ИОФАН

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Minkowski's Tensor
or Abraham's Tensor?

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Abstract

It is proved that the question in the articles title formulated by V.L. Ginsburg has an unambiguous answer, i.e. Abraham’s tensor. This statement is based on the approach developed by V.L. Ginsburg and V.A. Ugarov [1], UFN**118,** p 175 (1976).

The approach proposed in [1] is as follows.

(i) "The energy - momentum tensor in macroscopic electrodynamics is in a sense an auxiliary quantity. The fundamental quantities are the volume forces, energy density, and energy flux. Exactly, forces enter in the equations of medium motion and can be measured".

(ii) "It is desirable to uniformly obtain both forces and other expressions (the energy density, energy flux, and momentum density) based on the field equations".

(iii) "When discussing the energy conservation law, it is natural to turn to moving media, since the force acting on medium "works" only if the medium velocity is nonzero". Hence, even if the goal is determination of the force acting on matter at rest, nevertheless, it is necessary to consider a moving medium and in final results go to the limit v=0.

According to [1], "conservation laws cannot explicitly define entering quantities"; there are two possibilities: the force and field momentum are determined either by the Abraham formulas

$$\vec{f} = \vec{f}^{(G)} + \vec{f}^{(A)}, \quad \vec{g} = \frac{1}{4\pi c}[\vec{E} \times \vec{H}],$$

or, by the Minkowski formulas

$$\vec{f} = \vec{f}^{(G)}, \quad \vec{g} = \frac{1}{4\pi c}[\vec{D} \times \vec{B}]$$

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- $$f^{(G)} = \frac{1}{8\pi}[\nabla(\rho\frac{\partial\varepsilon}{\partial\rho}E^2 + \rho\frac{\partial\mu}{\partial\rho}H^2) -$$
$$-(E^2\nabla\varepsilon + H^2\nabla\mu)]$$
- $$f^{(A)} = \frac{\varepsilon\mu - 1}{4\pi c}\frac{\partial}{\partial t}[\vec{E}\times\vec{H}]$$

2. Implicitness of Abraham's tensor. We cannot agree with this statement. In our opinion, the approach proposed and us

Implicitness of Abraham's tensor. We cannot agree with this statement. Ehe approach proposed in [1] is an explicit approach in macroscopic electrodynamics. Therefore, we repeat the calculations of all major quantities made in [1] with the exception that we will not put on ε, μ ε, μ

and the medium velocity v any constraints (only v << *c*). We think that most important restriction in [1] which did not allow choosing the correct tensor was the assumption that the medium velocity must be constant in time. But if we rule out the constraints to only stationary motion of medium, all quantities are defined consistently and unambiguously. For a stationary medium, the energy density, energy flux density, momentum density, force density, and stress tensor are determined by the formulas

$$W = \frac{1}{8\pi}(\varepsilon E^2 + \mu H^2), \qquad \vec{S} = \vec{S}^{(\vec{P})},$$

$$\vec{g} = \frac{1}{c^2}\vec{S}^{(\vec{P})}, \qquad f = f^{(G)} + f^{(A)},$$

$$\sigma_{ij} = \sigma_{ij} = \frac{1}{4\pi}[\varepsilon E_i E_j - \frac{1}{2}(\varepsilon - \rho\frac{\partial\varepsilon}{\partial\rho})E^2\delta_{ij}]$$

$$+\frac{1}{4\pi}[\mu H_i H_j - \frac{1}{2}(\mu - \rho\frac{\partial\rho}{\partial\rho})H^2\delta_{ij}]$$

Hence 4-tensor $T_{\mu\nu}$ - is the of energy – momentum of the electromagnetic field in a stationary medium is the symmetric Abraham's tensor

$$T_{ij} = \sigma_{ij}, \quad T_{4j} = T_{j4} = -\frac{i}{c} S_j^{(P)} =$$

$$= -\frac{i}{4\pi} [\vec{E} \times \vec{H}]_j, \quad T_{44} = W$$

The average force acting on bounded electron or the force acting on an atom

$$\vec{f} = \frac{1}{4}\alpha_0 \nabla E_0^2 + \frac{\alpha_0}{2c}\frac{\partial}{\partial t}\mathrm{Re}[\vec{E}_0^* \times \vec{H}_0] +$$

$$+\frac{1}{2}\frac{\partial \alpha_0}{\partial \omega}\left\{\frac{\omega}{c}\mathrm{Re}[\frac{\partial \vec{E}_0^*}{\partial t} \times \vec{H}_0] + \mathrm{Im}(\frac{\partial \vec{E}_0^*}{\partial t}\nabla)\vec{E}_0\right\}$$

$$\alpha_0 \delta_{ij} = -\frac{2}{\hbar}\sum_n \frac{\omega_{n0}(d_i)_{0n}(d_j)_{no}}{\omega^2 - \omega_{n0}^z}$$

The force acting on a classical nonrelativistic oscillator (model of atom)

$$\vec{f} = \frac{\alpha}{4}\nabla E^2 + \frac{\alpha}{2}\frac{d}{dt}\mathrm{Re}[E_0^* \times H_0] +$$

$$+\frac{1}{2}\frac{d\alpha}{d\omega}\{\frac{\omega}{c}\mathrm{Re}[\frac{d\vec{E}_0^*}{dt} \times \vec{H}_0] + \mathrm{Im}(\frac{d\vec{E}_0^*}{dt}\nabla)\vec{E}_0\}$$

$$\alpha(\omega) = -\frac{e^2}{m(\omega^2 - \omega_0^2)}$$

Conclusion. In conclusion let us repeat that the approach proposed in [1] is a consistent approach in macroscopic electrodynamics. If time dependence of velocity of medium is taken into account then this approach allows making an explicit choise in favor of Abraham’s tensor. V.L. Ginzburg thought that only experiment will give the answer to existence of Abraham’s force and to the correctness of his tensor. In 70-ies this opinion was quite justified that was connected to very small magnitude of Abraham’s force in compare to Helmholtz’s force. Indeed, the ration of these forces is

$$\frac{f^{(A)}}{f^{(G)}} \sim \frac{L_0/\lambda}{\omega\tau_0} \approx \frac{L_0}{c\tau_0}$$

- Here τ_0 – **is temporary inhomogenuity and** L_0 – **is spatial inhomogenuity of the field amplitudes. In 70-ies this ration was negligibly small in experiments and this was the reason of Ginsburg's pessimism.** Nowadays **this ration is one order higher and it may occur that Abraham's force could appear to be much larger. Therefore the experimental verification of V.L. Ginzburg and V.A. Ugarov approach today is quite realizable**.