

A sunset scene with a bright sun low on the horizon, casting a warm orange glow over a vast sea of white and grey clouds. The clouds are layered and textured, with some darker patches of land or mountains visible in the distance.

# Towards quantum theory of chiral magnetic effect

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**Ginzburg Conference 2012  
Moscow, Russia  
31/05/2012**

It is often useful to think about QFT vacuum as about  
a *medium*



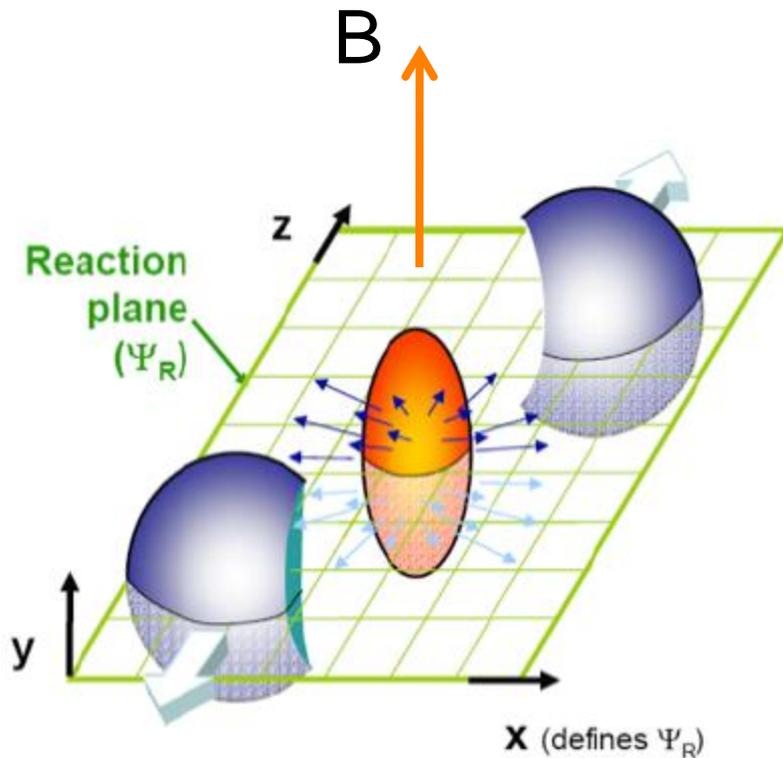
Αἶθήρ

There is a huge hierarchy of dynamical scales in Nature characterizing the vacua of different sectors of the SM (and beyond).

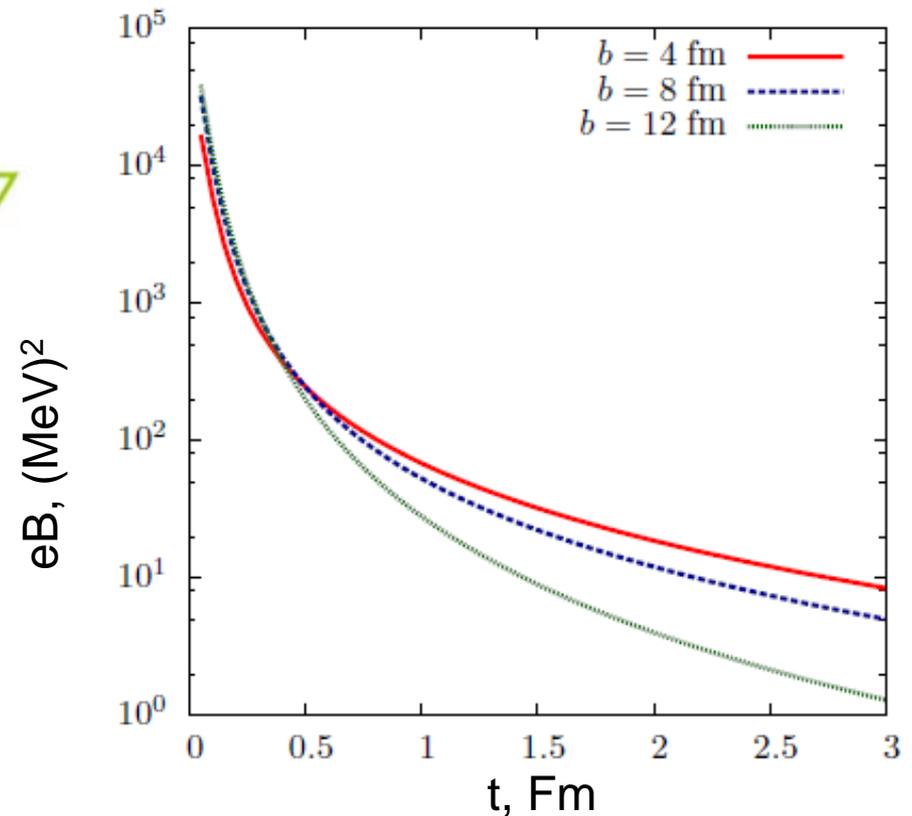
To study the properties of these (and actually of any) media one uses two main approaches:

- Send test particles
- Put external conditions (fields, temperature, density etc)

Heavy ions collision experiments provide an interesting interplay of these strategies because the matter created after the collision of electrically charged ions is hot ( $T \neq 0$ ), dense ( $\mu \neq 0$ ) and experience strong abelian fields in the collision region ( $B \neq 0$ ).



*Kharzeev, McLerran, Warringa, '07*



# Comparison of magnetic fields



The Earth's magnetic field 0.6 Gauss

A common, hand-held magnet 100 Gauss



The strongest steady magnetic fields achieved so far in the laboratory  $4.5 \times 10^5$  Gauss

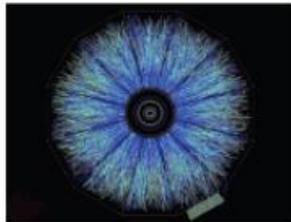
The strongest man-made fields ever achieved, if only briefly  $10^7$  Gauss



Typical surface, polar magnetic fields of radio pulsars  $10^{13}$  Gauss

Surface field of Magnetars  $10^{15}$  Gauss

<http://solomon.as.utexas.edu/~duncan/magnetar.html>



**Heavy ion collisions: the strongest magnetic field ever achieved in the laboratory**

Off central Gold-Gold Collisions at 100 GeV per nucleon

$e B(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$

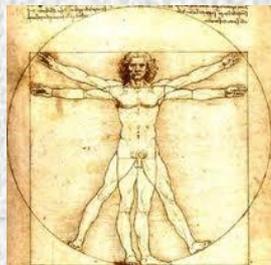
*(slide from D.Kharzeev)*

Of particular interest is a question about  
*the fate of discrete symmetries*  
under such extreme conditions

**Macro**

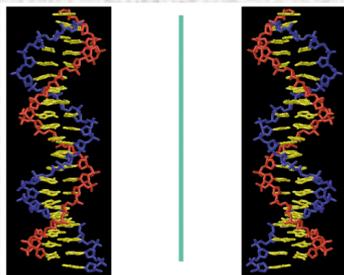
**Micro**

**C**

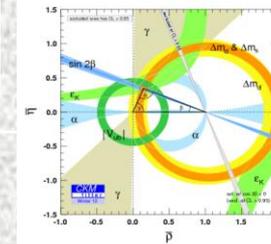
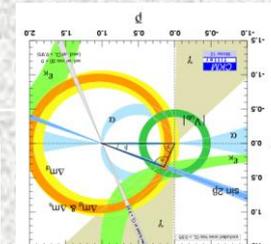
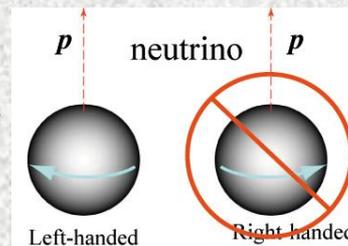


**Matter dominance**

**P**



**Chirality**



**T**

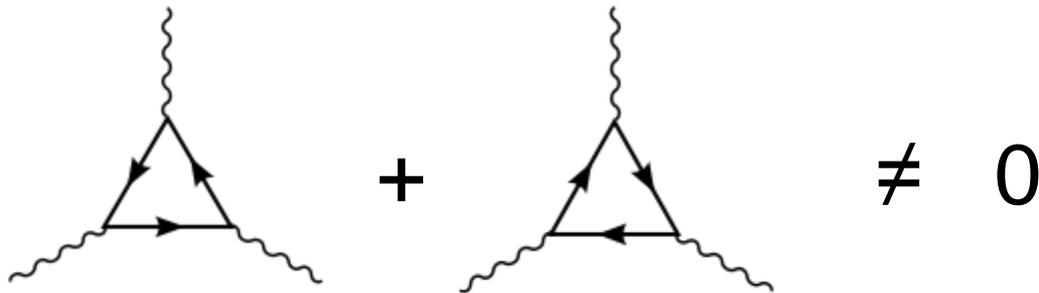


**Arrows of time**

QCD Lagrangian (without  $\theta$ -term) is invariant under **P**-, **C**- and **T**-transformations

$$\mathcal{L} = \sum_q \bar{\psi}_{q,a} (i\gamma^\mu \partial_\mu \delta_{ab} - g_s \gamma^\mu t_{ab}^C \mathcal{A}_\mu^C - m_q \delta_{ab}) \psi_{q,b} - \frac{1}{4} F_{\mu\nu}^A F^{A\mu\nu}$$

**C**-invariance holds at finite temperature but gets broken at finite density:



*no Furry theorem at  $\mu \neq 0$*

Vacuum expectation value of any local P-nonconserving observable has to vanish in vector-like theories such as QCD:

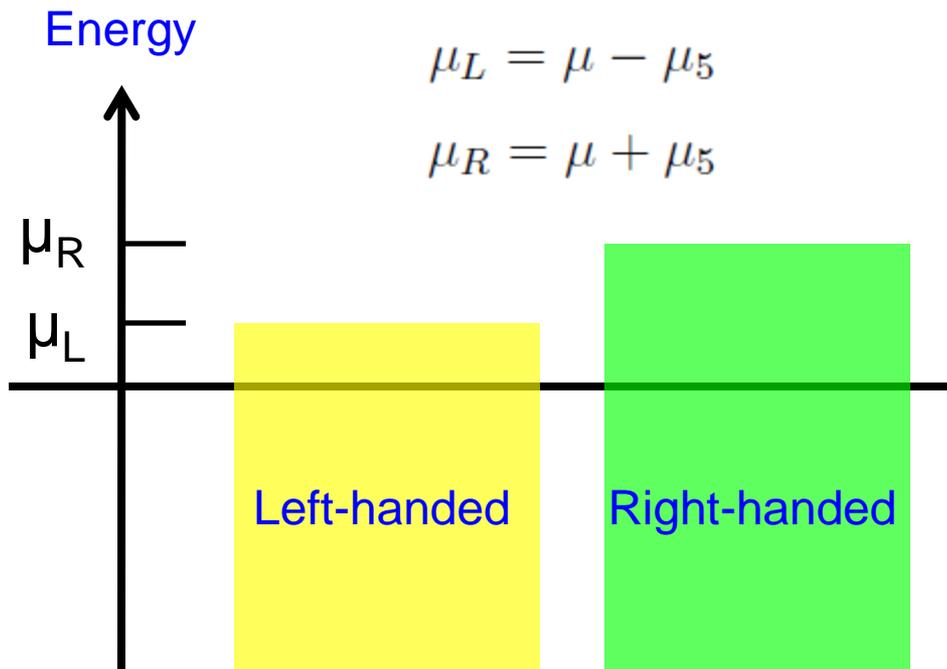
$$\langle \bar{\psi} \gamma^5 \psi \rangle = 0 \quad \langle \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu} \rangle = 0$$

*C. Vafa, E. Witten, '84*

## A seminal suggestion:

*Kharzeev, Pisarski, Tytgat, '98; Halperin, Zhitnitsky, '98;  
Kharzeev, '04; Kharzeev, McLerran, Warringa '07;  
Kharzeev, Fukushima, Warringa '08*

Possible experimental manifestations of  
**chiral magnetic effect**  
in heavy ion collisions ?



$$\mathbf{j} = \frac{e^2}{2\pi^2} \mu_5 \mathbf{B}$$

Many complementary ways to derive (Chern-Simons, linear response, triangle loop etc). At effective Lagrangian level

$$\mu_5 \sim \dot{\theta}$$

**Robust theoretical effect**

## Important questions:

1. What is CME in real QCD?

*There is no such thing as  $\mu_5$  in QCD Lagrangian*

2. What is quantum meaning of CME?

$$\langle \mathbf{j} \rangle = 0$$

3. Can CME be used to explain the observed charge asymmetries?

*There might be alternative explanations*

*(see, e.g. [Asakawa, Majumder, Müller '10](#))*

## P-parity violation probed by local order parameters

In QFT one usually has  $\langle \Omega | \mathcal{O}(x) | \Omega \rangle \propto c \cdot \Lambda^{d_{\mathcal{O}}}$

1. Macroscopic quantities out of microscopic ones  
*"VIII LL volume out of II LL volume"*  $\odot |\Omega\rangle = |\Phi\rangle \otimes |\phi\rangle$

$$j_{\mu}(x) = \bar{\psi}\gamma_{\mu}\psi(x) \leftrightarrow J_{\mu} \propto \langle \Phi | \int dx \rho_V(x) j_{\mu}(x) | \Phi \rangle$$

2. The medium: if at rest  $u^{\nu} = T \cdot (1, 0, 0, 0)$

3. The medium with applied external field:

$$\langle \phi | J_{\mu} | \phi \rangle \propto u^{\nu} F_{\mu\nu}$$

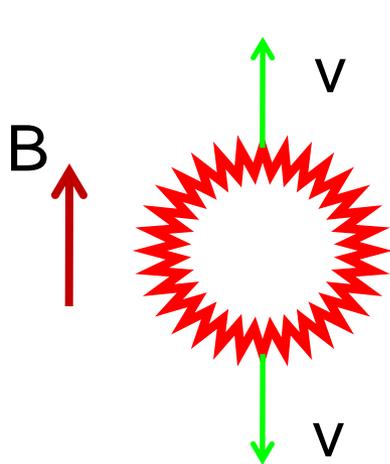
Local parity violation:  $P\mathcal{O}P^\dagger = -\mathcal{O}(x)$

and  $\langle \Omega | \mathcal{O}(x) | \Omega \rangle \neq 0$  e.g.  $\langle \Omega | \psi^\dagger \gamma^5 \psi | \Omega \rangle \neq 0$

In a medium with applied external field one can have

$$\langle \phi | J_\mu \partial J^5 | \phi \rangle \propto u^\nu \tilde{F}_{\mu\nu}$$

P-odd correlation between charge density and divergence of axial current in *moving medium in magnetic field*:



$$\langle \phi | J_0 \partial J^5 | \phi \rangle \propto \mathbf{v} \cdot \mathbf{B}$$

If  $\partial J^5$  is uniform in the volume of the fireball, charge density changes the sign crossing the reaction plane

# P-parity violation induced by measurement

Simple example:  $V(x) = V(-x)$

$$\langle x \rangle = \int x dx |\psi_0(x, t)|^2 = 0$$

$$\langle x \rangle = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^N x_i = 0$$

$$x_i \neq 0 \quad \leftarrow$$

Event-by-event P-parity violation?

In QM individual outcome (“event”) has no meaning

$$\langle x^2 \rangle = \int x^2 dx |\psi_0(x, t)|^2 \neq 0$$

Lets us take  $V(q) = V(-q)$  for  $q = (x, y, z)$  but not invariant under reflections of separate coordinates.

As soon as one starts to monitor P-odd observable, e.g.

$$\int \mathcal{D}y(t) \rightarrow \int \mathcal{D}y(t) \exp \left( -\kappa \int_0^T (y(t) - \bar{y}(t))^2 dt \right)$$

the amplitude takes the form:

$$U(q''; q') = \int \mathcal{D}p \int \mathcal{D}q \exp \left( \frac{i}{\hbar} \int_0^T (p\dot{q} - H(p, q)) dt - \kappa \int_0^T (y(t) - \bar{y}(t))^2 dt \right)$$

where the error bar is given by  $\Delta y \propto (\kappa T)^{-1/2}$

$$\langle x(T) \rangle = X[\bar{y}(t), \kappa] \neq 0$$

With the measuring device turned off  $X[\bar{y}(t), 0] = 0$

Just one more example – nonlinear electrodynamics:

$$L(\mathcal{F}, \mathcal{G}) = -\mathcal{F} + \sum_{n,m=1} a_{nm} \cdot \mathcal{F}^{n+1} \mathcal{G}^{2m}$$

$$\mathcal{F} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \qquad \mathcal{G} = \frac{1}{4} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

The simplest model for slow T-noninvariant measurement:

$$L = -\mathcal{F} + \frac{1}{2} \xi(t) \mathcal{G}^2$$

Then the equations of motion read

$$\partial_{\mu} \left( F^{\mu\nu} + \xi(t) \cdot \mathcal{G} \cdot \tilde{F}^{\mu\nu} \right) = j^{\nu}$$

For stationary fields:  $\text{rot } \mathbf{B} = \mathbf{j} + \mathbf{j}_{\chi}$  with the chiral current:

$$\mathbf{j}_{\chi} = \dot{\xi}(t) (\mathbf{E} \mathbf{B}) \mathbf{B}$$

Amplitude to click for pseudoscalar Unruh – DeWitt detector coupled to topological charge density:

$$\mathcal{A} \propto i \int d\tau \langle 1 | \mu(\tau) | 0 \rangle \cdot \langle \Psi^- | G_{\mu\nu} \tilde{G}^{\mu\nu}(x(\tau)) | \Psi^+ \rangle$$

Total probability proportional to the Wightman function:

$$F(E) = \int d\tau \int d\tau' e^{-iE(\tau-\tau')} \langle \Psi^+ | G_{\mu\nu} \tilde{G}^{\mu\nu}(x(\tau)) G_{\mu\nu} \tilde{G}^{\mu\nu}(x(\tau')) | \Psi^+ \rangle$$

vanishes for inertial observer in the vacuum  $|\Psi^+\rangle = |0\rangle$

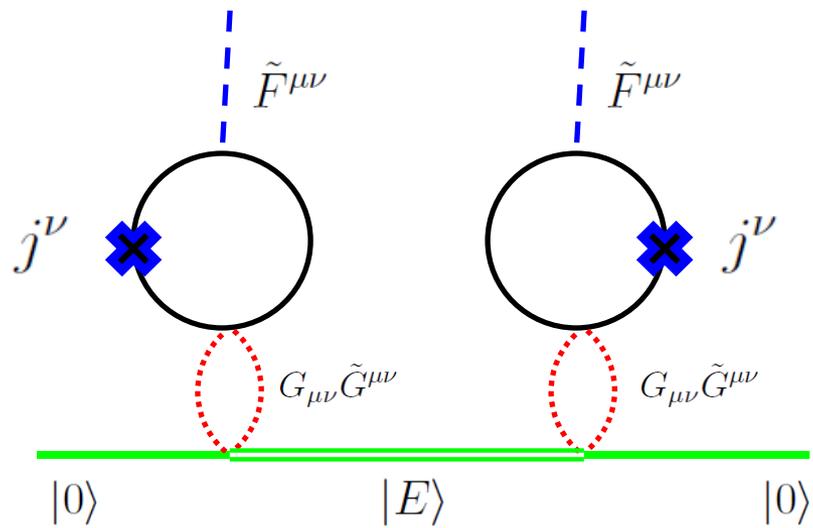
It **does not**, however, if external electromagnetic field is present:

$$|\Psi^+\rangle = e^{i \int dx j_\mu(x) A^\mu(x)} |0\rangle$$

due to the fact that

$$\langle 0 | j_\alpha(u) j_\beta(v) G_{\mu\nu} \tilde{G}^{\mu\nu}(x) | 0 \rangle \neq 0$$

$$P \propto \sum_E$$



Using the language of decoherence functionals and filter functions

$$\Psi[\alpha] = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu \alpha[\bar{\psi}, \psi, A] e^{iS_{QCD}}$$

one can define distribution amplitude for the current and some P-odd quantity

$$\alpha_\eta[\bar{\psi}, \psi] = \int \mathcal{D}\lambda f_\eta[\lambda] e^{i \int dx \lambda(x) (\eta(x) - \partial j_r^5(x))}$$

$$\alpha_J[\bar{\psi}, \psi] = \int \mathcal{D}\kappa f_J[\kappa] e^{i \int dx \kappa(x) (J(x) - n_\mu j_r^\mu(x))}$$

$$\Psi[\alpha_\eta, \alpha_J] = \int \mathcal{D}\lambda f_\eta[\lambda] e^{i \int dp \lambda(p) \eta(-p)} \int \mathcal{D}\kappa f_J[\kappa] e^{i \int dp \kappa(p) J(-p)} \cdot e^{-i \int dp \Sigma_2(p)}$$

In Gaussian approximation (i.e. keeping only at most quadratic terms) one can get:

$$\Psi[\alpha_\eta, \alpha_J] = \int \mathcal{D}\lambda f_\eta[\lambda] e^{i \int dp \lambda(p) \eta(-p)} \int \mathcal{D}\kappa f_J[\kappa] e^{i \int dp \kappa(p) J(-p)} \cdot e^{-i \int dp \Sigma_2(p)}$$

$$\begin{aligned} \Sigma_2(p) = & \frac{1}{2} \lambda(p) \Pi_r^5(p) \lambda(-p) + \frac{1}{2} \kappa(p) n_\mu \Pi_r^{\mu\nu}(p) n_\nu \kappa(-p) + \\ & + \text{Tr}[Q^2] \frac{e^2}{4\pi^2} \lambda(p) \kappa(-p) p_\alpha n_\mu \tilde{F}^{\alpha\mu} \end{aligned}$$

where the anomaly equation has been used

$$\int dx_1 dx_2 e^{i(q_1 x_1 + q_2 x_2)} \langle T \{ j_\mu(x_1) j_\nu(x_2) \partial j^5(0) \} \rangle = -\text{Tr}[Q^2] \frac{e^2}{4\pi^2} \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta$$

Euclidean current:

$$\int d\eta J_\mu(x, \eta) = \int d\eta \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_\mu j_\mu(x) \tilde{\delta}(\eta - \int_V dy \partial j^5(y)) e^{-S_{QCD}}$$

is defined with the Gaussian detector function

$$\tilde{\delta}(\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\lambda \exp(-\lambda^2 l^2 / 2 + i\lambda\eta)$$

Taking into account that for singlet current

$$\partial j^5 = -\text{Tr} [Q^2] \frac{N_c}{12\pi^2} F \tilde{F} - \frac{N_f}{16\pi^2} \text{Tr} G \tilde{G}$$

one gets in Gaussian approximation

$$J_\mu(x, \eta) = -\text{Tr} [Q^2] \frac{N_c}{12\pi^2} \frac{\eta e^{-\eta^2/2L^2}}{\sqrt{2\pi L^6}} \cdot \left[ \int \frac{d^4 q}{(2\pi)^4} e^{iqx} f_V(q) i q^\nu \right] \cdot \tilde{F}_{\mu\nu}$$

where  $L^2 = l^2 + \langle n_V^2 \rangle$  ;  $\chi = \lim_{V \rightarrow \infty} \langle n_V^2 \rangle / V N_f^2$

$$n_V = \int_V d^4 y \partial j^5 = -\frac{N_f}{16\pi^2} \int_V d^4 y \text{Tr} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

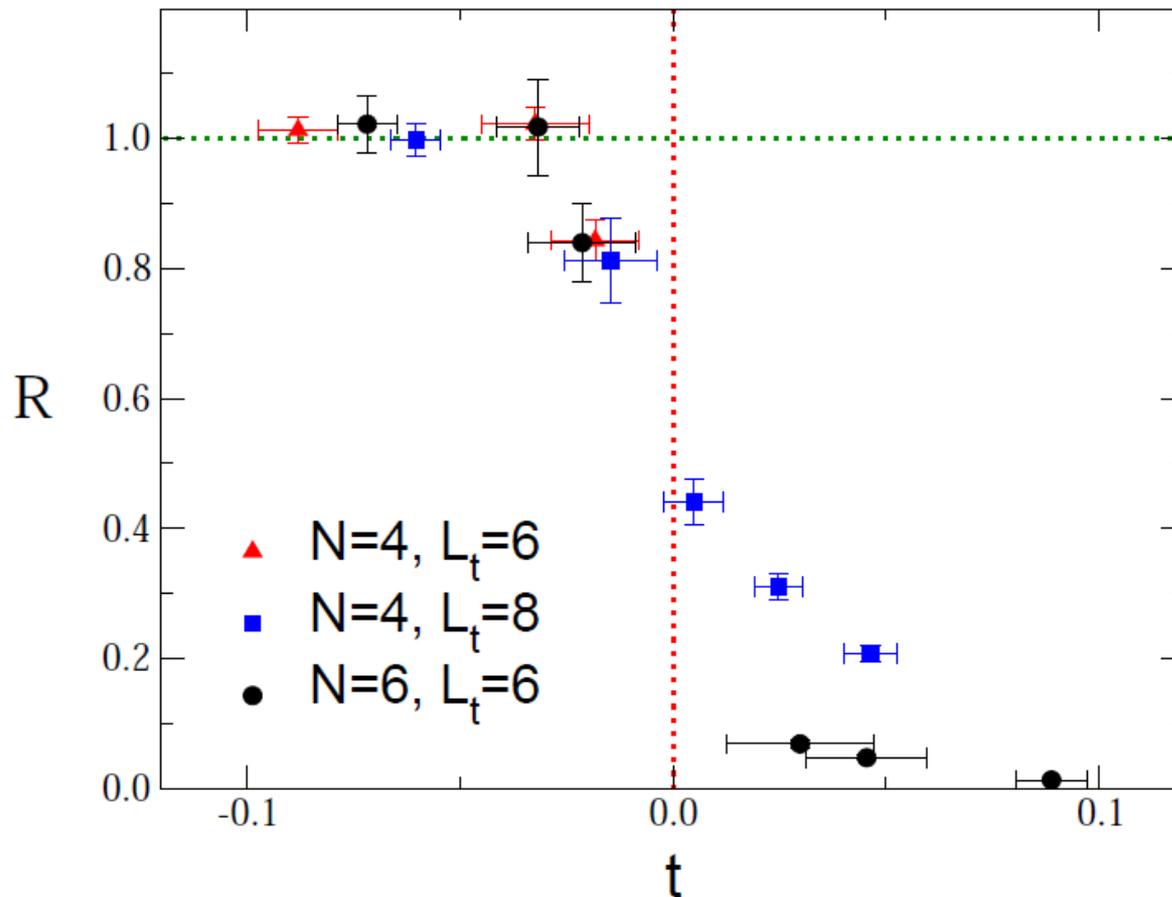
Maximal current is reached by  $J^{max}(x, \eta \sim L) \propto \frac{B}{\tau L^2}$

and flows only inside the volume  $V$

Subtle interplay of abelian and nonabelian anomalies.

Topological susceptibility sharply drops at  $T > T_c$

*Del Debbio, Panagopoulos, Vicari, '04*



## P-odd × P-odd contributions to P-even observables

The object of main interest is polarization operator

$$\Pi_{\mu\nu}^{(M)}(x, x') = i \langle T \{ j_\mu(x) j_\nu(x') \} \rangle_{F,T}$$

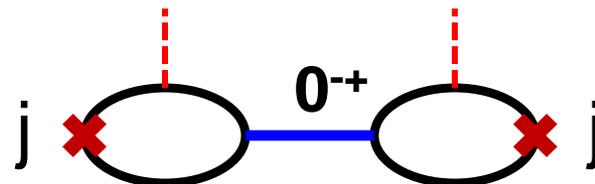
since it encodes information about  $\langle Q_V^2 \rangle$  where

$$Q_V = \int_V d\mathbf{x} j_0(x)$$

Confinement phase, large distances, weak fields

$$\langle j(x) j(x') \rangle \propto e^{-m_\rho |x-x'|} + C(B) \cdot e^{-m_\pi |x-x'|}$$

Clear similarity with vector-axial mixing at finite T  
(*Dey, Eletsky, Ioffe, '90*)



We are mostly interested in contributions from the states of negative parity

$$\langle \Omega | j_i j_k | \Omega \rangle \rightarrow \sum_A \langle \Omega | j_i | A \rangle \langle A | j_k | \Omega \rangle$$

Instead of addressing spatial current components, we consider Euclidean fluctuation pattern for electric charge:

$$\langle Q_V^2 \rangle_{st} = -T \int \frac{d\mathbf{q}}{(2\pi)^3} |F_V(\mathbf{q})|^2 \Pi_{44}(\mathbf{q}, 0)$$

where  $F_V(\mathbf{q})$  is spatial form-factor, and only the lowest Matsubara frequency contributes in the static limit

It is worth reminding that quantum fluctuations is a finite volume effect (UV-protected by gauge invariance):

$$\langle Q_V^2 \rangle_{B=0, T=0} \propto \Pi'(0) \cdot V_4^{-1/2}$$

To make contact with the observable distribution

$$\frac{dN_{\pm}}{d\phi} \propto 1 + 2v_{1,\pm} \cos \phi + 2v_{2,\pm} \cos 2\phi + 2a_{\pm} \sin \phi + \dots$$

we consider  $\left\langle \int \frac{d(N_+ - N_-)}{d\phi} d\phi \int \frac{d(N'_+ - N'_-)}{d\phi'} d\phi' \right\rangle_e$

and expand  $\langle Q_V^2 \rangle$  in harmonics:  $F_V(\mathbf{q}) = \int dx_1 e^{iq_1 x_1} \int_0^{\rho} \rho d\rho \int_0^{2\pi} d\phi e^{i\bar{q}\rho}$

$$\langle Q_V^2 \rangle = \dots + \int_0^{2\pi} d\phi \sin \phi \int_0^{2\pi} d\phi' \sin \phi' \langle (q_V^a)^2 \rangle + \dots$$

The result for the asymmetry  $N^2 \cdot (\langle (a_+ - a_-)^2 \rangle_e - \langle (v_{1,+} - v_{1,-})^2 \rangle_e)$

$$\langle q_V^2 \rangle = \langle (q_V^a)^2 \rangle - \langle (q_V^{v_1})^2 \rangle = - \sum_{\alpha, \beta = \pm} \alpha \beta \cos(\phi_{\alpha} + \phi_{\beta}') \quad \text{reads:}$$

$$\langle q_V^2 \rangle = T \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{q_3^2 - q_2^2}{q_3^2 + q_2^2} \left| \int dx_1 e^{iq_1 x_1} \int_0^{\rho} \rho J_1(\hat{q}\rho) d\rho \right|^2 \Pi_{44}(\mathbf{q}, 0)$$

Tensor structure of polarization operator is fixed by current conservation, Bose symmetry and generalized Furry's theorem (thus we neglect finite density effects)

$$\Pi_{\mu\nu}(q, u, F) = \Pi_{\mu\nu}(q, -u, -F) \quad q^\mu \Pi_{\mu\nu}(q) = q^\nu \Pi_{\mu\nu}(q) = 0$$

$$\Pi_{\mu\nu}(q) = \Pi_{\nu\mu}(-q) \quad \Pi_{\mu\nu}(q, u, F) = \sum_{i=1}^6 \pi^{(i)} \cdot \Psi_{\mu\nu}^{(i)}$$

*Baier, Katkov, '75*

*Perez Rojas,*

*Shabad, '79*

$$\Psi_{\mu\nu}^{(1)} = q^2 \delta_{\mu\nu} - q_\mu q_\nu$$

$$\Psi_{\mu\nu}^{(2)} = (q^2 u_\mu - q_\mu (uq))(q^2 u_\nu - q_\nu (uq))$$

$$\Psi_{\mu\nu}^{(3)} = (uq)(q_\mu F_{\nu\rho} q^\rho - q_\nu F_{\mu\rho} q^\rho + q^2 F_{\mu\nu})$$

$$\Psi_{\mu\nu}^{(4)} = (u_\mu F_{\nu\rho} q^\rho - u_\nu F_{\mu\rho} q^\rho + (uq) F_{\mu\nu})$$

$$\Psi_{\mu\nu}^{(5)} = F_{\mu\rho} q^\rho F_{\nu\sigma} q^\sigma$$

$$\Psi_{\mu\nu}^{(6)} = (q^2 \delta_{\mu\rho} - q_\mu q_\rho) F_\alpha^\rho F^{\alpha\sigma} (q^2 \delta_{\sigma\nu} - q_\sigma q_\nu)$$

Of special interest for us is a tensor  $\Psi_{\mu\nu}^{(7)} = \tilde{F}_{\mu\rho} q^\rho \tilde{F}_{\nu\sigma} q^\sigma$

$$q^2 \Psi_{\mu\nu}^{(7)} = (q^2 F^2 / 2 - (qF)^2) \Psi_{\mu\nu}^{(1)} + q^2 \Psi_{\mu\nu}^{(5)} + \Psi_{\mu\nu}^{(6)}$$

With conventional notation  $q_\perp = (q_1, q_2)$ ,  $q_\parallel = (q_3, q_4)$

$$\Pi_{\parallel}(q) = \pi^{(Q)} \cdot \Psi_{\parallel}^{(1)} + \pi^{(T)} \cdot \Psi_{\parallel}^{(2)} + \tilde{\pi}^{(F)} \cdot \Psi_{\parallel}^{(7)}$$

↑                      ↑                      ↑

“quantum”                      “thermal”                      “j ∥ B”

↓                      ↓                      ↓

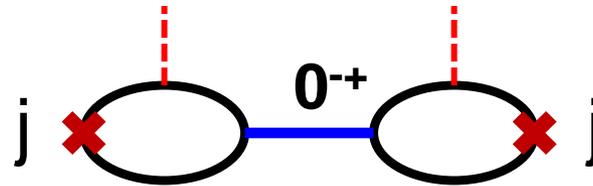
“j ⊥ B”

$$\Pi_{\perp}(q) = \pi^{(Q)} \cdot \Psi_{\perp}^{(1)} + \pi^{(T)} \cdot \Psi_{\perp}^{(2)} + \pi^{(F)} \cdot \Psi_{\perp}^{(5)}$$

$$\delta_B \langle j_3 j_3 \rangle = \tilde{\pi}^{(F)} \times \tilde{F}_{3\rho} p^\rho \times \tilde{F}_{3\sigma} p^\sigma$$

P-even = P-even × axial × axial

## Model example № 1: weak fields, confinement phase



$$\int dx \int dy e^{iq_1 x + iq_2 y} \langle 0 | \text{Tr} \{ j_\mu(x) j_\nu(y) \} | \pi^0(q) \rangle = \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_\pi(q^2, q_1^2, q_2^2)$$



$$\langle 0 | j_\mu(-q) | \pi^0(q) \rangle_F = i e q^\rho \tilde{F}_{\rho\mu} \mathcal{F}_\pi(q^2, q^2, 0)$$

On-shell the formfactor is fixed by the anomaly:  $\mathcal{F}_\pi(m_\pi^2, 0, 0) = -\frac{N_c}{12\pi^2 F_\pi}$

$$\tilde{\pi}^{(F)}(q^2) = \sum_{\phi=\pi,\eta,\eta'} \frac{|\mathcal{F}_\phi(q^2, q^2, 0)|^2}{q^2 - m_\phi^2}$$

$$\langle q_V^2 \rangle = \gamma \left( \frac{eB}{F_\pi} \right)^2 T R^3 \quad \gamma = 1.6 \cdot 10^{-4} \quad \text{for Gbc}$$

## Model example № 2: free fermions in magnetic fields

Since polarization operator is computed for  $B, T \neq 0$

(Alexandre, '00) explicitly, one can extract  $\tilde{\pi}^{(F)}$

$$\tilde{\pi}^{(F)} = -\frac{1}{(4\pi)^2} \frac{1}{eB} \int_{\epsilon}^{\infty} du \int_{-1}^{+1} dv \left( (1-v^2) \coth \bar{u} + f_{\perp}(\bar{u}, v) \right) \exp(-\phi^{(0)})$$

This formfactor becomes dominant at low  $T$ , large  $B$

$$\Pi_{44} \rightarrow q_3^2 (eB)^2 \tilde{\pi}^{(F)} \rightarrow -\frac{eB}{4\pi^2} e^{-\frac{q_{\perp}^2}{2|eB|}} \int_{-1}^1 dv \frac{(1-v^2)q_3^2}{4m^2 + (1-v^2)q_3^2}$$

Increase with  $B$  versus  $q$ -dependence suppression

*competing*  *effects!*

Saturation:  $\langle q_V^2 \rangle = \gamma' \cdot RT$  where  $\gamma' = 4.1 \cdot 10^{-2}$  for Gbc

# Conclusions

- Chiral magnetic effect is a robust theoretical result with many possible applications in high energy and condensed matter physics
- To construct a consistent quantum theory of CME one needs to understand an intricate relation between deconfinement and decoherence
- Future studies of heavy ion collisions (RHIC@BNL, ALICE@CERN, FAIR@GSI, NICA@JINR) have great discovery potential and will surely deepen our understanding of strong-electromagnetic interplay

Backup

# LHC experiments: testing symmetries



**General purpose - everything with high enough  $p_T$**

Electroweak gauge symmetry breaking pattern: Higgs boson or New Physics?

Space-time symmetries: extra dimensions, black holes, KK-states?

Supersymmetry: particles – superpartners? Dark matter?



**Enigma of flavor**

**CP**-symmetry violation: new sources?  
Baryon asymmetry of the Universe.  
Indirect search of superpartners.



**New state of matter**  
**- new symmetries?**

Chiral symmetry of strong interactions:  
pattern of restoration? Deconfinement.  
**P**-parity violation as interplay of strong  
and electromagnetic interactions?

M.Giovannini, M.Shaposhnikov, '97

M.Giovannini, M.Shaposhnikov, '97

*Kharzeev, Pisarski, Tytgat, '98;*

*Halperin, Zhitnitsky, '98;*

The condition of (non)locality is important.

Elementary local operators and their products – locality operationally means  $a \sim \Lambda^{-1}$  (there is no problem to have nonlocal  $P$ -odd matrix elements)

$$\chi(0) = \int d^4x \langle G\tilde{G}(x)G\tilde{G}(0) \rangle \propto F^2 m_\eta^2, \quad \int_V d^4x \langle G\tilde{G}(x) \rangle = 0$$

The medium  $\rightarrow$  finite (de)coherence length  $\rightarrow$  real time Minkowskii dynamics of fluctuations

There is an old deep question behind the above analysis: if the (microscopic) current non-conservation is anomalous - how is this fact encoded in equations for macroscopic, effective currents?

*See recent turn in Son, Surowka, '10*

$$\int \mathcal{D}A_\mu \rightarrow \int \mathcal{D}A_\mu \delta \left( n^\mu(x) \tilde{F}_{\mu\nu}(x) \right)$$

$$L = \dot{q}^2 - V(q)$$

$$\langle O_- \rangle_d = \text{Tr } \rho_d \cdot O_- = 0$$

$$|\psi_+\rangle = P |\psi_+\rangle \quad ; \quad |\psi_-\rangle = -P |\psi_-\rangle$$

$$|\psi\rangle = c_+ |\psi_+\rangle + c_- |\psi_-\rangle$$

$$\langle O_- \rangle = \langle \psi | O_- | \psi \rangle = 2\text{Re} \left\{ c_+^* c_- \langle \psi_+ | O_- | \psi_- \rangle \right\}$$

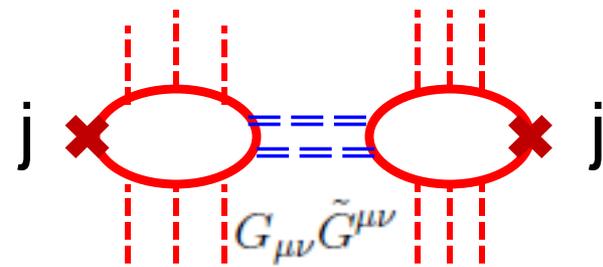
$$\rho \rightarrow \rho_d = |c_+|^2 |\psi_+\rangle \langle \psi_+| + |c_-|^2 |\psi_-\rangle \langle \psi_-|$$

Deconfinement phase, strong fields:

If Larmor radius is much smaller than  $\Lambda_{\text{QCD}}$  no quark degrees of freedom can propagate in transverse direction

$$\langle j(x)j(x') \rangle \propto e^{-eB(x-x')_{\perp}^2/2}$$

Thus correlations in transverse plane are entirely due to gluon degrees of freedom



Current correlations have been studied on the lattice  
(*Buividovich, Chernodub, Luschevskaya, Polikarpov; '09*)  
and analytically

(*Fukushima, Kharzeev, Warringa; '09*)