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Event horizon: Interfacing the Classical and the Quantum

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Abstract

A central idea of this talk is that the black hole horizon is an appropriate geometrical layout required to interface the classical and quantum realms. Forming the black hole horizon is attended with the transition from the classical regime of evolution to the quantum one. This statement is based on the following criterion for discriminating between the classical and the quantum: creations and annihilations of particle-antiparticle pairs are impossible in the classical reality but possible in the quantum reality. Technically, one can switch from the classical picture of field propagation in flat spacetime to the quantum picture by changing the overall sign of the spacetime signature. To describe a selfgravitating object at the final stage of its classical evolution, it is pertinent to use either Foldy–Wouthuysen representation of the Dirac equation in curved spacetimes or Gozzi classical path integral. In both approaches, maintaining the dynamics in the classical regime is controlled by supersymmetry.

Four views on the physical reality

First • ***Our world is essentially classical***

Quantum phenomena could be in principle accounted for by invoking an appropriate set of hidden variables

This view goes back to Schroedinger, de Broglie, and Einstein

In the 50s, Bohm translated it into a specific model

In this century, `t Hooft speculated that the ontology of the Planck region is governed by deterministic laws

Second. (Currently this is the superior paradigm.)

• ***Our world is fundamentally quantum***

There is no classical object whose existence is firmly certified, and whose individuality is preserved in time

Example: the present speaker is a superposition of alive and dead Kosyakov

The existence of the universe as a well-definite object is a mere appearance

If the famous Bell inequality is shown to be violated, then deterministic descriptions with hidden variables are necessarily ***nonlocal***

Third. (Copenhagen interpretation)

• ***Our world is partly classical and partly quantum***

Fourth. (Duality, holographic correspondence, etc.)

• ***A given realm may appear both as classical and quantum, but these two looks pertain to space-times of nearby dimensions***

Our concern here is with the third view. This raises the natural question of what is a demarcation line between the classical and the quantum. Naively, all quantum phenomena are associated with small distances, and classical phenomena are thought of as macroscopic. But what is the criterion for "smallness"? Are giant molecules small or large? What's about such quantum phenomena as superconductivity and superfluidity whose characteristic scale lengths are macroscopic? Normally, classical and quantum phenomena are mixed. Meanwhile there is a natural layout which has a clear demarcation line separating the classical from the quantum. I mean the ***event horizon of black holes***

The classical and the quantum

What is the criterion for discriminating between the classical and the quantum? It is common to see the statement that a quantum phenomenon is what is described by an expression containing $\hbar$'s, while a classical one is free of this dependence. However, our interest here is with the ***conceptual*** difference of classical and quantum objects and their behavior.

Let us compare the respective properties of ***particles*** and ***fields*** in the classical and quantum pictures.

A convenient framework bringing together the classical and quantum treatments is provided by the path integral approach

A quantum-mechanical particle can be described by

$$K(\mathbf{z}_f, T|\mathbf{z}_i, 0) = \int [\mathcal{D}q]\, \exp\left[i\int_0^T dt\, L(q, q')\right]. \qquad (1)$$

Whatever the kind of the world line $z^\mu(\tau)$ passing through the points $x^\mu = (0, \mathbf{z}_i)$ and $x^\mu = (T, \mathbf{z}_f)$, it contributes to the Feynman path integral provided that the Lagrangian L is real, and expression (1) is well defined for this path. To illustrate, the Poincare–Planck Lagrangian

$$L = -m\sqrt{\dot{z}^2} \qquad (2)$$

is ***real*** and ***finite*** only for ***timelike*** paths. If $z^\mu(\tau)$ is a null curve, then $L = 0$. If $z^\mu(\tau)$ is spacelike, then L is complex-valued. Since the imaginary part of L can be both positive and negative values, (1) is ***ill-defined***.

By contrast, the Brink–Deser–Zumino–Di Vecchia–Howe Lagrangian

$$L = -\frac{1}{2}\left(\eta\,\dot{z}^2 + \frac{m^2}{\eta}\right) \qquad (3)$$

is ***real*** and ***finite*** for all ***timelike***, ***null***, and ***spacelike*** curves.

We may restrict our consideration to timelike paths if we discriminate between ***smooth*** and ***piecewise smooth*** curves, notably, Λ- or V-shaped curves. One may think of a -shaped timelike curve as either path of a ***single*** particle moving from the remote past to a cusp point and then back in time or a history of the ***annihilation*** of a ***pair*** that occurs at this cusp point. Likewise, a -shaped curve is interpreted as either path of a ***single particle*** that runs initially from the far future to a cusp point and then again to the future or a history of the ***birth*** of a ***pair*** at this point. Any Λ- or V-shaped path passing through the end points contributes to the Feynman path integral (1).

Quantum description leaves room for both particles and antiparticles together with their creation and annihilation

Classical particles are governed by the ***principle of least action***. It can be formulated for smooth timelike and null world lines. However, it defies unambiguous formulation for V- and Λ-shaped world lines. Any spacelike hyperplane intersects a Λ-shaped curve twice or fails to intersect this curve at all. The same is true for V-shaped curves. Although the classical picture allows the coexistence of particles and antiparticles, ***creations*** and ***annihilations***, represented by V- and Λ-shaped world lines, are ***banned***.

Therein lies the fundamental difference between the quantum and classical viewpoints on particles: creations and annihilations of pairs are permissible in the quantum reality and impermissible in the classical reality

With V- and Λ-shaped world lines it is possible to form ***closed*** paths for virtual particles. The above statement translates into the well-known criterion: the classical picture is displayed as a ***tree*** diagram, whereas the quantum picture is represented by ***loops***. Usual derivations of this criterion are based on comparison of the powers of Planck's constant $\hbar$ entering in different terms of a perturbation series. The separation into trees and loops need not be related to the dilemma of whether or not the $\hbar$ present in the expressions. Physically, to discriminate between loops and trees we must decide between the feasibility of creations and annihilations of pairs and veto on these processes.

Hawking radiation associated with the pair creation and annihilation processes near the black hole horizon is a characteristically quantum phenomenon

The key difference between the classical and quantum manifestations of the same field is due to the different ***boundary conditions***. Consider first a massless scalar field in flat spacetime. The Fourier transform of the ***retarded*** Green's function

$$\widetilde{D}_{\mathrm{ret}}(k) = -\frac{1}{k^2 + 2ik_0\epsilon} = \frac{1}{\mathbf{k}^2 - (k_0 + i\epsilon)^2} \qquad (4)$$

gives an accurate account of how this field propagates in classical theory. If the integration over the variable $\varkappa = |\mathbf{k}|$ is carried out, then the poles at

$$\varkappa = \pm\sqrt{k_0^2} \pm i\epsilon$$

are avoided by the path of integration.

The propagation of a free massless field in quantum theory is described by the Feynman propagator

$$\widetilde{D}_F(k) = -\frac{1}{k^2 + i\epsilon} \ , \qquad (5)$$

which obeys the causal boundary condition. It follows the prescription for avoiding the poles

$$\omega = \pm\sqrt{\mathbf{k}^2} \mp i\epsilon \ , \qquad (6)$$

where ω denotes k_0. Exact propagators of interacting fields are given by the spectral Kallen–Lehmann representation

$$\widetilde{D}_c(k) = -\frac{1}{k^2 + i\epsilon} + \int_{4m^2}^{\infty} \frac{d\sigma(\mu^2)}{\mu^2 - k^2 - i\epsilon} \ , \qquad (7)$$

where the lower limit $4m^2$ represents the mass gap in the spectrum of this theory, and $\sigma(\mu^2)$ is a polynomial-bounded monotone nondecreasing function which takes into account loop contributions. $\widetilde{D}_c$ obeys the causal boundary condition.

In order to bridge the gap between the retarded and causal boundary conditions, we "euclideanize" both descriptions

Assuming that the integrand decreases sufficiently fast as $\varkappa$ approaches infinity, it is possible to rotate the integration path in a clockwise direction by $\frac{\pi}{2}$ in the complex $\varkappa$-plane without crossing the poles. This operation is similar to the Wick rotation. Introducing the variable $\mathbb{K}_j = i\mathbf{k}_j$ makes the length squared of k^μ positive definite:

$$k_E^2 = k_0^2 + \mathbb{K}^2 \ . \tag{8}$$

Analytical continuation of space variables to the imaginary axes

$$\mathbb{X}^j = -i\mathbf{x}^j \ , \tag{9}$$

performed together with analytical continuation in **k**-space, introduces the Euclidean metric

$$dx_E^2 = dx_0^2 + d\mathbb{X}^2 \ . \tag{10}$$

If we carry out the Wick rotation of the integration path of the Feynman propagator in a counterclockwise direction by

$\frac{\pi}{2}$ in the complex ω-plane without crossing the poles which is equivalent to introducing $k_4 = ik_0$ then the length squared of k^μ becomes negative definite:

$$k_E^2 = -\left(k_4^2 + \mathbf{k}^2\right) . \tag{11}$$

Analytical continuation of time variable to the imaginary axis

$$x_4 = -ix^0 \ , \tag{12}$$

performed together with the Wick rotation, introduces the Euclidean metric

$$dx_E^2 = -\left(dx_4^2 + d\mathbf{x}^2\right) . \tag{13}$$

We ***change the overall sign of the spacetime signature in the classical description of field propagation for it to be treated as the quantum description of field propagation***. Two Lorentzian metrics of opposite signatures can always be analytically continued to two Euclidean line elements of opposite sign, such as those shown in (10) and (13).

Taken alone, the sign of the Lorentzian metric is of no particular importance, its choice is a matter of convention However, if this overall sign is changed as one passes from some region of spacetime to a contiguous region, then this change of sign is evidence of switching from the classical to quantum regime of field propagation.

Such is the case for contiguous regions inside and outside the event horizon of a Schwarzschild black hole. The Schwarzschild metric describing an isolated spherically symmetric stationary black hole reads

$$ds^2 = \left(1 - \frac{r_S}{r}\right) dt^2 - \left(1 - \frac{r_S}{r}\right)^{-1} dr^2 - r^2 d\Omega \ . \quad (14)$$

Here, $d\Omega$ is the round metric in S^2, and $r_S = 2M$ is the Schwarzschild radius which represents the event horizon of this black hole. In the Schwarzschild exterior $r > r_S$, the Killing vector field $X = \partial_t$ is interpreted as the asymptotic time translation. In the Schwarzschild interior $r < r_S$, *r* is a time coordinate, and the integral lines of the vector field $X = \partial_r$ are incomplete timelike geodesics which terminate at $r = 0$. Once the Euclideanization has been performed, the regions inside and outside the boundary $r = r_S$ take the Euclidean metrics of the type of (10) and (13), respectively. What happens to the physical reality at the surface of the collapsing star when the Schwarzschild radius $r = r_S$ is crossed? Does the classical picture give way to the quantum picture?

Locally $r = r_S$ is a perfectly regular surface. The singularity at $r = r_S$ is a mere coordinate singularity in the original Schwarzschild coordinate frame. In some other coordinates, the metric is smooth at $r = r_S$. However, ***globally***, $r = r_S$ acts as a point of no return. Every light cone tilts over at this point, so that the roles of *t* and *r* are interchanged.

In summary ***the event horizon of a Schwarzschild black hole shows a clear demarcation between spacetime regions characterized by opposite signatures. This geometrical layout provides an explicit scheme for interfacing the classical and the quantum***.

We pass over charged black holes described by the Reissner – Nordstrom solution [which can be obtained from (14) by replacing $2M$ with $2M - q^2/r$ where q is the charge of the hole]. The major conclusion that such objects are suitable to studying a classical-quantum phase transition still stands.

The Foldy–Wouthuysen picture

L. L. Foldy and S. A. Wouthuysen, "On the Dirac theory of spin-$\frac{1}{2}$ particles and its nonrelativistic limit," *Phys. Rev*. **78** (1950) 29.

There is a unitary transformation that ***diagonalizes*** the free Dirac Hamiltonian H_0 with respect to ***positive*** and ***negative energies***,

$$U_{\text{FW}}^{-1} H_0 U_{\text{FW}} = \begin{pmatrix} \sqrt{-\nabla^2 + m^2} & 0 \\ 0 & -\sqrt{-\nabla^2 + m^2} \end{pmatrix} = \beta |H_0|$$

The standard representation of Dirac matrices is assumed,

$$\alpha^i = \begin{pmatrix} 0 & \sigma^i \\ \sigma^i & 0 \end{pmatrix}, \qquad \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$$

Positive energy states are ascribed to a Dirac ***particle*** while states of ***negative*** energy are attributed to its ***antiparticle***. Thus, the free Dirac equation

$$i\frac{\partial}{\partial t}\psi = \left[i\left(\vec{\alpha}\cdot\nabla\right) + \beta m\right]\psi \tag{15}$$

is unitarily equivalent to a pair of two-component equations

$$i\frac{\partial}{\partial t}\chi = \beta\sqrt{-\nabla^2 + m^2}\,\chi\ . \tag{16}$$

A separation of positive- and negative-energy states is still possible in the presence of an external time-independent magnetic field of arbitrary strength $\mathbf{B}$. Case found

K. M. Case, “Some generalizations of the Foldy–Wouthuysen transformation,” *Phys. Rev.* **95** (1954) 1323.

the Foldy–Wouthuysen transformation of the Dirac equation into

$$i\frac{\partial}{\partial t}\chi = \beta\sqrt{(i\nabla - e\mathbf{A})^2 - e\left(\vec{\sigma}\cdot\mathbf{B}\right) + m^2}\,\chi\ , \tag{17}$$

where $\mathbf{A}$ is the vector potential of this field $\mathbf{B} = \mathrm{curl}\,\mathbf{A}$. The energy gap of the Dirac sea is not penetrated in the constant magnetic field because this field leaves the energy of the Dirac particle unchanged.

However, the separation is not possible with time-dependent electromagnetic fields and scalar potentials; the positive- and negative-energy solutions may mix when the interaction is sufficiently strong

E de Vries "Foldy–Wouthuysen transformations and related problems" *Fortschr Phys* **18** (1970) 149

B. Thaller, *The Dirac Equation* (Berlin, Springer, 1992).

The energy gap of the Dirac sea is no longer insuperable. Creations and annihilations are made possible.

A Dirac particle manifests itself as a classical entity only in the case that the Hamiltonian can be diagonalized with respect to positive and negative energies.

In a ***curved*** spacetime the Dirac equation can be written

Yu. N. Obukhov, "Spin, gravity, and inertia," *Phys. Rev. Lett*. **86** (2001) 192; gr-qc/0012102.

in the Foldy–Wouthuysen form for ***stationary metrics***

$$ds^2 = V^2(\mathbf{r})\, dt^2 - W^2(\mathbf{r})\, d\mathbf{r}^2 \ , \tag{18}$$

where V and W are arbitrary functions of spatial coordinates $\mathbf{r} = (x, y, z)$. Schwarzschild geometry is a particular case of (18). When employing isotropic coordinates, the metric (14) takes the form (18) with

$$V = \left(1 - \frac{r_{\rm S}}{4r}\right)\left(1 + \frac{r_{\rm S}}{4r}\right)^{-1}, \quad W = \left(1 + \frac{r_{\rm S}}{4r}\right)^2. \tag{19}$$

To be more specific, Dirac particles in curved backgrounds are governed by the covariant Dirac equation

$$\left(i\gamma^{\hat{\alpha}} D_{\hat{\alpha}} - m\right)\psi = 0\ , \tag{20}$$

where, D_α is the spinor covariant derivative

$$D_{\hat{\alpha}} = e^{\mu}{}_{\hat{\alpha}}\left(D_\mu + \frac{1}{4}\Gamma_\mu\right), \tag{21}$$

$\Gamma_\mu = \frac{1}{4}[\gamma^{\hat{\alpha}}, \gamma^{\hat{\beta}}]\, e^{\nu}{}_{\hat{\alpha}}\, e_{\hat{\beta}\mu;\nu} = \frac{1}{2}\omega_{\mu\hat{\alpha}\hat{\beta}}\,\gamma^{\hat{\alpha}}\gamma^{\hat{\beta}}$ is the gravitational gauge potential.

If the covariant Dirac equation (21) can be brought into the form

$$i\frac{\partial\psi}{\partial t} = H\psi \tag{22}$$

with

$$H = \beta m - \frac{i}{2}\left[(\vec{\alpha}\cdot\nabla)F + F(\vec{\alpha}\cdot\nabla)\right], \quad F = \frac{V}{W}\ , \tag{23}$$

then there is a unitary transformation $H_{\rm FW} = U_{\rm FW} H U^{\dagger}_{\rm FW}$ such that

$$H_{\rm FW} =$$

$$\frac{1}{2}\left(\sqrt{\bar{H}^2} + \beta\sqrt{\bar{H}^2}\beta\right) + \frac{1}{2}\left(\sqrt{\bar{H}^2} - \beta\sqrt{\bar{H}^2}\beta\right)\gamma_5\beta\ , \tag{24}$$

$$\bar{H}^2 = m^2 V^2 - F\,\nabla^2\,F + \frac{1}{2}\,F\,(\nabla\vec{f}) - \frac{1}{4}\,\vec{f}^2 -$$

$$i\,F\,\vec{\Sigma}\cdot\left[\vec{f}\times\nabla - \gamma_5\beta m\,\vec{\phi}\right]. \qquad (25)$$

Here, $\vec{\Sigma}$ is the spin matrix $\vec{\Sigma} = \frac{1}{2}\,i\,\vec{\gamma}\times\vec{\gamma}$, and $\vec{f} = \nabla F$, $\vec{\phi} = \nabla V$.

The feasibility of a Foldy–Wouthuysen transformation is another way of stating that the system enjoys the property of supersymmetry

R. P. M. Romero, M. Moreno, and A. Zentella, "Supersymmetric properties and stability of the Dirac sea," *Phys. Rev*. D **43** (1991) 2036.

The ***origin of the Foldy–Wouthuysen picture*** for a Dirac particle in an external electromagnetic field is related to the ***existence*** of a ***supercharge***.

If the Dirac sea is ***stable***, then the ***positive***- and ***negative-energy*** solutions are ***supersymmetric*** partners of each other. When the ***supersymmetry*** is ***broken***, it is ***impossible*** to obtain an exact ***block-diagonalized Hamiltonian*** for this system.

These reasonings can be extended to ***curved spacetimes***.

S. Heidenreich, T. Chrobok, H.-H. v. Borzeszkowski, "Supersymmetry, exact Foldy–Wouthuysen transformation, and gravity," *Phys. Rev*. D **73** (2006) 044026.

A supercharge can be constructed for a relatively ***wide class*** of ***stationary metrics***, including that defined in (18).

The ***Foldy–Wouthuysen transformed Hamiltonian*** $H_{\rm FW}$ is proportional to the ***square root*** of the ***super-Hamiltonian***

$$H_{\rm FW} = \beta\sqrt{Q^2}\ . \tag{26}$$

For the metric (18), Q is given by

$$Q = \frac{1}{2}\{\vec{\alpha}\cdot p, F\} + JV\ . \tag{27}$$

Here, J is the involution operator $J = i\gamma_5\beta$ (a Hermitian and unitary operator, $J^\dagger = J$, $JJ^\dagger = 1$, which anticommutes with both the Hamiltonian and the β matrix, $JH + HJ = 0$, $J\beta + \beta J = 0$).

Consider a ***self-gravitating*** Dirac field ψ which arranges itself into a spherically symmetric ***collapsing wave packet***. Let the total mass of the wave packet be equal to m, the parameter that enters in the definition of the Schwarzschild radius $r_{\rm S} = 2m$. Before a black hole state settles down, the ψ is assumed to model an astrophysical collapsing object in the Schwarzschild spacetime.

Our interest here is with the “last stage” of evolution of the wave packet ***just before its shrinking down to below the horizon***. At this stage, the wave packet ψ is governed by the ***diagonalized Hamiltonian*** (24).

It is desirable to find an exact solution to this problem.

This solution should exhibit a singular point after which this classical regime of evolution is no longer valid.

The singularity is just the point in which the ***supersymmetry*** must be ***violated***.

For more detail see

B. P. Kosyakov, "Black holes: interfacing the classical and the quantum," *Found. Phys*. **38** (2008) 678

The classical path integral

E. Gozzi, "Hidden BRS invariance in classical mechanics," *Phys. Lett*. **201** (1988) 525

E. Gozzi, M. Reuter, and W. D. Thacker, "Hidden BRS invariance in classical mechanics. II," *Phys. Rev*. D 40 (1989) 3363

A. A. Abrikosov, Jr., E. Gozzi, and D. Mauro, "Geometric dequantization," *Ann. Phys*. (N. Y.) **317** (2005) 24; quant-ph/0406028

The ***probability amplitude*** of finding a ***classical system*** at a phase space point $\phi^a_f = (q^a_f, p^a_f)$ at time $t_f = T$ if it was at $\phi^a_i = (q^a_i, p^a_i)$ at time $t_i = 0$ is given by

$$K(\phi^a_f, T|\phi^a_i, 0) = \int [\mathcal{D}\phi]\, \delta\left[\phi^a - \phi^a_{\rm cl}\,(T;\phi_i, 0)\right]. \qquad (28)$$

Here $\phi^a_{\rm cl}$ is the ***solution*** to the ***classical equation of motion***

$\dot{\phi}^a = \omega^{ab}\partial_b H$, with ω^{ab} being a symplectic matrix, and *H* the Hamiltonian of this system. The integration is over the all phase space paths with fixed end points ϕ_i and ϕ_f.

Since

$$\delta(\phi - \phi_{\rm cl}) = \delta\left(\dot{\phi}^a - \omega^{ab}\partial_b H\right) \det\left(\delta^a{}_b\partial_t - \omega^{ac}\partial_c\partial_b H\right),$$

one may take the Fourier transform of the Dirac delta and exponentiate the determinant using an even Grassmannian variable λ_a and odd variables c^a and $\bar{c}_a$ to yield

$$K(\phi_f, T|\phi_i, 0) = \int [\mathcal{D}\phi]\, \mathcal{D}\lambda\, \mathcal{D}c\, \mathcal{D}\bar{c}\, \exp\left(i\int_0^T dt\, \widetilde{\mathcal{L}}\right). \quad (29)$$

Here

$$\widetilde{\mathcal{L}} = \lambda_a\dot{\phi}^a + i\bar{c}_a\dot{c}^a - \lambda_a\omega^{ab}\partial_b H - i\bar{c}_a\omega^{ad}\partial_d\partial_b H c^b . \quad (30)$$

If we define two anticommuting partners of *t*, $\bar{\theta}$ and θ, and assemble the variables ϕ, λ, $\bar{c}$, c into a single combination of ***supersymmetric phase space coordinates***

$$Q(t,\theta,\bar{\theta}) = q(t) + \theta c^q + \bar{\theta}\bar{c}_p + i\bar{\theta}\theta\lambda_p\ , \quad (31)$$

$$P(t,\theta,\bar{\theta}) = p(t) + \theta c^p - \bar{\theta}\bar{c}_q - i\bar{\theta}\theta\lambda_q\ , \quad (32)$$

then (29) takes a compact and elegant form

$$K(Q_f, T|Q_i, 0) =$$

$$\int [\mathcal{D}Q]\, \mathcal{D}P\, \exp\left[i\int_0^T idt\, d\theta\, d\bar{\theta}\ L(Q,P)\right], \quad (33)$$

where L is the ***usual*** Lagrangian $L(q,p) = p\dot{q} - H(q,p)$ of this system. Equation (33) bears the formal similarity to the quantum path integral

$$K(q_f, T|q_i, 0) = \int [\mathcal{D}q]\,\mathcal{D}p\,\exp\left[i\int_0^T dt\,L(q,p)\right]. \quad (34)$$

In fact, (33) derives from (34) by replacing the phase space coordinates q, p with the super phase space coordinates Q, P and extending the integration over t to an integration over the supertime $(it, \theta, \bar{\theta})$.
Consider a gravitating fluid in a Schwarzschild background, which arranges itself into a ***collapsing ball***. The canonical theory of classical perfect fluids is well studied

R. Jackiw, V. P. Nair, S. -Y. Pi, and A. P. Polychronakos, "Perfect fluid theory and its extension," *J. Phys*. A **37** (2004) R327; hep-ph/0407101.

If we ***extend*** this theory to ***curved*** spacetimes, construct its ***supersymmetric version*** by substituting the phase space for the super phase space (31)–(32), and write the classical path integral (33), then working out this integral, we will find that the resulting expression exhibits ***self-denial*** of classical physics at some point. The ***supersymmetry*** structure of (33) is automatically ***broken*** at this ***critical point***.

A further interesting issue is the relation between ***Hawking radiation*** and ***gravitational anomalies***

S. Robinson and F. Wilczek, "Relation between Hawking radiation and gravitational anomalies," *Phys. Rev. Lett*. **95** (2005) 011303; gr-qc/0502074v3.

In order to avoid a ***breakdown*** of ***general covariance*** at the ***quantum*** level, the total flux in each outgoing partial wave of a quantum field in a black hole background must be equal to the flux of a (1+1)-dimensional blackbody at the Hawking temperature. A ***gravitational anomaly*** arising for a chiral scalar field in spherically symmetric spacetimes with an event horizon,

$$D_\mu T^\mu_{\ \nu} = \frac{1}{96\pi\sqrt{-g}}\,\epsilon^{\beta\delta}\partial_\delta\partial_\alpha\Gamma^\alpha_{\ \nu\beta}\ , \qquad (35)$$

can be ***cancelled*** by a thermal flux of the form of blackbody radiation with the Hawking temperature $T = \kappa/(2\pi)$.

An anomaly is attributed to non-invariance of the quantum path integral measure under the symmetry transformation involved. The ***classical*** path integral ***measure*** is larger than the quantum measure because it includes auxiliary fields $\lambda, c, \bar{c}$. This aids in cancellating the anomaly and regaining the associated symmetry. For a fermion coupled with a gauge field, it was shown that the way these auxiliary fields

E. Gozzi, D. Mauro, A. Silvestri, "Chiral anomalies via classical and quantum functional methods," *Int. J. Mod. Phys.* **A 20** (2005) 5009; hep-th/0410129.

transform compensates exactly the Jacobian which arises from the transformation of the fields appearing in the quantum measure, so that the chiral anomaly is absent at the classical level. How does this mechanism for avoiding the breakdown of general covariance cease to be true in the act of forming a black hole due to the supersymmetry violation in the classical path integral, which leaves behind it the only possibility for keeping the system to be diffeomorphism invariant – to launch Hawking radiation?

Conclusions

- .***Criterion for comparing the classical and the quantum***: ***creations*** and ***annihilations*** of particle-antiparticle pairs are ***forbidden*** in the ***classical*** picture, but ***possible*** in the ***quantum*** picture

- Changing the ***overall sign of the spacetime signature*** in the ***classical*** description of ***field propagation*** renders it the ***quantum*** description of field propagation

- The ***event horizon*** of a Schwarzschild black hole is a boundary which ***demarcates*** the classical and the quantum

- A self-gravitating object at the last stage of its classical evolution, just before its shrinking down to below the horizon, can be described using the ***Foldy–Wouthuysen representation*** of the Dirac equation in curved spacetimes and ***Gozzi's*** classical ***path integral*** technique

- In both descriptions, maintaining the dynamics in the ***classical regime*** is ***controlled*** by ***supersymmetry***

- Finding the Foldy–Wouthuysen dynamics for a collapsing wave packet ψ or calculating Gozzi's path integral for a gravitationally collapsing fluid will indicate a ***critical point*** where a ***self-destruction*** of this classical machinery occurs. The ***supersymmetry*** undergoes a ***breakdown*** at this point