



**Ginzburg Conference on Physics**

May 28 - June 2, 2012  
Lebedev Institute

Alexander Vikman



**31.05.2012**





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# G-Bounce

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# THIS TALK IS BASED ON

- *Imperfect Dark Energy from Kinetic Gravity Braiding*<sup>1</sup>  
**arXiv:1008.0048**, JCAP 1010:026, 2010
- *The Imperfect Fluid behind Kinetic Gravity Braiding*<sup>2</sup>  
**arXiv:1103.5360**, JHEP 1111 (2011) 156
- *G-Bounce*,<sup>3</sup> **arXiv:1109.1047**, JCAP 1111:021, 2011

IN COLLABORATION WITH

**Cédric Deffayet**,<sup>1</sup> **Damien Easson**,<sup>3</sup>  
**Oriol Pujolàs**,<sup>1,2</sup> **Ignacy Sawicki**<sup>1,2,3</sup>



# PLAN

- Motivation
- General class of models - “perfect” imperfect fluids
- G - Bounce
- Open problems



# ORIGIN OF THE UNIVERSE?

- Was there a beginning of time i.e. of the quasiclassical universe? Was there a strong quantum gravity époque in our past?
- If there was a beginning, was the universe collapsing or expanding immediately afterwards?
- If the universe experienced an early period of inflation, which *all* observations currently ***perfectly*** confirm, what happened ***before inflation***? Indeed, *inflation* is not past-complete - Borde, Guth, Vilenkin (2001), *Vilenkin's talk today*



CAN ONE CONSTRUCT  
A **STABLE**  
(WITH REAL SOUND SPEED AND WITHOUT GHOSTS)  
**CLASSICAL MODEL**  
WHERE A  
**SPATIALLY FLAT**  
FRIEDMANN UNIVERSE  
**BOUNCES:**  
GOES FROM COLLAPSE TO EXPANSION **???**



## OBSTACLE:

TO BOUNCE ONE HAS TO VIOLATE  
NULL ENERGY CONDITION (NEC),

$$T_{\mu\nu}n^{\mu}n^{\nu} \geq 0$$

BUT NORMAL FIELDS AND PERFECT  
FLUIDS CANNOT DO IT!



*go to imperfect fluids i.e. to even less canonical fields!*



**GO G!**



# G STANDS FOR GALILEON

*Nicolis, Rattazzi, Trincherini, (2008)*

In current literature: *Galileons* = scalar-tensor theories with  
*higher derivatives in the action*  
*but* with equations of motion which are *all* of the  
*second order - NO Ostrogradsky's ghosts*



*the most general theory of this type was derived*  
by *Horndeski (1974)*, and rederived by *Deffayet, Gao, Steer, Zahariade (2011)*

*International Journal of Theoretical Physics, Vol. 10, No. 6 (1974), pp. 363-384*

## **Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space**

GREGORY WALTER HORNDESKI

*Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario,  
Canada*

*Received: 10 July 1973*



## Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space

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Department of Applied Mathematics, University of Waterloo, Waterloo, Ontario,  
Canada

Received: 10 July 1973

$$\begin{aligned} \mathcal{L} = & K + \mathcal{G} \phi_{;\mu}^{;\mu} + \\ & + \mathcal{G}_2 R + \mathcal{G}'_2 \left[ (\phi_{;\mu}^{;\mu})^2 - \phi_{;\mu}^{;\nu} \phi_{;\nu}^{;\mu} \right] + \\ & + \mathcal{G}_3 G_{\nu}^{\mu} \phi_{;\mu}^{;\nu} - \frac{1}{6} \mathcal{G}'_3 \left[ (\phi_{;\mu}^{;\mu})^3 - 3 \phi_{;\lambda}^{;\lambda} \phi_{;\mu}^{;\nu} \phi_{;\nu}^{;\mu} + 2 \phi_{;\alpha}^{;\mu} \phi_{;\beta}^{;\alpha} \phi_{;\mu}^{;\beta} \right] \end{aligned}$$

*Kobayashi, Yamaguchi, Yokoyama (2011)*

where we have 4 free functions!

$K(\phi, X)$  and  $\mathcal{G}_i(\phi, X)$

$$\mathcal{G}'_i = \frac{\partial \mathcal{G}_i}{\partial X}$$

$$X \equiv \frac{1}{2} g^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi$$



# WHY IS THIS **G** INTERESTING?

One can **stably violate** the most basic of the classical energy conditions-Null Energy Condition (NEC):  $T_{\mu\nu}n^\mu n^\nu \geq 0$   
which in cosmology reduces to:  $p + \varepsilon \geq 0$  or  $\dot{H} \leq 0$



*Pandora's  
box*





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*Pandora's  
box*

- *Healthy and testable  
Phantom Dark Energy,*  
*Deffayet, Pujolas, Sawicki, AV, 2010*
- *Bouncing Cosmology  
for a spatially flat Friedmann universe*  
*Creminelli, Nicolis, Trincherini 2010;  
Easson, Sawicki, AV;  
Taotao Qiu, Evslin, Cai, Li, Zhang 2011*
- *Superinflation with blue spectra of  
gravity waves*  
*Kobayashi, Yamaguchi, Yokoyama 2010*





# SIMPLEST INTERESTING SUBSECTOR OF GALILEONS /HORNDESKI'S THEORIES -

## *Kinetic Gravity Braiding*

$$S_\phi = \int d^4x \sqrt{-g} [K(\phi, X) + G(\phi, X) \square \phi]$$

kinetic mixing / braiding  
 $\frac{\partial \phi}{\partial g}$

*Armendariz-Picon, Damour, Mukhanov, Steinhardt 1999/2000*

*Pujolàs, Deffayet, Sawicki, AV, 2010*

where  $X \equiv \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$

**Minimal** coupling to gravity  $S_{\text{tot}} = S_\phi + S_{\text{EH}}$

However, derivatives of the metric are coupled to the derivatives of the scalar, provided  $G_X \neq 0$

No “Galilean symmetry”!  ~~$\partial_\mu \phi \rightarrow \partial_\mu \phi + c_\mu$~~



# EQUATION OF MOTION I

$$L^{\mu\nu} \nabla_\mu \nabla_\nu \phi + (\nabla_\alpha \nabla_\beta \phi) Q^{\alpha\beta\mu\nu} (\nabla_\mu \nabla_\nu \phi) + Z - G_X R^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi = 0$$

**Braiding**

**EOM is of the second order:**  $L_{\mu\nu}$ ,  $Q^{\alpha\beta\mu\nu}$ ,  $Z$

**constructed from field and it's first derivatives**

$Q^{\alpha\beta\mu\nu}$  is such that EOM is a 4D Lorentzian generalization of the Monge-Ampère Equation, always *linear* in  $\ddot{\phi}$



SHOULD  $\phi$  BE FUNDAMENTAL?  
NO NOT AT ALL  
 $\phi$  CAN MODEL SOME  
HYDRODYNAMICS !

$K(X)$  for equation of state,  $G(X)$ : transport coefficient



# EQUATION OF MOTION II

- **Shift-Charge Noether Current:** - interpret as “particle” current  $J_\mu$
- **New Equivalent Lagrangian:**  $\mathcal{P}$  pressure!
- **Equation of motion is a “conservation law”:**



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$$\nabla_\mu J^\mu = \mathcal{P}_\phi$$



# EQUATION OF MOTION II

- **Shift-Charge Noether Current:** - interpret as “particle” current  $J_\mu$

$$J_\mu = (\mathcal{L}_X - 2G_\phi) \nabla_\mu \phi - G_X \nabla_\mu X$$

- **New Equivalent Lagrangian:**  $\mathcal{P}$  pressure!

- **Equation of motion is a “conservation law”:**

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- **New Equivalent Lagrangian:**  $\mathcal{P}$  pressure!

$$\mathcal{P} = K - 2XG_\phi - G_X \nabla^\lambda \phi \nabla_\lambda X$$

- **Equation of motion is a “conservation law”:**

$$\nabla_\mu J^\mu = \mathcal{P}_\phi$$



# IMPERFECT FLUID

## FOR TIMELIKE GRADIENTS

- **Four velocity :**  $u_\mu \equiv \frac{\nabla_\mu \phi}{\sqrt{2X}} \Rightarrow \phi$  is an ***internal clock***

- **projector:**  $\perp_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$

- **Time derivative:**  $(\dot{\phantom{x}}) \equiv \frac{d}{d\tau} \equiv u^\lambda \nabla_\lambda$

- **Expansion :**  $\theta \equiv \perp^\lambda_\mu \nabla_\lambda u^\mu = \dot{V}/V$   
↑  
**comoving volume**

*Shift-symmetry*  
 $\phi \rightarrow \phi + c$   
*violates*  
 $\phi \rightarrow -\phi$   
*and introduces*  
***arrow of time***



# EFFECTIVE MASS & CHEMICAL POTENTIAL

$$\kappa \equiv 2XG_X$$

- charge density:  $n \equiv J^\mu u_\mu = n_0 + \kappa\theta$   
“Braiding”
- energy density:  $\mathcal{E} \equiv T^{\mu\nu} u_\mu u_\nu = \mathcal{E}_0 + \theta\dot{\phi}\kappa$
- effective mass per shift-charge / chemical potential:

$$m \equiv \left( \frac{\partial \mathcal{E}}{\partial n} \right)_{V, \phi} = \sqrt{2X} = \dot{\phi}$$

c.f. Schutz 1970



# SHIFT-CURRENT AND “DIFFUSION”

$$J_{\mu} = n u_{\mu} - \frac{\kappa}{m} \perp_{\mu}^{\lambda} \nabla_{\lambda} m$$

**“Diffusion”**

§ 59, *L&L*, vol. 6

$$\kappa \equiv 2XG_X$$

Is a “diffusivity”/  
transport coefficient

*Particle / charge current is not parallel to energy flow!*



# DIFFUSION OF CHARGE?

For incompressible motion  $\theta \equiv 0$   
equation of motion is:

$$\dot{n} = -\bar{\nabla}_{\mu} \left( \mathfrak{D} \bar{\nabla}^{\mu} n \right) + \mathfrak{D} a^{\mu} \bar{\nabla}_{\mu} n$$

where the diffusion constant:

c.f. § 59, *L&L*, vol. 6, p 232

$$\mathfrak{D} \equiv -\frac{\kappa}{n_m m}$$

4-acceleration:

$$a^{\mu} \equiv \dot{u}^{\mu}$$

spatial gradient:

$$\bar{\nabla}_{\mu} \equiv \perp_{\mu}^{\nu} \nabla_{\nu}$$



# IMPERFECT FLUID ENERGY-MOMENTUM TENSOR

- **Pressure**

$$\mathcal{P} = P_0 - \kappa \dot{m}$$

- **Energy Flow**

*No Heat Flux!*

$$q_\mu = -\kappa \perp_\mu^\lambda \nabla_\lambda m = m \perp_\mu^\lambda J_\lambda$$

- **Energy Momentum Tensor**

$$T_{\mu\nu} = \mathcal{E} u_\mu u_\nu - \perp_{\mu\nu} \mathcal{P} + 2u_{(\mu} q_{\nu)}$$

*Solving for  $\dot{m}$  for small gradients or small  $\kappa$  one obtains “bulk viscosity” but change of frame results in a perfect fluid with vorticity up to  $\mathcal{O}(\kappa^2)$*



# ENERGY CONSERVATION IN COMOVING VOLUME

Energy conservation:  $u_\nu \nabla_\mu T^{\mu\nu} = 0$



$$dE = -\mathcal{P}dV + md\mathcal{N}_{\text{dif}}$$

Euler relation:  $\mathcal{E} = mn - P_0$



Momentum conservation:

$$\perp_{\mu\nu} \nabla_\lambda T^{\lambda\nu} = 0$$



# COSMOLOGY

$$q_{\mu} = 0 \quad \text{and} \quad \theta = 3H$$

**Friedmann Equation:**

$$H^2 = \kappa m H + \frac{1}{3} (\mathcal{E}_0 + \rho_{\text{ext}})$$

$$r_c^{-1} = \kappa m \quad \text{“crossover” scale in DGP}$$



# EQUATION OF MOTION IN COSMOLOGY (CHARGE CONSERVATION)

$$\dot{n} + 3Hn = \mathcal{P}_\phi$$

**If there is shift-symmetry then**

$$\mathcal{P}_\phi = 0$$


$$n \propto a^{-3}$$



# ACTION FOR THE COSMOLOGICAL PERTURBATIONS

$$S_2 = \int d^3x dt A \left[ \dot{\zeta}^2 - \frac{c_s^2}{a^2} (\partial_i \zeta)^2 \right]$$

where  $A = \frac{X a^3}{(H - m\kappa/2)^2} D$

$$D = \frac{\mathcal{E}_m - 3H\kappa}{m} + \frac{3}{2} \kappa^2$$



Controls “ghosts”  $D > 0$  No ghosts!



# SOUND SPEED

$$c_s^2 = \frac{\mathcal{P}_m + 2\dot{\kappa} + \kappa (4H - \kappa m/2)}{\mathcal{E}_m - 3\kappa (H - \kappa m/2)} \neq \frac{\dot{\mathcal{P}}}{\dot{\mathcal{E}}}$$

The relation between the equation of state, the sound speed and the presence of ghosts is very different from the *k-essence* & perfect fluid.



A manifestly stable *Phantom* ( $w_X < -1$ ) is possible even with a *single* degree of freedom and *minimal* coupling to gravity



# G BOUNCE IDEA I

- Consider matter with constant equation of state - radiation, dust, spatial curvature etc...  $p_{\text{ext}} = w\rho_{\text{ext}}$  with  $w = \text{const}$
- Shift-symmetric Lagrangian for the scalar field  $K(\not{\phi}, X)$  and  $G(\not{\phi}, X)$
- Phase space is two dimensional  $(m, \rho_{\text{ext}})$
- Go to new coordinates  $(m, H)$  by solving the Friedmann equation
- Integral of motion i.e. 1st integral

$$I(m, H) = \frac{n^{1+w}}{\rho_{\text{ext}}}$$



# G BOUNCE IDEA II

- In new coordinates  $(m, H)$  at  $H = 0$  pose conditions

$$\rho_{\text{ext}} > 0, D > 0, \dot{H} > 0, c_s^2 > 0$$

- These conditions are on the range of chemical potential  $\Delta m$  and on the equation of state  $K(m)$  along with the transport coefficient  $\kappa(m)$  or on  $K(X)$  and  $G(X)$



**“Healthy” Bounce!**



IS IT POSSIBLE  
TO SATISFY  
ALL THESE CONDITIONS ?







# SIMPLE HIERARCHY

if  $K(m)$  and  $\kappa(m)$  satisfy the hierarchy:

$$K > mK_m > \frac{1}{2} (m\kappa)^2 > 0 > \frac{1}{2} m^2 K_{mm} > -\frac{3}{4} (m\kappa)^2$$

then at  $H = 0$  :

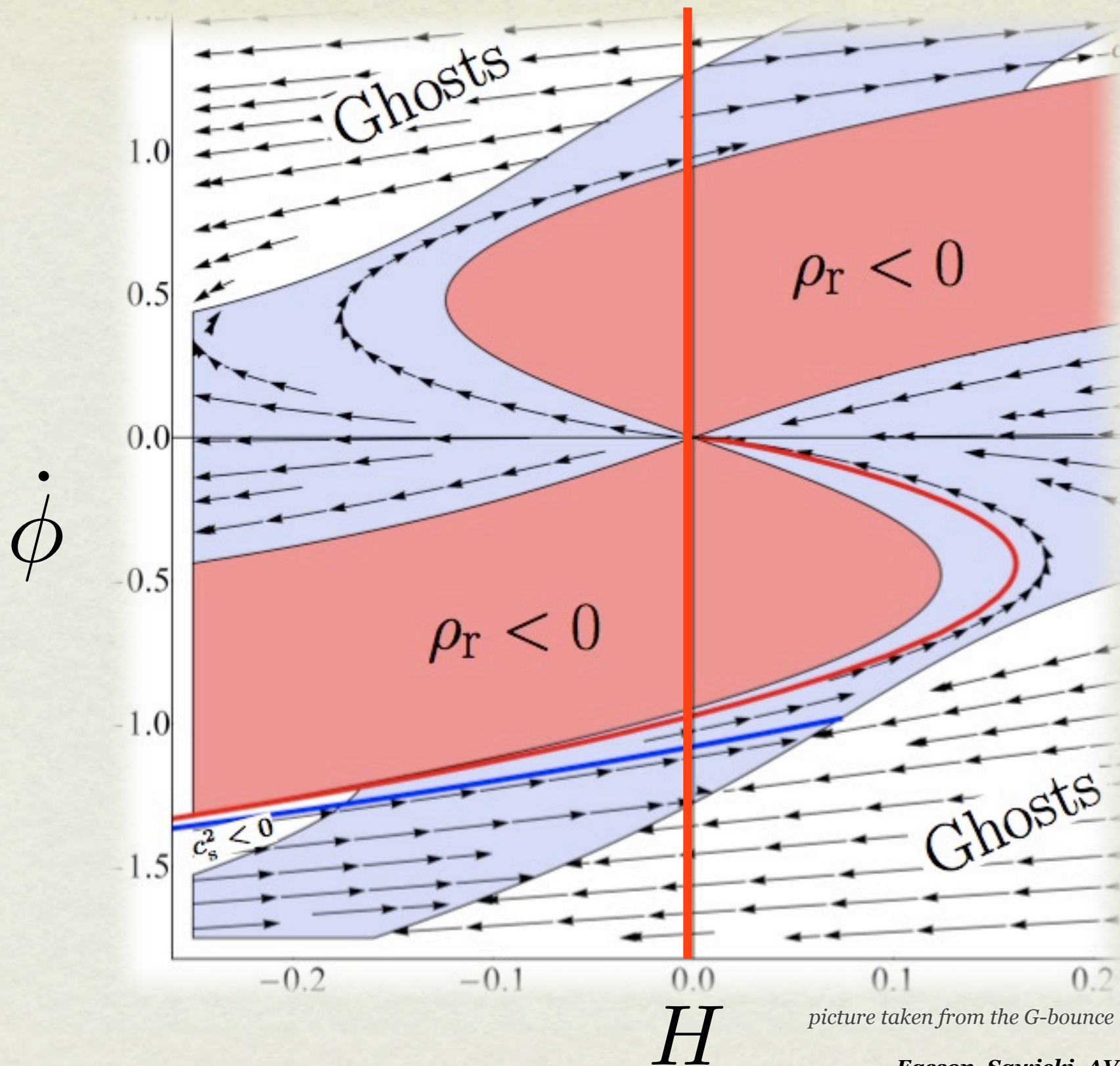
$$\rho_{\text{ext}} > 0, D > 0, \dot{H} > 0, c_s^2 > 0$$



**“Healthy” Bounce!**

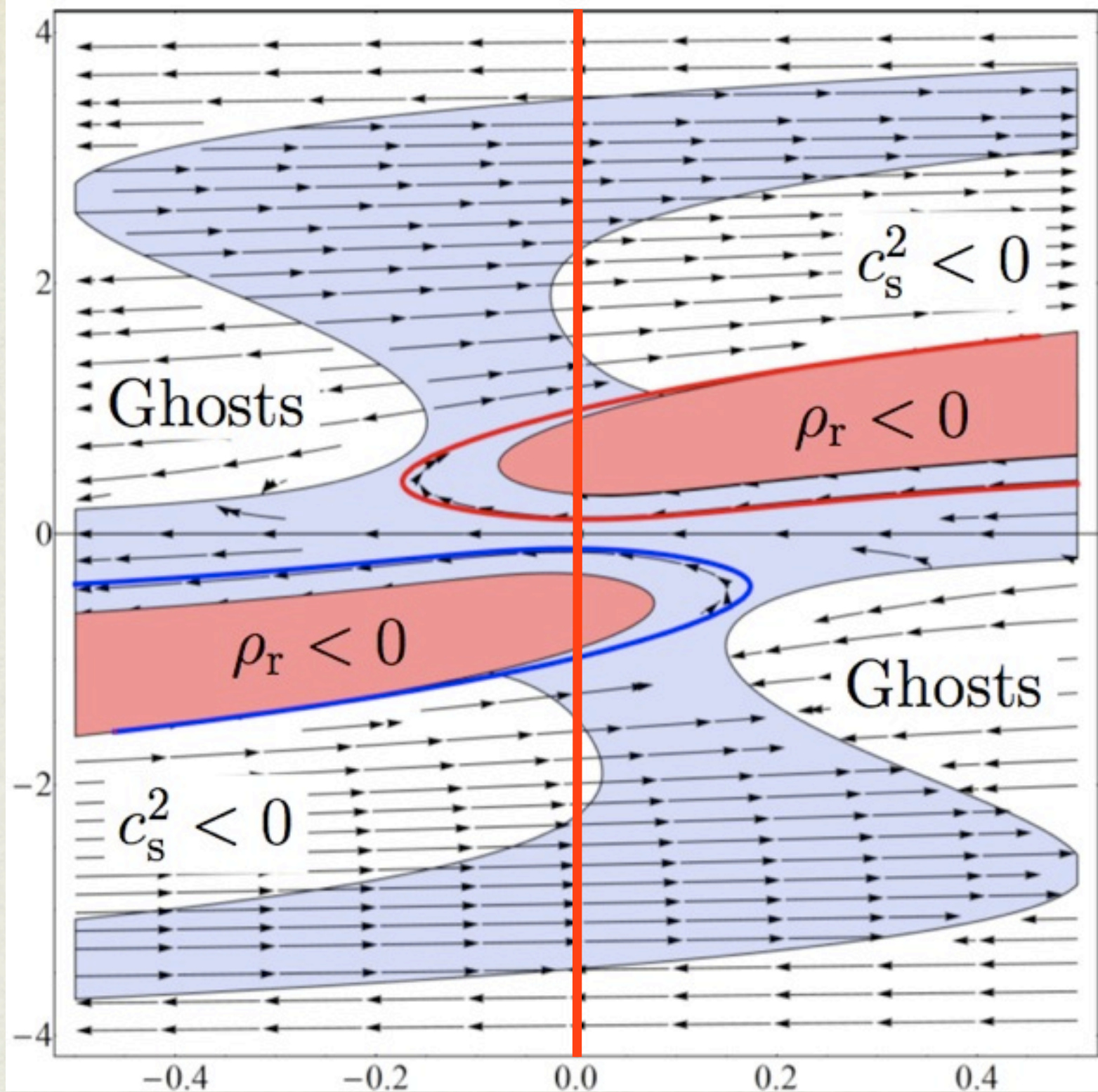
does not  
depend on  
external equation  
of state  $w$ !





**G-Bounce:**  $\mathcal{L} = X - \alpha X^3 + \varkappa X \square \phi + \text{Radiation } \rho_r$



$\dot{\phi}$ 

picture taken from the G-bounce

Easson, Sawicki, AV (2011)

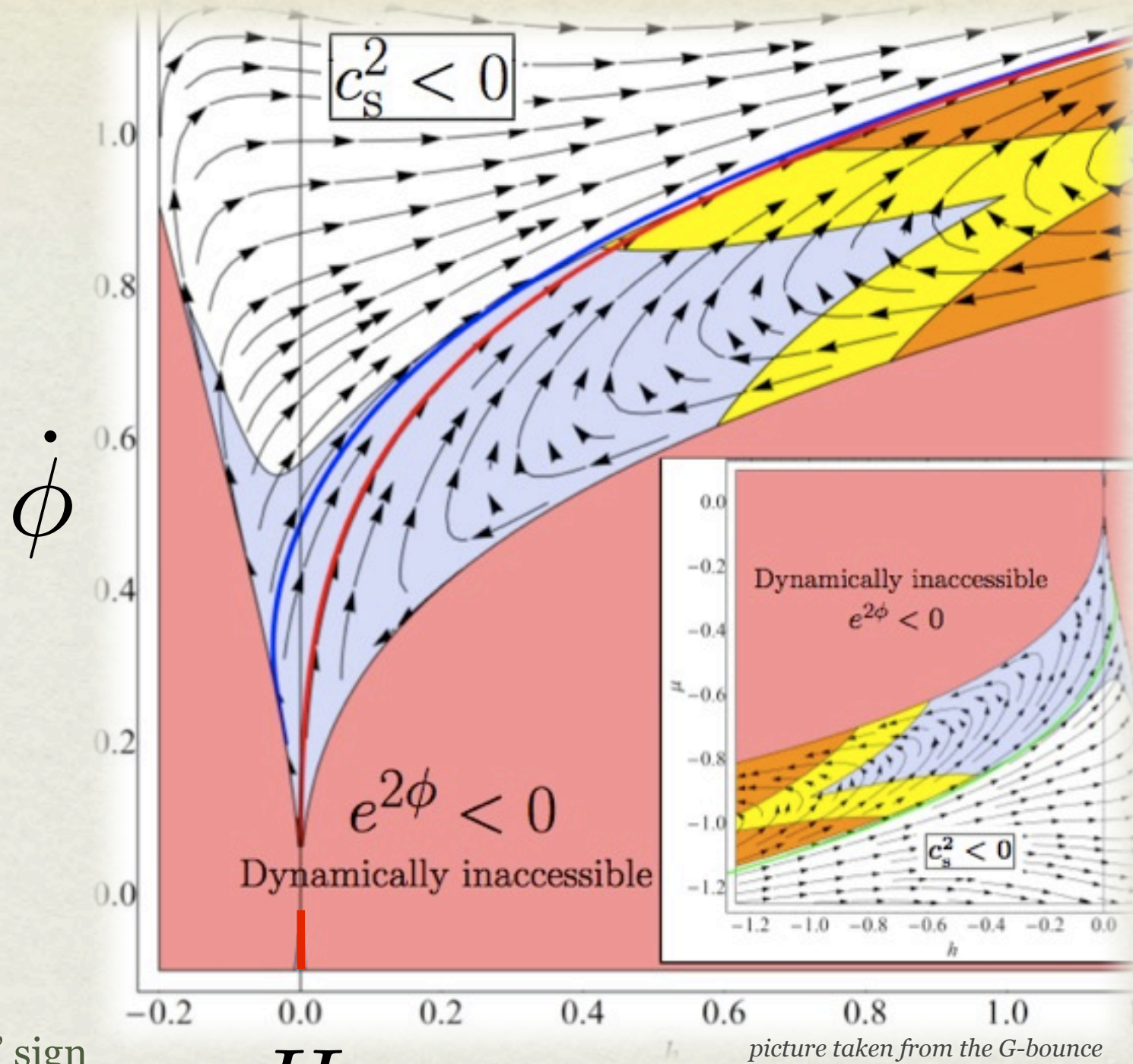
 $H$ 

$$\kappa = 2\beta X + \gamma X^2 \quad K = -\Lambda + X - X^3 + \mathbf{Radiation} \quad \rho_r$$



CAN ONE GENERALIZE  
THIS APPROACH FOR  
SYSTEMS WITHOUT  
SHIFT-SYMMETRY ?





picture taken from the G-bounce

**Galilean Genesis: Creminelli, Nicolis, Trincherini (2010)**

wrong "ghostly" sign

"Conformal Galileon":

$$\mathcal{L} = -2f^2 e^{2\phi} X + \frac{2f^3}{\Lambda^3} X^2 + \frac{2f^3}{\Lambda^3} X \square \phi$$

where:  $X \equiv \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$  and  $\Lambda, f$  are constants



# OPEN QUESTIONS

- Strong coupling?
- Possible anisotropy? Too strong tachyonic / Jeans instabilities?
- Can one arrange a cyclic i.e. oscillating evolution?
- Can one avoid all singularities and troubles for the past?
- Perturbations?
- *Any realistic scenarios? Smooth transition to standard cosmology and inflation?*



THANKS A LOT FOR  
YOUR ATTENTION!