Gravitational lensing in presence of plasma

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Outline

- Gravitational Lensing: Introduction
- Gravitational Lensing in Plasma

- Gravitational Lensing:

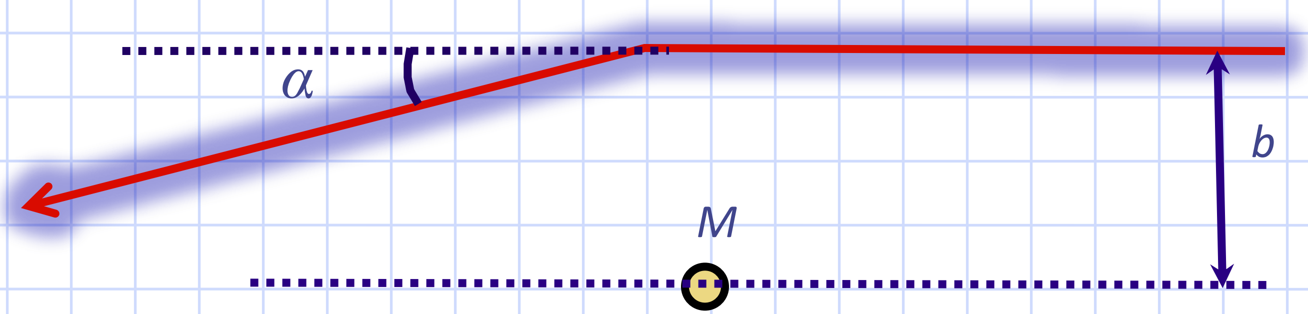
wide range of phenomena connected with deflection of light by gravity

General Relativity predicts that a light ray passing near a spherical body of mass *M* with impact parameter *b*, is deflected by the “Einstein angle”:

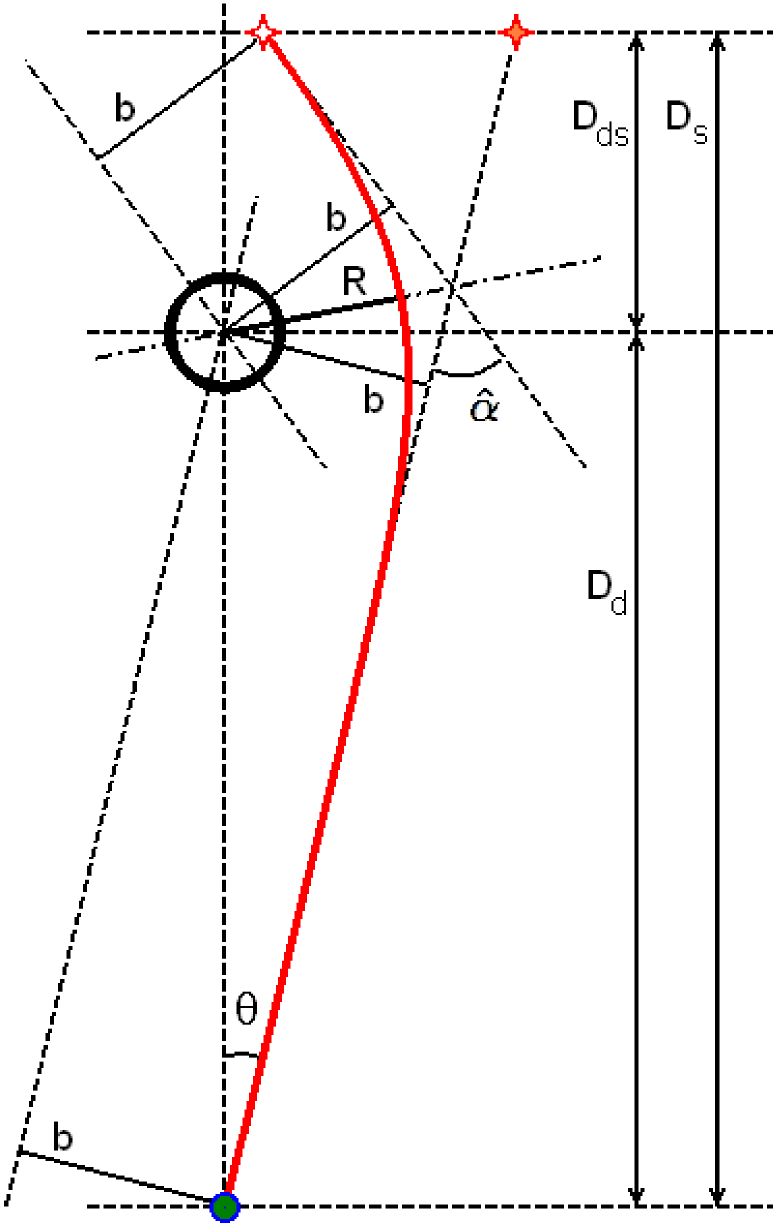
$$\hat{\alpha} = \frac{4GM}{c^2 b} = \frac{2R_S}{b}$$

provided the impact parameter *b* is much larger than the corresponding Schwarzschild radius R_S:

$$b \gg R_S = \frac{2GM}{c^2}$$



- The GL theory usually deals with a **geometrical optics** in vacuum and uses a notion of the **deflection angle**.
- Basic assumption is an approximation of a **weak deflection** angle of a photon. In the most astrophysical situations related with GL approximation of weak deflection is well satisfied.
- Deflection angle of photon in vacuum does not depend on photon frequency (or energy). So GL in vacuum is **achromatic**.
- Exact expression for deflection angle in Schwarzschild metric can be expressed via elliptic integrals



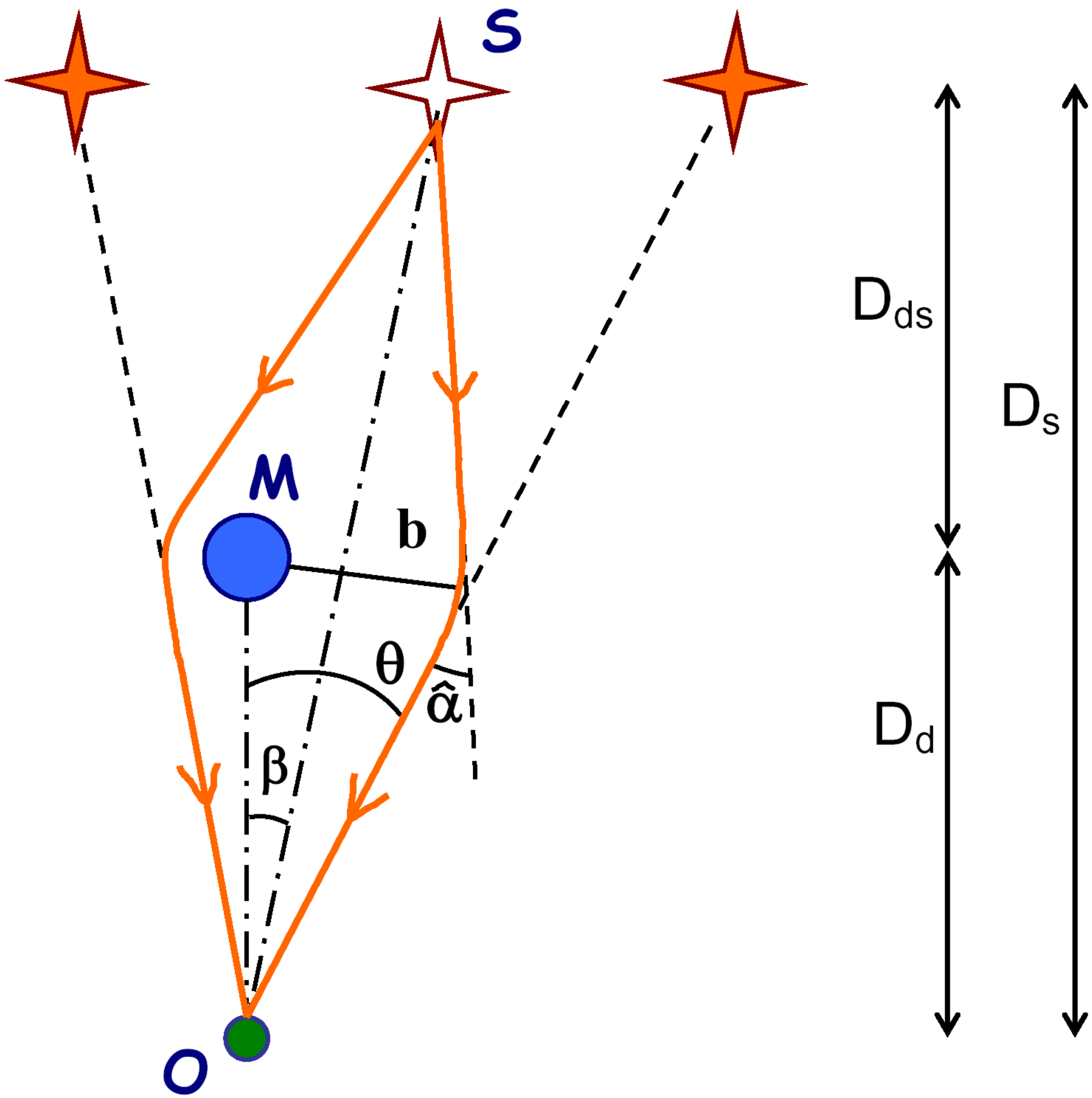
Example of calculation of the trajectory of the photon.

GL changes angular position of source.

Light ray from the
SOURCE
deflected by angle α by the point-mass gravitational
LENS
with the Schwarzschild radius R_S
goes to the
OBSERVER.

The observer sees the
IMAGE
of the source at angular position θ, which is different from the source position.
R is the closest point of trajectory to gravitating center, it is usually referred as the distance of the closest approach,
b is the impact parameter of the photon.

Gravitational lensing in vacuum, Schwarzschild point-mass lens



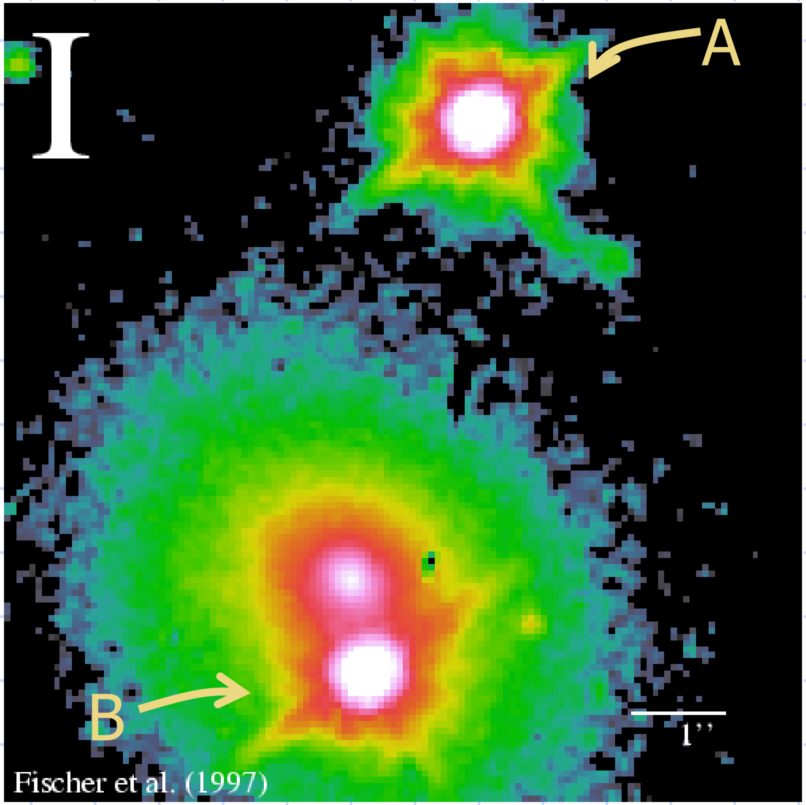
At this picture there is the example of the simplest model of Schwarzschild point-mass lens.

Observer sees two point images of source instead of one single real point source.

Rays from source which are not in plane of Source-Lens-Observer will not go to observer.

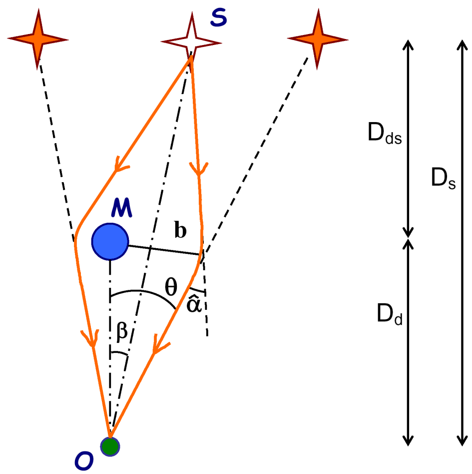
Multiple images!

The first observational example of gravitational lens (1979)



QSO 0957+561

- Maximum separation – 6.1 arcsec
- Image redshift – 1.41
- Lens redshift – 0.36
- B/A = 2/3



If source is not point, but has finite size, two images are not points, but arcs.

In case of perfect alignment, observer sees image in form of ring - Einstein (Einstein-Chwolson) ring

B838 SIDNEY LIEBES, JR.

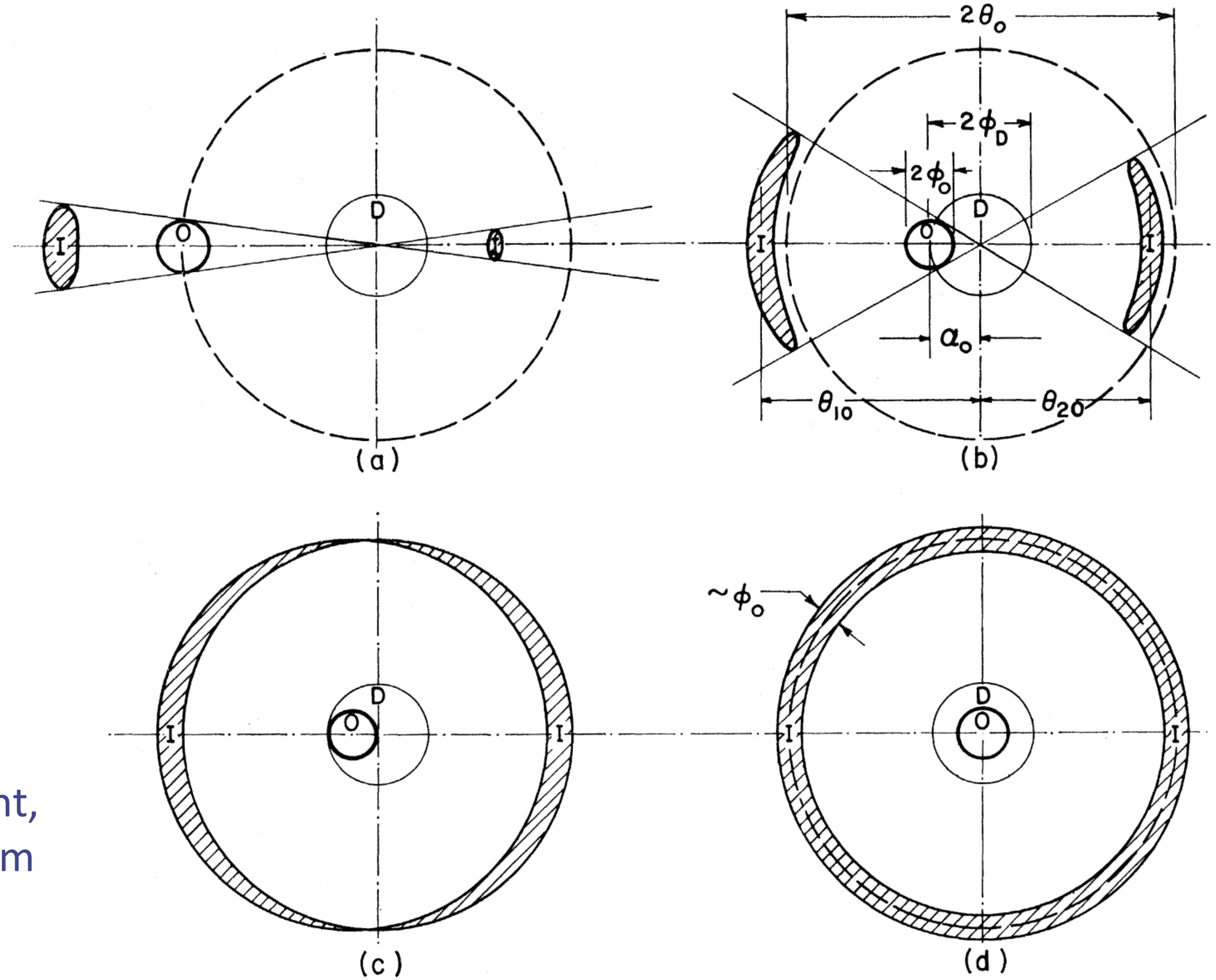


FIG. 2. Transformation of the image configuration I of the object O as the object moves from left to right behind the deflector D.

B1938+666, Einstein Ring

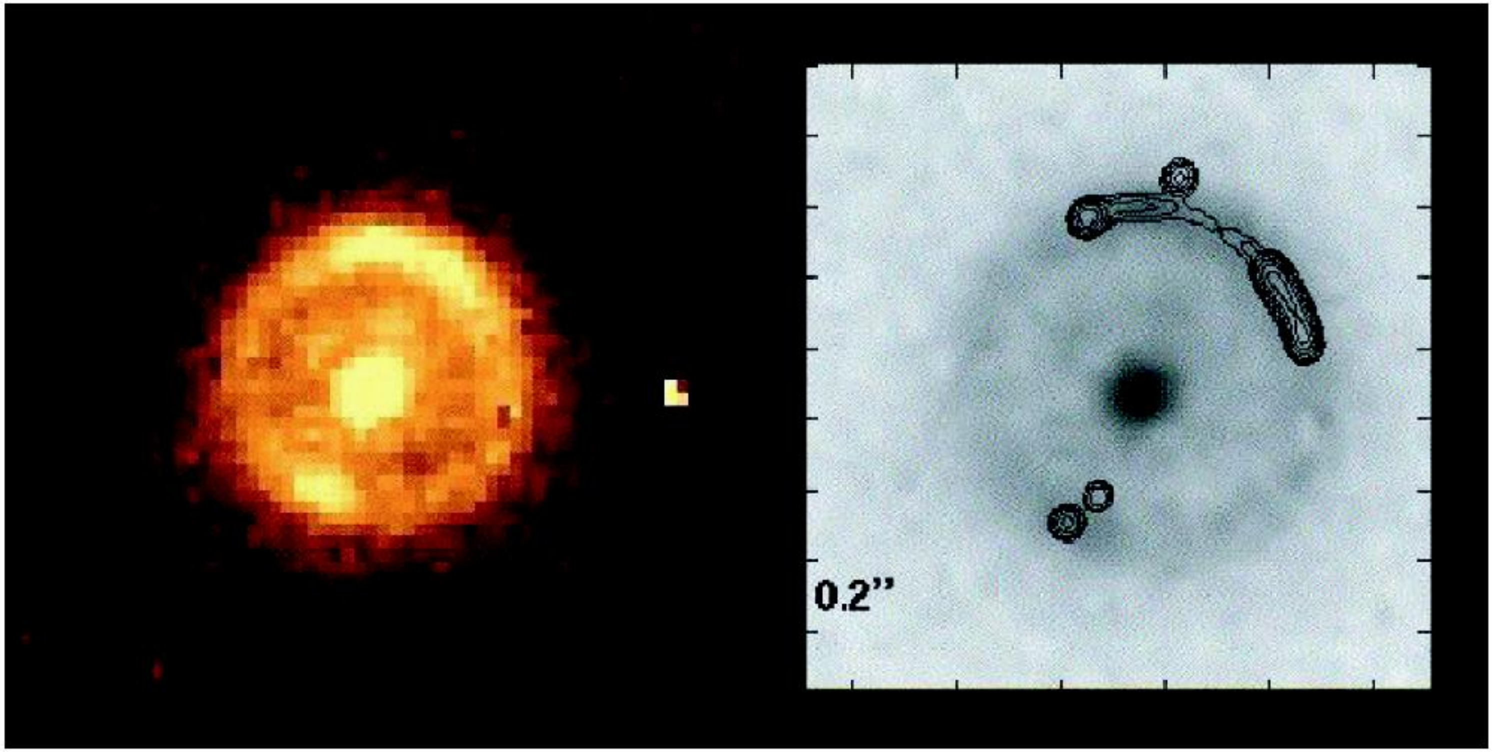
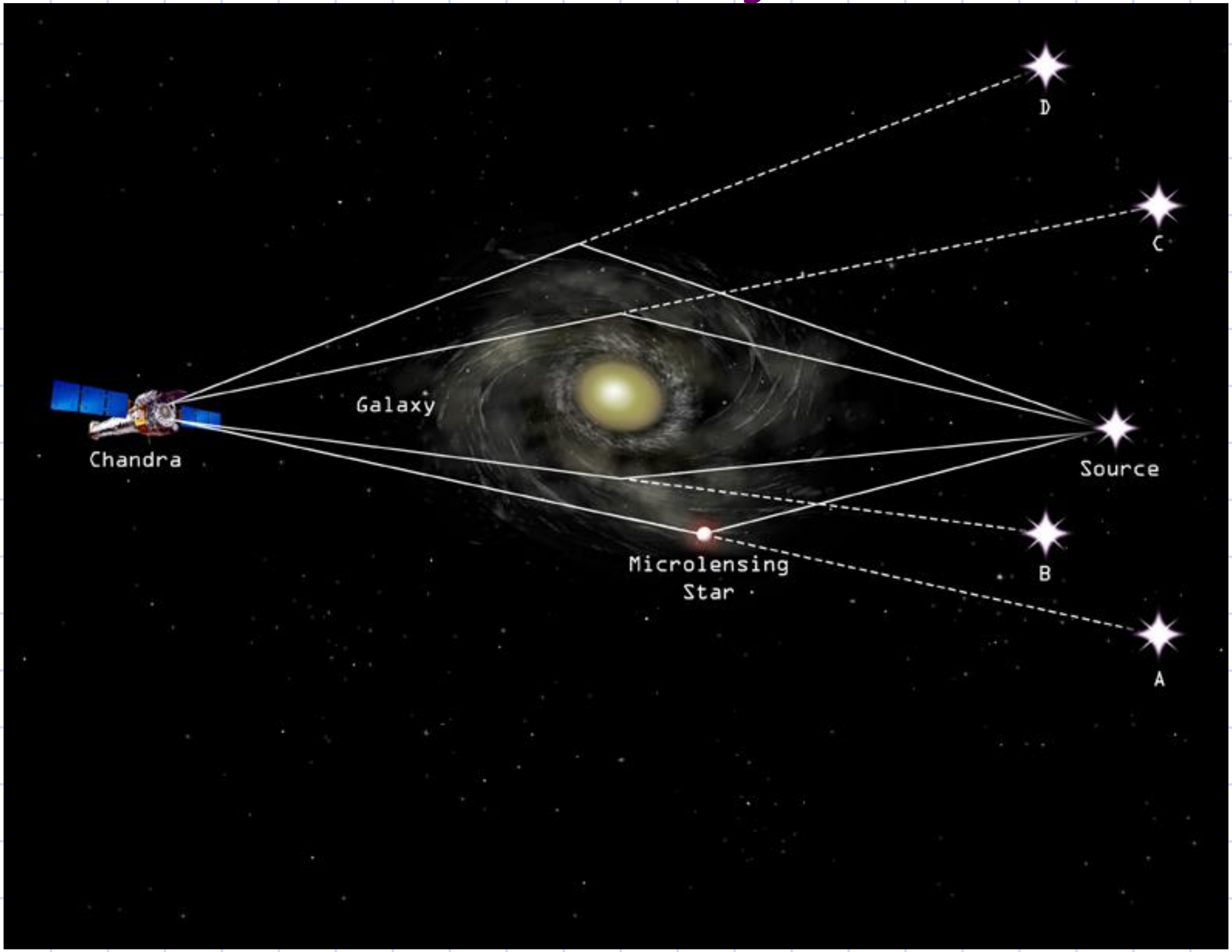


Fig. 8. The gravitational lens system B 1938+666. The *left panel* shows a NICMOS@HST image of the system, clearly showing a complete Einstein ring into which the Active Galaxy is mapped, together with the lens galaxy situated near the center of the ring. The *right panel* shows the NICMOS image as gray-scales, with the radio observations superposed as contours. The radio source is indeed a double, with one component being imaged twice (the two images just outside and just inside the Einstein ring), whereas the other source component has four images along the Einstein ring, with two of them close together (source: L.J. King, see King et al. 1998)

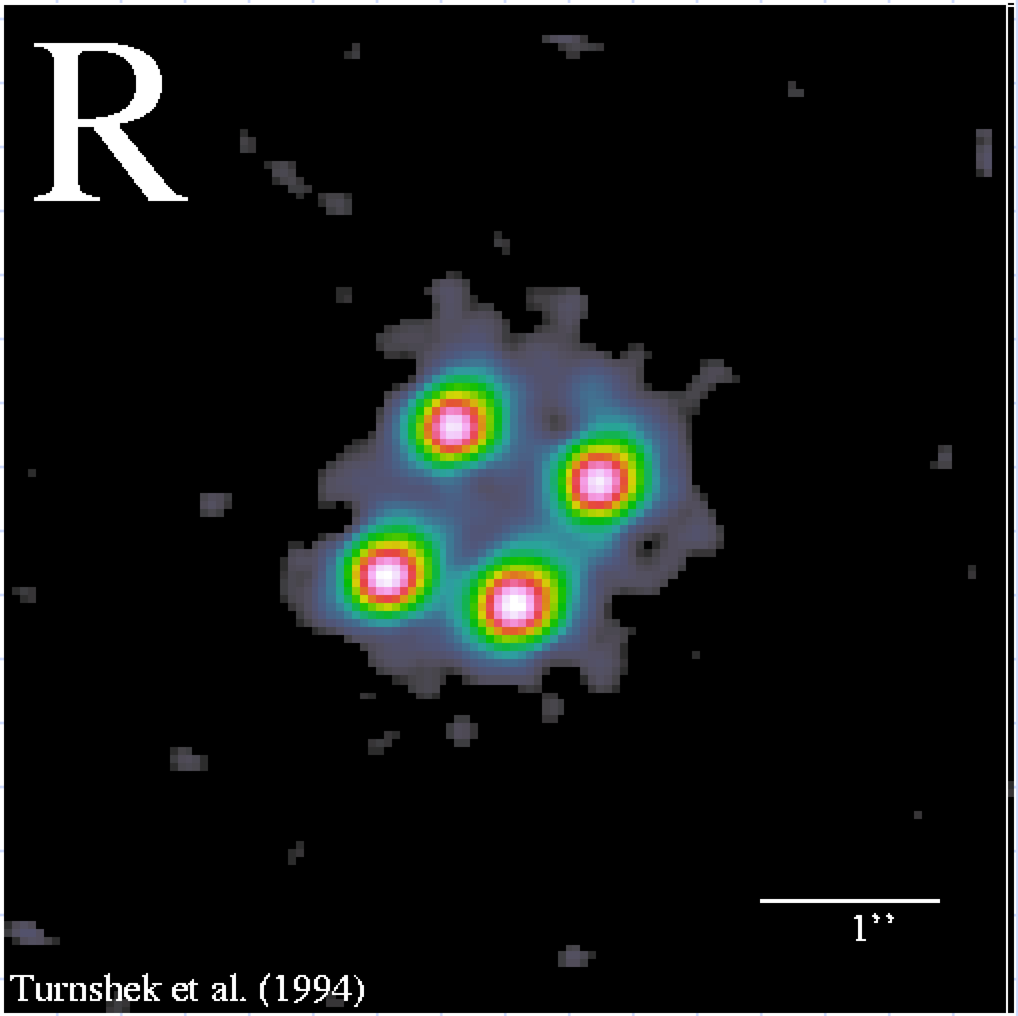
Cloverleaf Quasar (H1413+117)
Illustration of Gravitational Lensing Effect



If GL is not point and has distribution of mass, with broken symmetry, observer can see several images of the same source.

http://chandra.harvard.edu/photo/2004/h1413/more.html

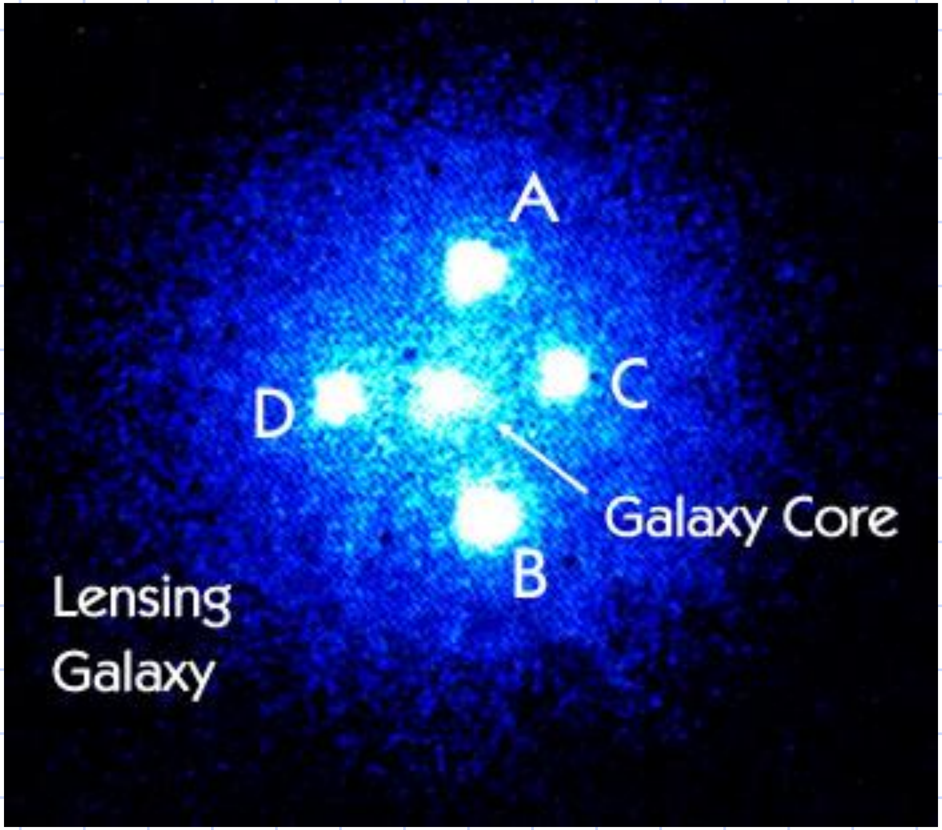
Cloverleaf Quasar (H1413+117)



- Hubble Optical Image of Cloverleaf Quasar

http://chandra.harvard.edu/photo/2004/h1413/more.html

The Einstein Cross



- G2237+0305
- 1985, Huchra and others
- It is observed: four QSO images arrayed around the nucleus of the galaxy.
- The model: 1988, Schneider and others

Summary of Multiply Imaged Systems

http://www.cfa.harvard.edu/castles/

Gravitational Lens Data Base - Mozilla Firefox

http://www.cfa.harvard.edu/castles/

Summary of Multiply Imaged Systems

See also the Summary of Binary Quasars table below.

#	Image	Lens Name	G	z_s	z_l	RA (J2000)	Dec (J2000)	E(B-V)	m_s (mag)	m_l (mag)	F_{GHz} (mJy)	N_{im}	size (")	dt (days)	sigma (km/s)
1		Q0047-2808	A	3.60	0.48	00:49:41.89	-27:52:25.7	0.016		I=20.05		4ER	2.7		229±15
2		HE0047-1756	A	1.66	0.41	00:50:27.83	-17:40:8.8	0.022	I=16.52/2	I=18.96		2	1.44		
3		HST01247+0352	C			01:24:44.4	+03:52:00	0.029	I=24.13/2	I=21.86		2	2.20		
4		HST01248+0351	C			01:24:45.6	+03:51:06	0.029				2	0.74		
5		B0128+437	B	3.124		01:31:13.405	+43:58:13.14	0.082			F_5=48	4	0.55		
6		PMNJ0134-0931	A	2.216	0.77	01:34:35.67	-09:31:02.9	0.031	I=18.96/4	I=19.31	F_5=529	5R	0.73		
7		Q0142-100	A	2.72	0.49	01:45:16.5	-09:45:17	0.031	I=16.47/2	I=18.72	F_5~1	2	2.24		
8		QJ0158-4325	A	1.29		01:58:41.44	-43:25:04.20	0.015	I=17.39/2	I=18.91	F_8<0.2	2	1.22		
9		B0218+357	A	0.96	0.68	02:21:05.483	+35:56:13.78	0.068	I=19.28/2	I=20.06	F_5=1209	2ER	0.34	10.5±0.4	
10		HE0230-2130	A	2.162	0.52	02:32:33.1	-21:17:26	0.022	I=18.00/4	I=20.39		4	2.05		
11		SDSS0246-0825	A	1.68		02:46:34.11	-08:25:36.2	0.026	I=16.97/2	I=20.81		2	1.19		
12		CFRS03.1077	B	2.941	0.938	03:02:30.9	+00:06:02.1	0.098		I=20.36		2ER	2.1		256±19
13		J0332-2756	A		0.617	03:32:38.22	-27:56:52.9	0.008	V=-1.95/6	V=20.74		2E	3.64		

Done

The magnification factor

The flux from source is changed due to GL, because angular size of source is changed.

Angular size of each image is different from angular size of source, and can be larger and smaller.

The surface brightness I for an image is identical to that of the source in the absence of the lens. The flux of an image of an infinitesimal source is the product of its surface brightness and the solid angle $\Delta\omega$ it subtends on the sky.

The magnification μ is the ratio of the flux of an image to the flux of the unlensed source:

$$\mu = \frac{\Delta\omega}{(\Delta\omega)_0}$$

Modern situation

Many effects:

- Angular position change
- Multiple imaging
- Magnification (change of flux)
- Distortion (change of form)
- Time delay (geometrical delay + Shapiro delay)
  Note: independent way to get Hubble constant!

Now gravitational lensing is a powerful astrophysical tool for investigations of distant objects, a distribution of dark matter and large scale structure, the cosmic microwave background, discovery of planet and checking of the General Relativity.

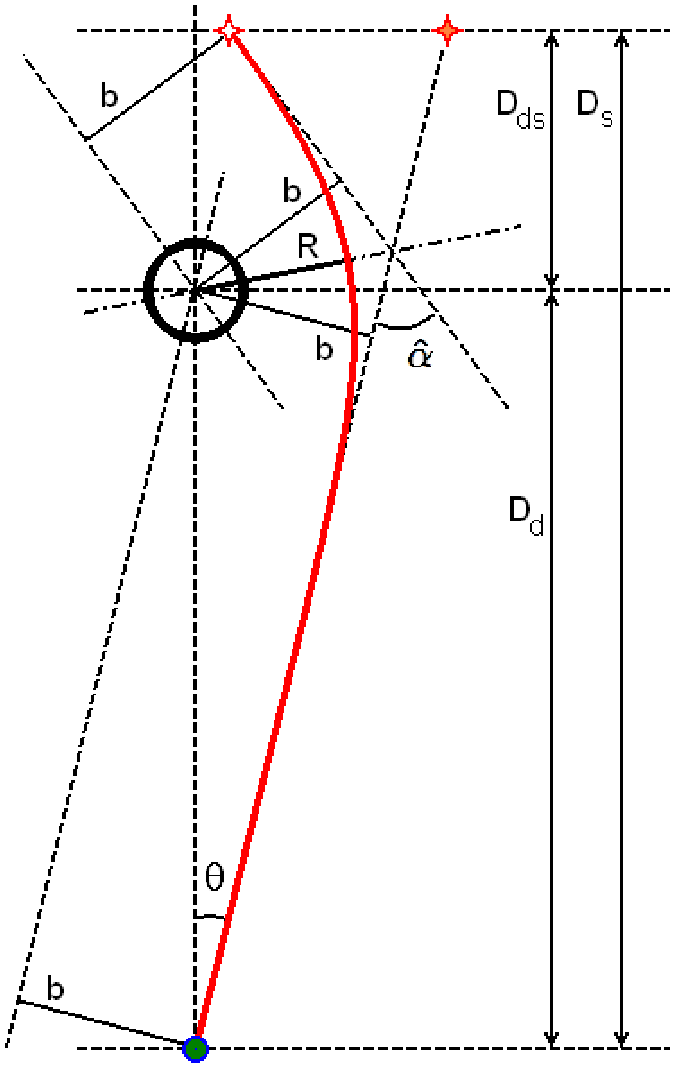
«Ordinary» theory of gravitational lensing:
1) Small deflection angles ($\alpha \ll 1$)
2) Vacuum, lensing is achromatic

Possible ways to expand the usual consideration are:
1) to go beyond the weak deflection limit.
If the photon impact parameter is close to its critical value, photon which goes from infinity can perform several turns around the central object and then go to infinity. In this case deflection angle is not small.
2) to consider medium instead vacuum.
In cosmic space the light rays propagate through the plasma, so the main interest is to consider how the deflection angle is changed in presence of the plasma.
New effects, chromatic lensing.

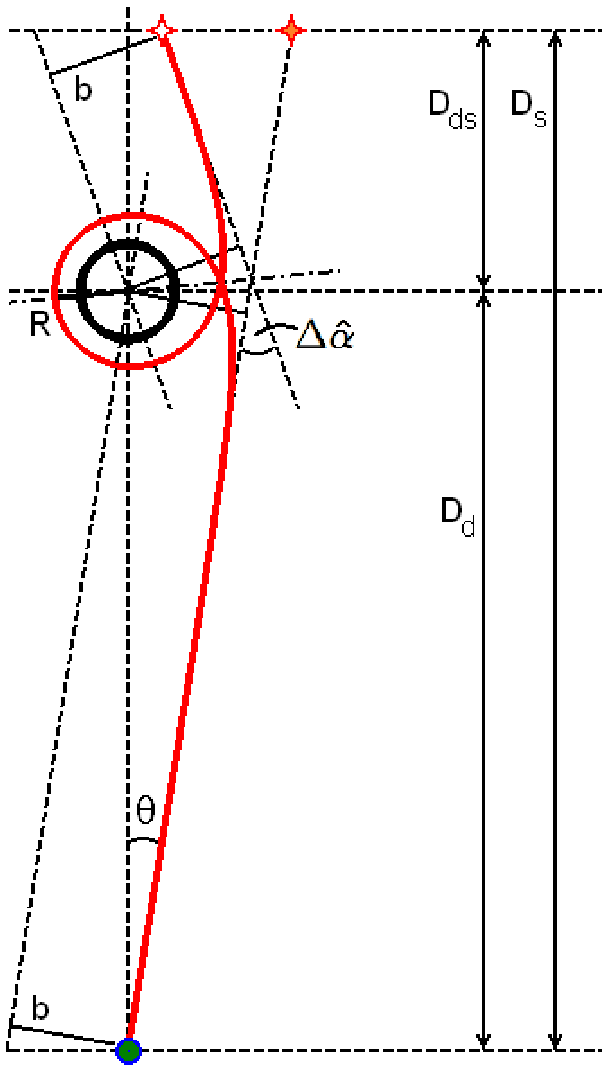
Recent new branches of gravitational lensing:
1) Gravitational lensing in case of strong deflection
2) Gravitational lensing in presence of plasma

Scheme of weak deflection and strong deflection lensing

Weak deflection, small deflection angle, ordinary image



Strong deflection, large deflection angle, relativistic image



Images are drawn by Oleg Tsupko

The strong deflection limit

For relativistic images the weak deflection approximation is not working. We need to use the exact expression.

But if the value of the impact parameter is close to the critical value, we can also use approximate analytical formula for deflection angle (strong deflection limit), and it works very well!

Deflection angle as a function of distance of the closest approach *R*

$$\hat{\alpha} = -2\ln\frac{R-3M}{36\left(2-\sqrt{3}\right)M} - \pi$$

Deflection angle as a function of distance of the impact paremeter *b*

$$\hat{\alpha} = -\ln\left(\frac{b}{b_{cr}} - 1\right) + \ln[216(7-4\sqrt{3})] - \pi$$

Darwin (1959); Bozza et al (2001)

The strong deflection limit

Example: Relativistic Rings

G. S. Bisnovatyi-Kogan and O. Yu. Tsupko, Astrophysics, 51, 99 (2008).

The GL beyond weak deflection approximation is very popular now, for different theories of gravity!

But at present time the relativistic images can not be observed due to small separation from BH and very small magnification (flux).

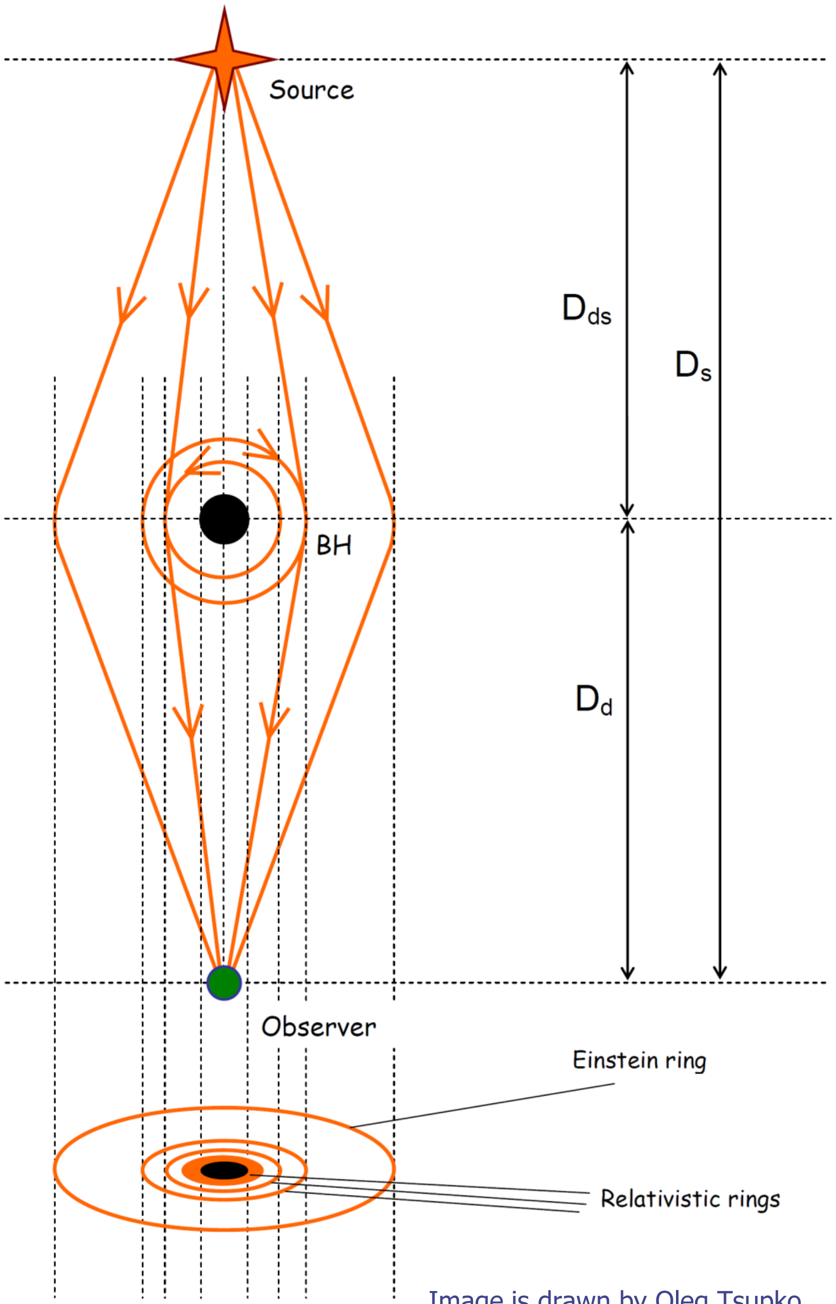


Image is drawn by Oleg Tsupko

- Gravitational lensing in plasma

Gravitational lensing in presence of plasma, previous results

Another way to expand the usual gravitational lens theory is to consider a medium instead vacuum. In cosmic space the light rays propagate through the plasma.

The consideration of the photon deflection in presence of gravitating center and plasma around it was considered in classical book of problems (Lightman, Press, Price, Teukolsky, 1979). In application to gravitational lensing it was discussed in details in book of Bliokh, Minakov (1989) where gravitational lensing by the gravitating body with surrounding spherically symmetric plasma distribution was considered.

When we work in the frame of geometrical optics what is usual for gravitational lensing, we can characterize the properties of the medium by using of the refractive index. In the inhomogeneous medium (the refractive index depends explicitly on the space coordinates) the photon moves along curved trajectory, it is a refraction. The effect of refractive deflection of light has no relation to relativity and gravity, and takes place only in non-homogeneous media. Bliokh and Minakov have performed study of gravitational lensing in plasma in approximation, as just sum of two effects separated from each other: vacuum deflection due to gravitation of point mass, and refractive deflection due to non-homogeneity of the plasma.

A rigorous treatment of the light bending in gravity and plasma requires an answer to the question:

is the gravitational deflection of the light rays in the medium the same as in vacuum?

Gravitational deflection in presence of plasma

A general theory of the geometrical optic in the curved space-time, in arbitrary medium, is presented in the book: J.L. Synge, Relativity: the General Theory, North-Holland Publishing Company, Amsterdam, 1960.
Also see book of Perlick (2000).

Let us consider a static space-time assuming a small perturbation h_{ik} of flat metric. Let us consider, in this gravitational field, a static inhomogeneous plasma with a refraction index n, which depends on the space location x^α and the frequency of the photon $\omega(x^\alpha)$:

Refraction index of plasma:

$$n^2 = 1 - \frac{\omega_e^2}{[\omega(x^\alpha)]^2}, \quad \omega_e^2 = \frac{4\pi e^2 N(x^\alpha)}{m}.$$

We denote $\omega(\infty) = \omega$, e is the charge of the electron, m is the mass of the electron, $N(x^\alpha)$ is the electron concentration in the inhomogeneous plasma, ω_e is the electron plasma frequency in the plasma.

Gravitational deflection in presence of plasma

Refraction index of plasma:

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We denote $\omega(\infty) = \omega$, e is the charge of the electron, m is the mass of the electron, $N(x^\alpha)$ is the electron concentration in the inhomogeneous plasma, ω_e is the electron plasma frequency in the plasma.

We have shown for the first time, that the gravitational deflection in homogeneous plasma differs from the vacuum deflection angle, and depends on frequency of the photon:

new result

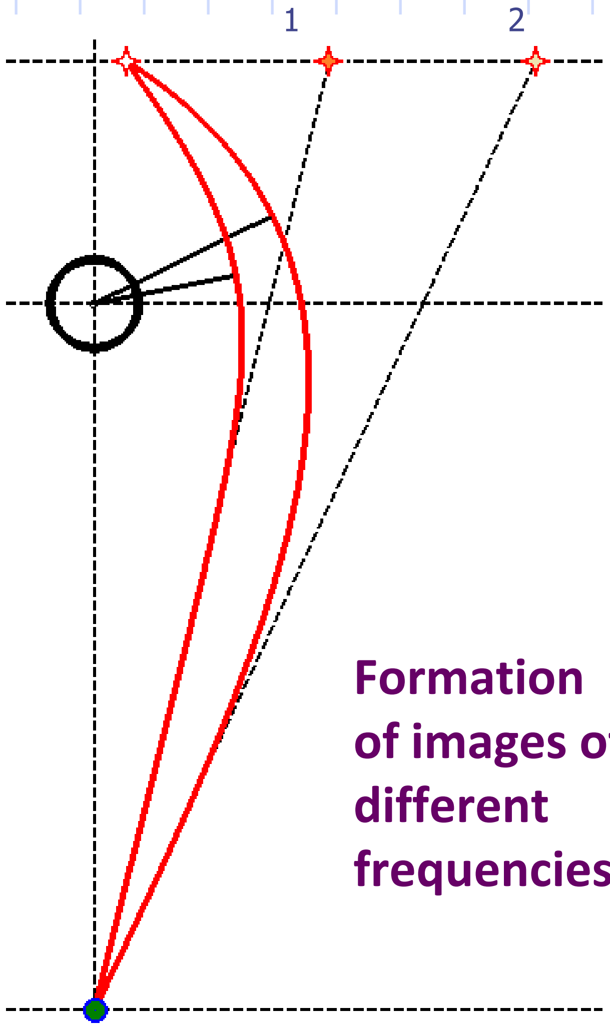
$$\hat{\alpha} = \frac{2R_S}{b} = \frac{4GM}{c^2 b}$$

in vacuum

$$\boxed{\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)}$$

in homogeneous plasma may be much larger

That resembles the properties of the refractive prism spectrometer. The strongest action of this spectrometer is for the frequencies slightly exceeding the plasma frequency, what corresponds to very long radiowaves.
We called this effect “gravitational radiospectrometer”.



Formation of images of different frequencies

$$\boxed{\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)}$$

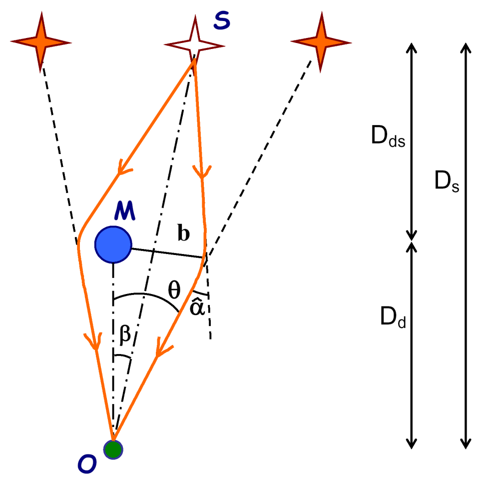
Gravitational deflection in homogeneous plasma:
Photons of different frequencies will be deflected by different angles!

The photons of smaller frequency are deflected by a larger angle by the gravitating center.

The first ray has the photon frequency which is much larger than plasma frequency, and the trajectory can be computed with using the vacuum equations. The another ray has the photon frequency which is close to plasma frequency. In this case plasma effects are significant, and the trajectory should be computed with using plasma equations.

Gravitational radiospectrometer

Instead of two concentrated images with complicated spectra, we will have two line images, formed by the photons with different frequencies, which are deflected by different angles.

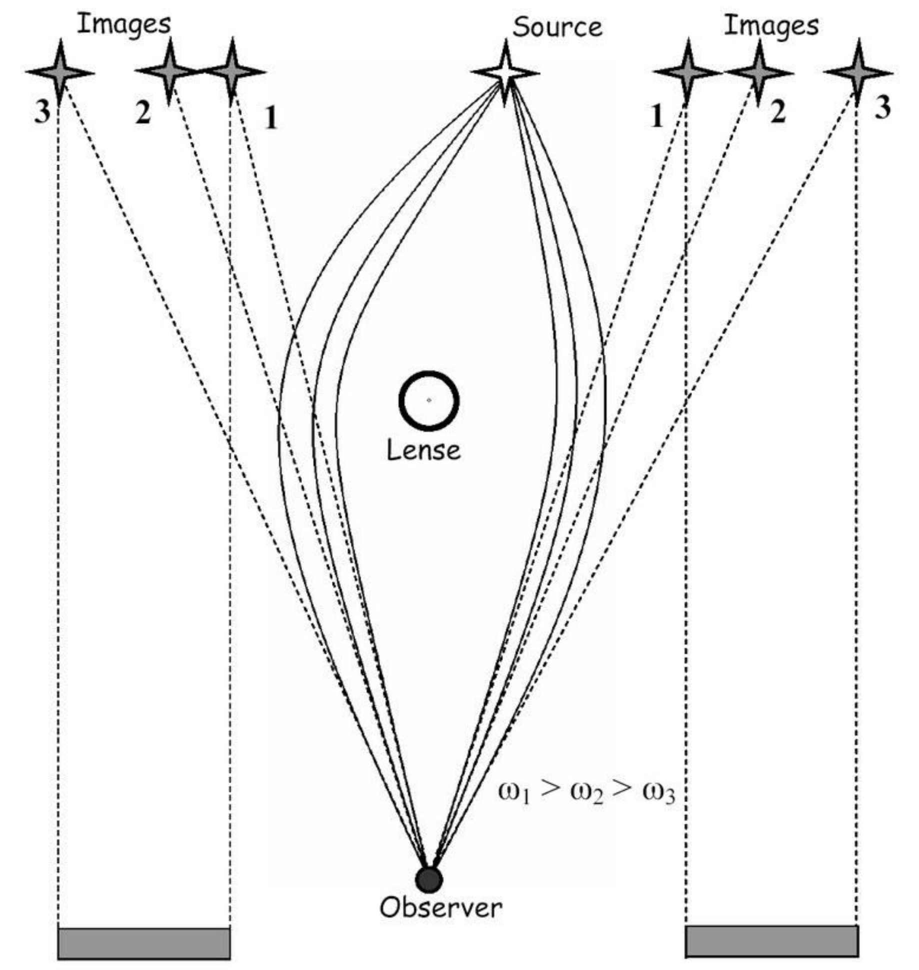


vacuum

$$\lambda_1 < \lambda_2 < \lambda_3$$

$$v_{gr1} > v_{gr2} > v_{gr3}$$

$$\alpha_1 < \alpha_2 < \alpha_3$$



homogeneous plasma

Observations

The typical angular separation between images of the source depends on deflection law. A difference between the angular separation of similar images in vacuum and in plasma is defined as

$$\frac{\Delta\theta_0}{\theta_0} = \frac{\theta_0^{pl} - \theta_0}{\theta_0} = \frac{1}{4}\frac{\omega_e^2}{\omega^2} \simeq 2.0 \cdot 10^7 \frac{N_e}{\nu^2}$$

where ν is the photon frequency in Hz, $\omega = 2\pi\nu$

This formula gives the difference between the deviation angle of the radio wave with the frequency ν, and the optical image, which may be described by the vacuum formula.

Observations at RadioAstron

http://www.radioastron.ru/

Let us estimate possibility of the observation of this effect by the planned project Radioastron. The Radioastron is the VLBI space project led by the Astro Space Center of Lebedev Physical Institute in Moscow. The payload is the Space Radio Telescope, based on the spacecraft Spektr-R.

The observed angular separation of quasar images is usually around 1 arcsec.

For the lowest frequency of the Radioastron, $\nu = 327 \cdot 10^6$ Hz, the angular difference between the vacuum and the plasma images is about 10^{-5} arcsec, when the plasma density on the photon trajectory, in the vicinity of the gravitational lens, N_e is $\sim 5 \cdot 10^4$ cm^{-3}.

$\omega_e^2/\omega^2 \approx 10^{-4}$

Change of magnification!

- Gravitational Lensing also leads to magnification.
- In vacuum: ratios of fluxes $F_{1opt}/F_{2opt} = F_{1rad}/F_{2rad}$, because lensing in vacuum is achromatic.
- In plasma:
- Another law of angle deflection (significant for radio waves)
- -> another formula for magnification
- -> ratios of magnifications can be different for optical band and for radio band
- -> ratios of fluxes can be different for optical and for radio bands if light rays forming images travel through plasma with different density:
- $F_{1opt}/F_{2opt} \neq F_{1rad}/F_{2rad}$
- It can give information about properties of plasma around lens.

All plasma effects in gravitational lensing are very small and their observations are scarcely possible at the moment. Different kinds of absorption, as well as refraction properties of the non-uniform plasma contaminate this effect. Special conditions should exist for possibility to detect it observationally. We have made estimates of the optical depth due to Thomson scattering and free-free absorption during the process of gravitational lensing in a plasma.

Analogy between a photon in plasma and a massive particle in vacuum

Kulsrud, Loeb, 1992; Broderick, Blandford, 2003a,b

The photon moves in gravitational field and in homogeneous plasma exactly like a massive particle with the following parameters

$$m_{eff} = \frac{\hbar}{c^2}\,\omega_e, \quad E_{eff} = \hbar\,\omega(x^\alpha), \quad v_{eff} = v_{gr} = n\,c,$$

analogy. A probe massive particle passing with the velocity v near a spherical body with the mass M, with the impact parameter $b \gg R_S$, deflects to the angle α_m defined as (Misner et al. 1973; Lightman et al. 1979)

$$\alpha_m = \frac{R_S}{b}\left(1 + \frac{1}{\beta^2}\right), \quad \beta = \frac{v}{c}.$$

Using in this formula

$$v_{gr} = c\,n = [1 - (\omega_e^2/\omega^2)]^{1/2}$$

we obtain the same formula for the photon deflection in the homogeneous plasma

$$\hat{\alpha} = \frac{R_S}{b}\left(1 + \frac{1}{1 - (\omega_e^2/\omega^2)}\right)$$

Gravitational lensing in non-homogeneous plasma:

Our approach, based on equations of Synge (1960, the general theory of the geometrical optic in the curved space-time, in isotropic dispersive medium), allows us simultaneously consider and take into account the two effects:

gravitational deflection in plasma, which is different from the vacuum case (new effect)

It depends on the photon frequency. It takes place both in homogeneous and inhomogeneous plasma

+

the non-relativistic effect (refraction) connected with the plasma inhomogeneity

It depends on the photon frequency because the plasma is dispersive medium. But this angle equals to zero if the plasma is homogeneous

Deflection by non-uniform plasma distribution

M(b) is the projected mass enclosed by the circle of the radius b (impact parameter), in another words it is the mass inside cylinder with radius b

$$\hat{\alpha}_b = \frac{4GM(b)}{c^2 b} + \frac{2GM(b)\,b}{c^2\omega^2}\int_0^\infty \frac{\omega_e^2}{r^3}\,dz +$$

$$+ \frac{K_e b}{\omega^2}\int_0^\infty \frac{1}{r}\frac{dN(r)}{dr}\,dz + \frac{K_e b}{\omega^4}\int_0^\infty \frac{\omega_e^2}{r}\frac{dN(r)}{dr}\,dz =$$

$$= \hat{\alpha}_1 + \hat{\alpha}_2 + \hat{\alpha}_3 + \hat{\alpha}_4.$$

$$\omega_e^2 = \frac{4\pi e^2 N(x^\alpha)}{m} = K_e N(x^\alpha)$$

α_1 is the vacuum gravitational deflection,
α_2 is the correction to the vacuum gravitational deflection, due to presence of plasma,
α_3 is the refraction deflection due to inhomogeneity of plasma,
α_4 is the correction to the third term. We are interested mainly in the effects, described by the terms α_1, α_2, α_3.

Here is an expansion with approximation that the refractive index n is close to unit (near vacuum case).

Singular isothermal sphere

$$\rho(r) = \frac{\sigma_v^2}{2\pi G r^2}$$

Here σ_v is a one-dimensional velocity dispersion (for stars in galaxies or for galaxies in clusters of galaxies).

$$\hat{\alpha}_1 = 4\pi \frac{\sigma_v^2}{c^2}$$

Using (64), we obtain the deflection angles $\hat{\alpha}_2$ and $\hat{\alpha}_3$:

$$\hat{\alpha}_2 = \frac{2}{3} \frac{\sigma_v^2}{c^2} \frac{K_e}{\kappa m_p} \frac{\sigma_v^2}{G\omega^2 b^2},$$

$$\hat{\alpha}_3 = -\frac{K_e b}{\omega^2 \kappa m_p} \frac{\sigma_v^2}{\pi G} \int\limits_0^\infty \frac{dz}{(z^2+b^2)^2} = -\frac{1}{4} \frac{K_e}{\kappa m_p} \frac{\sigma_v^2}{G\omega^2 b^2}.$$

For the ratio of the angles $\hat{\alpha}_2$ and $\hat{\alpha}_3$ we have:

$$\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| = \frac{8}{3} \frac{\sigma_v^2}{c^2}.$$

In this configuration the nonuniform plasma deflection effects are much stronger, than the gravitational plasma effects, and have an opposite direction.

Non-singular isothermal gas sphere

$$\rho(r) = \frac{\sigma_v^2}{2\pi G(r^2 + r_c^2)} = \frac{\rho_0}{\left(1 + \frac{r^2}{r_c^2}\right)^2}, \quad \rho_0 = \frac{\sigma_v^2}{2\pi G r_c^2}$$

$$\hat{\alpha}_1 = 4\pi \frac{\sigma_v^2}{c^2} \frac{\sqrt{b^2 + r_c^2} - r_c}{b}$$

Introducing the mass of the uniform core $M_c = \frac{4\pi}{3}\rho_0 r_c^3$, and its gravitational radius $R_{sc} = \frac{2GM_c}{c^2}$, we obtain the ratio of these angles as

$$\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| = \frac{3R_{sc}}{2r_c} \quad (r_c \gg b), \qquad \left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| = \frac{2R_{sc}}{r_c} \quad (r_c \ll b). \tag{79}$$

In the realistic cases $\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| \ll 1$ in (67),(79), because spheres have $\sigma_v \ll c$, and $R_{sc} \ll r_c$. Besides, these relations are needed for the stability of isothermal spheres, see Bisnovatyi-Kogan & Zeldovich (1969). Therefore in this configuration the nonuniform plasma deflection effects are much stronger, than the gravitational plasma effects, and have an opposite direction.

Plasma sphere around a black hole

Let us consider a black hole of mass M_0, surrounded by the electron-proton plasma. We will consider a case, when we can neglect the self-gravitation of the plasma particles, compared to the gravity of a central black hole. Let us find, in the Newtonian approximation, a density distribution of the plasma in the gravitational field a central point mass M_0.

M_0. The equation of hydrostatic equilibrium for spherically symmetric mass distribution of a isothermal gas with the equation of state $P = \rho \Re T$ in the field of the central mass is (Binney & Tremaine 1987; Chandrasekhar 1939)

$$\frac{\Re T}{\rho} \frac{d\rho}{dr} = -\frac{GM_0}{r^2}.$$

$$\rho(r) = \rho_0 \, e^{\frac{GM_0}{\Re T}\left(\frac{1}{r} - \frac{1}{r_0}\right)} = \rho_0 \, e^{B\left(\frac{1}{r} - \frac{1}{r_0}\right)}, \quad B = \frac{GM_0}{\Re T}$$

$$\left| \frac{\hat{\alpha}_2}{\hat{\alpha}_3} \right| = \frac{2 \Re T}{c^2}$$

While the temperature of the non-self-gravitating sphere may have arbitrary values, the plasma effects may be comparable, and even less than the GR plasma effects. It is due to the fact, that with increasing temperature the plasma density can become arbitrary uniform, with corresponding decreasing of non-uniform plasma effect for refraction. We have used Newtonian non-relativistic description of the gas sphere, but from the arguments listed above it is clear, that this conclusion remains valid also in the correct relativistic consideration.

Plasma in a galaxy clusters

In a galaxy cluster the electron distribution may be more homogeneous due to large temperature of electrons. An appropriate approach for this case is to consider a singular isothermal sphere as a model for the distribution of the gravitating matter, neglecting the mass of plasma, and to find a plasma density distribution from the solution of the equation of the hydrostatic equilibrium of plasma in the gravitational field of a singular isothermal sphere.

$$\rho_{gr}(r) = \frac{\sigma_v^2}{2\pi G r^2} \qquad \frac{\Re T}{\rho}\frac{d\rho}{dr} = -\frac{2\sigma_v^2}{r} \qquad \rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-s}, \qquad s = \frac{2\sigma_v^2}{\Re T}$$

Under the condition $s \ll 1$ what corresponds to $2\sigma_v^2 \ll \Re T$, the expressions are simplified to

$$\hat{\alpha}_2 = 2\pi \frac{\sigma_v^2}{c^2\omega^2}\frac{K_e}{m_p}\rho_0 \left(\frac{r_0}{b}\right)^s,$$

$$\hat{\alpha}_3 = -\pi \frac{\sigma_v^2}{\Re T \omega^2}\frac{K_e}{m_p}\rho_0 \left(\frac{r_0}{b}\right)^s,$$

$$\left|\frac{\hat{\alpha}_2}{\hat{\alpha}_3}\right| = \frac{2\Re T}{c^2}.$$

If relativistic plasma is present in a galaxy cluster, for example in jets from AGNs, the plasma GR effects may be larger than the effects of the nonuniform plasma. If the distribution of plasma is not spherically symmetric, there may be distribution of plasma with the density gradient opposite to the direction of the gravitational force, for example, in the presence of rotation. In this situation the angles $\hat{\alpha}_2$ and $\hat{\alpha}_3$ may be of the same sign.

The gravitational effect in plasma may be identified when plasma non-uniformity is not prevailing, what is possible in relativistic plasma, kT ~ m_e c^2

Conslusions about gravitational deflection in medium

- In vacuum the gravitational deflection equals to the Einstein angle.
- If medium is homogeneous (refraction index n is constant, it does not depend on space coordinates) and not dispersive (refraction index does not depend on the photon frequency), the gravitational deflection angle is the same as in vacuum.
- If medium is homogeneous but dispersive (refraction angle does not depend on space coordinates but depends on frequency), the gravitational deflection angle is different from the Einstein angle.
- Example: plasma.
- If medium is non-homogeneous (depends on space coordinates), we will have also refraction deflection.

Conclusions about Lensing in Plasma

1. In the homogeneous plasma gravitational deflection differs from vacuum (Einstein) deflection angle and depends on frequency of the photon. So gravitational lens in plasma acts as a gravitational radiospectrometer.

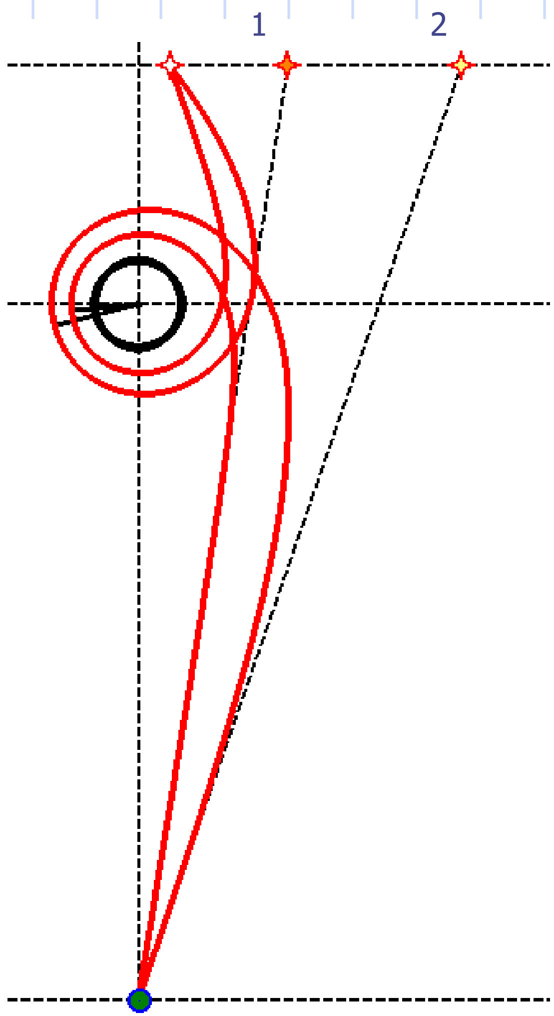
2. Plasma effects leads to changing of angular separation between images and ratios of fluxes, if we compare optical and radio waves. It leads to: in the observations the spectra of two images may be different in the long wave side due to different plasma properties along the trajectory of the images. It can give information about properties of plasma around lens.

3. We have carried out the calculations for models with the nonuniform plasma distribution: singular and nonsingular isothermal sphere; for hot gas inside the gravitational field of a black hole, and of a cluster of galaxies.

4. For different gravitational lens models we compare the corrections to the vacuum lensing due to the gravity effect in plasma (alpha2), and due to the plasma inhomogeneity (alpha3). We have shown that the plasma correction to gravitational deflection can be important in the case of a hot gas in the gravitational field of a galaxy cluster.

1. Bisnovatyi-Kogan G.S., Tsupko O.Yu. Gravitational radiospectrometer // Gravitation and Cosmology. 2009. V.15. N.1. P.20. arXiv:0809.1021v2 [astro-ph] 19 Sep 2008
2. Bisnovatyi-Kogan G.S., Tsupko O.Yu. Gravitational lensing in a non-uniform plasma, Mon. Not. R. Astron. Soc. 404, 1790–1800 (2010). arXiv:1006.2321v1 [astro-ph.CO]

Strong deflection + homogeneous plasma



Formation of relativistic images of different frequencies

Gravitational lensing in homogeneous plasma. Scheme of formation of the relativistic images of different frequencies. The photons of smaller frequency are deflected by a larger angle by the gravitating center. Instead of the image with all frequencies the observer see a line image, which consists of the images of different frequencies. At this picture we present only two of such images. The first ray has the photon frequency which is much larger than plasma frequency, and the trajectory can be computed with using the vacuum equations. The another ray has the photon frequency which is close to plasma frequency. In this case plasma effects are significant, and the trajectory should be computed with using plasma equations.

V. Perlick, Ray Optics, Fermat's Principle, and Applications to General Relativity, Springer-Verlag (Berlin Heidelberg New York), 2000.
O. Yu. Tsupko and G. S. Bisnovatyi-Kogan (2012), in preparation

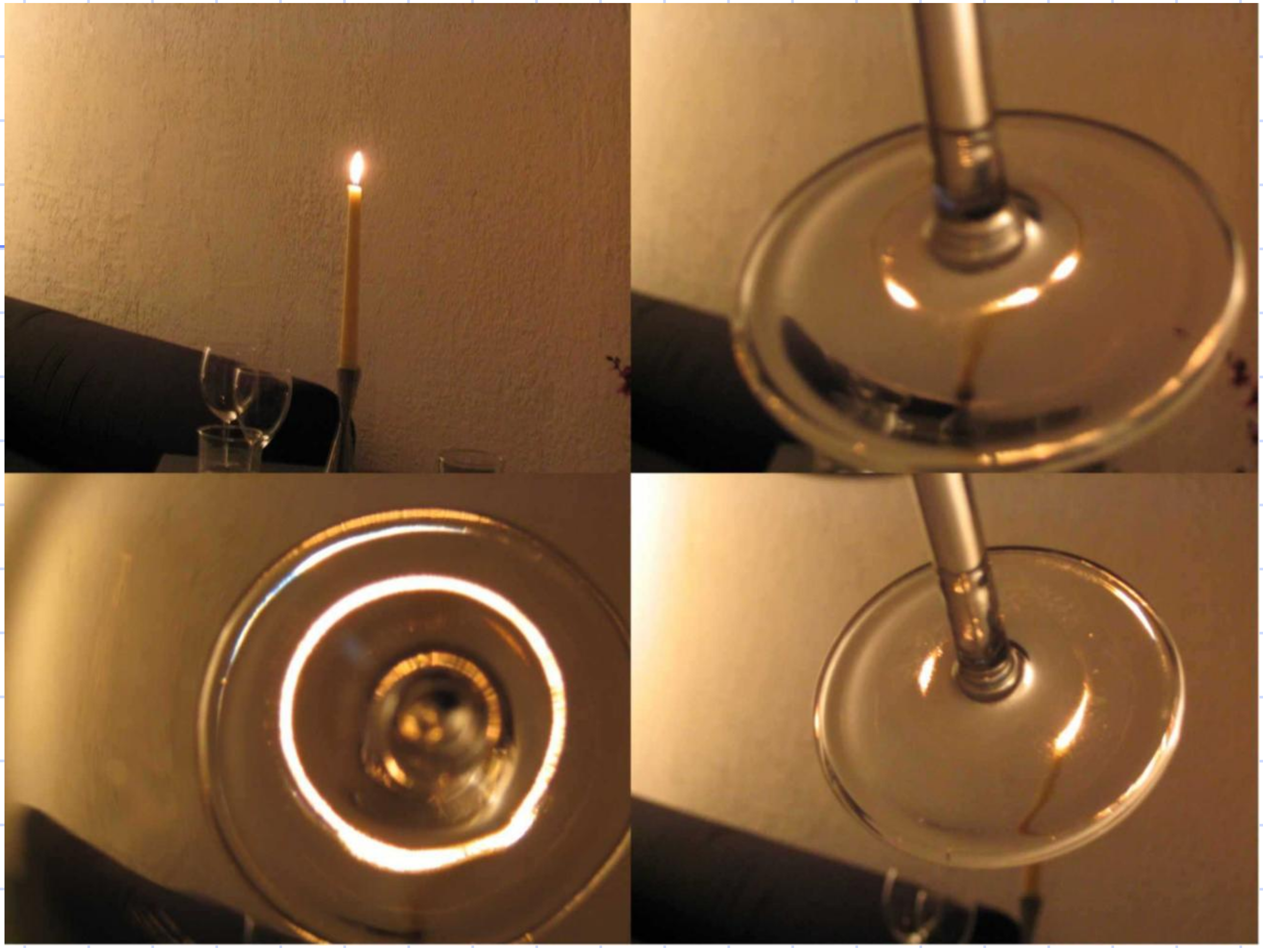
$$\hat{\alpha}(R,x) = -2\sqrt{\frac{1+x}{2x}}\ln\left[z(x)\,\frac{R-r_M}{r_M}\right] - \pi\,,$$

$$\text{where} \quad z(x) = \frac{9x-1+2\sqrt{6x(3x-1)}}{48x}\,,$$

$$r_M = 6M\,\frac{1+x}{1+3x}\,, \quad x \equiv \sqrt{1-\frac{8}{9E^2}} = \sqrt{1-\frac{8\omega_e^2}{9\omega^2}}\,.$$

$$\hat{\alpha}(b,x) = -\sqrt{\frac{1+x}{2x}}\ln\left[\frac{2\,z^2(x)}{3x}\,\frac{b-b_0}{b_0}\right] - \pi\,.$$

$$b_0 = \sqrt{3}\,r_M\,\sqrt{\frac{1+x}{3x-1}}$$



Optical analogy to illustrate the gravitational lensing phenomenon. The optical properties of the stem of a wineglass are similar to those of a typical galaxy scale lens. Viewed through a wineglass, a background compact source such as distant candle (top left), can reproduce the quad (top right), Einstein ring (bottom left), and double (bottom rights) configurations observed in gravitational lensing. Image courtesy of P. Marshall.