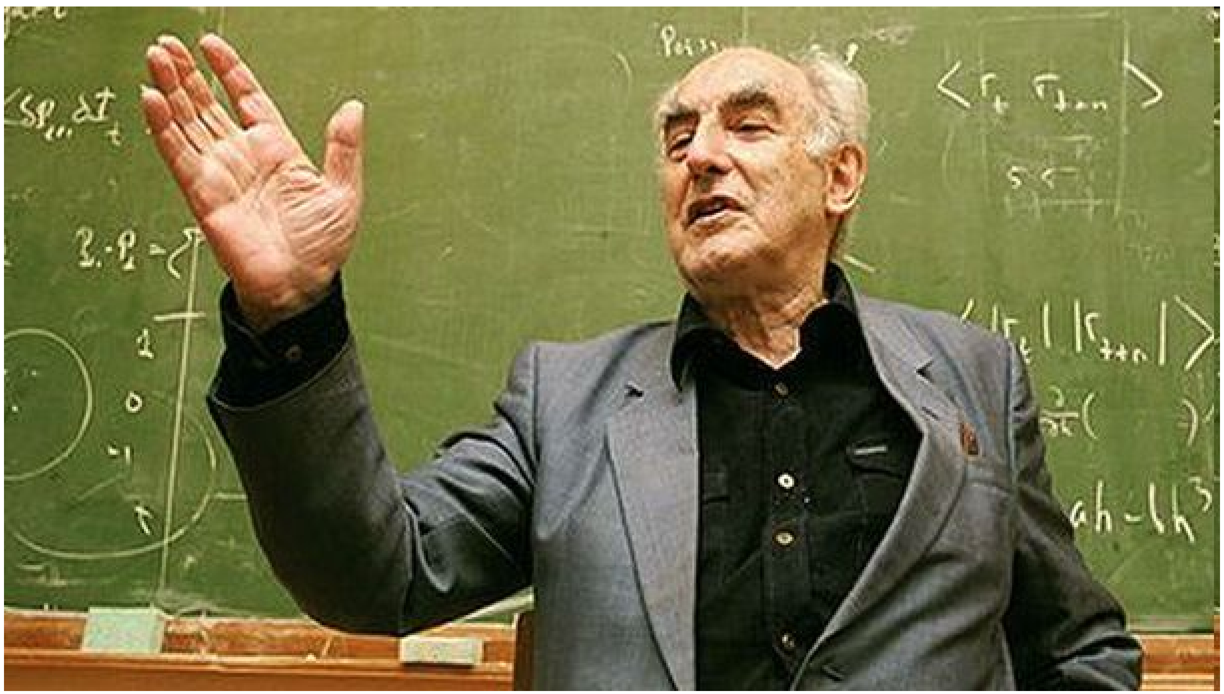
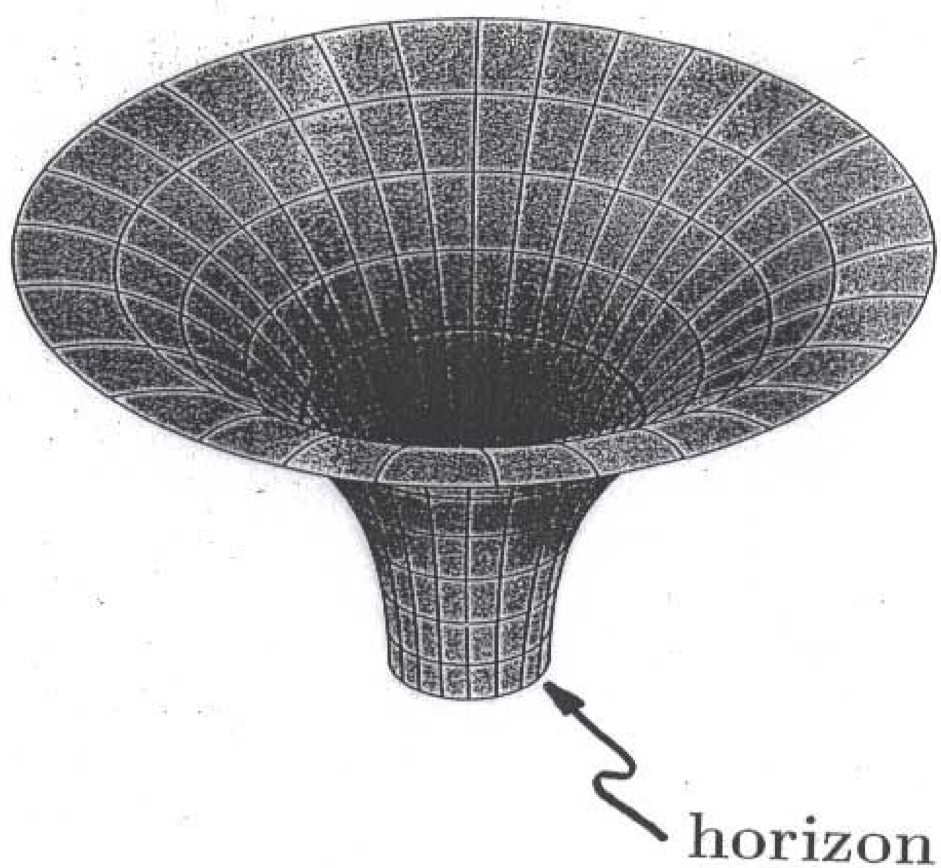
Ginzburg Conference on Physics, 2012



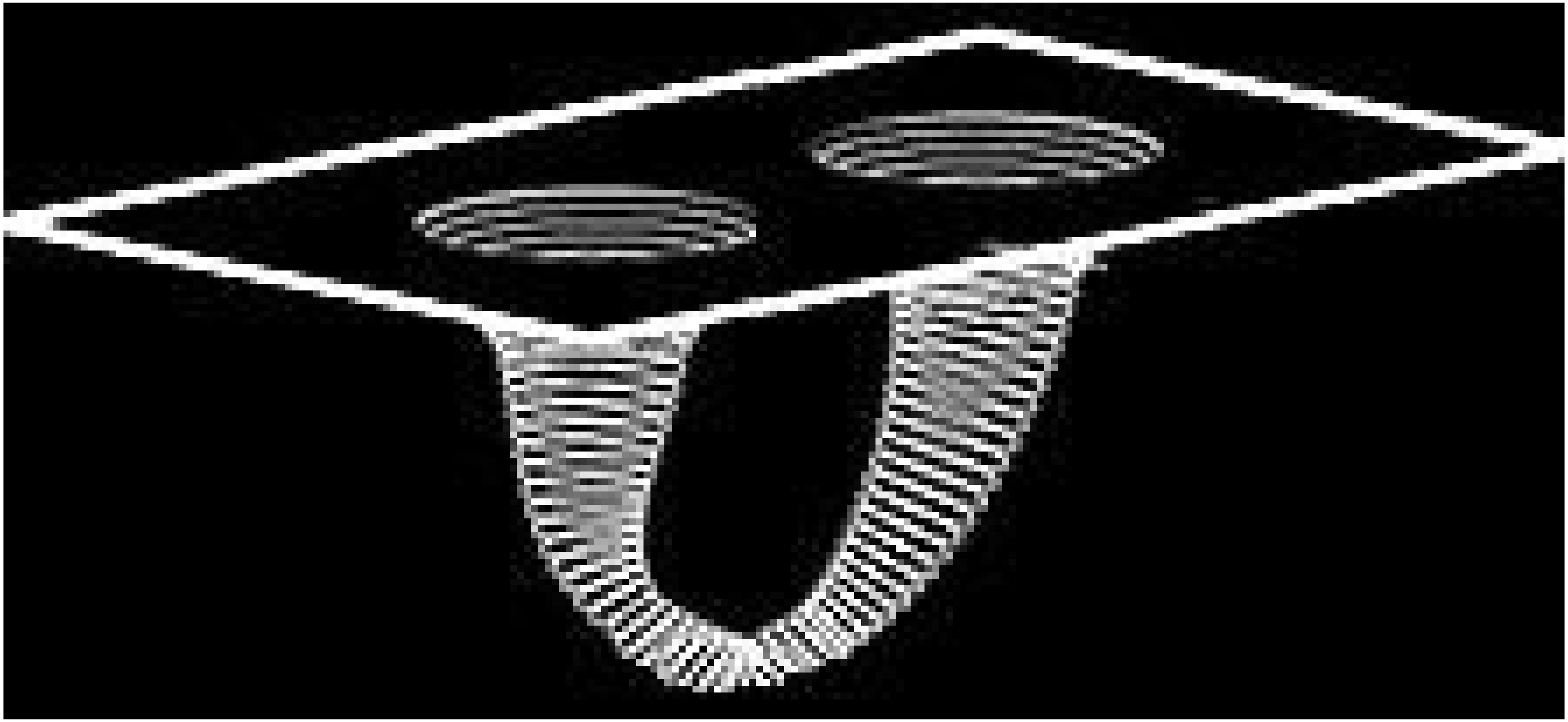
Stability Analysis of Wormhole

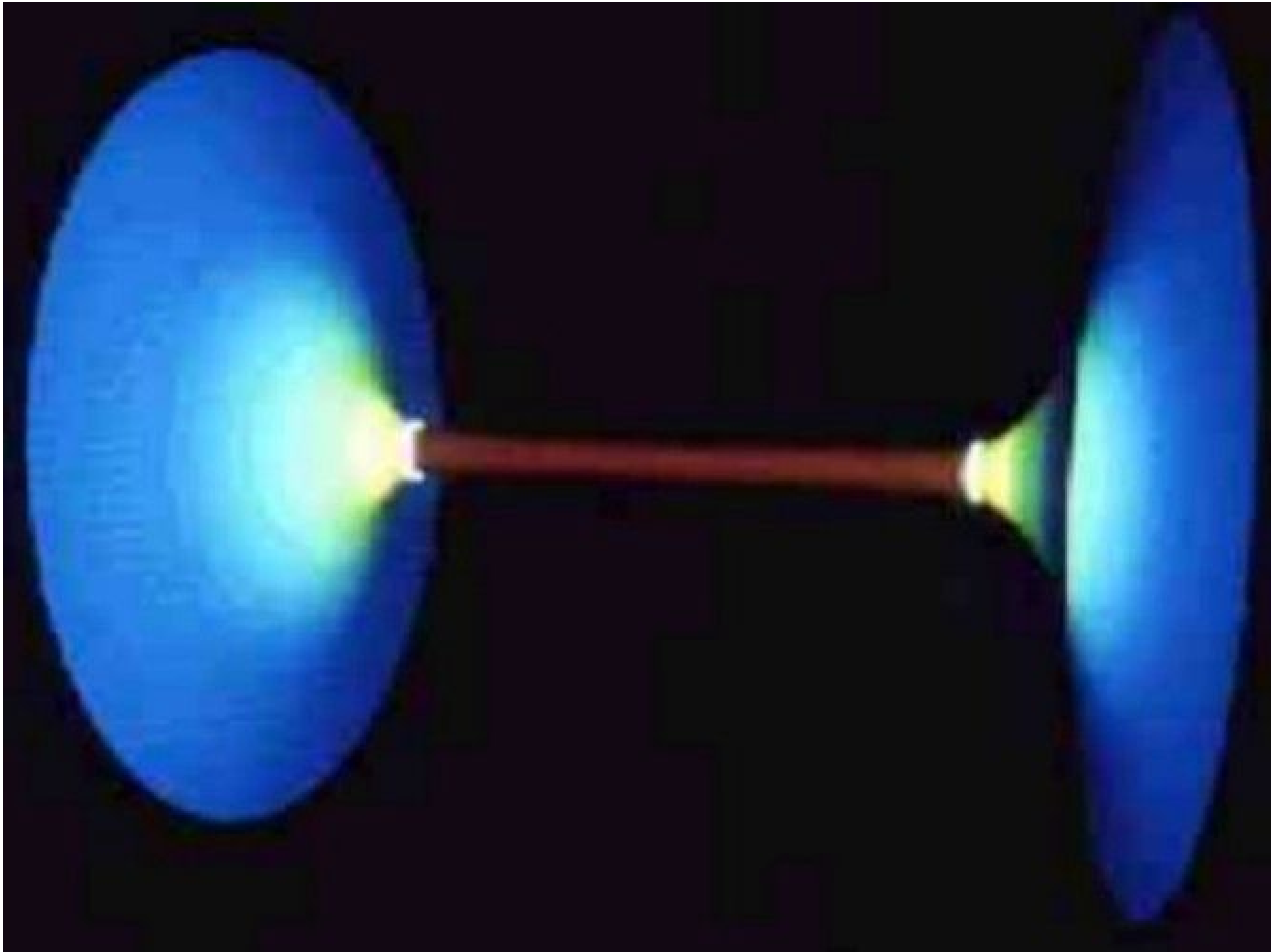
I. D. Novikov and A. A. Shatskiy

SPACE IS CURVED IN THE VICINITY OF A BLACK HOLE



Two-dimensional Analogy of a Wormhole

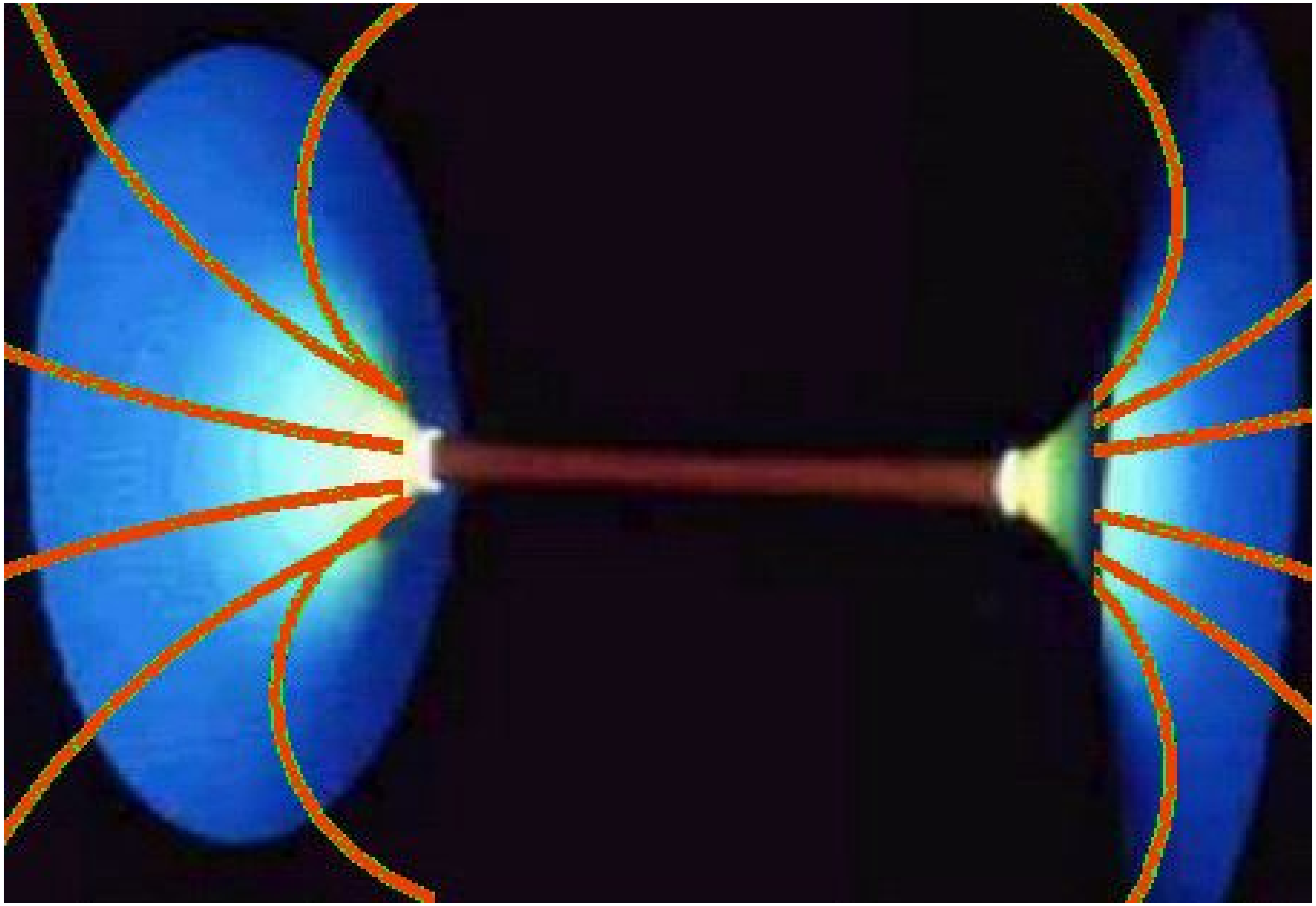




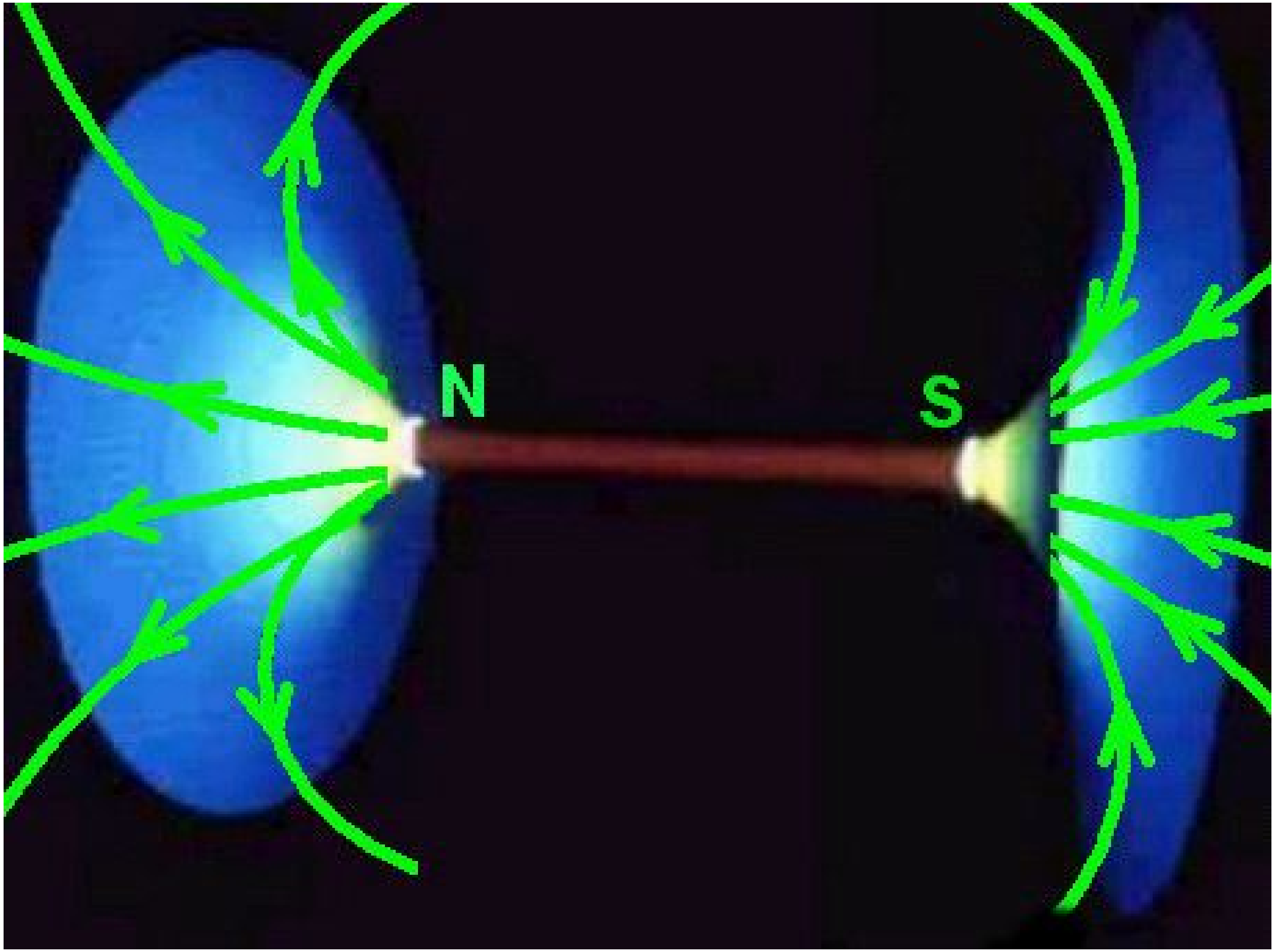
In order for a static wormhole can exist the presence of the exotic matter in the throat of the wormhole is needed.

Depending on the type of the matter wormholes can be of different types.

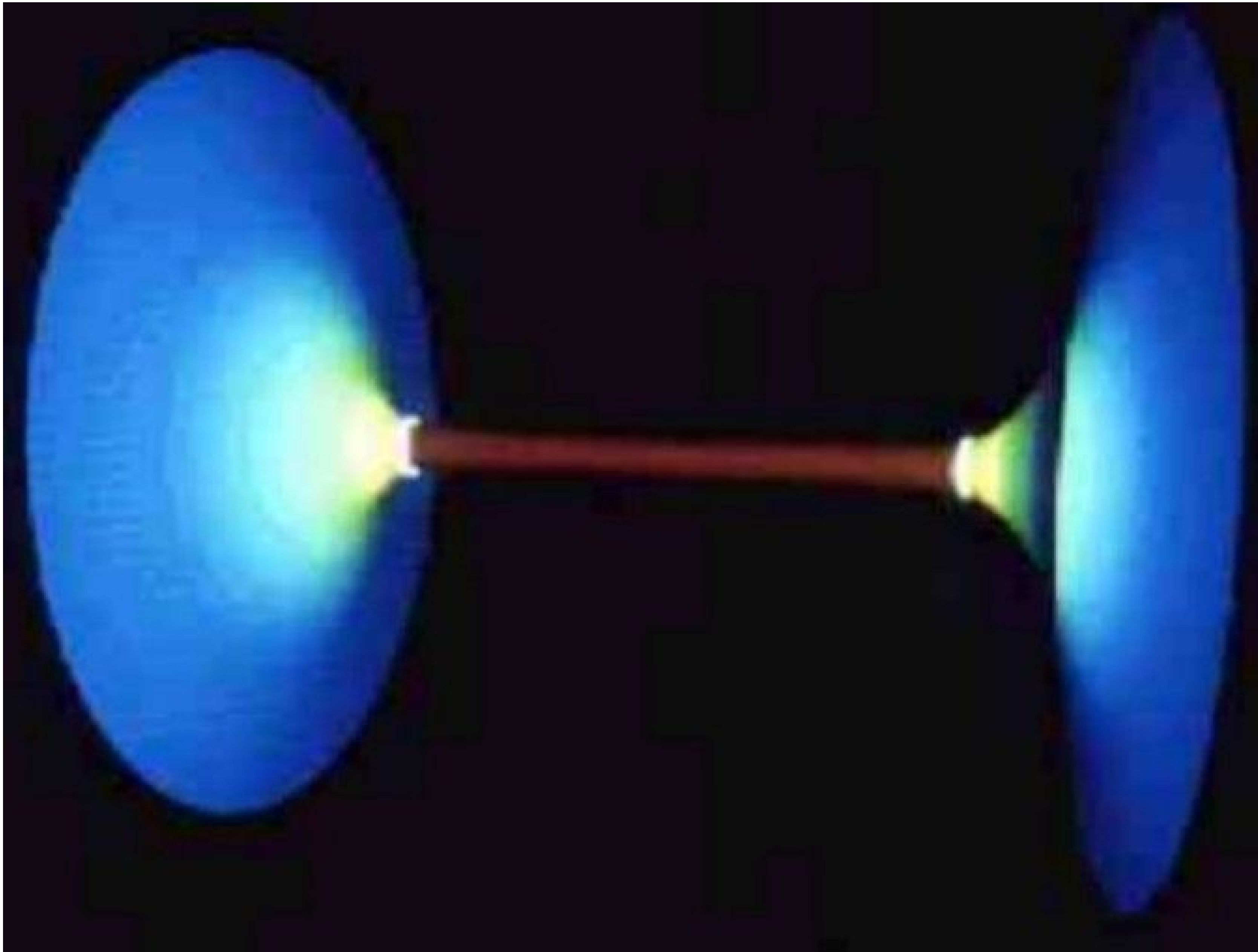
For example Morris-Thorne-Ellis-Bronnikov wormhole with a scalar field



or wormholes with the magnetic field



or Brance-Dikke Wormholes



The question arises:
Are such objects stable?

We investigated this question
with respect to the small perturbations.

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dx^2 - e^{\mu} d\Omega^2$$

The linearized Einstein equation corresponding to this metric can be written as

$$\delta T_{\tau}^{\tau} = \qquad f = \frac{\lambda - \eta - 3x\eta' + x\lambda'}{\xi^2} - \eta'' + \frac{2\eta + \lambda}{\xi^4},$$

$$\delta T_R^R = -hf = f + \ddot{\eta} + \frac{\eta(1 - x^2)}{\xi^4} + \frac{\lambda x^2}{\xi^4} - \frac{x(\nu' + \eta')}{\xi^2} = 0,$$

$$-\delta T_0^0 = hf = \frac{\nu'' + \eta'' - \ddot{\lambda} - \ddot{\eta}}{2} + \frac{2\eta - \lambda}{\xi^4} + \frac{x(\eta' - \lambda'/2 + \nu'/2)}{\xi^2},$$

$$\delta T_{\tau}^{R} = \qquad \dot{\eta}' + x(\dot{\eta} - \dot{\lambda})/\xi^2 = 0.$$

$$8\pi\varepsilon \equiv \frac{-2q^2}{(q^2 + x^2)^2} + f(x,t), \quad e^{\mu} \equiv (q^2 + x^2)e^{\eta(x,t)}, \quad \xi^2 \equiv (q^2 + x^2), \quad 8\pi p = hf.$$

If the perturbations grow with time,
the wormhole is unstable,

and if they are damping or oscillating
– the wormhole is stable.

In the first papers (Armendariz-Picon) it was concluded that the wormholes with scalar field are stable.

Unfortunately this work have been erroneous.

In the future, numerous attempts to find a stable wormhole failed.

We have set the task to find a stable model of the wormhole.

To simplify the problem, we carried out a study on the stability only with respect to the radial modes (which, probably, are the most unstable of all).

In addition, we added a special type of pressure.

Namely we hypothesized that the additional pressure is proportional to the f (deviation of the energy density from its equilibrium value)

$$P \sim \Delta\varepsilon \, r^4/(r_0^2+r^2)^2$$

All parameters of the perturbations can be written as:

$$\eta(z,t) = \sum_{n=0}^{\infty} \Psi_n(z) \exp(i w_n t)$$

Here z – the new radial coordinate

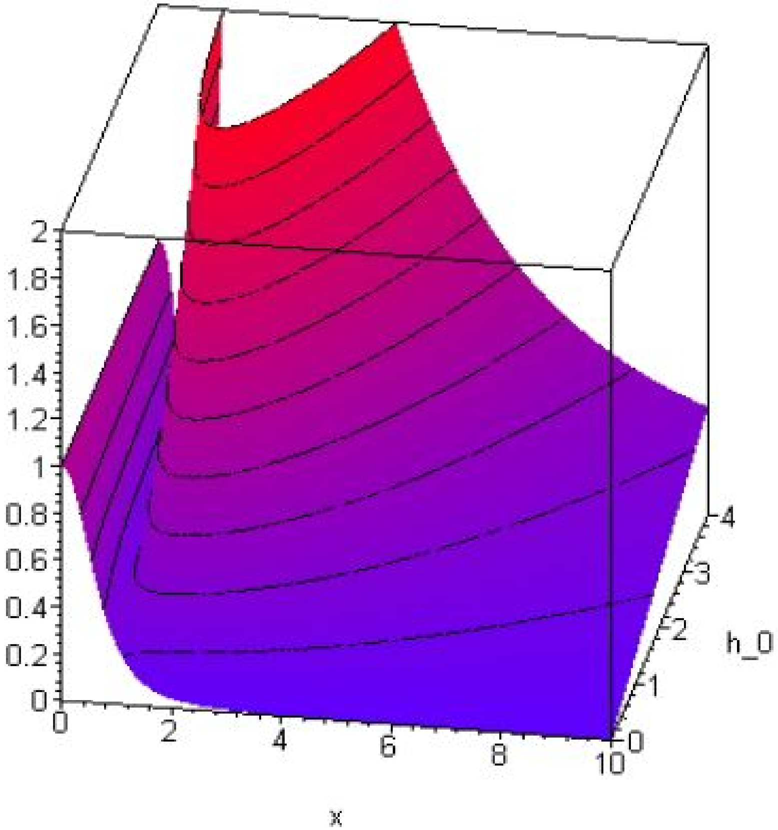
and w_n – frequency oscillation of the n-harmonic.

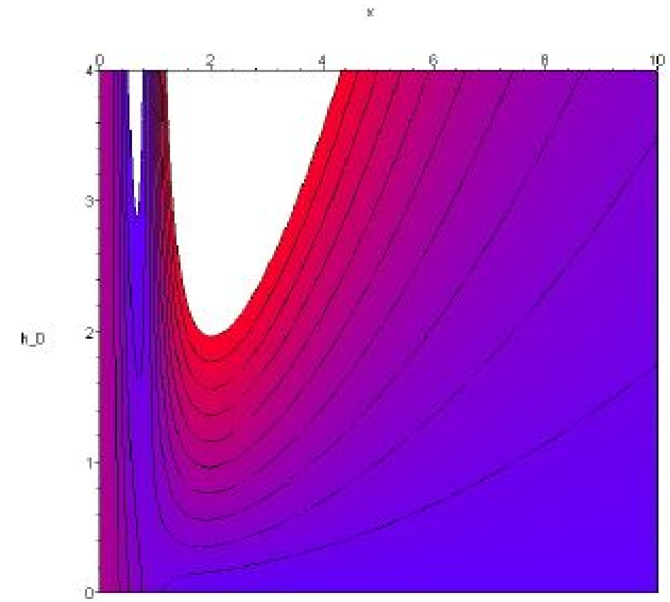
If w_n are real, the wormhole is stable !

The final equation for the perturbations lead to

$$\Psi_{n,zz} + w_n^2 \Psi_n - U(z)\Psi_n = 0$$

Her U(z) is the effective potencial.
This equation is similar to the stationary Schrodinger equation where w_n^2 are the eigenvalues





Shape of the surface of the potential $U(x, h_0)$.
It can thus be seen that the range $0 < h_0 < 2.8$ corresponds to the domain $U > 0$ for any values of x.

CONCLUSIONS:

We proved that the model of a stationary and traversable wormhole that would be stable against small spherical perturbations (stable against spherical modes) could be constructed in terms of general relativity.

Thank you !