Supergravity Divergences and Puzzles

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Gell-Mann's Totalitarian Principle:

"Everything not forbidden is compulsory"

So why are expected UV divergences not occurring on schedule in maximal supergravity?

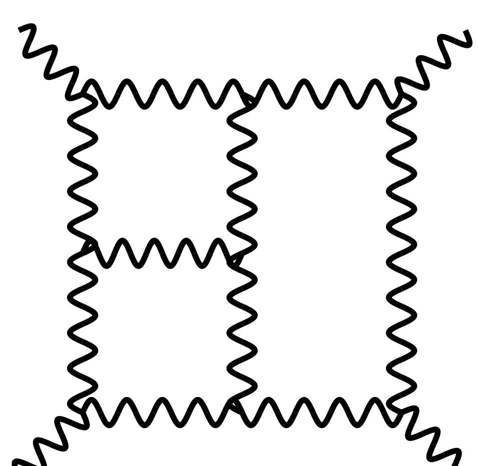
Are miracles happening?

Ultraviolet Power Counting in Gravity

- Simple power counting in gravity and supergravity theories leads to a naïve degree of divergence

$$\Delta = (D-2)L + 2$$

in D spacetime dimensions. So, for D=4, L=3, one expects Δ=8. In dimensional regularization, only logarithmic divergences are seen ($1/\varepsilon$ poles, ε=D-4), so 8 powers of momentum would have to come out onto the external lines of such a diagram.



- It has been recognized since the earliest days of supergravity that counterterms would have to be invariant under local supersymmetry. The dangerous counterterms are those that do not vanish subject to the classical equations of motion.

- Local supersymmetry implies that the pure-curvature part of such a D=4, 3-loop divergence candidate must be built from the square of the Bel-Robinson tensor

Deser, Kay & K.S.S 1977

$$\int \sqrt{-g}\, T_{\mu\nu\rho\sigma} T^{\mu\nu\rho\sigma}\,, \qquad T_{\mu\nu\rho\sigma} = R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta} R_{\rho\alpha\sigma\beta} + {}^{*}R_{\mu}{}^{\alpha}{}_{\nu}{}^{\beta}\, {}^{*}R_{\rho\alpha\sigma\beta}$$

- The consequences of supersymmetry for the ultraviolet structure are not restricted to the requirement that counterterms be supersymmetric invariants.
- There exist more powerful "nonrenormalization theorems" in superspace (where $\int d\theta\, \theta = 1$, $\int d\theta = 0$) the most famous of which *excludes* infinite renormalization of chiral invariants in D=4, N=1 supersymmetry, given in N=1 superspace by holomorphic integrals over just *half* the superspace: $\int d^4x d^2\theta\, W(\phi(x,\theta,\bar{\theta}))\ ,\quad \bar{D}\phi = 0$ (as compared to full superspace $\int d^4x d^4\theta L(\phi,\bar{\phi})$)
- However, extended SYM and supergravity theories do not all have formalisms with all supersymmetries linearly realized "off-shell" in superspace. So the power of such nonrenormalization theorems is limited to the off-shell linearly realizable subalgebra.

- The degree of “off-shell” supersymmetry is the maximal supersymmetry for which the algebra can close without use of the equations of motion.
- Knowing the extent of this off-shell supersymmetry is tricky, and may involve formulations (*e.g.* harmonic superspace) with infinite numbers of auxiliary fields. Galperin, Ivanov, Kalitsin, Ogievetsky & Sokatchev
- For maximal N=4 Super Yang-Mills and maximal N=8 supergravity, the off-shell realizable supersymmetry has been believed since the 1980’s, based upon a linearized analysis, to be at least *half* the full supersymmetry of the theory. So at that time the first generally allowed counterterms were expected to have “1/2 BPS” structure as compared to the full supersymmetry of the theory.

- Of course, there are other symmetries in supergravity beside diffeomorphism invariance and supersymmetry. In particular, D=4, N=8 supergravity also has a rigid nonlinearly realized E_7 symmetry. At leading order, this symmetry is realized by constant shifts of the 70 scalars, which take their values in the coset space $E_7/SU(8)$.

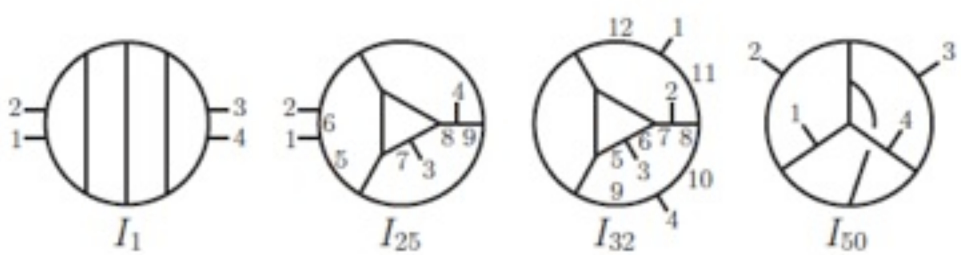
- The 1/2 BPS R^4 candidate satisfies at least the *minimal* requirement of invariance under such constant shifts of the 70 scalars because, at the leading 4-particle order, the integrand may be written such that every scalar field is covered by a derivative.

Unitarity-based calculations

Bern, Carrasco, Dixon, Johansson, Roiban et al. 2007 ... 2011

- The calculational front has made substantial progress since the late 1990s.

- These have led to unanticipated and surprising cancellations at the 3- and 4-loop orders, yielding new lowest possible orders for the super Yang-Mills and supergravity divergence onset.



plus 46 more topologies

Max. SYM first divergences, current lowest possible orders (for spacetime dimensions $\in \mathbb{Z}$).

Dimension D	10	8	7	6	5	4
Loop order L	1	1	2	3	6?	∞
BPS degree	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
Gen. form	$\partial^2 F^4$	F^4	$\partial^2 F^4$	$\partial^2 F^4$	$\partial^2 F^4$	finite

Blue: known divergences

Max. supergravity first divergences, current lowest possible orders (for spacetime dimensions $\in \mathbb{Z}$).

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6?	5?
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{4}$
Gen. form	$\partial^{12} R^4$	$\partial^{10} R^4$	R^4	$\partial^4 R^4$	$\partial^6 R^4$	$\partial^{12} R^4$	$\partial^4 R^4$

Algebraic Renormalization and Ectoplasm

Dixon; Howe, Lindstrom & White; Piguet & Sorella; Hennaux; Stora;
Baulieu & Bossard; Voronov 1992; Gates, Grisaru, Knut-Whelau, & Siegel 1998
Berkovits and Howe 2008; Bossard, Howe & K.S.S. 2009

- The construction of supersymmetric invariants is isomorphic to the construction of cohomologically nontrivial closed forms in superspace: $I = \int_{M_0} \sigma^* \mathcal{L}_D$ (where σ^* is a pull-back to a section of the projection map down to the purely bosonic "body" subspace M_0) is invariant if $\mathcal{L}_D$ is a closed form in superspace, and it is nonvanishing only if $\mathcal{L}_D$ is nontrivial.
- Using the BRST formalism, one can handle all gauge symmetries including space-time diffeomorphisms by the nilpotent BRST operator s. The invariance condition for $\mathcal{L}_D$ is $s\mathcal{L}_D + d_0\mathcal{L}_{D-1} = 0$, where d_0 is the usual bosonic exterior derivative. Since $s^2 = 0$ and s anticommutes with d_0, one obtains using Poincaré's lemma $s\mathcal{L}_{D-1} + d_0\mathcal{L}_{D-2} = 0$, etc.

- Solving the BRST Ward identities thus becomes a cohomological problem. Note that the supersymmetry ghost is a commuting field. One needs to study the cohomology of the nilpotent operator $\delta = s + d_0$, whose components $\mathcal{L}_{D-q,q}$ are (*D*-*q*) forms with ghost number *q*, *i.e.* (*D*-*q*) forms with *q* spinor indices. The spinor indices are totally *symmetric* since the supersymmetry ghost is *commuting*.

- For gauge-invariant supersymmetric integrands, this establishes an isomorphism between the cohomology of closed forms in superspace (aka "ectoplasm") and the construction of BRST-invariant counterterms.

Cohomological non-renormalization

- Spinorial cohomology allows one to derive non-renormalization theorems for counterterms: the cocycle structure of candidate counterterms must match that of the classical action.
  - For example, in maximal SYM, this leads to non-renormalization theorems ruling out the F^4 counterterm that was otherwise expected at L=4 in D=5.
  - Similar non-renormalization theorems exist in supergravity, but their study is complicated by local supersymmetry and the density character of counterterm integrands.

Duality invariance constraints

cf. also Broedel & Dixon 2010

- Maximal supergravity has a series of duality symmetries which extend the automatic GL(11-D) symmetry obtained upon dimensional reduction down from D=11, *e.g.* E_7 in the N=8, D=4 theory, with the 70=133-63 scalars taking their values in an $E_7/SU(8)$ coset target space.

Bossard, Hillman & Nicolai 2010

- The N=8, D=4 theory can be formulated in a manifestly E_7 covariant (but non-manifestly Lorentz covariant) formalism. Anomalies for SU(8), and hence E_7, cancel. Marcus 1985

- Combining the requirement of continuous duality invariance with the spinorial cohomology requirements gives further powerful restrictions on counterterms.

Supergravity Densities

- In a curved superspace, an invariant is constructed from the top (pure "body") component in a coordinate basis:

$$I = \frac{1}{D!}\int d^D x\, \varepsilon^{m_D\ldots m_1}\; E_{m_D}{}^{A_D}\cdots E_{m_1}{}^{A_1}\; L_{A_1\ldots A_D}(x,\theta=0)$$

- Referring this to a preferred "flat" basis and identifying $E_M{}^A$ components with vielbeins and gravitinos, one has, *e.g.* in D=4

$$I = \frac{1}{24}\int \big(e^a{}_\wedge e^b{}_\wedge e^c{}_\wedge e^d\; L_{abcd} + 4e^a{}_\wedge e^b{}_\wedge e^c{}_\wedge \psi^{\underline{\alpha}} L_{abc\underline{\alpha}} + 6e^a{}_\wedge e^b{}_\wedge \psi^{\underline{\alpha}}{}_\wedge \psi^{\underline{\beta}}\; L_{ab\underline{\alpha}\underline{\beta}}$$
$$+4e^a{}_\wedge \psi^{\underline{\alpha}}{}_\wedge \psi^{\underline{\beta}}{}_\wedge \psi^{\underline{\gamma}} L_{a\underline{\alpha}\underline{\beta}\underline{\gamma}} + \psi^{\underline{\alpha}}{}_\wedge \psi^{\underline{\beta}}{}_\wedge \psi^{\underline{\gamma}}{}_\wedge \psi^{\underline{\delta}} L_{\underline{\alpha}\underline{\beta}\underline{\gamma}\underline{\delta}}\big)$$

  - Thus the "soul" components of the cocycle also contribute to the local supersymmetric covariantization.

- Since the gravitinos do not transform under the D=4 E_7 duality, the L_{ABCD} form components have to be *separately* duality invariant.

- At leading order, the $E_7/SU(8)$ coset generators of E_7 simply produce *constant shifts* in the 70 scalar fields, as we have seen. This leads to a much easier check of invariance than analyzing the full spinorial cohomology problem.

Howe, K.S.S. & Townsend 1981

- Although the pure-body (4,0) component L_{abcd} of the R^4 counterterm has long been known to be shift-invariant at lowest order (since all 70 scalar fields are covered by derivatives), it is harder for the fermionic "soul" components to be so, since they are of lower dimension.

- Thus, one finds that the maxi-soul (0,4) $L_{\alpha\beta\gamma\delta}$ component is *not* invariant under constant shifts of the 70 scalars. Hence the D=4, N=8, 3-loop R^4 1/2 BPS counterterm is not E_7 duality invariant, so it is ruled out as an allowed counterterm.

Bossard, Howe & K.S.S. 2010

Elvang & Kiermeier 2010;
Bossard, Howe & K.S.S 2010;
Beisert, Elvang, Freedman, Kiermaier, Morales & Stieberger 2010

- Similar duality & supersymmetry/ectoplasm analysis shows that the 1/4 BPS $\partial^4 R^4$ candidate in D=4, L=5, as well as the 1/8 BPS $\partial^6 R^4$ candidate in D=4, L=6 are ruled out.
  - D=4 3-loop R^4 invariants in N=5 and N=6 supergravities are similarly ruled out.
  - Candidate divergences in D>4 dimensions are also ruled out, e.g. the 1/8 BPS $\partial^6 R^4$ candidate in D=5, L=4, as confirmed by explicit calculation.

    Bern, Carrasco, Dixon, Johansson & Roiban 2009
  - Some candidates are *not* ruled out, e.g. the D=8, L=1 R^4 candidate, which is found to *occur* by explicit calculation.

Linearized versus nonlinear invariants in superspace

- An unanticipated consequence of the counterterm studies has been the recognition that not all on-shell supergravity invariants have a natural expression in superspace at the full nonlinear level, either as a subsurface BPS type integral or as a full superspace integral.

- For example, the R^4 counterterm has a 1/2 BPS form at linearized order (with just 4-point terms), but attempts to generalize this to the full nonlinear level fail.

de Haro, Sinkovics & Skenderis 2003
Berkovits & Howe 2003

- All invariants can be viewed as integrals over pull-backs of closed forms in superspace, however. The relevant question then is the structure of their cocycles and whether they respect duality invariances.

- Another puzzling feature of full nonlinear invariants is the way the apparent BPS structure can differ between a linearized invariant and the full nonlinear invariant. The candidate $\partial^8 R^4$ invariant at L=7, D=4 illustrates this.
- At linearized order, this $\Delta = 16$ invariant appears to be writable as a $\int d^{32}\theta$ full superspace integral. It also passes the linearized test for E_7 invariance.
  - The question then arises which manifestly covariant and manifestly duality invariant expression this could be.
  - The natural L=7 suggestion would be the full volume of superspace,

$$\int d^4x d^{32}\theta E(x,\theta)$$

E_7 invariant counterterms are long known to exist for L>7:
Howe & Lindstrom 1981
Kallosh 1981

  - This is manifestly fully invariant under superdiffeomorphisms and under E_7 duality transformations.

Vanishing Volume

Bossard, Howe, K.S.S. & Vanhove 2011

- The 7-loop situation, however, turns out to be more complex than anticipated: the superspace volume actually *vanishes* on-shell.
- Explicitly integrating out the volume $\int d^4x d^{32}\theta E(x,\theta)$ using the superspace constraints implying the classical field equations would be an ugly task.
  - However, using an on-shell implementation of harmonic superspace together with a superspace implementation of the normal-coordinate expansion, one can nonetheless see that it vanishes on-shell for all D=4 supersymmetry extensions N:

$$\int d^4x d^{32}\theta E(x,\theta) = 0 \qquad \text{on-shell}$$

1/8 BPS E_7 invariant candidate notwithstanding

Bossard, Howe, K.S.S. & Vanhove 2011

- Despite the vanishing of the full N=8 superspace volume, one can nonetheless use the harmonic superspace formalism to construct a different manifestly E_7 -invariant but 1/8 BPS candidate:

$$I^8 := \int d\mu_{(8,1,1)}\, B_{\alpha\dot\beta}\, B^{\alpha\dot\beta}$$

- At the leading 4-point level, this invariant of generic $\partial^8 R^4$ structure can be written as a full superspace integral with respect to the linearized N=8 supersymmetry. It cannot, however, be rewritten as a non-BPS full-superspace integral at the nonlinear level.

- Non-BPS full-superspace and manifestly E_7 -invariant candidates exist in any case from 8 loops onwards.

Current outlook: maximal supergravity

- So far, things look(ed) pretty much under control from a purely field-theoretic analysis: what is prohibited does not occur, and what is not prohibited has occured, as far as one could see. So Gell-Mann should not be protesting, so far.
- As far as one knows, the first acceptable D=4 counterterm for maximal supergravity still occurs at L=7 loops ($\Delta = 16$).
- The current divergence expectations for maximal supergravity are consequently:

Dimension D	11	10	8	7	6	5	4
Loop order L	2	2	1	2	3	6	7
BPS degree	0	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	0	$\frac{1}{8}$
Gen. form	$\partial^{12}R^4$	$\partial^{10}R^4$	R^4	∂^4R^4	∂^6R^4	$\partial^{12}R^4$	∂^8R^4

Blue: known divergences

Green: anticipated divergences

Puzzles

- Not everything is perfect in this picture, however.
A puzzle has appeared in an unexpected sector: D=4, N=4 supergravity at L=3. The expected R^4 divergence (Δ=8) does not occur in that case. Bern, Davies, Dennen & Huang 2012
- Yet, the L=7 candidate counterterm of N=8 supergravity has a natural 1/4 BPS analogue here:

$$I^4 = \int d\mu_{(4,1,1)} \, B_{\alpha\dot{\beta}} \, B^{\alpha\dot{\beta}}$$

  - Expanding the content of this N=4 invariant at linearized level, one finds a leading R^4 structure undressed by the $SL(2,\mathbb{R})/U(1)$ complex scalar field: it is perfectly duality invariant. Bossard, Howe, K.S.S. & Vanhove 2011
  - Moreover, there is no known problem with the cocycle structure of this invariant.

- Some aspects of this N=4 case:

  Marcus 1985

  - There are anomalies in the U(1) R-symmetry. These destroy the $SL(2,\mathbb{R})$ duality symmetry. But this could only make matters worse, allowing $\int d^4x d^{16}\theta E f(\phi)$ full-superspace type invariants. But perhaps these start at amplitude point levels higher than 4, so Bern et al. would not have found them yet.

  Tourkine & Vanhove 2012

  - Genus-1 and genus-2 asymmetric-orbifold string analysis gives an explanation why R^4 divergences should not appear in analogous N=4 supergravity models coupled to $4 \le n_v \le 22$ vector multiplets. However, such matter-coupled models are already divergent at L=1, so there are subdivergence subtractions to worry about. Fischler 1979

- Perhaps what is going on is that Bern et al. are instructing us that one can achieve more off-shell supersymmetry than we realized. The N=1, D=10 supergravity theory can be formulated off-shell at the linearized level with all 16 supersymmetries manifest.

Howe, Nicolai & Van Proeyen 1982

  - Reducing this to D=4 gives N=4 supergravity coupled to 6 N=4 Maxwell multiplets.
  - However, one does not know how to formulate N=4 super-Maxwell with all 16 supersymmetries off-shell, even in harmonic superspace.
  - Maybe, however, one can realize 6 of them off-shell. This could perhaps be done in harmonic superspace.

Bets

- People may remember that Zvi Bern and I have had a series of bets on divergence onset orders, payable in bottles of wine. I have not done too well so far:

Current bets are more modest:

Classy Barolo,
paid out for
N=8, L=4 in 2009

so now:

Chateau
Le Grand Vostok
Anapa

or

Chapel Down Flint Dry
Tenterden