Critical Chaos

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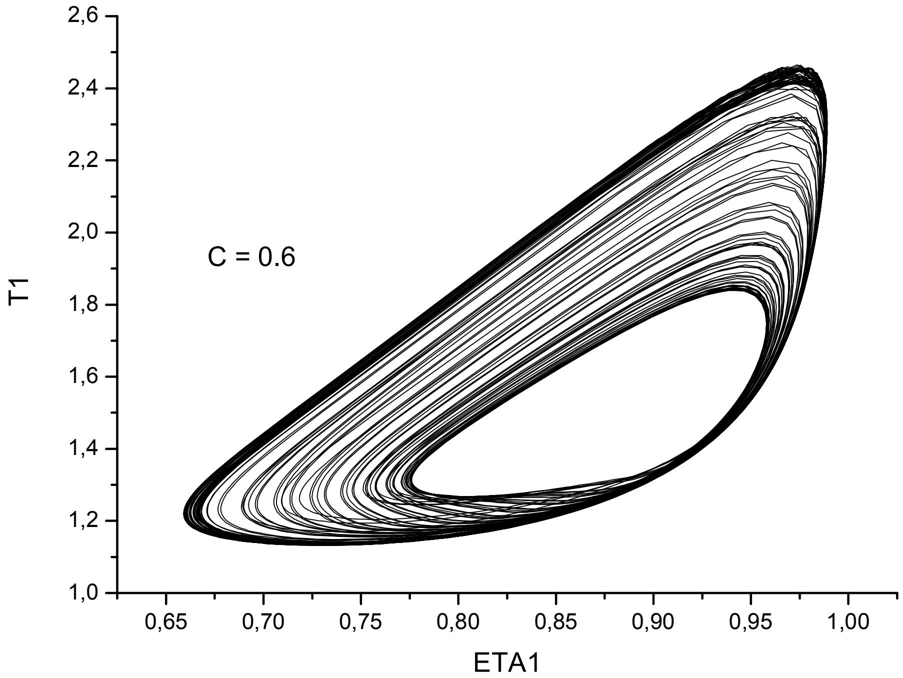
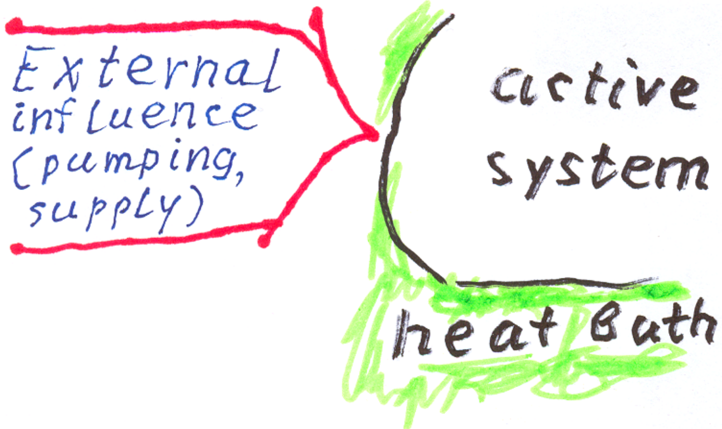
There are two species of chaos, critical and dynamical. Trajectories of dynamical chaos are locally smooth, while for the critical kind they are of Brownian type. Critical chaos exists near bifurcations where unavoidable noise is amplified. The pulsating spectrum is independent of initial noise, soft modes being prevail. Onset of soft modes betokens catastrophe.

The following example is illustrating. Analytical results show that in the Edward Lorenz set with parameters close to the values of $\beta = 8/3$, $\sigma = 10.2$, $r = 30.2$ (chaos boundary) stable limit cycles should exist. However, the cycles were not obtained by numerical simulation. Moreover, there always exists such a parameter region where, according to a rigorous theory, stable cycles can occur, and yet they escape detection by numerical means, whatever the accuracy of numerical analysis.

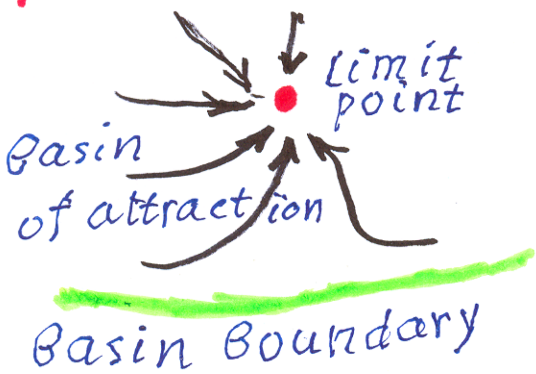
Such a failure is due to the noise. Any weak noise becomes strong as parameters approach the boundary, i.e. bifurcation.



C = 0.6
T1
ETA1
2,6
2,4
2,2
2,0
1,8
1,6
1,4
1,2
1,0
0,65
0,70
0,75
0,80
0,85
0,90
0,95
1,00



space of states



Bifurcation:

distance between the limit point and basin boundary decreases

↓

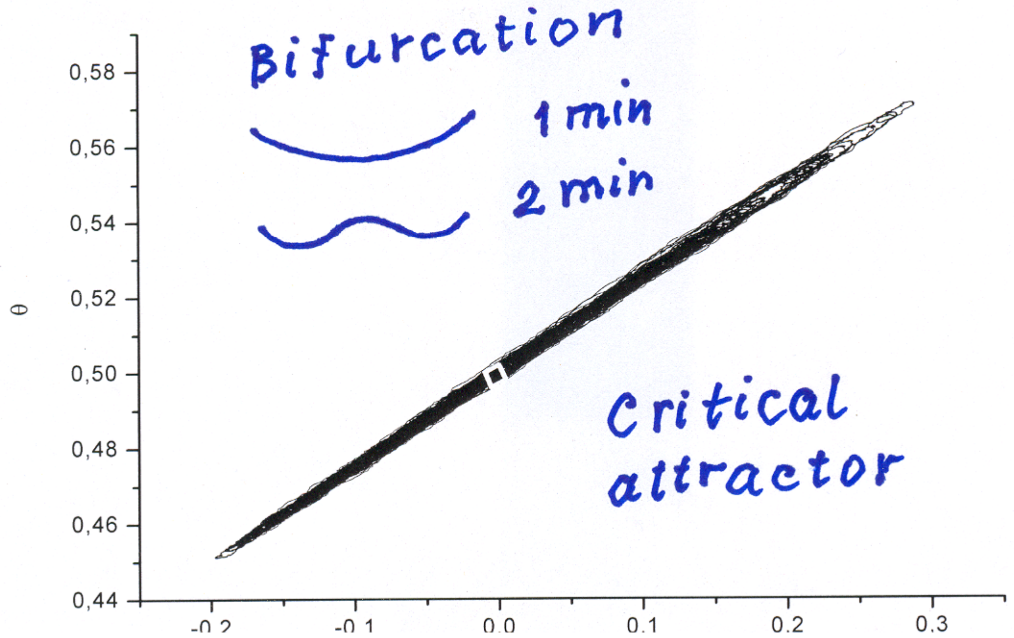
restoring force for deviations decreases

↓

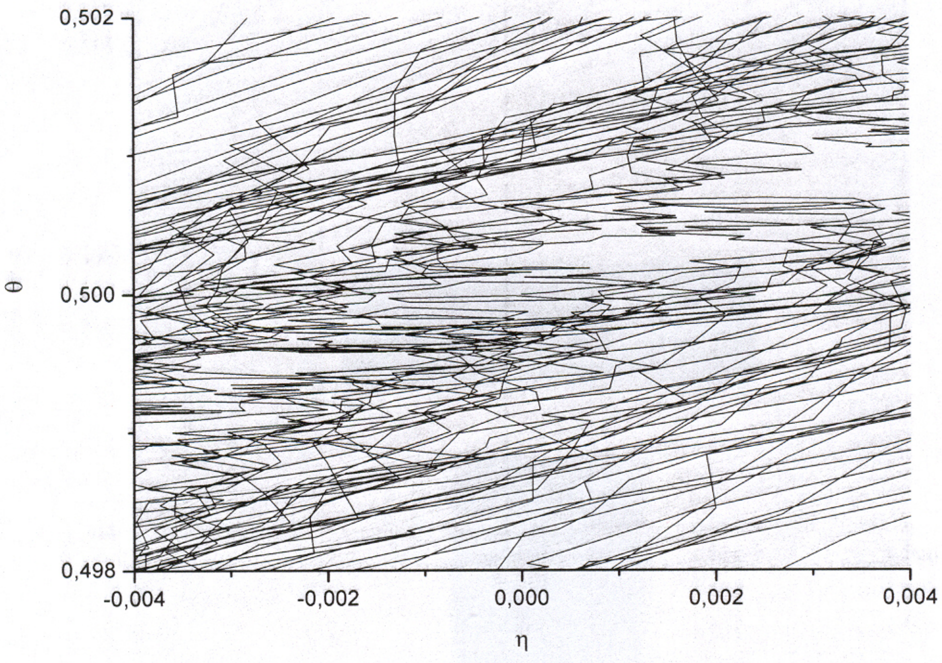
weak noise becomes strong

↓

Langevin equations


Bifurcation
1 min
2 min
Critical attractor
θ
0,58
0,56
0,54
0,52
0,50
0,48
0,46
0,44
-0.2
-0.1
0.0
0.1
0.2
0.3

Brownian trajectory



В случае одной переменной уравнение для отклонений

$$dx/dt = -\lambda x + y(t).$$

$y(t)$

-- источник отклонений.

При начальном условии

$x(t \to -\infty) = 0,$

имеем

$$x(t) = e^{-\lambda t} \int_{-\infty}^{t} y(t')e^{\lambda t'} dt',$$

компонента Фурье

$$x_\omega = \frac{y_\omega}{\lambda - i\omega},$$

так что вещественная и мнимая части восприимчивости

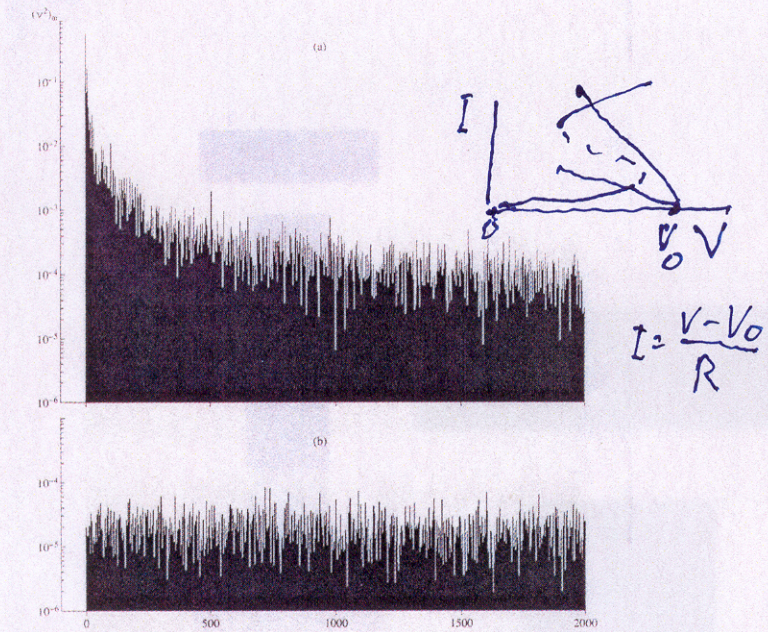
$$\mathrm{Re}\,\sigma = \frac{\lambda}{\lambda^2 - \omega^2}, \qquad \mathrm{Im}\,\sigma = \frac{\omega}{\lambda^2 - \omega^2}.$$

Около бифуркации,

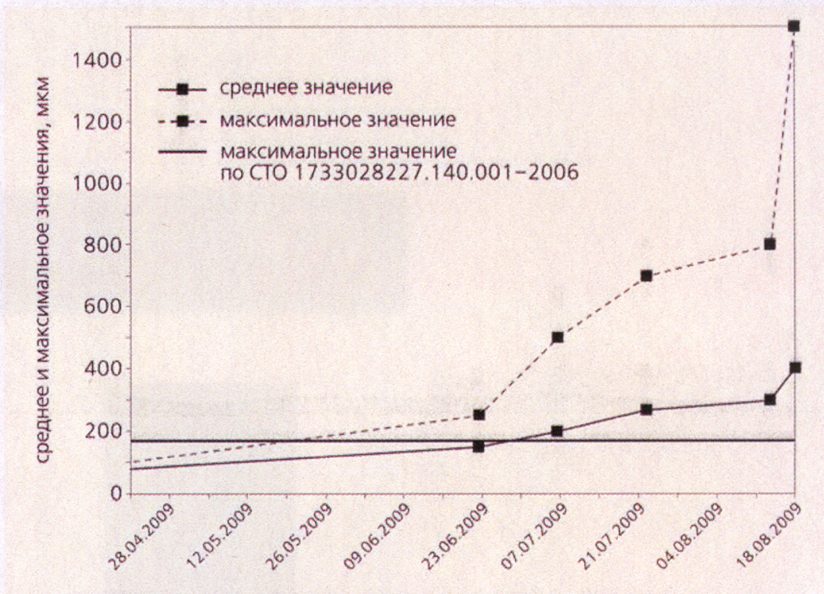
$\lambda \to 0,$

они расходятся на низких частотах,

$\omega < \lambda.$



$$I = \frac{V - V_0}{R}$$



среднее значение
максимальное значение
максимальное значение
по СТО 1733028227.140.001–2006
среднее и максимальное значения, мкм
0
200
400
600
800
1000
1200
1400
28.04.2009
12.05.2009
26.05.2009
09.06.2009
23.06.2009
07.07.2009
21.07.2009
04.08.2009
18.08.2009