Dark Energy

Ginzburg Conference on Physics
Lebedev Institute, Moscow

David Polarski

LCC, Université Montpellier 2

May 31 2012

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- "Big Bang" cosmology: Universe is expanding (recession of galaxies)

- Possible in the framework of General Relativity:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

- Redshift:

$$1 + z = \frac{a_0}{a} = \frac{\lambda_0}{\lambda}$$

- Spectacularly confirmed: Cosmic Microwave Background (CMB), Primordial Nucleosynthesis

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- "Big Bang" cosmology: Universe is expanding (recession of galaxies)

- Possible in the framework of General Relativity:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

- Redshift:

$$1 + z = \frac{a_0}{a} = \frac{\lambda_0}{\lambda}$$

- Spectacularly confirmed: Cosmic Microwave Background (CMB), Primordial Nucleosynthesis

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- "Big Bang" cosmology: Universe is expanding (recession of galaxies)
- Possible in the framework of General Relativity:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

- Redshift:

$$1 + z = \frac{a_0}{a} = \frac{\lambda_0}{\lambda}$$

- Spectacularly confirmed: Cosmic Microwave Background (CMB), Primordial Nucleosynthesis

- "Big Bang" cosmology: Universe is expanding (recession of galaxies)

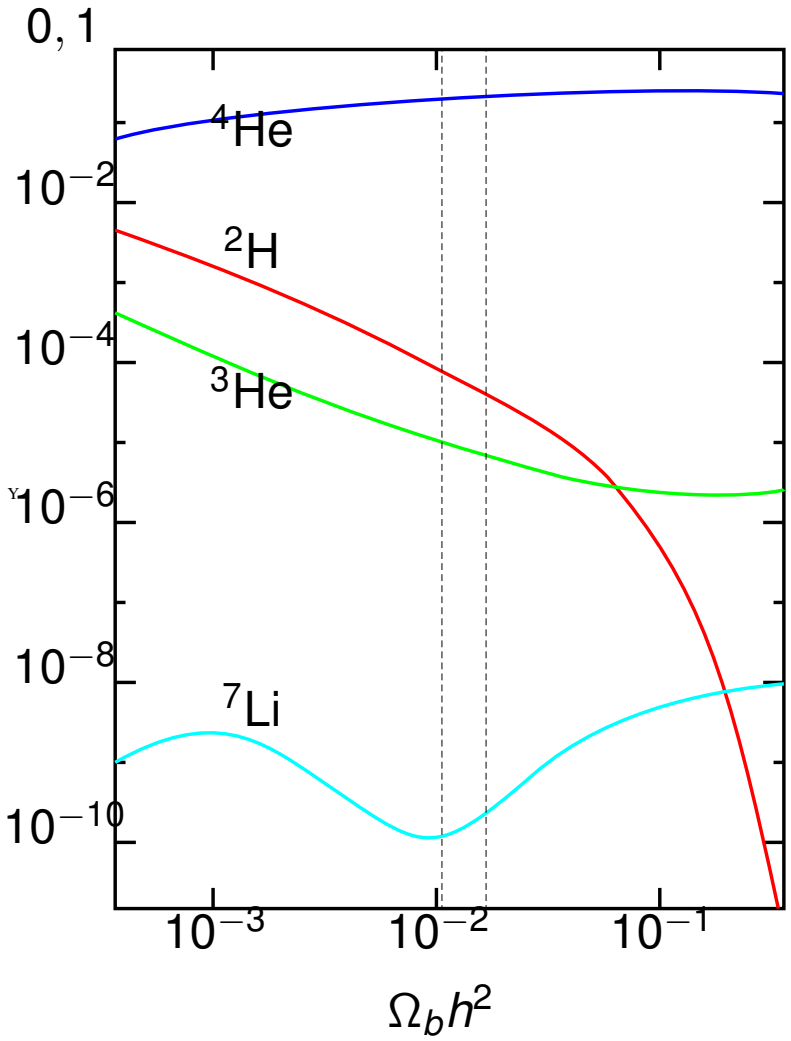
- Possible in the framework of General Relativity:

$$ds^2 = dt^2 - a^2(t)\, d\ell^2$$

- Redshift:

$$1 + z = \frac{a_0}{a} = \frac{\lambda_0}{\lambda}$$

- Spectacularly confirmed: Cosmic Microwave Background (CMB), Primordial Nucleosynthesis



"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

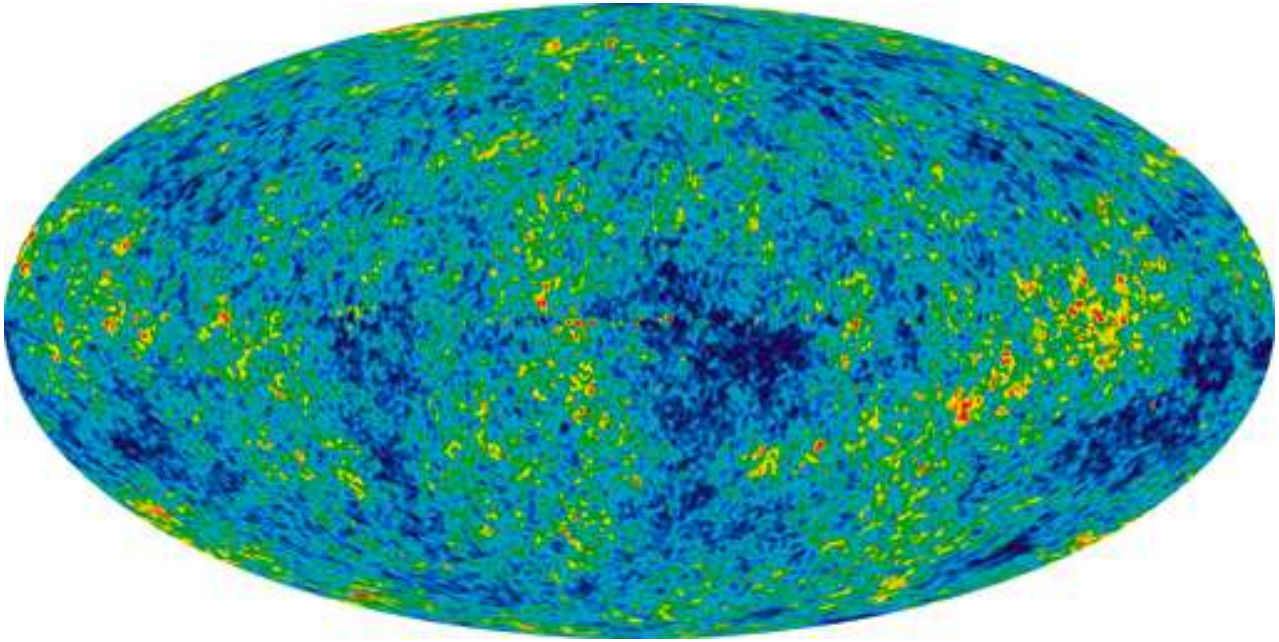
Chameleon models

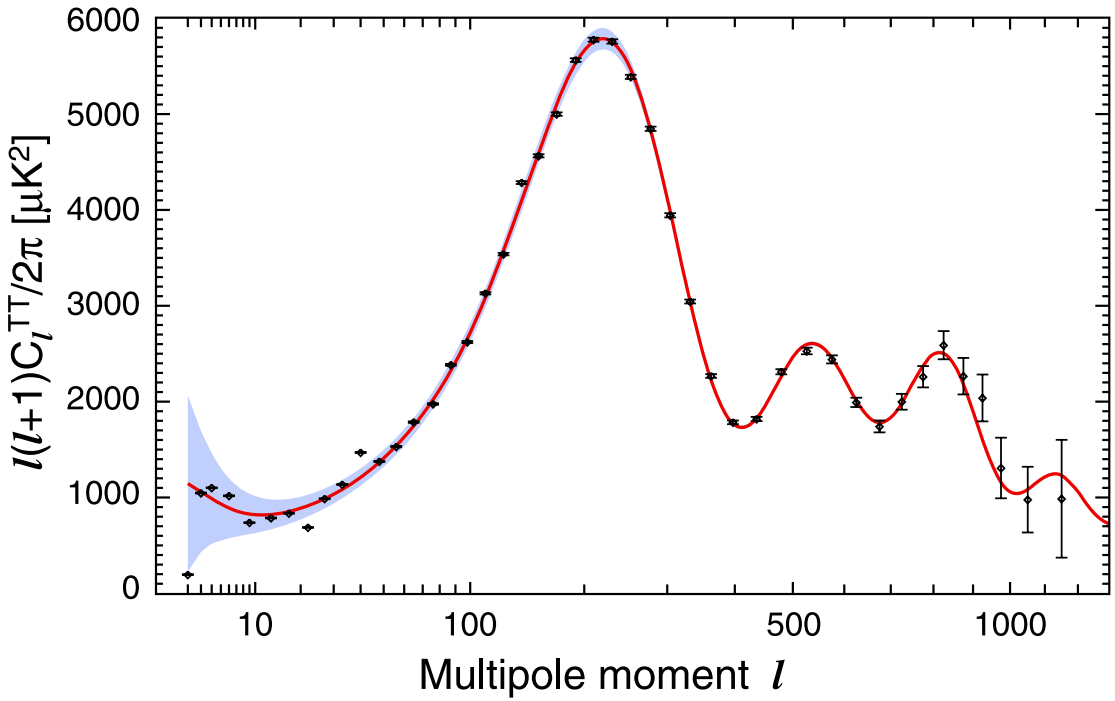
Growth function, growth index

Future of our universe?

Observations

Outlook





Dark Energy

David Polarski

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- In "standard" Big Bang cosmology: decelerated expansion

Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\,\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- $p_m = 0, \quad p_r = \frac{1}{3} > 0 \quad \Rightarrow \quad \ddot{a} < 0$

- $\frac{\dot{a}}{a} \equiv H \qquad H_0 = 74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- In "standard" Big Bang cosmology: decelerated expansion

  Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\,\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- $p_m = 0, \quad p_r = \frac{1}{3} > 0 \quad \Rightarrow \quad \ddot{a} < 0$

- $\frac{\dot{a}}{a} \equiv H \qquad H_0 = 74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- In "standard" Big Bang cosmology: decelerated expansion

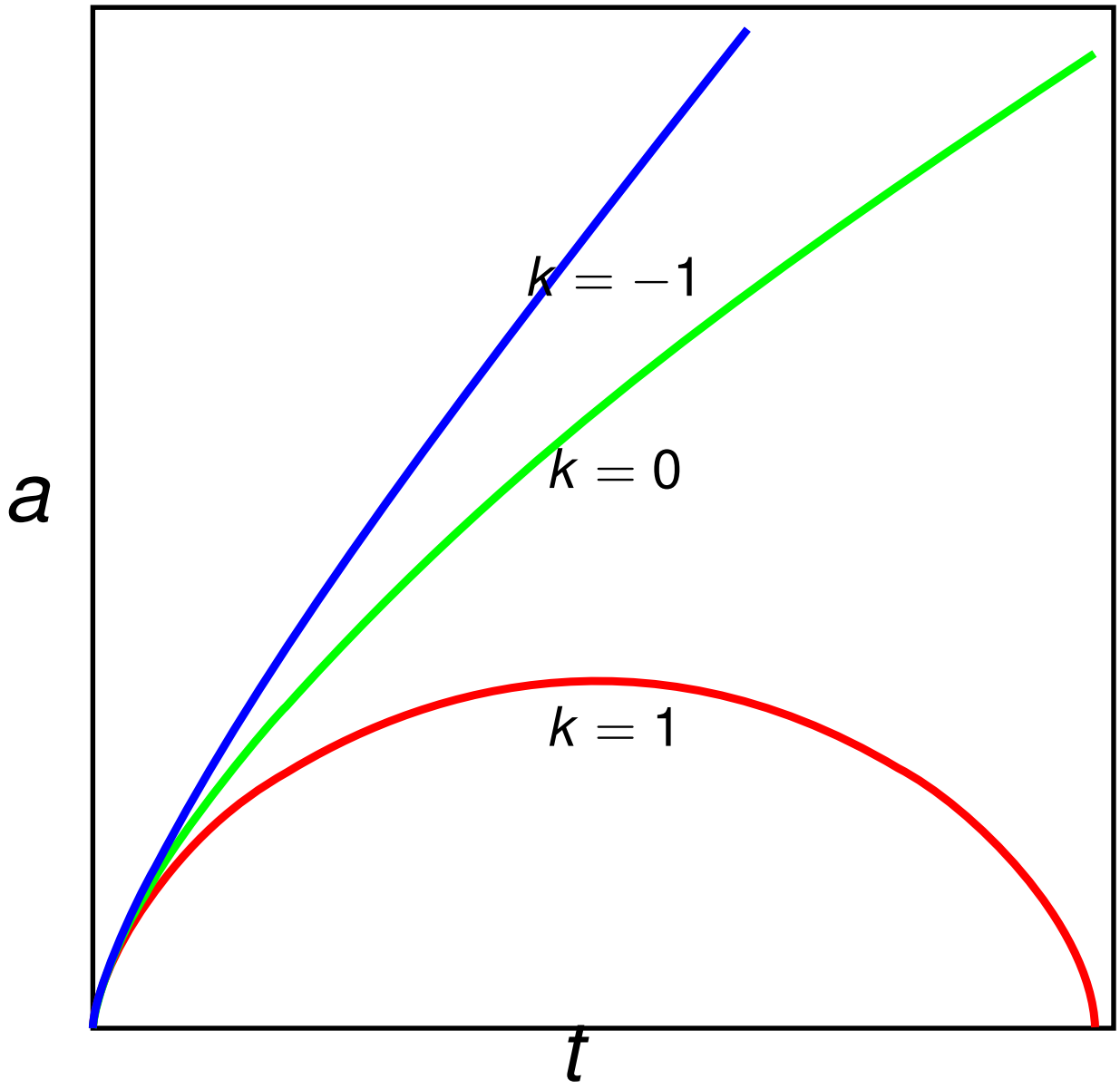
  Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\,\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

- $p_m = 0, \quad p_r = \frac{1}{3} > 0 \quad \Rightarrow \quad \ddot{a} < 0$

- $\frac{\dot{a}}{a} \equiv H \qquad H_0 = 74\ \mathrm{km\ s^{-1}\ Mpc^{-1}}$



"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \qquad m - M = 5 \log d_L + 25$$

$$d_L(z) = c\,(1+z)\,H_0^{-1}\,|\Omega_{k,0}|^{-\frac{1}{2}}\,\mathcal{S}\left(|\Omega_{k,0}|^{\frac{1}{2}}\,\int_0^z \frac{dz'}{h(z')}\right)$$

- Expansion no longer as in standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad z \sim 0.5$$

- Whet is the origin of this accelerated expansion ?

- We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \qquad m - M = 5 \log d_L + 25$$

$$d_L(z) = c\,(1+z)\,H_0^{-1}\,|\Omega_{k,0}|^{-\frac{1}{2}}\,\mathcal{S}\left(|\Omega_{k,0}|^{\frac{1}{2}}\,\int_0^z \frac{dz'}{h(z')}\right)$$

- Expansion no longer as in standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad z \sim 0.5$$

- Whet is the origin of this accelerated expansion ?

- We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \qquad m - M = 5 \log d_L + 25$$

$$d_L(z) = c\,(1+z)\,H_0^{-1}\,|\Omega_{k,0}|^{-\frac{1}{2}}\,\mathcal{S}\left(|\Omega_{k,0}|^{\frac{1}{2}}\,\int_0^z \frac{dz'}{h(z')}\right)$$

- Expansion no longer as in standard cosmology

$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad z \sim 0.5$$

- Whet is the origin of this accelerated expansion ?

- We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Dark Energy paradigm comes from observations: SNIa Luminosity-distances

$$\mathcal{F} = \frac{L}{4\pi d_L^2} \qquad m - M = 5 \log d_L + 25$$

$$d_L(z) = c\,(1+z)\,H_0^{-1}\,|\Omega_{k,0}|^{-\frac{1}{2}}\,\mathcal{S}\left(|\Omega_{k,0}|^{\frac{1}{2}}\,\int_0^z \frac{dz'}{h(z')}\right)$$

- Expansion no longer as in standard cosmology

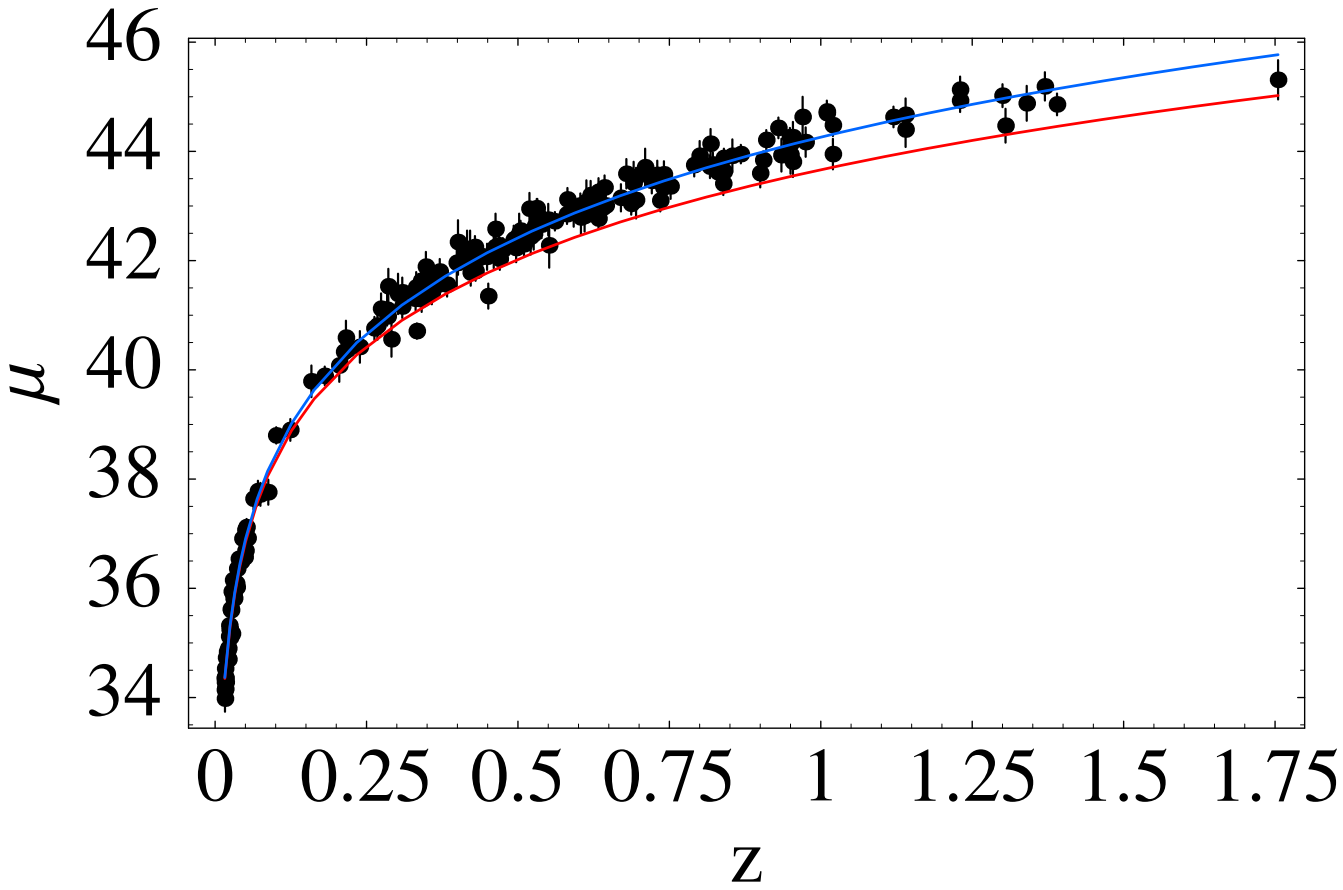
$$\ddot{a} < 0 \rightarrow \ddot{a} > 0 \qquad z \sim 0.5$$

- Whet is the origin of this accelerated expansion ?

- We are not really unhappy...

$$\Omega_{m,0} \approx 0.3, \quad \Omega_{DE,0} \approx 0.7, \quad \Omega_{k,0} \approx 0$$

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook



$\Lambda CDM \quad \Omega_{m,0} = 0.3 \quad \Omega_{k,0} = 0$

EdS

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Introduce Dark Energy (DE)

$$\begin{aligned} 3H^2 &= 8\pi G(\rho_m + \rho_{DE}) \\ -2\dot{H} &= 8\pi G(\rho_m + \rho_{DE} + p_{DE}) \end{aligned}$$

- $h^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\,f(z)$

$$f(z) = \exp\left[3\int_0^z dz'\,\frac{1+w_{DE}(z')}{1+z'}\right]$$

- $w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} < -\frac{1}{3}\,\Omega_{DE}^{-1}$ accelerated expansion

Dark Energy

David Polarski

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Introduce Dark Energy (DE)

$$\begin{aligned} 3H^2 &= 8\pi G(\rho_m + \rho_{DE}) \\ -2\dot{H} &= 8\pi G(\rho_m + \rho_{DE} + p_{DE}) \end{aligned}$$

- $h^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\,f(z)$

$$f(z) = \exp\left[3\int_0^z dz'\,\frac{1+w_{DE}(z')}{1+z'}\right]$$

- $w_{DE} = \frac{p_{DE}}{\rho_{DE}} < -\frac{1}{3}\,\Omega_{DE}^{-1}$ accelerated expansion

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Introduce Dark Energy (DE)

$$\begin{aligned} 3H^2 &= 8\pi G(\rho_m + \rho_{DE}) \\ -2\dot{H} &= 8\pi G(\rho_m + \rho_{DE} + p_{DE}) \end{aligned}$$

- $h^2(z) = \left(\frac{H(z)}{H_0}\right)^2 = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\,f(z)$

$$f(z) = \exp\left[3\int_0^z dz'\,\frac{1+w_{DE}(z')}{1+z'}\right]$$

- $w_{DE} \equiv \frac{p_{DE}}{\rho_{DE}} < -\frac{1}{3}\,\Omega_{DE}^{-1}$ accelerated expansion

- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Cosmological constant Λ: remarkable simplicity!

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho_m + \frac{\Lambda}{3}$$

- Conceptual problem : $\Lambda \sim 10^{-122} l_{Pl}^{-2}$
- There are also observational problems
- $w_\Lambda = -1$, and ρ_Λ is exactly constant
- Other models have generically: $w_{DE}(z)$!!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Quintessence: (minimally coupled) scalar field $\phi(t)$

$$\begin{aligned} \rho_\phi &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p_\phi &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned}$$

$$w_\phi = \frac{p_\phi}{\rho_\phi} = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}$$

$$-1 \leq w_\phi \leq 1 \Leftrightarrow \rho_\phi + p_\phi \geq 0$$

No “phantom”!

“Big Bang” Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

- $$V = -G_{\text{eff}}\,\frac{M_1\, M_2}{r} \qquad \text{massless}\ \ \Phi\ \ \text{field}$$

$$G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD} + 3}\right) \qquad G_N = \frac{G_*}{F}$$

- $G_{\text{eff},0} \simeq G_{N,0}$

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

- $V = -G_{\text{eff}}\,\frac{M_1\, M_2}{r}$ massless Φ field
- $G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD}+3}\right)$ $G_N = \frac{G_*}{F}$
- $G_{\text{eff},0} \simeq G_{N,0}$

- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

- $$V = -G_{\text{eff}}\,\frac{M_1\ M_2}{r} \qquad \text{massless}\ \ \Phi\ \ \text{field}$$

$$G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD} + 3}\right) \qquad G_N = \frac{G_*}{F}$$

- $G_{\text{eff},0} \simeq G_{N,0}$

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- $L = \frac{1}{16\pi G_*}\left(F(\Phi)\, R - Z\partial_\mu\Phi\partial^\mu\Phi - 2U(\Phi)\right) + L_m(g_{\mu\nu})$
- Brans-Dicke parametrization

$$F(\Phi) = \Phi \qquad Z(\Phi) = \frac{\omega_{BD}(\Phi)}{\Phi}$$

Another choice

$$F(\Phi) = \text{arbitrary} \qquad Z = 1 \Leftrightarrow \omega_{BD} > 0$$

$$\omega_{BD} = \frac{F}{(dF/d\Phi)^2} > -\frac{3}{2} \qquad \omega_{BD,0} > 4 \times 10^4$$

- $$V = -G_{\text{eff}}\,\frac{M_1\ M_2}{r} \qquad \text{massless}\ \ \Phi\ \ \text{field}$$

$$G_{\text{eff}} = G_N\left(1 + \frac{1}{2\omega_{BD} + 3}\right) \qquad G_N = \frac{G_*}{F}$$

- $G_{\text{eff},0} \simeq G_{N,0}$

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- $3FH^2 = 8\pi G_* \rho_m + \frac{\dot{\Phi}^2}{2} + U - 3H\dot{F}$

$-2F\dot{H} = 8\pi G_* \rho_m + \dot{\Phi}^2 + \ddot{F} - H\dot{F}$

Define ρ_{DE} and p_{DE}:

$$3\left(H^2 + \frac{k}{a^2}\right) = 8\pi G_{N,0}\left(\rho_m + \rho_{DE}\right)$$
$$-2\left(\dot{H} - \frac{k}{a^2}\right) = 8\pi G_{N,0}\left(\rho_m + \rho_{DE} + p_{DE}\right)$$

- $h^2(z) = \Omega_{m,0}(1+z)^3 + \Omega_{DE,0} f(z) + \Omega_{k,0}(1+z)^2$

$$f(z) = \exp\left[3\int_0^z dz' \frac{1 + w_{DE}(z')}{1+z'}\right]$$

- $3FH^2 = 8\pi G_* \rho_m + \frac{\dot{\Phi}^2}{2} + U - 3H\dot{F}$

  $-2F\dot{H} = 8\pi G_* \rho_m + \dot{\Phi}^2 + \ddot{F} - H\dot{F}$

  Define ρ_{DE} and p_{DE}:

$$3\left(H^2 + \frac{k}{a^2}\right) = 8\pi G_{N,0}\left(\rho_m + \rho_{DE}\right)$$
$$-2\left(\dot{H} - \frac{k}{a^2}\right) = 8\pi G_{N,0}\left(\rho_m + \rho_{DE} + p_{DE}\right)$$

- $h^2(z) = \Omega_{m,0}\,(1+z)^3 + \Omega_{DE,0}\, f(z) + \Omega_{k,0}\,(1+z)^2$

$$f(z) = \exp\left[3\int_0^z dz' \, \frac{1 + w_{DE}(z')}{1+z'}\right]$$

Growth of matter perturbations is modified:

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

$$h^2\ \delta_m'' + \left(\frac{(h^2)'}{2} - \frac{h^2}{1+z}\right)\delta_m' = \frac{3}{2}(1+z)\frac{G_{\rm eff}}{G}\ \Omega_{m,0}\ \delta_m$$

Perturbations $\delta_m(z)$ must be consistent with background expansion!

Some “DE” clustering

“Big Bang” Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Modifying gravity ?

  Example : $R \rightarrow f(R)$

$$f(R) = R - \lambda R_c \left(1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right)$$

- Growth of perturbations

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

- $G_{\rm eff} = G_{\rm eff}(z, k) \Leftrightarrow V(r) = -\frac{G_*}{F}\frac{M_1\ M_2}{r}\ \left(1 + \frac{1}{3}\ e^{-mr}\right)$

  $F \equiv \frac{df(R)}{dR}$)

- Modifying gravity ?

  Example : $R \rightarrow f(R)$

$$f(R) = R - \lambda R_c \left(1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right)$$

- Growth of perturbations

$$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

- $G_{\rm eff} = G_{\rm eff}(z, k) \Leftrightarrow V(r) = -\frac{G_*}{F}\frac{M_1\ M_2}{r}\ \left(1 + \frac{1}{3}\ e^{-mr}\right)$

  $F \equiv \frac{df(R)}{dR}$)

- Modifying gravity ?

  Example : $R \rightarrow f(R)$

  $$f(R) = R - \lambda R_c \left(1 - \left(1 + \frac{R^2}{R_c^2}\right)^{-n}\right)$$

- Growth of perturbations

  $$\ddot{\delta}_m + 2H\dot{\delta}_m - 4\pi G_{\rm eff}\ \rho_m\ \delta_m = 0$$

- $G_{\rm eff} = G_{\rm eff}(z, k) \Leftrightarrow V(r) = -\frac{G_*}{F}\frac{M_1\ M_2}{r}\ \left(1 + \frac{1}{3}\ e^{-mr}\right)$

  $F \equiv \frac{df(R)}{dR}$)

- $L = \frac{R}{16\pi G_*} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) + L_m \left[\Psi_m; A^2(\phi) \, g_{\mu\nu} \right]$

- $A^2 = e^{2\beta\phi/M_{PL}}$ $\qquad V = M^4 \, e^{(\frac{M}{\phi})^n}$

  $M \ll \phi \ll M_{PL} \rightarrow V$ is like Λ!

- $G_{\rm eff}(z,k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r} \left(1 + 2\,\beta^2\, e^{-m_\phi r}\right)$

  m_ϕ is too large, no influence on cosmological scales!

- Interacting dark sector

- $L = \frac{R}{16\pi G_*} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) + L_m \left[\Psi_m; A^2(\phi) \, g_{\mu\nu} \right]$

- $A^2 = e^{2\beta\phi/M_{PL}}$ $\qquad\qquad$ $V = M^4 \, e^{(\frac{M}{\phi})^n}$

  $M \ll \phi \ll M_{PL} \rightarrow V$ is like Λ!

- $G_{\rm eff}(z,k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r} \left(1 + 2\,\beta^2\, e^{-m_\phi r}\right)$

  m_ϕ is too large, no influence on cosmological scales!

- Interacting dark sector

- $L = \frac{R}{16\pi G_*} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) + L_m \left[\Psi_m; A^2(\phi) \, g_{\mu\nu} \right]$

- $A^2 = e^{2\beta\phi/M_{PL}}$ $\qquad\qquad$ $V = M^4 \, e^{(\frac{M}{\phi})^n}$

  $M \ll \phi \ll M_{PL} \rightarrow V \;\; \text{is} \;\; \text{like} \;\; \Lambda!$

- $G_{\text{eff}}(z, k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r} \left(1 + 2 \, \beta^2 \, e^{-m_\phi r}\right)$

  m_ϕ is too large, no influence on cosmological scales!

- Interacting dark sector

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- $L = \frac{R}{16\pi G_*} - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi - V(\phi) + L_m \left[\Psi_m; A^2(\phi) \, g_{\mu\nu} \right]$

- $A^2 = e^{2\beta\phi/M_{PL}}$ $\qquad\qquad$ $V = M^4 \, e^{(\frac{M}{\phi})^n}$

  $M \ll \phi \ll M_{PL} \rightarrow V$ is like Λ!

- $G_{\text{eff}}(z, k) \Leftrightarrow V(r) = -G_* \frac{M_1 M_2}{r} \left(1 + 2 \, \beta^2 \, e^{-m_\phi r}\right)$

  m_ϕ is too large, no influence on cosmological scales!

- Interacting dark sector

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2}(1 - 3\, w_{\rm eff})\, f = \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$.
  Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
  It can be very different in modified gravity models!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2}(1 - 3\, w_{\rm eff})\, f = \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$.
  Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
  It can be very different in modified gravity models!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook

- Matter perturbations can be characterized by the "growth function" $f = \frac{d \ln \delta}{d \ln a} \equiv \frac{d \ln \delta}{dx}$

$$\frac{df}{dx} + f^2 + \frac{1}{2}(1 - 3\, w_{\rm eff})\, f = \frac{3}{2}\frac{G_{\rm eff}}{G}\Omega_m$$

- A convenient "parameterization" $f = \Omega_m^{\gamma}$. Actually

$$\delta_m(z, k) \Leftrightarrow \gamma = \gamma(z, k)$$

- In ΛCDM: $\gamma \simeq 0.55$
  It can be very different in modified gravity models!

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

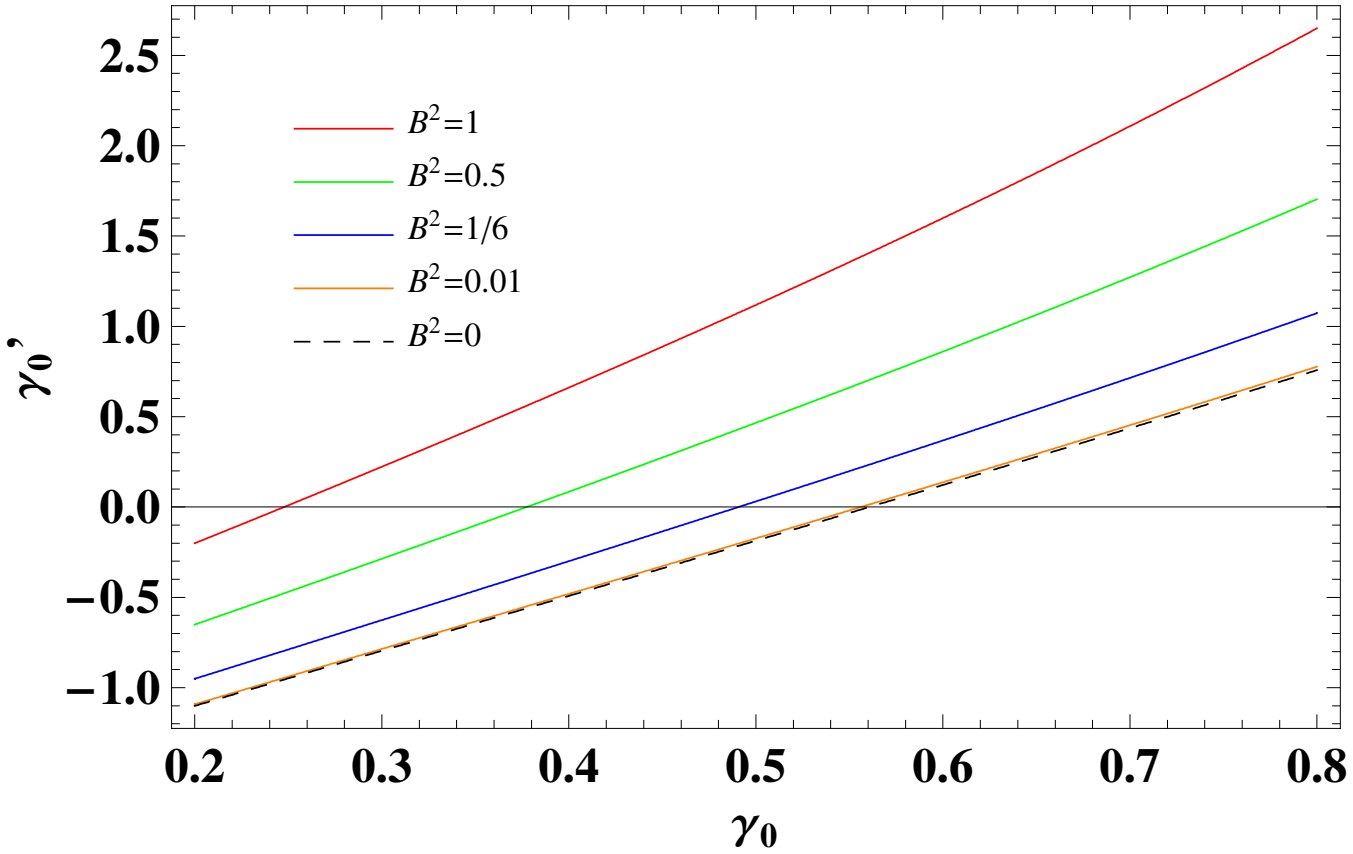
Growth function, growth index

Future of our universe?

Observations

Outlook

$$C\left(\gamma_0(k),\ \gamma_0'(k),\ \Omega_{m,0},\ w_{\mathrm{eff},0},\ \frac{G_{\mathrm{eff},0}(k)}{G} \equiv 1 + 2B^2\right) = 0$$



$$\Omega_{m,0} = 0.29 \qquad w_{DE,0} = -1$$

Dark Energy

David Polarski

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

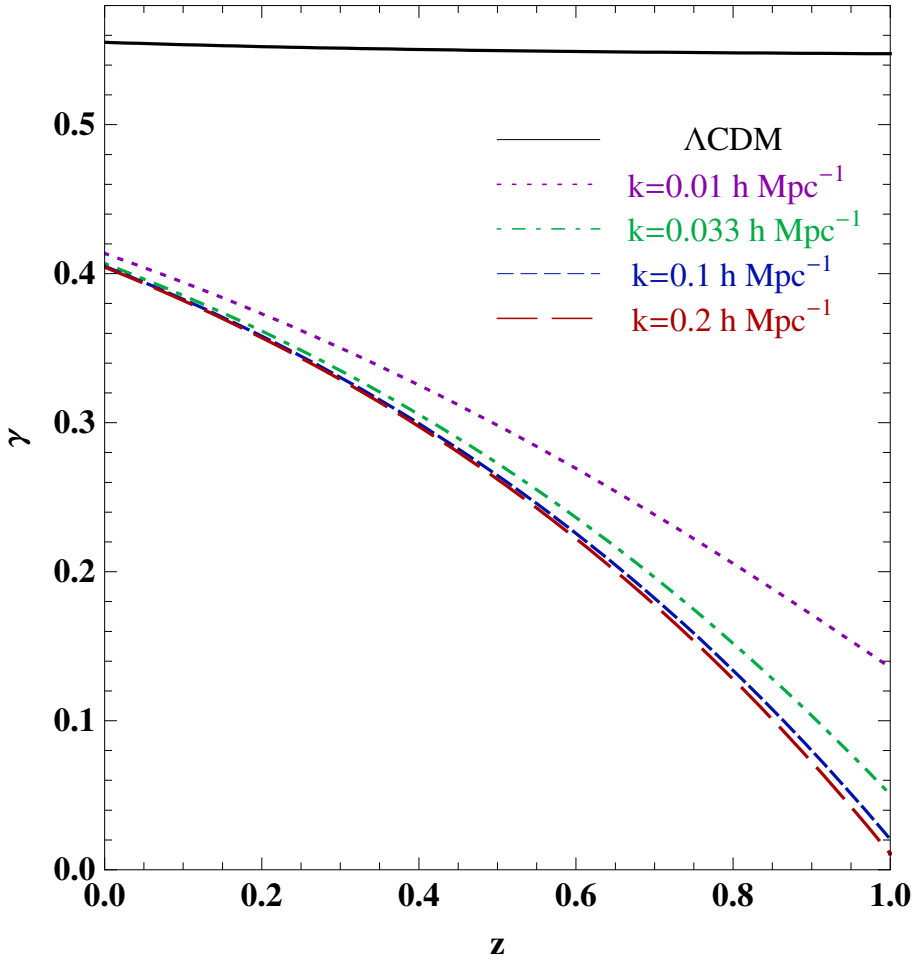
Growth function, growth index

Future of our universe?

Observations

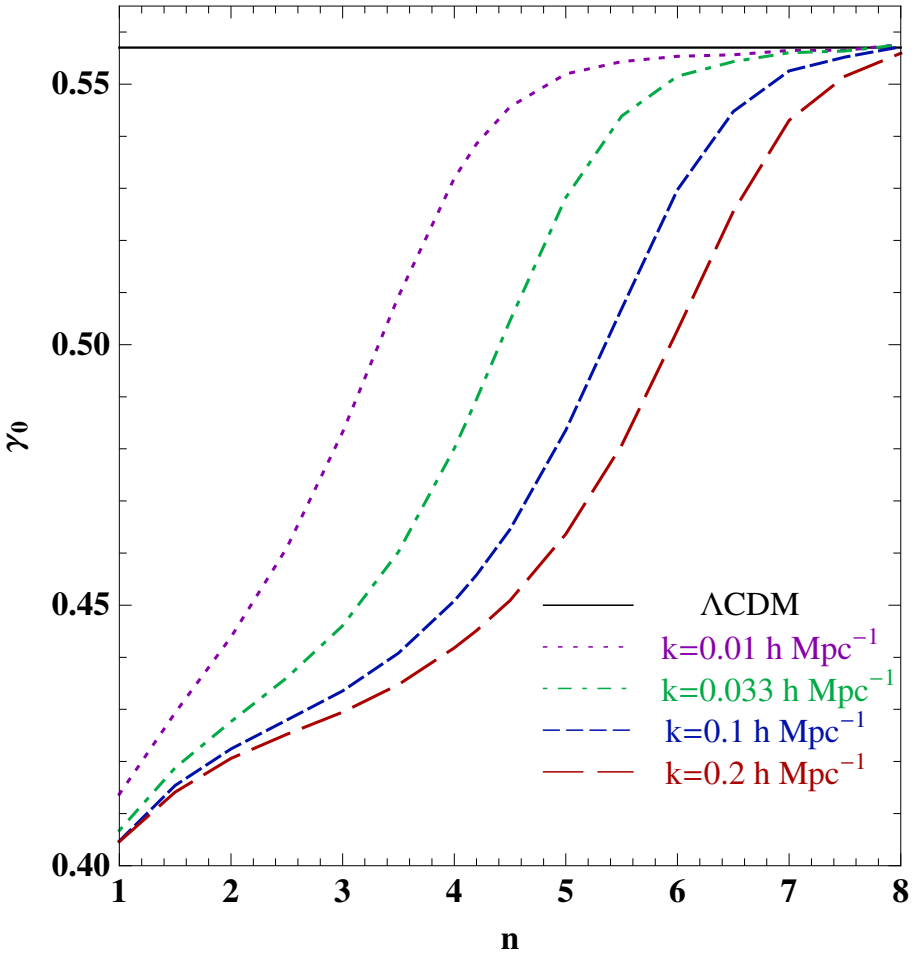
Outlook

$$f(R) = R - \lambda R_c \frac{x^{2n}}{x^{2n}+1} \qquad x \equiv \frac{R}{R_c}$$



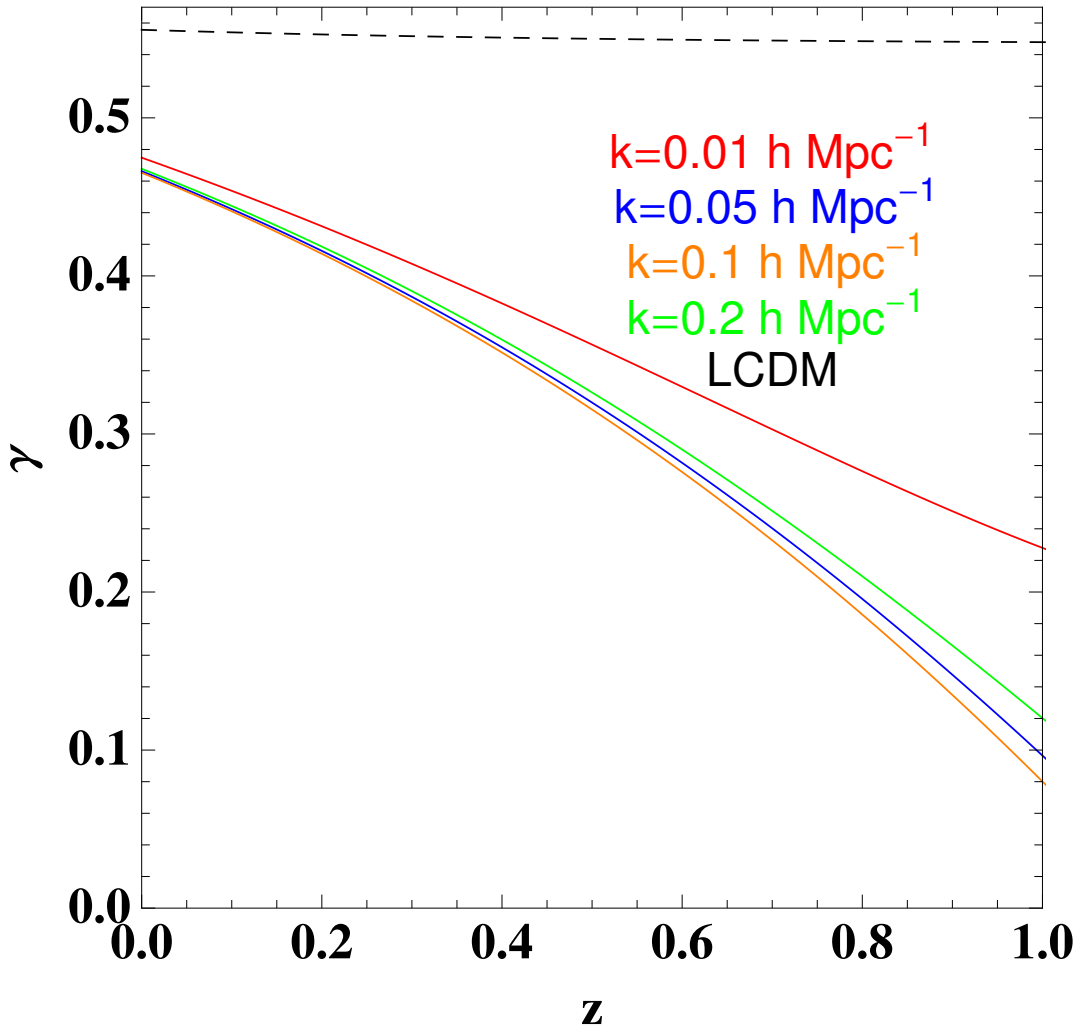
$$n = 1, \quad \lambda = 1.55$$

$$f(R) = R - \lambda R_c \frac{x^{2n}}{(x^{2n}+1)} \qquad x \equiv \frac{R}{R_c}$$

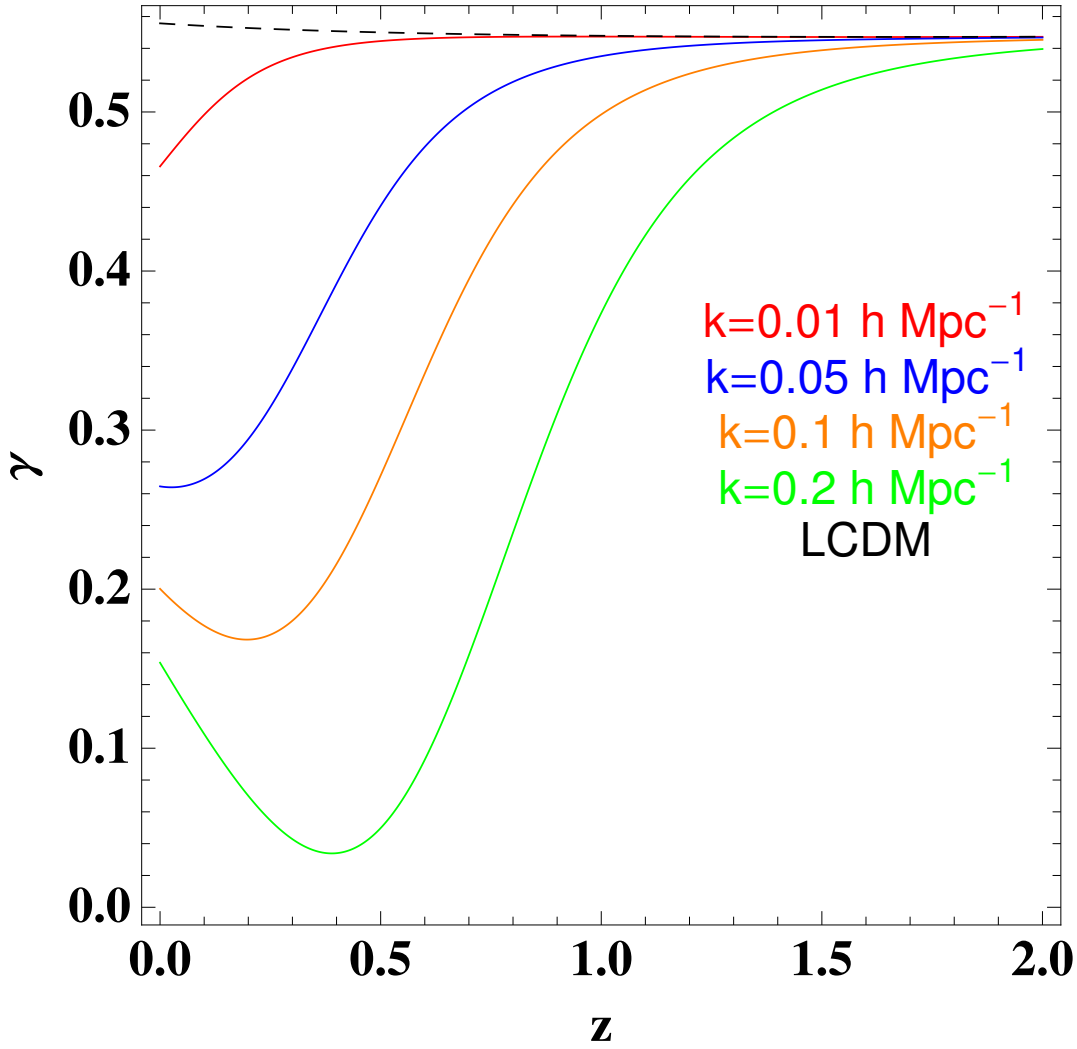


$$\lambda = 1.55$$

$$V(\phi) = M^4[1 - \mu(1 - e^{-\phi/M_{\rm pl}})^n]$$

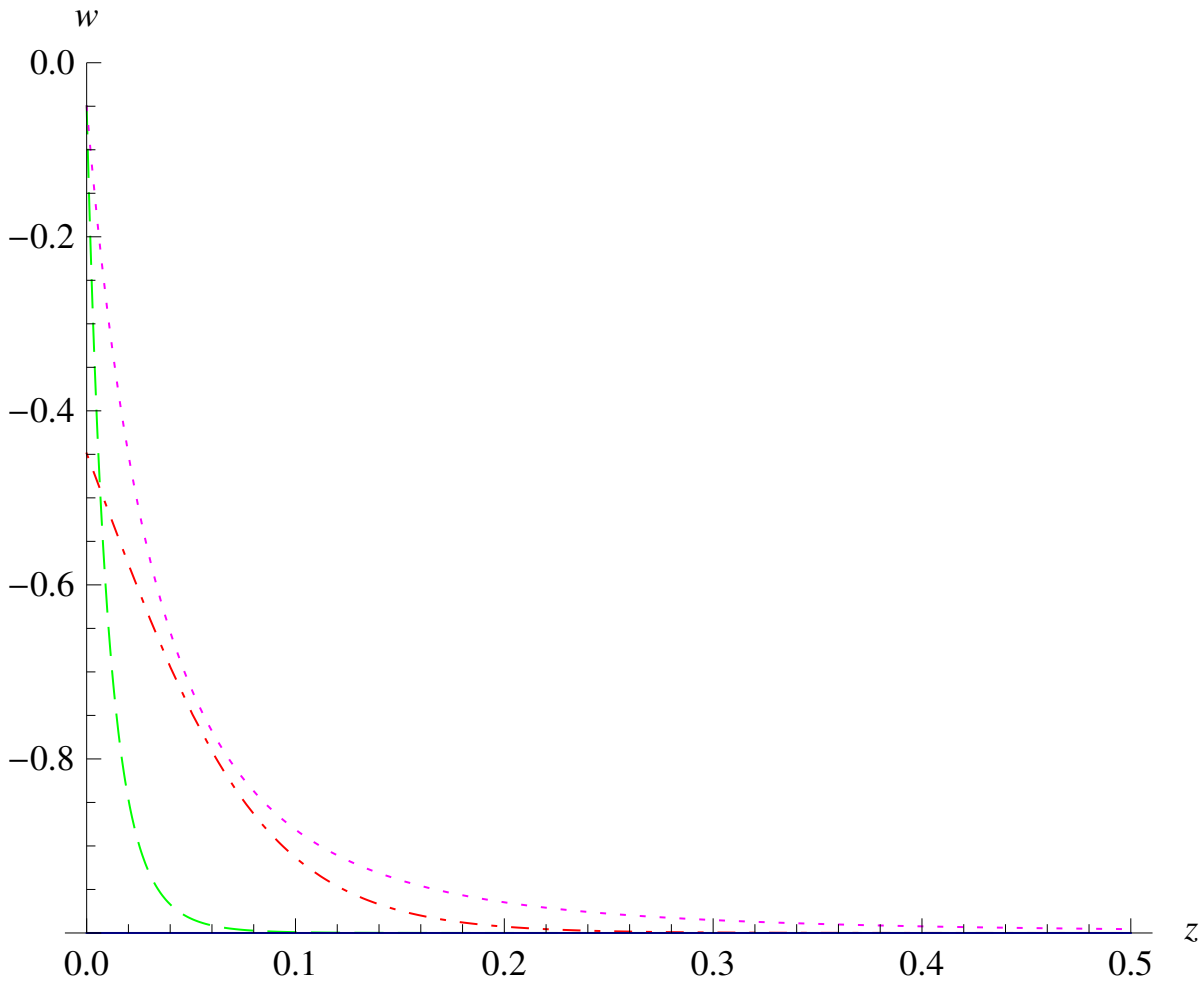


$$n = 0.8 \quad \mu = 0.5 \quad \beta = 1/\sqrt{6}$$

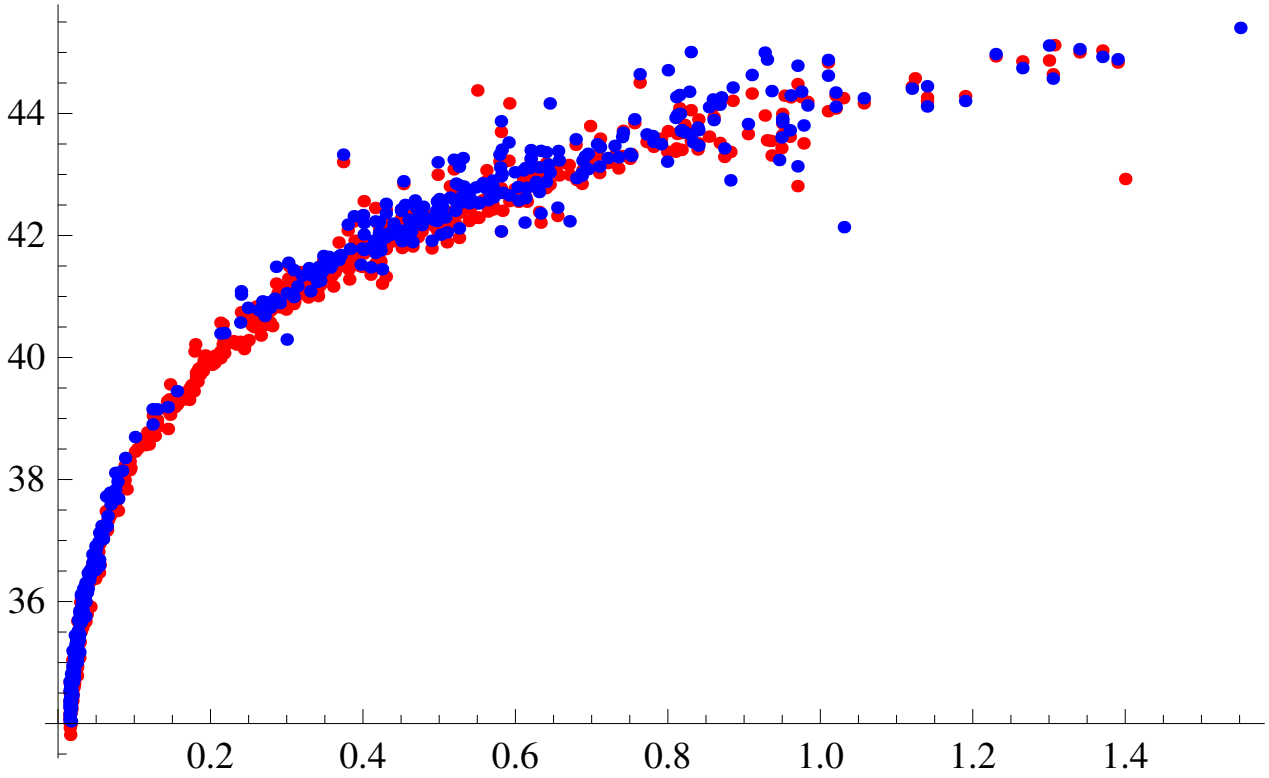


$n = 0.7 \quad \mu = 0.05 \quad \beta = 1$

- Universe future is undetermined...forever?



"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook



"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- We need complementary probes:

  Supernovae

  Clusters

  Weak lensing

  Baryon Acoustic Oscillations

  Cosmic Microwave Background

  Gamma Ray Bursts?

  Gravitational waves?

  Redshift drift?

- ...theoretical tools: parametrizations, Fisher matrices, Bayesian approach,..

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

- We need complementary probes:

  Supernovae

  Clusters

  Weak lensing

  Baryon Acoustic Oscillations

  Cosmic Microwave Background

  Gamma Ray Bursts?

  Gravitational waves?

  Redshift drift?

- ...theoretical tools: parametrizations, Fisher matrices, Bayesian approach,..

"Big Bang" Cosmology

Universe Expansion

Dark Energy

First models

Quintessence

Scalar-tensor DE models

Modified gravity

Chameleon models

Growth function, growth index

Future of our universe?

Observations

Outlook

The art of inducing accelerated expansion

Smooth component in GR

Λ? $w(z)$? $w < -1$?
Quintessence? Chaplygin? Tachyons?... Clustering?

Modifying gravity ?

Scalar-tensor? $f(R)$? Galileon? Massive gravity?
Modified growth of perturbations

Challenging cosmological framework ?

What is the universe future ...and past ?

Observations will single out viable models

Deep problems induce exciting solutions and challenging questions!

"Big Bang" Cosmology
Universe Expansion
Dark Energy
First models
Quintessence
Scalar-tensor DE models
Modified gravity
Chameleon models
Growth function, growth index
Future of our universe?
Observations
Outlook