Conformal mechanisms for generation of cosmological density perturbations

Maxim Libanov

INR RAS, Moscow

In collaboration with
S. Mironov, V. Rubakov,
S. Ramazanov

May 29, 2012

Lebedev Institute

Outline

- **Introduction**
- **Two ways of getting flat spectrum of field perturbations**
  - Why do they have the same outcome?
- **Properties at non-linear level**
- **Two sub-scenarios and their predictions:**
  - Statistical anisotropy
  - Non-Gaussianity
- **Summary**

Introduction

- Primordial scalar perturbations: Gaussian (or nearly Gaussian) random field $\zeta(\vec{x})$
  - $\zeta(\vec{x})$ obeys Wick theorem
  - This suggests the origin: enhanced *vacuum fluctuations of some (almost) free quantum field*
- Flat or nearly flat power spectrum

$$\langle\zeta(\vec{k})\zeta(\vec{k}')\rangle = \frac{1}{4\pi k^3}\mathcal{P}(k)\delta(\vec{k}+\vec{k}')$$

$$\mathcal{P} \propto k^{n_s-1}$$

  - Flat power spectrum, $n_s = 1$ Harrison '01, Zeldovich '72
    consistent with observations within $2 \div 3\sigma$
  - Small red tilt favored, $n_s - 1 \simeq -0.4$

There must be some symmetry behind this property

- Inflation: symmetry of de Sitter space-time, $SO(4,1)$

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$

Symmetry: spatial dilatations supplemented by time translations

$$\vec{x} \to \lambda \vec{x}\,, \quad t \to t - \frac{1}{2H} \log \lambda$$

Inflation automatically generates nearly flat spectrum.

- Alternative: conformal symmetry $SO(4,2)$

Conformal group includes dilatations, $x^\mu \to \lambda x^\mu$.

$\Longrightarrow$ No scale, good chance for flatness of spectrum

First mentioned by Antoniadis, Mazur, Mottola' 97

Concrete models: Rubakov' 09

Creminelli, Nicolis, Trincherini' 10

What if our Universe started off from or passed through
an unstable conformal state
and then evolved to much less symmetric state we see today?

- Further motivation: Alternatives to inflation:
  - Contraction - Bounce - Expansion
  - Start up from static state

Creminelli et.al.'06; '10

Difficult, but not impossible.

How to generate density perturbations with nearly flat spectrum?

Two ways of getting flat scalar spectrum

Way # 1: conformal rolling

Rubakov'09

Conformal $\oplus$ global symmetry instead of de Sitter symmetry

NB: Main requirement: long evolution before the hot stage. But otherwise insensitive to regime of cosmological evolution. Can work at inflation and its alternatives.

Model:

$$S = S_{G+M} + S_{\varphi}$$

S_{G+M} - gravity $\oplus$ dominating matter

S_{φ} - conformal complex scalar field φ with negative quartic potential. Spectator until late epoch.

$$S = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\varphi^*\partial_\nu\varphi + \frac{R}{6}|\varphi|^2 - (-h^2|\varphi|^4)\right]$$

Conformal symmetry. Global symmetry U(1).

$\varphi = 0$ - unstable state with unbroken conformal symmetry.

NB: Conformal symmetry explicitly broken at large fields. To be discussed later.

Homogeneous and isotropic Universe,

$$ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2]$$

In terms of the field $\chi(\eta, \vec{x}) = a(\eta)\varphi(\eta, \vec{x}) = \chi_1 + i\chi_2$, evolution is Minkowskian,

$$\eta^{\mu\nu}\partial_\mu\partial_\nu\chi - 2h^2|\chi|^2\chi = 0$$

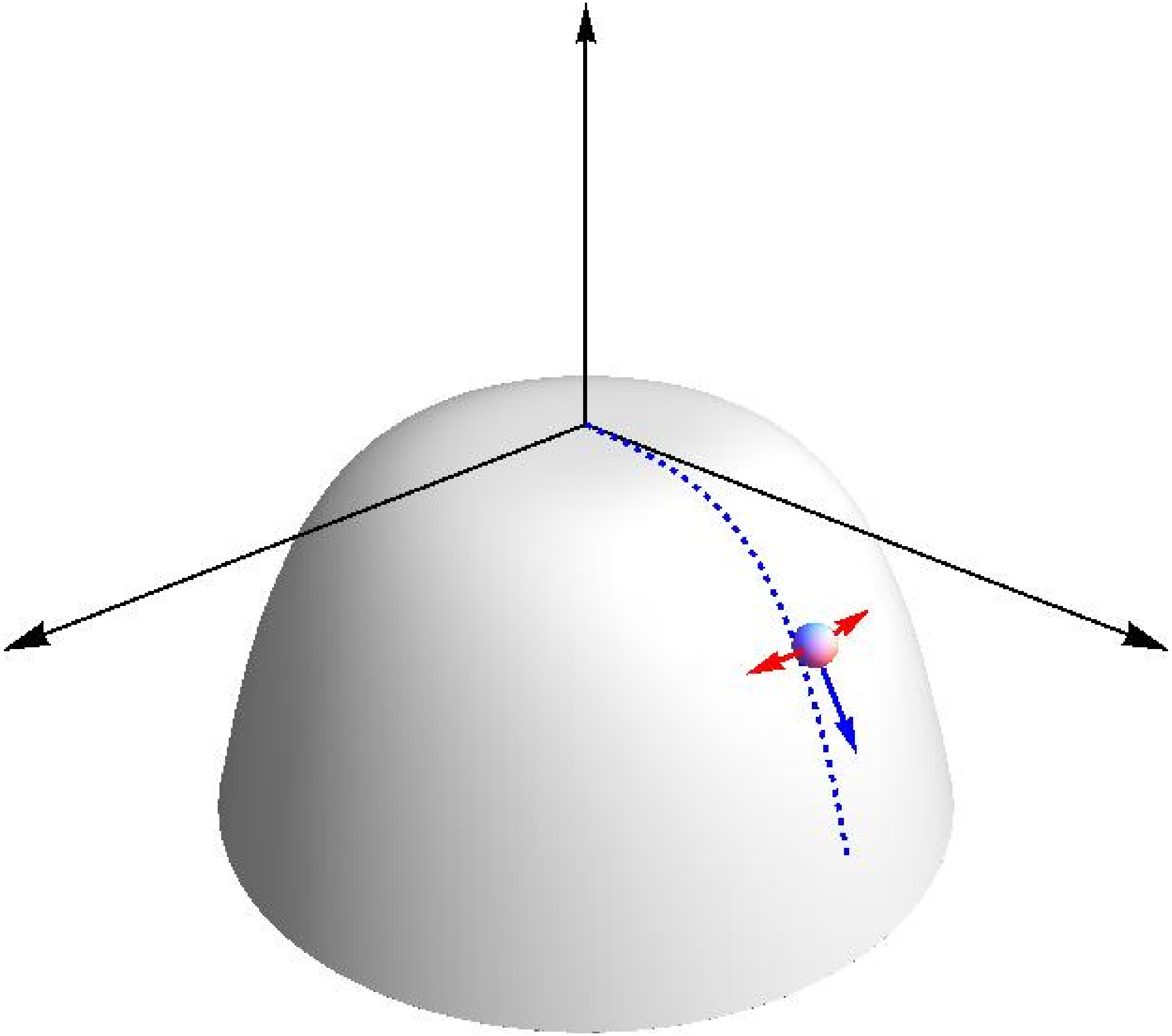
Homogeneous background solution

Attractor (real without loss of generality)

$$\chi_c(\eta) = \frac{1}{h(\eta_* - \eta)}$$

η_* = constant of integration, end time of roll.

NB: Particular behavior $\chi_c \propto (\eta_* - \eta)^{-1}$ dictated by conformal symmetry.



Fluctuations of Im χ automatically have flat spectrum

Linearized equation for fluctuation $\delta\chi_2 \equiv \mathrm{Im}\chi$. Mode of 3-momentum k:

$$\frac{d^2}{d\eta^2}\delta\chi_2 + k^2\delta\chi_2 - 2h^2\chi_c^2\delta\chi_2 = 0$$

[recall $h\chi_c = 1/(\eta_* - \eta)$]

Regimes of evolution:

- Early times, $k \gg 1/(\eta_* - \eta)$, short wavelength regime, χ_c negligible, free Minkowskian field

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} e^{-ik\eta} A_{\vec{k}} \; + \; \text{h.c.}$$

- Late times, $k \ll 1/(\eta_* - \eta)$, long wavelength regime, term with χ_c dominates,

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{1}{k(\eta_* - \eta)} \cdot A_{\vec{k}} \;+\; h.c.$$

✓ Phase of the field φ freezes out:

$$\delta\theta = \frac{\delta\chi_2}{\chi_c} = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} \cdot \frac{h}{k} \cdot A_{\vec{k}} \;+\; h.c.$$

✓ Power specrum of phase is flat:

$$\langle\delta\theta^2\rangle = \frac{h^2}{2(2\pi)^3}\int\frac{d^3k}{k^3} \Longrightarrow \mathcal{P}_{\delta\theta} = \frac{h^2}{(2\pi)^2}$$

✓ This is automatic consequence of global U(1) and conformal symmetry

- To see this, consider long wavelength regime: $\vec{k}$ negligible,
- equation for $\delta\chi_2$ is equation for spatially homogeneous perturbation.

  - χ_c is solution to full field equation, $e^{i\alpha}\chi_c$ also $\Longrightarrow$
  - $\delta\chi = i\alpha\chi_c$ is solution to perturbation equation $\Longrightarrow$

$$\delta\chi_2 : \quad e^{-ik\eta} \Longrightarrow C(k)\chi_c(\eta) = \frac{1}{k(\eta_* - \eta)}$$

NB: $1/k$ on dimensional grounds.

NB: In fact, equation for $\delta\chi_2$ is precisely the same as equation for minimally coupled massless scalar field in inflating Universe

Comments:

- Mechanism requires long cosmological evolution: need

$$(\eta_* - \eta) \gg 1/k$$

early times, short wavelength regime,
well defined vacuum of the field $\delta\chi_2$.

For $k \sim H_0$ this is precisely the requirement that the horizon problem is solved.

This is probably a pre-requisite for any mechanism that generates density perturbations

- Small explicit breaking of conformal invariance $\Longrightarrow$ tilt of the spectrum

Osipov, Rubakov'10

✓ Depends both on the way conformal invariance is broken and on the evolution of scale factor

Way # 2: Galilean Genesis

Creminelli, Nicolis, Trincherini '10

Begin with galileon field π, Lagrangian

$$\mathcal{L}_\pi = -f^2 e^{2\pi}\partial_\mu\pi\partial^\mu\pi + \frac{f^3}{\Lambda^3}\partial_\mu\pi\partial^\mu\pi \cdot + \frac{f^3}{2\Lambda^3}(\partial_\mu\partial^\mu\pi)^2$$

Conformally invariant. Under dilatations

$$e^{\pi(x)} \to \lambda e^{\pi(\lambda x)}$$

Universe begins from Minkowski space-time. Galileon rolls as

$$e^{\pi_c} = \frac{1}{H_G(t_* - t)}, \quad t < t_*,$$

where $H_G^2 = \frac{2\Lambda^3}{3f}$. Again dictated by conformal invariance.

Initial energy density is zero, then it slowly builds up,

$$H(t) = \frac{1}{3}\frac{f^2}{M_{Pl}^2}\frac{1}{H_G^2(t_* - t)^3}$$

until $(t_* - t_e) \sim H_G^{-1} \cdot f/M_{PL}$.

NB: Hubble parameter grows in time. Strong violation of all energy conditions. Yet fully consistent theory, no ghosts, tachyons, other pathologies.

At some point galileon is assumed to transmit its energy to conventional matter, hot epoch begins.

Galileon perturbations are not suitable for generating scalar perturbations.

Introduce another field θ of conformal weight 0,

$$L_\theta = e^{2\pi}(\partial_\mu\theta)^2 \quad \Longrightarrow \quad L_\theta = \frac{\text{const}}{(t_* - t)^2} \cdot (\partial_\mu\theta)^2$$

Dynamics of perturbations $\delta\theta$ in background π_c is exactly the same as in conformal rolling model.

Similarity is not an accident

Hinterbichler, Khouri '11

General setting:

- Effectively Minkowski space-time
- Conformally invariant theory
- Field ρ of conformal weight $\Delta \neq 0$

$\rho = \text{const} \cdot |\varphi|$ in conformal rolling model

$\rho = \text{const} \cdot e^{\pi}$ in Galilean Genesis; $\Delta = 1$ in both models.

Homogeneous classical solution

$$\rho_c(t) = \frac{1}{(t_* - t)^{\Delta}}$$

by conformal invariance.

NB: t is conformal time in conformal rolling scenario

- Another scalar field θ of conformal weight 0.
- Kinetic term dictated by conformal invariance (modulo field rescaling)

$$L_\theta = \rho^{2/\Delta}(\partial_\mu\theta)^2$$

- Assume potential terms negligible $\Longrightarrow$

Lagrangian in rolling background

$$L_\theta = \frac{1}{(t_* - t)^2} \cdot (\partial_\mu\theta)^2$$

Exactly like scalar field minimally coupled to gravity in de Sitter space, with t = conformal time, $a(t) = \mathrm{const}/(t_* - t)$.

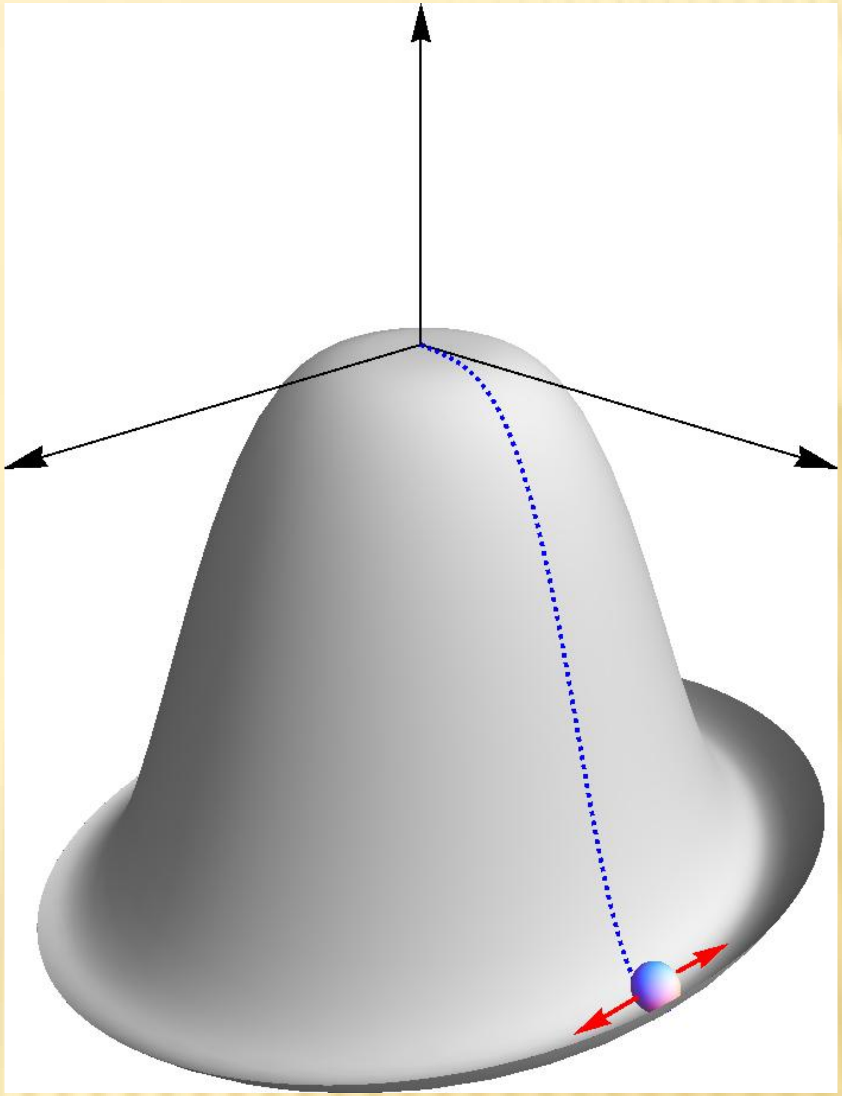
θ develops perturbations with flat power spectrum.

Use conformal rolling model in what follows for definiteness.

Reprocessing field perturbations into adiabatic perturbations

- Assume that conformal evolution ends up at some late time. Scalar potential actually has a minimum at large field.

  Modulus of the field φ freezes out at the minimum of the scalar potential. Assume that energy density of φ is negligible at that time (probably, unimportant).



There are at least two known mechanisms of converting phase perturbations into adiabatic perturbations.

Option # 1: phase θ as curvaton

- Let the phase θ be *pseudo*-Goldstone field interacting with matter
- Generically, phase θ ends up at a slope of its potential
- Perturbations δθ become density perturbations once δθ starts rolling down at radiation domination and in the end delivers its energy to hot medium.

Linde, Mukhanov' 97;
Enqvist, Sloth' 01; Lyth, Wands' 01; Moroi, Takahashi' 01;
K. Dimopoulos et.al.' 2003

Option # 2: Modulated decay

Dvali, Gruzinov, Zaldarriaga' 03

Kofman' 03;

better known in the context of modulated reheating

- Let masses and/or widths of some heavy particles X depend on θ,

$$M_X = M_0 + c_M\theta \quad \text{and/or} \quad \Gamma_X = \Gamma_0 + c_\Gamma\theta$$

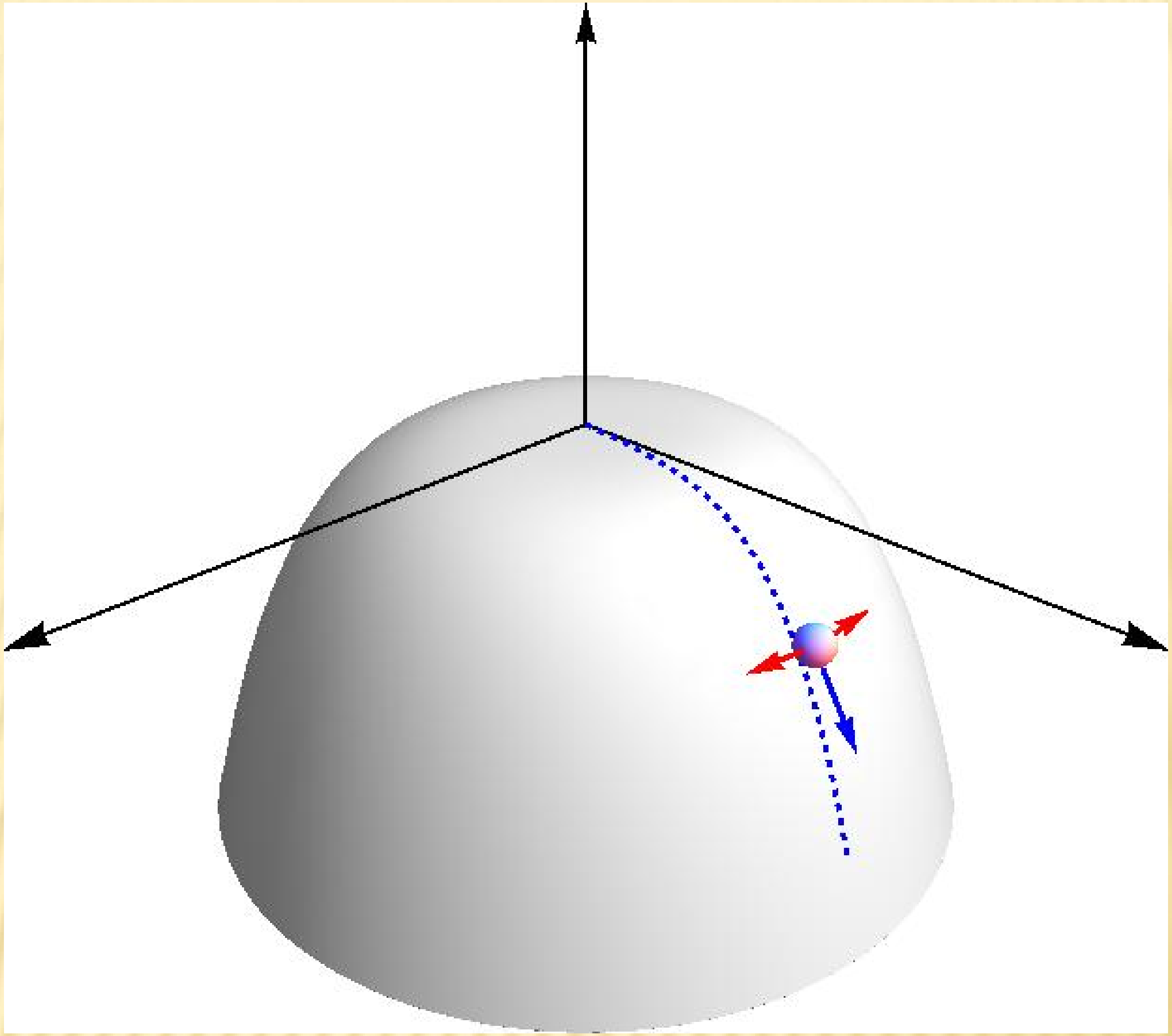
- Let there be intermediate matter dominated epoch, at which these particles are non-relativistic and dominate the expansion
- Then X-particles decay and deliver their energy to hot medium.
- Adiabatic perturbations are generated because mass and/or width is spatially inhomogeneous, $\delta M_X(\vec{x}) \propto \delta\theta(\vec{x})$

In either case

$$\zeta = \mathrm{const} \cdot \delta\theta + \text{possible non-linear terms}$$

- Adiabatic perturbations inherit shape of power spectrum and correlation properties from $\delta\theta$, plus possible additional non-Gaussianity.
- const < 1 and may be $\ll 1 \Longrightarrow \delta\theta \gg \zeta$ quite possible $\Longrightarrow h < 1$, but not necessarily $h \sim 10^{-4}$
- In any case, no relationship with tensor perturbations

Order h effects: back to conformal evolution



Peculiarity: radial perturbations.

- Linear analysis of perturbations of $\chi_1 = \mathrm{Re}\chi$ about the homogeneous real solution χ_c:

$$\frac{d^2}{d\eta^2}\delta\chi_1 + k^2\delta\chi_1 - 6h^2\chi_c^2\delta\chi_1 = 0$$

  - Recall $h\chi_c = 1/(\eta_* - \eta)$.
  - Again initial condition

$$\delta\chi_1 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} e^{i\vec{k}\vec{x} - ik\eta} B_{\vec{k}} \quad + \quad \text{h.c.}$$

  - But now the solution is

$$\delta\chi_1 = \frac{1}{4\pi}\sqrt{\frac{\eta_* - \eta}{2}} H_{5/2}{}^{(1)}\left[k(\eta_* - \eta)\right] \cdot B_{\vec{k}} \quad + \quad \text{h.c.}$$

- In long wavelength regime, $k \ll 1/(\eta_* - \eta)$,

$$\delta\chi_1 = \frac{3}{4\pi^{3/2}} \frac{1}{k^2\sqrt{k}(\eta_* - \eta)^2}$$

✓ Red spectrum:

$$\langle \delta\chi_1^2 \rangle \propto \int \frac{d^3k}{k^5}$$

✓ Large $\delta\chi_1$ at small $(\eta_* - \eta)$

[Recall $\chi_c = 1/[h(\eta_* - \eta)]$]

- Again by symmetry: now translations of conformal time:

$\chi_c \propto 1/(\eta_* - \eta) \Longrightarrow$ spatially homogeneous solution to perturbation equation $\delta\chi = \partial_\eta \chi_c$.

- ✓ Modulo field redefinition and notations, properties of galileon perturbations are exactly the same as properties of radial perturbations in conformal rolling scenario.

  M.L.,Mironov,Rubakov,'11

- ✓ Furthermore, these properties are unambiguously determined by conformal invariance

  M.L.,Mironov,Rubakov,'11

  Hinterbichler, Khouri' 11

- ✓ In fact, invariance with respect to dilatations is sufficient.

Hence, we are dealing with the whole class of models

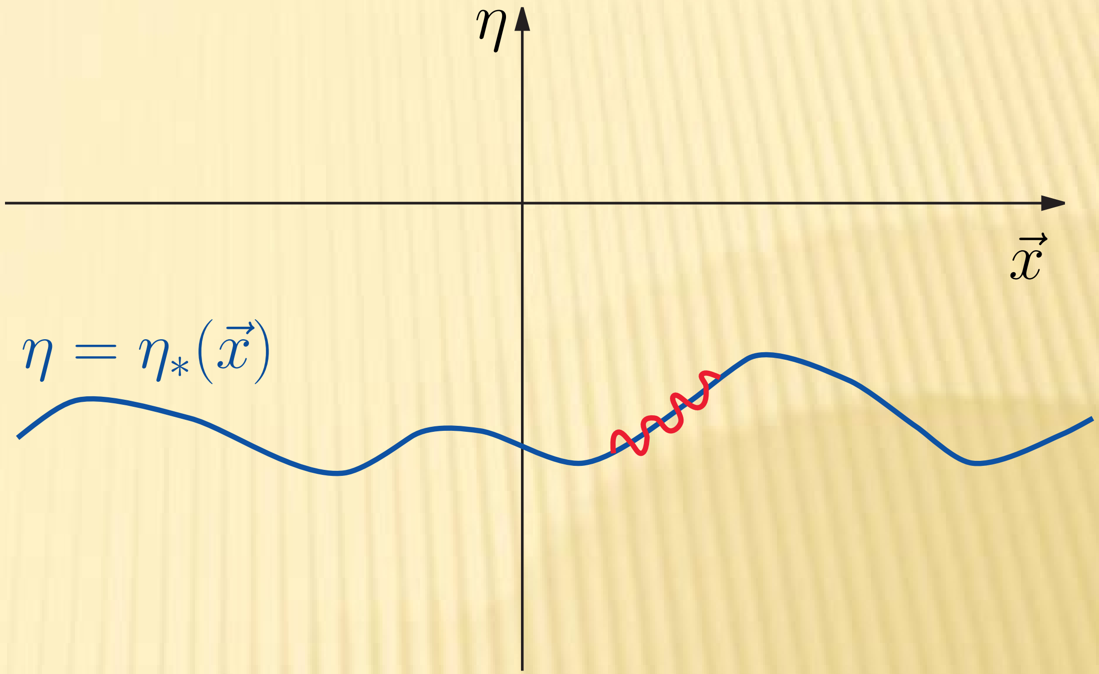
- Interpretation: shift

$$\eta_* \longrightarrow \eta_* + \delta\eta_*(\vec{x})$$

✓ Background for perturbations $\delta\chi_2$ = Imχ (in other words, for phase θ) is no longer spatially homogeneous.

✓ Red spectrum of $\delta\eta_*(\vec{x})$:

$$\sqrt{\mathcal{P}_{\delta\eta_*}} = \frac{3h}{2\pi k}$$



- Have to study perturbations of Imχ in *spatially inhomogeneous background*, slowly varying in space,

$$\chi_c = \frac{1}{h(\eta_*(\vec{x}) - \eta)}$$

- Back to equation for perturbations of $\delta\chi_2 = \mathrm{Im}\chi$

$$\frac{d^2}{d\eta^2}\delta\chi_2 - \frac{\partial^2}{\partial\vec{x}^2}\delta\chi_2 - \frac{2}{(\eta_*(\vec{x}) - \eta)}\delta\chi_2 = 0$$

- Initial condition as $\eta \to -\infty$:

$$\delta\chi_2 = \frac{1}{(2\pi)^{3/2}\sqrt{2k}} e^{i\vec{k}\vec{x} - ik\eta} A_{\vec{k}} \quad + \quad h.c.$$

- $\eta_*(\vec{x})$: long ranged field, derivative expansion appropriate
  - Zeroth order in $\partial_i\eta_*$: local shift of conformal time, $\delta\eta_*(\vec{x})$
  - First order in $\partial_i\eta_*$:

    $\eta_*(\vec{x}) - \eta = \eta_*(0) - (\eta - \vec{\partial}\eta_* \cdot \vec{x})$

    $\Longrightarrow$ local Lorentz boost with $\vec{v} = -\vec{\partial}\eta_*$;

    background is locally homogeneous and isotropic in a reference frame other than cosmic frame.

Solution to the first order: time shift and boost of the original solution

Potentially observable effects depend on what happens to phase perturbations after the end of conformal rolling stage.

NB: Once radial field $|\varphi|$ has settled down to minimum of its potential, phase θ is massless scalar field minimally coupled to gravity.

Two sub-scenarios

Sub-scenario # 1

- Phase perturbations superhorizon in conventional sense after end of conformal rolling stage.

  M.L., Rubakov, '10

- $\delta\theta$ remains frozen until the time it gets reprocessed into adiabatic perturbations $\Longrightarrow$

$$\mathcal{P}_\zeta \propto \mathcal{P}_{\delta\theta}$$

- No effect to the linear order in $\vec{v}$ **(!!)**
  Lorentz-invariance does the job.

- Derivative expansion to the second order: perturbative solution.

  Long wavelength regime:

$$\delta\theta = \frac{h}{(2\pi)^{3/2}\sqrt{2kq}} e^{i\vec{k}\vec{x}-ik\eta_*(\vec{x})} \left(1 - \frac{\pi}{2k}\frac{k_i k_j}{k^2}\partial_i\partial_j\eta_*\right) \cdot A_{\vec{k}} + \text{h.c.}$$

  Scalar power spectrum ($\vec{n} = \vec{k}/|\vec{k}|$)

$$\mathcal{P}(\vec{k}) = \mathcal{P}_0(k)\left(1 - \frac{\pi}{k} n_i n_j \partial_i\partial_j\eta_*\right)$$

  Statistical anisotropy due to constant in space tensor $\partial_i\partial_j\eta_*|_{\text{long wavelengths}}$

  $\Longrightarrow$ CMB correlators $\langle a_{l,m} a^*_{l\pm 2,m}\rangle$, etc.

  - Quadrupole of general form
  - Momentum dependence $1/k$

NB: Power spectrum of $\partial^2 \eta_*$ is blue $\Longrightarrow$

$$\langle (\pi \partial_i \partial_j \eta_*)^2 \rangle_{\text{long wavelengths}} \simeq \frac{9h^2}{4} \int_0^{H_0} kdk \simeq h^2 H_0^2$$

Statistical anisotropy effect on perturbations of wave vector k

$$\mathcal{P}(\vec{k}) = \mathcal{P}_0(k) \left(1 + \frac{hH_0}{k} w_{ij} n_i n_j \right)$$

(with $w_{ij} w_{ij} = 1$) $\Longrightarrow$ effect on CMB behaves as $1/l$

NB: From CMB

$$h^2 < 190$$

Ramazanov, Rubtsov, '12

Non-Gaussianity to order h^2

M.L., Mironov, Rubakov, '10,11

Over and beyond non-Gaussianity which can be generated when perturbations in θ are converted into adiabatic perturbations.

Invariance $\theta \to -\theta \Longrightarrow$ bispectrum vanishes.

Trispectrum fully calculated. Most striking property: singularity in folded limit:

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle = \text{const} \cdot \delta \left(\sum_{i=1}^{n} \vec{k}_i \right) \cdot \frac{1}{k_{12} k_1^4 k_3^4} \left[1 - 3 \left(\frac{\vec{k}_{12} \cdot \vec{k}_1}{k_{12} k_1} \right)^2 \right] + \left[\quad \vec{k}_1 \leftrightarrow \vec{k}_3 \right]$$

$$\vec{k}_{12} = \vec{k}_1 + \vec{k}_2 \to 0$$

This is in sharp contrast to single field inflation.

Origin: infrared enhancement of radial perturbations $\delta\chi_1$

Sub-scenario # 2

NB: more natural in conformal rolling model, less natural in Galilean Genesis.

- Phase perturbations sub-horizon in conventional sense after end of conformal rolling stage

M.L., Ramazanov, Rubakov, '11

Fairly generic feature of alternatives to inflation:
long stage of almost Minkowskian evolution

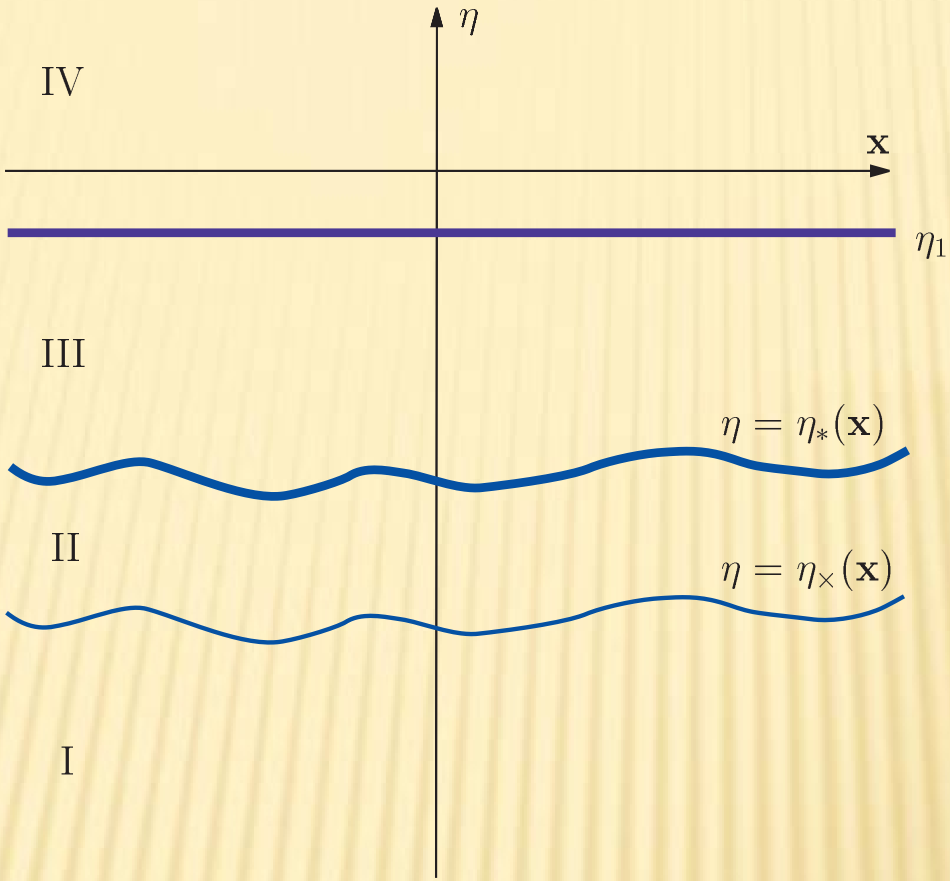
Example: contracting Universe. Need stiff equation of state, otherwise strong inhomogeneity and anisotropy towards the end of contraction (Belinsky-Lifshits-Khalatnikov phenomenon)
$\Longrightarrow$ Ekpyrotic models (in broad sense): at contraction

Erickson et. al. '03;

Garfinkle et. al. '08

$$a(t) = |t|^p, \quad p \ll 1$$

δθ evolves non-trivially before it becomes super-horizon and freezes out again.



- Need nearly Minkowskian evolution at intermediate stage III. Otherwise power spectrum becomes tilted!
- Two-fold effect of radial perturbations
  - Initial field $\delta\theta$ non-trivial
  - Cauchi hypersurface $\eta = \eta_*(\vec{x})$ non-trivial
- For given $\vec{k}$, phase perturbation after second freeze-out is a linear combination of waves coming from direction of $\vec{k}$ and from opposite direction and traveling distance $r = \eta_1 - \eta_*$ $\Longrightarrow$ Imprint on $\delta\theta(\vec{k})$ of random field $\vec{v}(\pm nr)$, which depends on $\vec{n}$ only,

$$\delta\theta = F[\vec{v}(\pm\vec{n}r)] \cdot A_{\vec{k}} + \text{ h.c.}$$

In particular,

$$\mathcal{P}_{\delta\theta}(\vec{k}) = \mathcal{P}_0 \left\{1 + n \left[\vec{v}(+\vec{n}r) - \vec{v}(-\vec{n}r)\right]\right\}$$

Non-trivial dependence on n. Statistical anisotropy with all even multipoles.

- Resulting statistical anisotropy

$$\mathcal{P}_\zeta(\vec{k}) = \mathcal{P}_\zeta^{(0)}(k)\left[1 + \mathcal{Q}\cdot w_{ij}\left(n_{k\,i}n_{k\,i} - \frac{1}{3}\delta_{ij}\right) + \text{higher multipoles}\right]$$

with $w_{ij}w_{ij} = 1$ and

$$\langle \mathcal{Q}^2 \rangle = \frac{675}{32\pi^2}h^2\,.$$

NB: multipoles $\mathcal{Q}$, etc., are independent of k $\Longrightarrow$ no suppression of effect on CMB at large l, unlike in sub-scenario # 1.

NB: From CMB

Ramazanov, Rubtsov, '12

$$h^2 < 0.045$$

- Non-Gaussianity

  Non-trivial part of tri-spectrum: dependence on directions of momenta

$$\langle\zeta(\vec{k})\zeta(\vec{k}')\zeta(\vec{q})\zeta(\vec{q}')\rangle = \frac{\mathcal{P}_\zeta^{(0)}(k)}{4\pi k^3}\frac{\mathcal{P}_\zeta^{(0)}(q)}{4\pi q^3}\delta(\vec{k}+\vec{k}')\delta(\vec{q}+\vec{q}')\cdot\left[1+F_{NG}(\vec{n}_k,\vec{n}_q)\right]$$

$$+ \quad \text{permutations}$$

  with

$$F_{NG} = \frac{3h^2}{\pi^2}\log\frac{\text{const}}{|\vec{n}_k-\vec{n}_q|}$$

NB: recall that power spectrum of $\vec{v}$ is flat $\Longrightarrow$ log behavior of F_{NG}.

To summarize:

- Flat (or nearly flat) spectrum of scalar perturbations may be a consequence of conformal symmetry (+ possibly global symmetry), rather than de Sitter symmetry
  - Models of this sort:

    (i) conformally coupled spectator complex scalar field with negative quartic potential

    (ia) minimally coupled dynamical complex scalar field with negative quartic potential

    ii Galilean Genesis
  - Properties of perturbations dictated by conformal invariance
  - Predictions are model-independent, at least to the leading non-linear level (modulo effects due to conversion of field perturbations into adiabatic perturbations)

- Peculiar property which has potentially observable consequences: fluctuations along rolling direction
  - Interpretation in terms of local time shift
- Interplay between phase perturbations, responsible for density perturbations in the end, and local time shift $\delta\eta_*(\vec{x}) \Longrightarrow$ non-trivial correlation properties of density perturbations

Sub-scenario # 1:

- Statistical anisotropy of quadrupole form

$$\mathcal{P}(\vec{k}) = \mathcal{P}_0(k)\left(1 + \frac{hH_0}{k} w_{ij} n_i n_j\right)$$

- Trispectrum singular in folded limit

$$\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \zeta_{\vec{k}_4} \rangle \propto \frac{1}{|\vec{k}_1 + \vec{k}_2|}$$

Sub-scenario # 2:

- Statistical anisotropy of a general form
- Non-Gaussianity of a peculiar kind
- Intrinsic negative tilt in power spectrum

What if the world started out conformal indeed?