MASSLESS LIMIT OF NONABELIAN HIGGS MODEL.

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It is well known that the massless limit of the global $U(1)$ invariant theory of massive neutral vector field interacting with the matter fields reproduces the results of quantum electrodynamics (QED). In the limit $\mu = 0$ the three dimensionally longitudinal component of the vector field decouples of all other excitations.

In the nonabelian case the situation is different. In the limit $\mu \to 0$ the massive Yang-Mills model does not pass to the massless Yang-Mills theory (A.A.Slavnov, L.D.Faddeev, Theoretical and Mathematical Physics 03(1970)18). It is natural to suggest that the massless limit can be achieved in the Higgs model, leading to the massless Yang-Mills field interacting with the massless Higgs meson.

However there is plenty of controversy in this question. Recently in the paper (R.Ferrari, hep-th/11065537) it was stated that in the limit $\mu \to 0$ the longitudinal component of the vector field does not decouple, but undergoes the metamorphosis to the massless scalar fields, corresponding to the Goldstone bosons of the Higgs model.

In this talk I argue that the hypothesis about decoupling of the longitudinal component of the vector field in the massless limit of the Higgs model is correct. The statement of R.Ferrari corresponds to another limit: massless Yang-Mills field interacting with the complex scalar doublet with real nonzero mass, when this mass vanishes.

The free massive vector field is described by the Lagrangian

$$L = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A\mu)(\partial_\mu A_\nu - \partial_\nu A_\mu) + \frac{m^2}{2}A_\mu^2 \qquad (1)$$

To diagonalize the Hamiltonian one uses the decomposition with respect to the local reper

$$A_i^a(\mathbf{k}) = e_i^1 a_a^1 + e_i^2 a_2^a + \frac{k_i k_0}{|k|m} a_3^a; \quad A_0^a(\mathbf{k}) = \frac{|k|}{m} a_3^a \qquad (2)$$

The third and zero components are singular in the limit $m \to 0$

In the Abelian case any matrix element $< n|S|l >$ where the state $|l >$ includes at least one longitudinal quanta vanishes. Indeed

$$< n|S|l >=< n|\partial_\mu J_\mu|l' >= 0 \tag{3}$$

where $J_\mu = \frac{\delta S}{\delta A_\mu} S^+$ is the current-source of the vector field, and $|l' >$ is some state vector.

Quantization of the Abelian vector field does not spoil the conservation of the current J_μ. Therefore the longitudinal component of the vector field decouples, and in the limit $m \to 0$ one obtains QED.

In the nonabelian case the quantization spoils the conservation of the current J_μ and the arguments used in the Abelian case do not work. It was shown by R.Feynman, that in the limit $\mu \rightarrow 0$ the Yang-Mills theory the unitarity condition

$$< l|S_2 + S_2^+|l >= \sum_n < l|S_1^+|n >< n|S_1|l > \qquad (4)$$

is violated already in the second order of perturbation theory. This paradox, as was explained in the paper (S.F.) is due to the fact that in the nonabelian case the longitudinal component of the vector field does not decouple and in the limit $\mu \rightarrow 0$ gives a nonzero contribution.

In the nonabelian case the limit of the Lagrangian when $\mu \to 0$ exists, but this Lagrangian is singular, and as was shown by L.D. Faddeev and V.N.Popov and by B.De Witt, should be quantized in a special way. The Higgs-Kibble model is described by the classical Lagrangian

$$L_{ef} = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + (D_\mu \varphi)^* (D_\mu \varphi) - \kappa[\varphi^* \varphi - \mu^2]^2 \qquad (5)$$

where $\varphi(x)$ is a complex doublet, conveniently parametrized by the Hermitean components as follows

$$\varphi = (\frac{i\varphi_1 + \varphi_2}{\sqrt{2}}; \frac{\sigma - i\varphi_3}{\sqrt{2}}) \qquad (6)$$

The Higgs model is gauge invariant and may be considered in different gauges. In the unitary gauge $\varphi^a = 0$ and the Lagrangian after the shift $\sigma \to \sqrt{2}\mu$ looks as follows

$$L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{m_1^2}{2}A^a_\mu A^a_\mu + \frac{1}{2}\partial_\mu\sigma\partial_\mu\sigma - \frac{m_2^2}{2}\sigma^2$$
$$+\frac{m_1 g}{2}\sigma A_\mu^2 + \frac{g^2}{8}\sigma^2 A_\mu^2 - \frac{g m_2^2}{4m_1}\sigma^3 - \frac{g^2 m_2^2}{32 m_1^2}\sigma^4$$
$$m_1 = \frac{\mu g}{\sqrt{2}}; \quad m_2 = 2\lambda\mu \tag{7}$$

In this gauge the spectrum of the model is obvious: nine massive excitations corresponding to massive vector field A_μ^a, and one scalar field σ. Goldstone bosons φ^a and dynamical gauge fields $c, \bar{c}$ are absent.

It is known that although this gauge is not manifestly renormalizable, for any invariant regularization all nonrenormalizable divergencies present in observable gauge invariant amplitudes cancel and to calculate a renormalized scattering amplitude it is sufficient to redefine the parameters entering the Lagrangian (10): masses, charge and wave function normalization of the fields.

The limit $\mu \to 0$ in the Higgs-Kibble model in the unitary gauge suffers the same decease as in the Yang-Mills theory: the limit of the Lagrangian exists, but is singular and requires a modification of the quantization procedure. It is also necessary to look for cancelation of nonrenormalizable divergencies in observable amplitudes. To avoid these complications we shall use not unitary, but Lorentz gauge $\partial_\mu A_\mu^a = 0$. The corresponding effective Lagrangian looks as follows

$$L_{ef} = -\frac{1}{4} F_{\mu\nu}^a F_{\mu\nu}^a + (D_\mu \varphi - \hat{\mu})^* (D_\mu \varphi - \hat{\mu}) - \kappa[(\varphi^* - \hat{\mu})(\varphi - \hat{\mu}) - \frac{\mu^2}{g^2}]$$
$$+\lambda^a \partial_\mu A_\mu^a + i\bar{c}^a \partial_\mu D_\mu c^a; \quad \hat{\mu} = (0, \frac{\mu\sqrt{2}}{g}). \; (8)$$

The spectrum of the theory in this gauge includes apart from physical quanta also unphysical component of the vector field, Faddeev-Popov ghosts and Goldstone bosons φ^a.

However the Lagrangian (11) is invariant with respect to the BRST transformations

$$
\begin{aligned}
\delta A_\mu^a &= (D_\mu c)^a \\
\delta\varphi^a &= \mu c^a + \frac{g}{2}\varepsilon^{abd}\varphi^b c^d + \frac{g}{2}\sigma c^a \\
\delta\bar{c}^a &= \lambda^a \\
\delta c^a &= g\varepsilon^{abd}c^b c^d \\
\delta\sigma &= -\frac{g}{2}\varphi^a c^a \\
\delta\lambda^a &= 0.
\end{aligned}
\tag{9}
$$

By the Noether theorem this invariance generates the conserved charge Q, and physical asymptotic states are selected by the condition

$$Q^0|\varphi>_{as}= 0 \tag{10}$$

where Q^0 is the asymptotic BRST charge. For any μ different from zero the invariance of the Lagrangian (11) with respect to the transformations (12) leads to decoupling of the nine physical vector field excitations and the excitation corresponding to the scalar field σ from all unphysical exitations.

Clearly the amplitudes, describing the transition from the transversally polarized state to another transversally polarized state is finite in the limit $\mu = 0$. It is also clear that the matrix elements between states including the longitudinal components of the vector field are singular in this limit.

Nevertheless, as we shall show, in the limit $\mu \longrightarrow 0$ the longitudinal states decouple and the scattering matrix is unitary in the space including only four dimensionally transversal polarizations and the massless Higgs scalar.

Let us consider the forward scattering between states which do not include longitudinal states.By the optical theorem the imaginary part of the amplitude of the forward scattering is proportional to the total cross section of the process. That means that the sum

$$\sum_l | < n|\tilde{S}|l > |^2 \sim Im < n|\tilde{S}|n > \qquad (11)$$

where the vectors $|n >$ do not include the longitudinal polarizations, and the vectors $|l >$ span the complete space where the the scattering matrix acts.

The r.h.s. of the equation (14) has a limit when $\mu \to 0$ and this limit coincides with the imaginary part of the corresponding amplitude in the massless theory. This theory is known to be unitary in the space including only transversal polarizations of the vector field and massless Higgs meson. So we conclude that the amplitudes $< n|\tilde{S}|m >= 0$ if the vector $|m >$ contains at least one longitudinally polarized quant.

That means that as it happens in the case of neutral vector meson longitudinal polarizations of the massive Yang-Mills field decouple in the limit $\mu \to 0$, and the resulting theory describes in this limit the massless Yang-Mills field interacting with the massless Higgs bosons.

A massless Yang-Mills field may also interact with the complex scalar doublet with real masses. This theory is also BRST-invariant, but in this case the scalar fields do not contribute to the asymptotic BRST charge:

$$\hat{Q}_B^0 = \int d^3k[(a_0^+ + a_3^+)c^- + c^+(a_0^- + a_3^-)] \tag{12}$$

Any vector satisfying his equation has a form

$$|\varphi >= |\varphi >_{ph} + Q_0|\chi > \tag{13}$$

where $|\varphi >_{ph}$ contains only three dimensionally transversal components of A_μ^a and the excitations corresponding to the complex doublet of the scalar mesons.

In the limit when the mass of the scalar mesons vanishes we have precisely the particle content described in the paper by R.Ferrari. However in the previous case in the limit $\mu \longrightarrow 0$ the longitudinal quanta decouples, whereas in the present case the charged scalar bosons interact with the massless Yang-Mills field even in the limit $\mu \longrightarrow 0$.