Phenomenological Heterotic Theory:

Standard Models, the Renormalization Group

And All That

Ginzburg Conference

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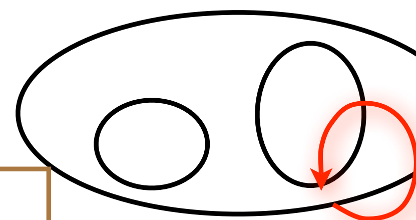
Smooth Heterotic Compactifications

- SU(4) Heterotic Standard Models

$D = 10, \quad g_{MN}, \quad A^a_M, E_8$

$X, D = 6$



$V \;, G = SU(4)$

“slope” stable

$W \;, F = \mathbb{Z}_3 \times \mathbb{Z}_3$

$\mathbb{R}^4$

$N = 1 \; SUSY$

$$H^1(V)^F \Rightarrow \; matter$$
$$H^1(V^*)^F \Rightarrow \; conjugate\ matter$$
$$H^1(\wedge^2 V)^F \Rightarrow \; Higgs$$
$$H^1(V \otimes V^*)^F \Rightarrow \; Bundle\ Moduli$$

$\mathbb{R}^4$ Theory Gauge Group:

$$G = SU(4) \Rightarrow \; E_8 \to Spin(10)$$

Choose the $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines to be Braun, He, Ovrut, Pantev 2006

$$\chi_Y = e^{iY_Y \frac{2\pi}{3}} \quad , \quad \chi_{B-L} = e^{iY_{B-L}\frac{2\pi}{3}}$$

where

$$Y_Y = 6Y \ , \quad Y_{B-L} = 3(B-L)$$

$\Rightarrow$

$$F = \mathbb{Z}_3 \times \mathbb{Z}_3 \Rightarrow \ Spin(10) \to \ SU(3)_C \times SU(2)_L \times U(1)_Y \times \underset{\text{gauged}}{U(1)_{B-L}}$$

$\mathbb{R}^4$ Theory Spectrum:

$n_r = (h^1(X, U_R(V)) \otimes \mathbf{R})^{\mathbb{Z}_3 \times \mathbb{Z}_3} \Longrightarrow$ **3** families of quarks/leptons

$$Q_L = (3,2,1,1), \quad u_R = (\bar{3},1,-4,-1), \quad d_R = (\bar{3},1,2,-1)$$

$$L_L = (1,2,-3,-3), e_R = (1,1,6,3), \qquad \nu_R = (1,1,0,3)$$

and **1** pair of Higgs-Higgs conjugate fields

$$H = (1,2,3,0), \quad \bar{H} = (1,\bar{2},-3,0)$$

under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$.

No further vector-like pairs or exotics.

That is, we get exactly the matter spectrum of the MSSM

with 3 right-handed neutrinos! In addition, there are

$n_1 = h^1(X, V \times V^*)^{\mathbb{Z}_3 \times \mathbb{Z}_3} = 13$ vector bundle moduli $\phi = (1, 1, 0, 0)$

Denote this low energy theory as a B-L MSSM.

However-

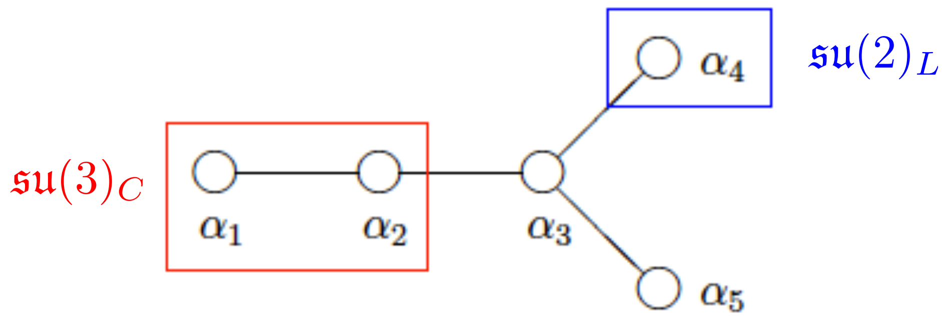
$$Tr(Y_Y Y_{B-L})_{\mathfrak{so}(10)} \neq 0 \quad \Rightarrow \text{initial } \mathrm{U(1)_Y U_{B-L}} \text{ operator mixing}$$

$$Tr(Y_Y Y_{B-L})_{3\oplus 2\oplus 1\oplus 1} \neq 0 \quad \Rightarrow U(1)_Y U_{B-L} \text{ mixing evolves with scale}$$

Greatly complicates the RG and low energy analysis!

Question: Are there other inequivalent choices of Wilson lines leading to a B-L MSSM with no U(1)U(1) kinetic mixing?

$\mathfrak{so}(10)$ Dykin Diagram $\cdot$



First, find the most general element of the Cartan subalgebra $\mathfrak{h} \subset \mathfrak{so}(10)$ that commutes with $\alpha^1, \alpha^2, \beta, \alpha^4$. The result is

$$H_{3\oplus 2} = a(H_1 + H_2 + H_3) + b(H_4 + H_5)$$

$\Rightarrow$ the elements of $\mathfrak{so}(10)$ that commute with $\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L$ form a two-dimensional subspace $\mathfrak{h}_{2\oplus 3} \subset \mathfrak{h}$. Any basis is of potential interest.

However,

$$H_1 + H_2 + H_3 \;, \quad H_4 + H_5$$

arise "naturally". We call this the "canonical basis" and explore its properties. One can identify

$$Y_{B-L} = \; 2(H_1 + H_2 + H_3) = 3(B-L)$$
$$Y_{T_{3R}} = \; H_4 + H_5 = 2(Y - \frac{1}{2}(B-L)) = 2T_{3R}$$

Now choose the $\mathbb{Z}_3 \times \mathbb{Z}_3$ Wilson lines to be

$$\chi_{T_{3R}} = e^{iY_{T_{3R}}\frac{2\pi}{3}} \;, \quad \chi_{B-L} = e^{iY_{B-L}\frac{2\pi}{3}}$$

Note that

$$\chi^3_{T_{3R}} = \chi^3_{B-L} = 1$$

$\Rightarrow$

$$Spin(10) \rightarrow \; SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$$

Canonical Spectrum:

The Spin(10) spectrum is determined from $H^1(X, U_R(V))$.

For $R = 16$

$$H^1(X, V) = RG^{\oplus 3}$$

where

$$RG = 1 \oplus \chi_1 \oplus \chi_2 \oplus \chi_1^2 \oplus \chi_2^2 \oplus \chi_1\chi_2 \oplus \chi_1^2\chi_2 \oplus \chi_1\chi_2^2 \oplus \chi_1^2\chi_2^2$$

and $\chi_1 = \chi_{T_{3R}}, \chi_2 = \chi_{B-L}$. Note that

$$h^1(X, V) = 27$$

$\Rightarrow$ 27 16 representations. The action of the Wilson lines on each 16 is

$$\begin{aligned} \mathbf{16} \;=\; & \chi_{T_{3R}}^2 \cdot \chi_{B-L}^2(\bar{\mathbf{3}}, \mathbf{1}, -1, -1) \oplus \chi_{T_{3R}} \cdot \chi_{B-L}^2(\bar{\mathbf{3}}, \mathbf{1}, 1, -1) \\ & \oplus 1 \cdot \chi_{B-L}(\mathbf{3}, \mathbf{2}, 0, 1) \oplus 1 \cdot 1(\mathbf{1}, \mathbf{2}, 0, -3) \oplus \chi_{T_{3R}}^2 \cdot 1(\mathbf{1}, \mathbf{1}, -1, 3) \\ & \oplus \chi_{T_{3R}} \cdot 1(\mathbf{1}, \mathbf{1}, 1, 3)\ . \end{aligned}$$

Then $(H^1(X, V) \otimes 16)^{\mathbb{Z}_3 \times \mathbb{Z}_3}$ consists of 3 families of quarks/leptons each transforming as

$$Q=(U,D)^T=(\mathbf{3},\mathbf{2},0,\frac{1}{3}),\quad u=(\bar{\mathbf{3}},\mathbf{1},-\frac{1}{2},-\frac{1}{3}),\quad d=(\bar{\mathbf{3}},\mathbf{1},\frac{1}{2},-\frac{1}{3})$$

$$L=(N,E)^T=(\mathbf{1},\mathbf{2},0,-1),\quad \nu=(\mathbf{1},\mathbf{1},-\frac{1}{2},1),\quad e=(\mathbf{1},\mathbf{1},\frac{1}{2},1)$$

under $SU(3)_C\times SU(2)_L\times U(1)_{T_{3R}}\times U(1)_{B-L}$.

For $R=10$

$$h^1(X,\wedge^2 V)=4$$

$\Rightarrow$ 4 10 representations. We find that $(H^1(X,\wedge^2 V)\otimes 10)^{\mathbb{Z}_3\times\mathbb{Z}_3}$

has 1 pair of Higgs-Higgs conjugate fields

$$H=(\mathbf{1},\mathbf{2},\frac{1}{2},0),\quad \bar{H}=(\mathbf{1},\mathbf{2},-\frac{1}{2},0)$$

That is,

- *When the two Wilson lines corresponding to the canonical basis are turned on simultaneously, the resulting low energy spectrum is precisely that of the MSSM–that is, three families of quark/lepton chiral superfields, each family with a right-handed neutrino supermultiplet, and one pair of Higgs-Higgs conjugate chiral multiplets. There are no vector-like pairs or exotic particles.*

Note in the above analysis that each quark/lepton and Higgs arises from a different 16 or 10 of $Spin(10)$.

Canonical Kinetic Mixing:

For arbitrary $U(1)_1 \times U(1)_2$

$$\mathcal{L}_{kinetic} = -\frac{1}{4}((F^1_{\mu\nu})^2 + 2\alpha F^1_{\mu\nu}F^{2\mu\nu} + (F^2_{\mu\nu})^2 + \dots)$$

For $U(1)_{T_{3R}} \times U(1)_{B-L}$, $(H_i|H_j) = \delta_{ij} \Longrightarrow$ the "Killing" bracket

$(Y_{T_{3R}}|Y_{B-L}) = 0 \quad \Rightarrow \quad Tr(Y_{T_{3R}}Y_{B-L})_{\mathfrak{so}(10)} = 0 \quad \Rightarrow$ no initial mixing

- *Since the generators of the canonical basis are Killing orthogonal in $\mathfrak{so}(10)$, the value of the kinetic field strength mixing parameter α must vanish at the unification scale. That is, $\alpha(M_u) = 0$.*

For arbitrary $U(1)_1 \times U(1)_2$

$$\mathcal{L}_{kinetic} = -\frac{1}{4}((F^1_{\mu\nu})^2 + 2\alpha F^1_{\mu\nu}F^{2\mu\nu} + (F^2_{\mu\nu})^2 + \dots)$$

with covariant derivative

$$D = \partial - iT^1 g_1 A^1 - iT^2 g_2 A^2$$

Redefining the gauge fields by rotation and rescaling one finds

$$\mathcal{L}_{kinetic} = -\frac{1}{4}((\mathcal{F}^1_{\mu\nu})^2) + (\mathcal{F}^2_{\mu\nu})^2)$$

with the covariant derivative in the "upper triangular" form

$$D = \partial - i(T^1, T^2) \begin{pmatrix} \mathcal{G}_1 & \boxed{\mathcal{G}_M} \\ 0 & \mathcal{G}_2 \end{pmatrix} \begin{pmatrix} \mathcal{A}^1 \\ \mathcal{A}^2 \end{pmatrix}$$

$$\mathcal{G}_1 = g_1, \quad \mathcal{G}_2 = \frac{g_2}{\sqrt{1-\alpha^2}}, \quad \mathcal{G}_M = \frac{-g_1\alpha}{\sqrt{1-\alpha^2}}$$

That is, kinetic mixing reappears as an off-diagonal $\mathcal{G}_M$ in the covariant derivative. Note that

$$\alpha \to 0 \quad \Rightarrow \quad \mathcal{G}_2 \to g_2 \,, \quad \mathcal{G}_M \to 0$$

The RGE for $\mathcal{G}_M$ is

$$\frac{d\mathcal{G}_M}{dt} = \frac{1}{16\pi^2}\beta_M$$

where

$$\beta_M = \mathcal{G}_2^2\mathcal{G}_M B_{22} + \mathcal{G}_M^3 B_{11} + 2\mathcal{G}_1^2\mathcal{G}_M B_{11} + 2\mathcal{G}_2\mathcal{G}_M^2 B_{12} + \mathcal{G}_1^2\mathcal{G}_2 B_{12}$$

and

$$B_{ij} = Tr(T^i T^j)_{3\oplus 2\oplus 1\oplus 1}$$

Assuming $\mathcal{G}_M = 0$ at the initial scale, it can only "regrow" from from the last term. $\Rightarrow$ kinetic mixing will vanish at all scales iff

$$Tr(T^1 T^2)_{3\oplus 2\oplus 1\oplus 1} = 0$$

For generic $U(1)_1 \times U(1)_2$ this is not the case.

However, for the canonical basis

$$Tr(Y_{T_{3R}}Y_{B-L})_{16} = 0$$

Furthermore,

$$[Y_{T_{3R}}]_{H,\bar{H}} = (1)\mathbf{1}_2 \oplus (-1)\mathbf{1}_2 \quad , \quad [Y_{B-L}]_{H,\bar{H}} = (0)\mathbf{1}_2 \oplus (0)\mathbf{1}_2$$

$\Rightarrow$

$$Tr(Y_{Y_{3R}}Y_{B-L})_{H\bar{H}} = 0$$

Conclusion:

- *The generators of the canonical basis are such that $Tr(T^1T^2) = 0$ when the trace is performed over the matter and Higgs spectrum of the MSSM. This guarantees that if the original kinetic mixing parameter vanishes, then α and, hence, $\mathcal{G}_M$ will remain zero under the RG at any scale. This property of not having kinetic mixing greatly simplifies the renormalization group analysis of the $SU(3)_C \times SU(2)_L \times U(1)_{T_{3R}} \times U(1)_{B-L}$ low energy theory.*

What about non-canonical bases? We can prove a theorem that

- *The only basis of $\mathfrak{h}_{3\oplus 2} \subset \mathfrak{h}$ for which $U(1)_{Y_1} \times U(1)_{Y_2}$ kinetic mixing vanishes at all values of energy-momentum is the canonical basis $Y_{T_{3R}}$, Y_{B-L} and appropriate multiples of this basis.*

Sequential Wilson Line Breaking

$\pi_1(X/(\mathbb{Z}_3 \times \mathbb{Z}_3)) = \mathbb{Z}_3 \times \mathbb{Z}_3$ $\Rightarrow$ 2 independent classes of non-contractible curves. $\Rightarrow$ each Wilson line has a mass scale $\propto r^{-1}$ of the curve it "wraps". Denote these by $M_{\chi_{T_{3R}}}, M_{\chi_{B-L}}$.

Three possibilities: $M_{\chi_{T_{3R}}} \simeq M_{\chi_{B-L}}$, $M_{\chi_{B-L}} > M_{\chi_{T_{3R}}}$ or $M_{\chi_{T_{3R}}} > M_{\chi_{B-L}}$. First consider

$\underline{M_{\chi_{B-L}} > M_{\chi_{T_{3R}}}}$:

Recall $Y_{B-L} = 2(H_1 + H_2 + H_3) = 3(B-L)$. In addition to $\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L$ this commutes with α_5 and, hence, $\mathfrak{su}(2)_R$.

$\Rightarrow$

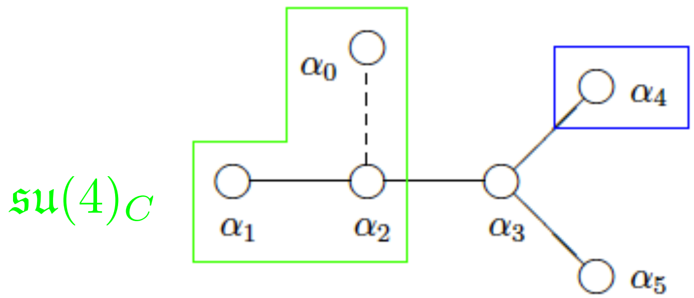
$$Spin(10) \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$$

Gauge group of the "left-right" model. At $M_I = M_{\chi_{T_{3R}}}$ $\chi_{T_{3R}}$ turns on and breaks $SU(2)_R \to U(1)_{T_{3R}}$.

Second, consider

$\underline{M_{\chi_{T_{3R}}} > M_{\chi_{B-L}}}$:

Recall $Y_{T_{3R}} = \;\; H_4 + H_5 = 2(Y - \frac{1}{2}(B-L)) = 2T_{3R}$. In addition to $\mathfrak{su}(3)_C \oplus \mathfrak{su}(2)_L$ this commutes with



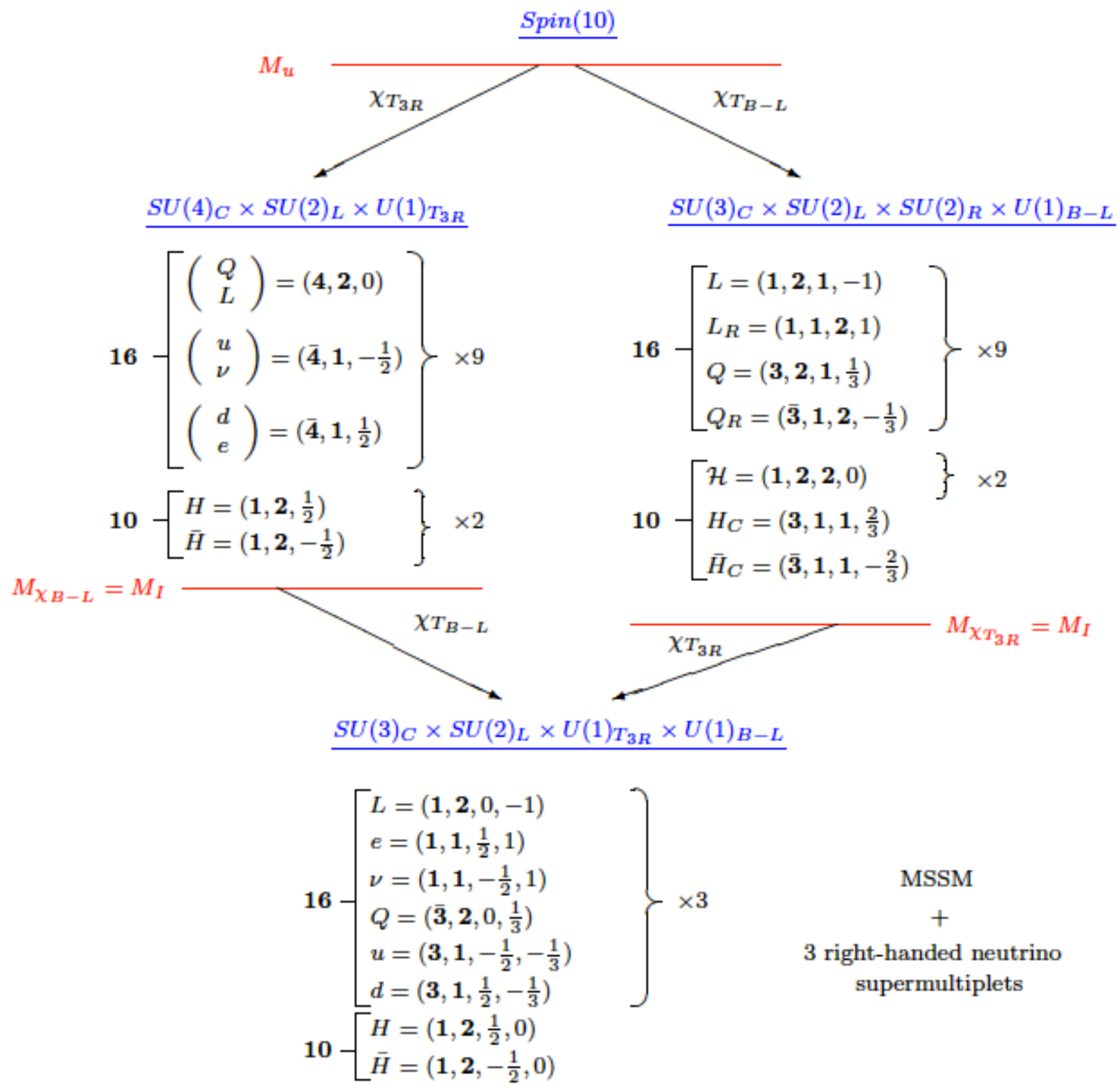
$\Rightarrow$

$$Spin(10) \to SU(4)_C \times SU(2)_L \times U(1)_{T_{3R}}$$

Gauge group of the "Pati-Salam" model. At $M_I = M_{\chi_{B-L}}$

χ_{B-L} turns on and breaks $SU(4)_C \to SU(3)_C \times U(1)_{B-L}$.

In each case, can compute the exact zero-mode spectrum in the intermediate region.



- The two sequential Wilson line breaking patterns of $Spin(10)$.

The 3R/B-L Breaking Scale

At a scale $M_{B-L} < M_I$ must spontaneously break

$$U(1)_{T_{3R}} \times U(1)_{B-L} \to U(1)_Y$$

Since $\mathcal{G}_M = 0$ in the canonical basis $\Longrightarrow$ the $U(1)_{T_{3R}} \times U(1)_{B-L}$ covariant derivative is

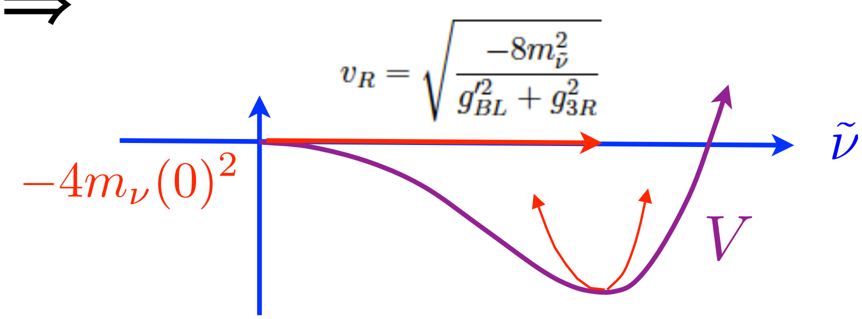
$$D = \partial - i\left(Y - \tfrac{1}{2}(B-L), \sqrt{\tfrac{3}{8}}(B-L)\right)\begin{pmatrix} g_{3R} & \boxed{0} \\ 0 & g_{BL} \end{pmatrix}\begin{pmatrix} W_R^0 \\ B_{B-L} \end{pmatrix}$$

The potential for a right-handed sneutrino $\tilde{\nu}$ is approximately

$$V = m_{\tilde{\nu}}^2|\tilde{\nu}|^2 + \frac{1}{8}(g_{BL}'^2 + g_{3R}^2)|\tilde{\nu}|^4$$

where $g'_{BL} = \sqrt{\frac{3}{2}}g_{BL}$ and $m_{\tilde{\nu}}^2$ is the soft ~~SUSY~~ parameter.

RG scaling $\Longrightarrow$



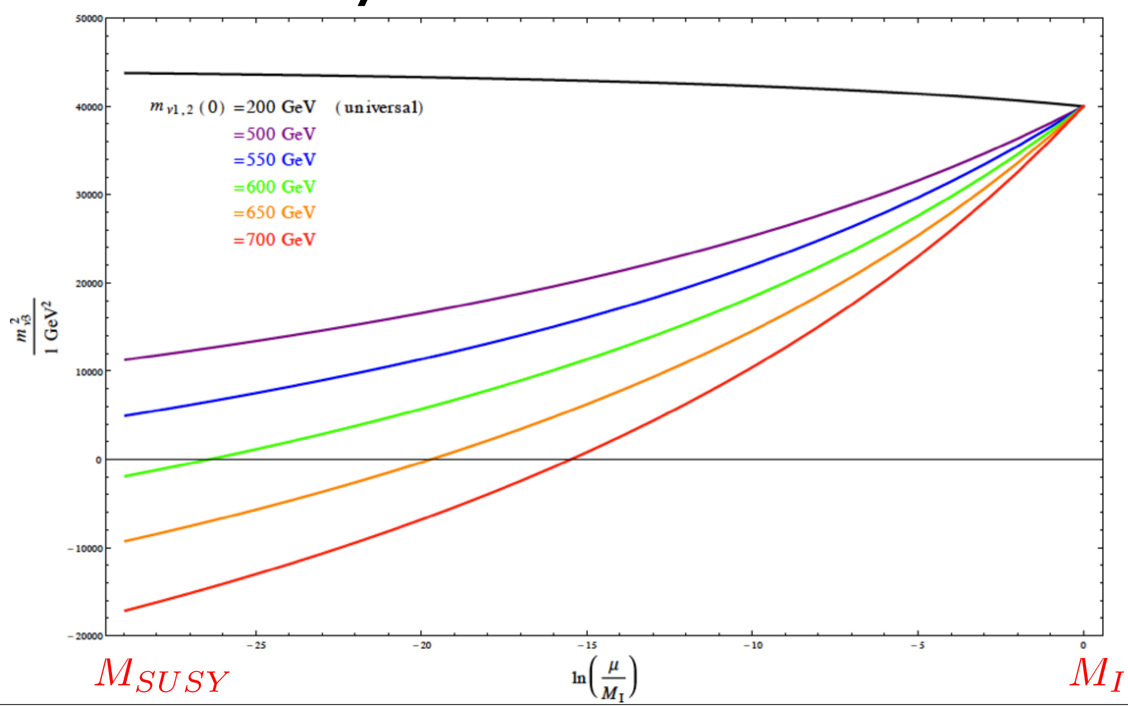
In the canonical basis, no kinetic mixing implies we can solve the sneutrino soft breaking mass RGEs analytically. For example, in the "left-right" model

$$16\pi^2 \frac{d}{dt} m_{\nu}^2 = -3 g_{B-L}^2 |M_{B-L}|^2 - 2 g_{I_3^R}^2 |M_{I_3^R}|^2 + \frac{3}{4} g_{B-L}^2 S_{B-L} - g_{I_3^R}^2 S_{I_3^R}$$

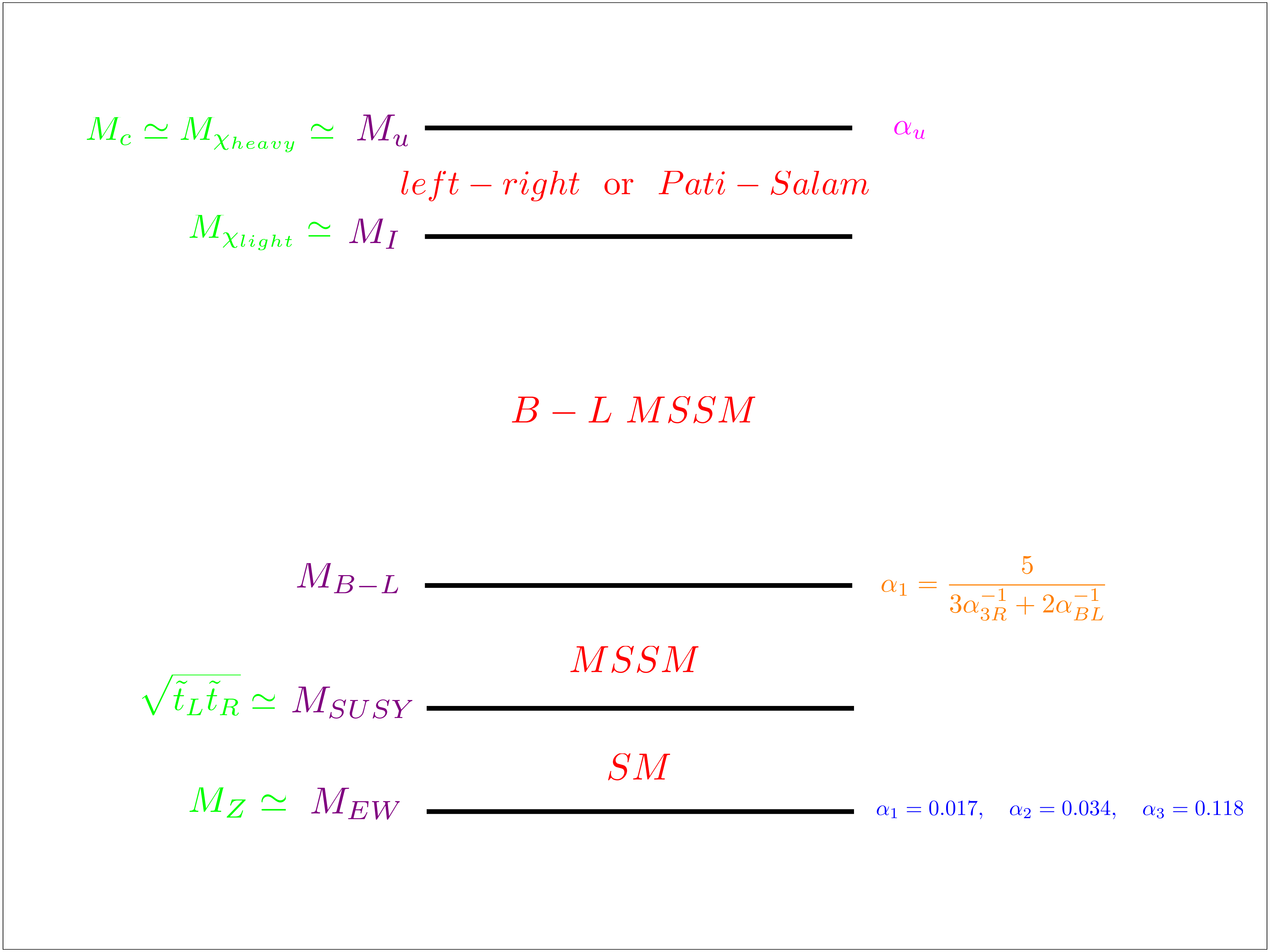
$$16\pi^2 \frac{d}{dt} S_{B-L} = 12 g_{B-L}^2 S_{B-L}$$

$$16\pi^2 \frac{d}{dt} S_{I_3^R} = 14 g_{I_3^R}^2 S_{I_3^R}$$

Taking $M_{1/2} = 200\ GeV$ and all soft masses universal except the first and second family sneutrinos $\Rightarrow$



$\leftarrow m_{\tilde{\nu}_3}^2(0) = (200\ GeV)^2$



$M_c \simeq M_{\chi_{heavy}} \simeq M_u$
α_u
$left-right$ or $Pati-Salam$
$M_{\chi_{light}} \simeq M_I$
$B-L\ MSSM$
M_{B-L}
$\alpha_1 = \frac{5}{3\alpha_{3R}^{-1} + 2\alpha_{BL}^{-1}}$
$MSSM$
$\sqrt{\tilde{t}_L \tilde{t}_R} \simeq M_{SUSY}$
SM
$M_Z \simeq M_{EW}$
$\alpha_1 = 0.017, \quad \alpha_2 = 0.034, \quad \alpha_3 = 0.118$

Example: Taking the "left-right" model, choosing

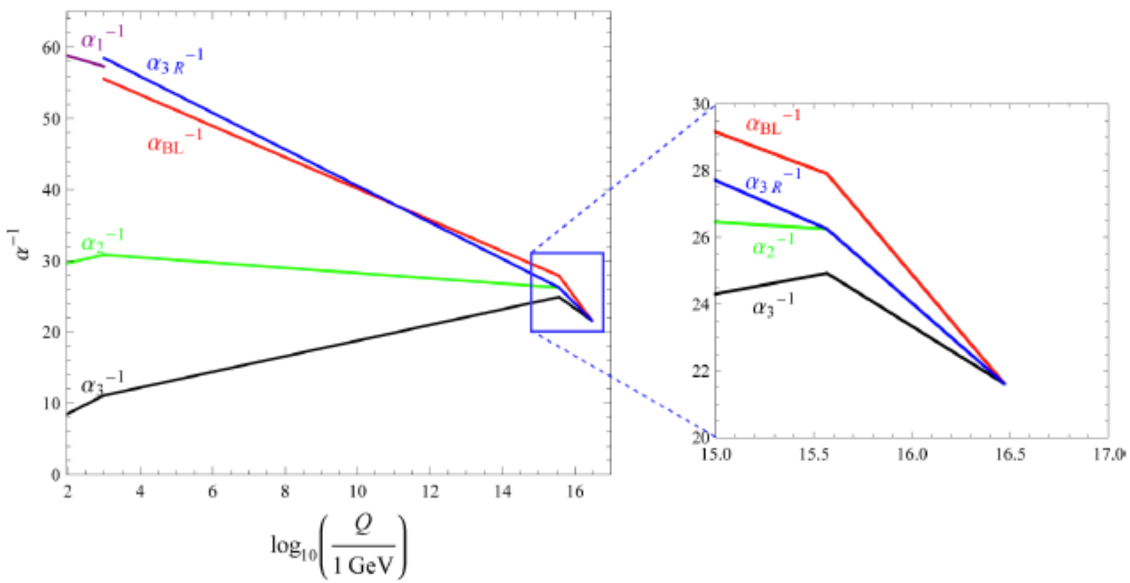
$$M_{SUSY} = 1\ TeV, \quad M_{B-L} = 1\ TeV$$

and enforcing gauge unification, we find

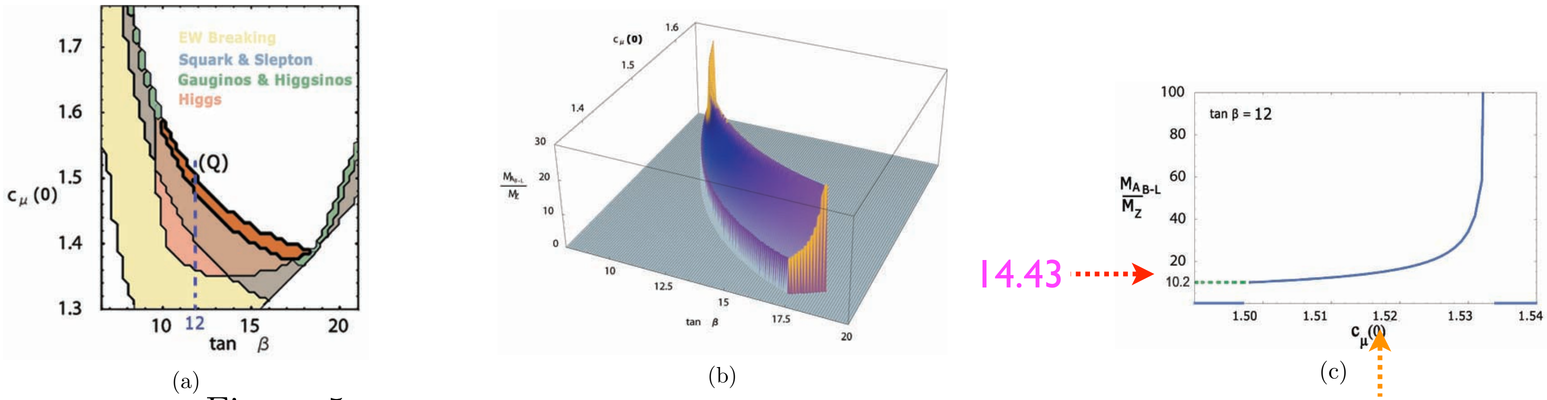
$$M_u = 3.0 \times 10^{16}\ GeV, \quad M_I = 3.7 \times 10^{15}\ GeV$$

$$\alpha_u = 0.046, \quad \alpha_{3R}(M_{B-L}) = 0.0171, \quad \alpha_{BL}(M_{B-L}) = 0.0180$$

The running gauge parameters are



- One-loop RGE running of the inverse gauge couplings, α_i^{-1} in the case of the left-right model with $M_{B-L} = 1\ TeV$ with an enlarged image of the intermediate region.



(a) (b) (c)

Figure 5: Plot (a) shows the $c_\mu(0)$-$\tan\beta$ plane corresponding to point (B) in Figure 2 with the phenomenologically allowed region indicated in dark brown. The mass spectrum at (Q) was presented in Table 3. A plot of the hierarchy M_{B-L}/M_Z over the allowed region is given in (b). Graph (c) shows the hierarchy as a function of $c_\mu(0)$ along the $\tan\beta = 12$ line passing through (Q).

Particle	Symbol	Mass [GeV]	Particle	Symbol	Mass [GeV]
Squarks	$\tilde{Q}_{1,2}$	850	Higgs	h^0	127
	$\tilde{t}_{1,2}, \tilde{b}_{1,2}$	775, 953		H^0	382
	$\tilde{b}_3^{(1)}, \tilde{b}_3^{(2)}$	670, 915		A^0	381
	$\tilde{t}_3^{(1)}, \tilde{t}_3^{(2)}$	456, 737		$H^\pm$	390
Sleptons	$\tilde{L}_{1,2}$	1255	Neutralinos	$\tilde{N}_1^0$	97
	$\tilde{\tau}_{1,2}$	1237		$\tilde{N}_2^0$	189
	$\tilde{\tau}_3^{(1)}, \tilde{\tau}_3^{(2)}$	1217, 1246		$\tilde{N}_3^0$	499
Charginos	$\tilde{\chi}^\pm, \tilde{\chi}'^\pm$	190, 510		$\tilde{N}_4^0$	509
Gluinos	$\tilde{g}$	712	Z'	$A_{B-L}, \tilde{A}_{B-L}$	1314, 1348

Table 3: The predicted spectrum at point (Q) in Figure 3. The tilde denotes the superpartner of the respective particle. The superpartners of left-handed fields are depicted by an upper case label whereas the lower case is used for right-handed fields. The mixing between the third family left- and right-handed scalar fields is incorporated.