk space-time is given by the well known [20], [21], [22] elementary magnetic fluxbrane solution of the theory (21):

$$ds^2_{(10)} = H^{-3/8}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + d\Omega_1^2) + H^{5/8}(dr^2 + r^2 d\Omega_4^2), \quad e^\varphi = e^{\varphi_\infty} H^{-1/4}, \tag{22}$$

$$F_{(4)} = Q_{(4)}\epsilon_4, \quad H = 1 + \left(\frac{L}{r}\right)^3, \quad L^3 = \frac{1}{3}Q_{(4)}e^{\varphi_\infty/4},$$

where $\mu, \nu = (0,1,2,3)$, $d\Omega_1^2$, $d\Omega_4^2$ are line elements of unit torus and 4-sphere correspondingly; ϵ_4 is volume form of unit 4-sphere; φ_∞ is the value of dilaton field at $r = \infty$; $Q_{(4)}$ is the 4-form charge. Fulfillment of the anisotropic Israel junction conditions demands fine tuning of the UV brane's density parameter σ and 4-form charge $Q_{(4)}$ (or according to (22) tuning of σ, φ_∞ and characteristic length of the throat L); Israel conditions also fix position of the UV boundary on the top of the throat at the value of isotropic coordinate $r = r_0 = L/2^{1/3}$.

To calculate the radion potential we, as it was said, shall use in (21) the bulk solution (22) but move the UV boundary (and hence change the upper limit of integration over r in (20)) from $r = r_0 = L/2^{1/3}$ fixed by junction conditions to the arbitrary position $\rho(x)$, slowly depending on 4-coordinates x^μ:

$$r_0 \to \rho(x), \tag{23}$$

$\rho(x)$ is called radion field which kinetic term is received from the brane's Nambu-Goto Action in (21). Integrating out 6 extra dimensions in (21) with account of (22), (23) gives effective Brans-Dicke type Action in 4 dimensions:

$$S^{(4)} = \int \left[\Phi(\rho)\tilde{R}^{(4)} - \frac{1}{2}\omega(\rho)\tilde{g}^{\mu\nu}\rho_{,\mu}\,\rho_{,\nu} - \widetilde{V}(\rho)\right]\sqrt{-\tilde{g}^{(4)}}d^{(4)}x, \tag{24}$$

where Brans-Dicke field $\Phi(\rho)$, kinetic term function $\omega(\rho)$ and potential $\widetilde{V}(\rho)$ are given in elementary functions.

The last step is to rescale 4-metric $\tilde{g}_{\mu\nu}$ in (23) to the Einstein-frame metric $g_{\mu\nu}$:

$$\tilde{g}_{\mu\nu} = \frac{M_{Pl}^2}{\Phi(\rho)}\, g_{\mu\nu}, \tag{25}$$

and to introduce the canonical scalar field $\psi(\rho)$:

$$\psi(\rho) = \frac{1}{L}\int_{r_0}^{\rho} \epsilon(\rho)\, d\rho = \int_{y_0}^{y} \epsilon(y)\, dy, \qquad y = \frac{\rho}{L}, \tag{26}$$

here the point $\rho = r_0$ of stable extremum of the radion effective potential at the top of the throat is chosen at $\psi = 0$; $y_0 = 2^{-1/3}$; $\epsilon(\rho)$ is expressed through functions $\Phi(\rho)$ and $\omega(\rho)$ in (24):

$$\epsilon^2(y) = L^2 \left[\frac{\omega(\rho)}{\Phi(\rho)} + 3\left(\frac{1}{\Phi}\frac{d\Phi}{d\rho}\right)^2\right] =$$
$$= 6\left(\frac{2}{3}\right)^{1/3} y^4 \left(\frac{y^5}{5} + \frac{y^2}{2}\right)^{-1}\left(1 + \frac{1}{y^3}\right)^{4/3} + 3y^8\left(\frac{y^5}{5} + \frac{y^2}{2}\right)^{-2}\left(1 + \frac{1}{y^3}\right)^2. \tag{27}$$

Inside the throat, i.e. at $\rho \ll L$ inverse dependence $\rho(\psi)$ is exponential:

$$\rho = L e^{\psi/c}, \qquad c = 2(18^{1/3} + 3)^{1/2}. \tag{28}$$

Finally the following scalar-tensor effective theory in 4 dimensions is received:

$$S^{(4)} = \int \left[M_{Pl}^2 R^{(4)} - (1/2) M_{Pl}^2 (\nabla\psi)^2 - \mu^4 V(\psi)\right] \sqrt{-g^{(4)}}\, d^{(4)}x, \tag{29}$$

μ is a calculable constant of dimensionality of mass - the characteristic of the looked for radion potential $V(\psi)$ which is taken dimensionless for convenience. Also radion field $\psi(\rho)$ is taken dimensionless (normalized to Planck mass).

Here is the promised exact analytical expression for potential V which reflects all its interesting features described above ($y(\psi) = \rho(\psi)/L$ must be calculated from (26), (27)):

$$V(\psi) \equiv K(y(\psi)) = \frac{F(y)}{(y^5/5 + y^2/2)^2}, \tag{30}$$

where

$$F(y) = y^3 \left[3 \left(\frac{2}{3} \right)^{1/3} (1 + y^3)^{1/3} + \frac{3}{2(1 + y^3)} - 4 \right]. \qquad (31)$$

It is easy to see that $F(y)$ (31) possesses zero minimum at $y = y_0 = 2^{-1/3}$, i.e. at $\psi = 0$ according to (26). As it was said this minimum is protected by potential barrier from decompactification at $\rho \gg L$ ($\psi \gg 1$) where potential again tends to zero. Dependence $V(\psi)$ (30) is depicted in Fig. 2 of [15].

At $\psi < 0$ there is steep slope of potential, and at $\psi \ll -1$, i.e. at $\rho \ll L$, potential relatively slow exponentially depends on ψ:

$$V_{-}(\psi) = (2^{7/3} 3^{2/3} - 10) e^{-\psi/c} \approx 0,48 \cdot e^{-0,21 \cdot \psi}, \qquad (32)$$

where c is given in (28).

This is perhaps the main result of the approach. Analyses performed in [14] shows that potential (32) meets the flatness and slow roll conditions of the early inflation and also permits to receive the number of e-foldings N_e during inflation demanded by the astrophysical observations ($N_e \approx 80 - 100$) [23], [24], [25]. It is also shown in [14] that validity condition of all the approach is satisfied, i.e. that the value of potential $\mu^4 V(\psi)$ in (29) is much below the Planck density M_{Pl}^4 in the region of the length of the throat where needed number of e-foldings during inflation is achieved.

4 Discussion

To Part I: Surely the results of this Part may be interesting only as a demonstration of the possibility to receive "seesaw" mass scale without seesaw mechanism and of the phenomenon of essentially different behavior in 4D experiments of seemingly symmetric right and left chiral components of 5D Dirac spinor (7). Hopefully these observations may be useful in more elaborated models incorporating basic features of SM.

Also the infinite tower (12) of the practically zero-mass "neutrinos" may come in confrontation with observations. The real challenge is to receive three generations observed in 4 dimensions from the spectrum of higher dimensional Fermions. Flux terms determining the dynamics of Fermions and their mass spectra may appear in different higher-dimensional models. Perhaps the throat-like background solution in the Type IIA supergravity with

$S^4 \otimes S^1$ base or background of Klebanov-Strassler model [26] with its rich flux structure will prove to be more promising for this direction of thought.

It is expected that in more elaborated models background fluxes in the extra-dimensional Fermi fields' equations may substitute Higgs scalar in forming the observed mass spectra of Fermions. As it was shown in Part II of this Report background flux may also substitute Higgs scalar in creating the mass gap in the spectrum of excitations of KK gauge field associated with isometries of extra subspace where flux "lives". It is evident that "mass generating" tool of the supergravity fluxes is less ambiguous than the tool of Higgs scalars introduced "by hand".

To Part II: The reason of appearance of mass of KK gauge field when there is background flux is trivial: n-form is a matter field charged as regards to KK gauge potentials and presence of vacuum condensate of a charged matter field always generate mass of the corresponding gauge field. Surely it must be repeated after the same comments to Part I that background supergravity solutions compatible with SM must be considered instead of the illustrative toy models of this Report or of [13].

Thus KK gauge fields can acquire masses in the fluxbrane solutions of the supergravity. Thereupon the question arises if the classical KK approach to gauge theories in low dimensions as non-diagonal components of the higher-dimensional metric is out of date? And more general question: if it is out of date nowadays the final note to [27] which says that in the 10-dimensional supergravity questions about observed physical phenomena may be translated into questions about properties of compact extra manifold? This approach met a number of difficulties including the difficulty of too small gauge coupling constants of KK gauge fields (this difficulty arises because of big volume of extra space as compared to Planck scale, and big volume of extra space is a demand of applicability of the string theory low-energy approximation).

Possibly the dual holography approach to low dimensional theories looks more promising. I don't know if the idea to generate masses of gauge fields with a tool of background fluxes may be applied in this approach.

It would be also interesting to study what may be the 4D dual CFT theory associated with D5 gauge theory (16) received with the KK reduction from 10 to 5 dimensions?

And some remark "aside" about possible correspondence of the classical KK and the dual holography approaches to the low-dimensional theories. Matter content of D4 theory received in KK approach, i.e. with intergration out of extra dimensions, is given by the normalized in extra space modes of bulk fields. Whereas in the dual holography approach the UV boundary values of the non-normalized modes of the same bulk fields are the sources of currents in the functional of quantum currents of dual CFT in 4 dimensions. Spectrum of bound states ("glueballs") of this functional and spectrum of normalized KK modes evidently coincide by definition of dual functional (this is true for KK modes which eigenfunctions are taken equal to zero at the UV boundary of the bulk). But what about the possible correspondence of coupling constants of the dual functional bound states (given by residues in the poles of functional) and coupling constants of KK modes (calculated from the higher-dimensional Action with a tool of conventional dimensional reduction with account of the overlapping of corresponding wavefunctions)? Perhaps their coincidence may be postulated as a sort of the *bootstrap holography principle* [13] - in line with the known "bootstrap" ambition to consider particles as bound states in S-matrix amplitudes of the same particles? Or perhaps by analogy with Schwinger's "Source theory" with its idea of identification of particles (quantum fields) and their sources (quantum currents) [28]? And may be this bootstrap holography principle will serve a sort of selection rule for the choice from plethora of possibilities of the higher-dimensional string-supergravity background vacuum states?

To Part III: The idea to use dynamical scalar associated with extra dimensions, interbrane distance in particular, as a candidate for inflaton is not a novel one (see e.g. [29], [30]). The very possibility to get in frames of this idea the exact analytical expression (30) for the scalar field potential possessing qualitatively the basic features demanded by the early inflation looks attractive. The basic difficulty of this approach is the lack of physical grounds for the appearance of the heavy local source which forms the UV boundary of the throat and for the choice of its dynamics. The simplest Nambu-Goto choice taken in Action (21) is crucial for the calculations of the paper. But the "simplest" does not mean "well grounded". On the other hand this difficulty is common for all Randall-Sundrum type models.

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