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Symmetrization of superconducting Coulomb pairing potential due to electron-phonon interaction

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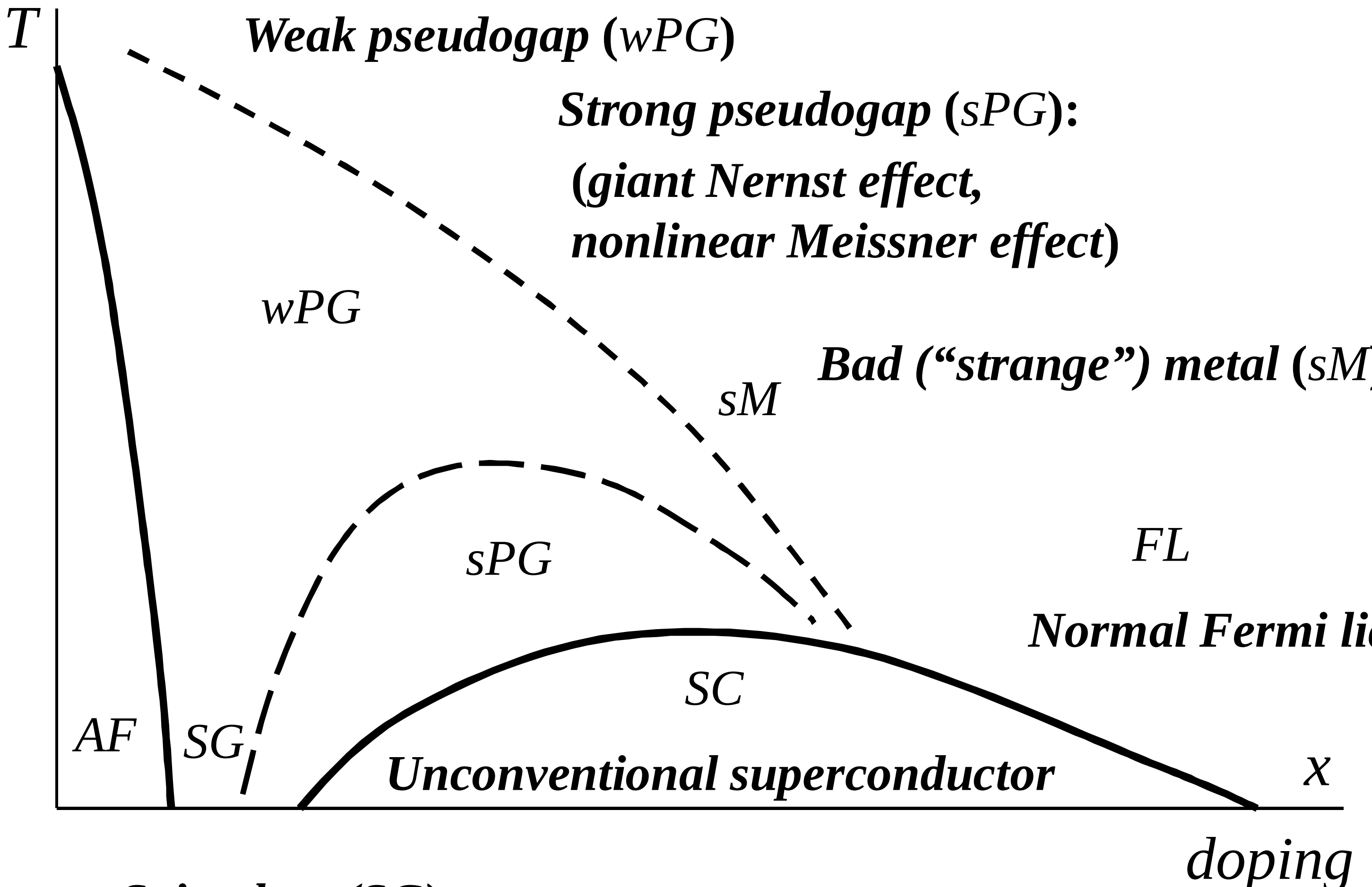
Outlook

- ***Phase diagram. High-T_c superconductivity of cuprates***
  - ***Isotope effect.***
  - ***Unconventional superconductivity of cuprates***
  - ***Fluctuations of the SC order in broad temperature region above T_c***
  - ***Optical conductivity: high-energy problem***
  - ***Checkerboard real space pattern***
- ***Fulde-Ferrell-Larkin-Ovchinnikov state***
- ***Superconducting pairing with large momentum (K-pairing)***
  - ***Generic nesting feature of the Fermi contour***
  - ***Quasi-stationary states (QSS) of K-pairs***
  - ***Topology of the SC order parameter: intrinsic lines of zeroes***
- ***K-pairing under Coulomb repulsion: basic corollaries***
  - ***QSS: broad region of SC fluctuations above T_c***
  - ***Electron-hole asymmetry: "high-energy" physics***
  - ***K-pair density wave: generic checkerboard pattern both below and above T_c***
  - ***Phonon-induced enhancement of T_c***

Phase diagram of cuprates

***Neel antiferromagnet** (AF)*



Spin glass (SG)

Isotope effect in cuprates

"In conventional superconductors, the large dependence of the superconducting transition temperature** (**T_c**) **on the isotope mass** (**$T_c \approx m^{-\alpha}$, with $\alpha \approx 0.5$**) **gives strong evidence for electron pairing by a phonon-mediated mechanism. The copper oxide superconductors … exhibit a small oxygen isotope effect** (**$\alpha \approx$ 0.02 …**)**, suggesting a mechanism more complex than simple phonon-mediated pairing…."

D. Zech et al., *Nature* **371**, 681 (1994)

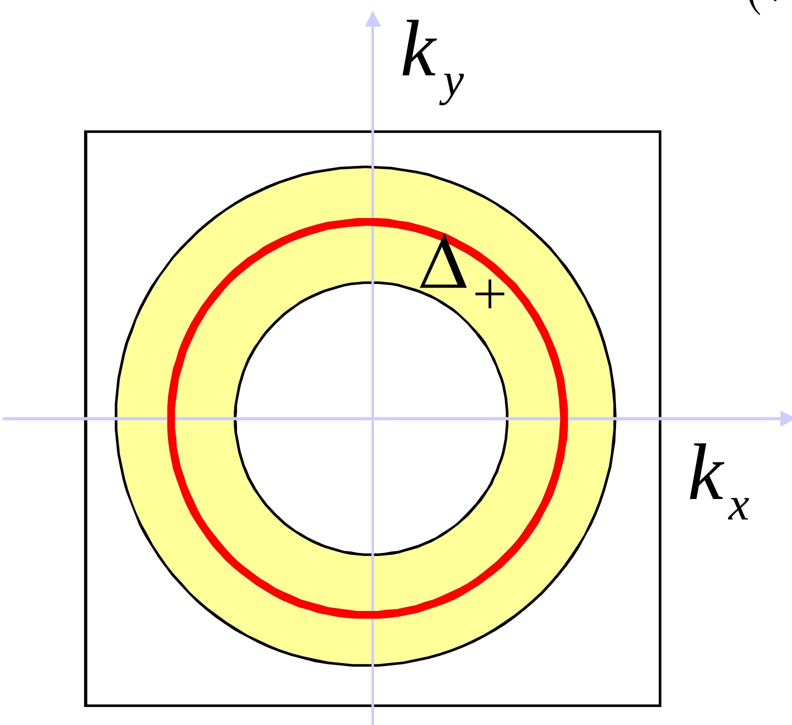
"The dominant mechanism of Cooper-pair formation in the copper oxide materials … must either be all electronic in nature or, conceivably, involve the phonons in a way much more sophisticated than in BCS theory."

A.J. Leggett, *Nature Physics* **2**, 134 (2006)

BCS s-wave superconductivity

Self-consistency equation

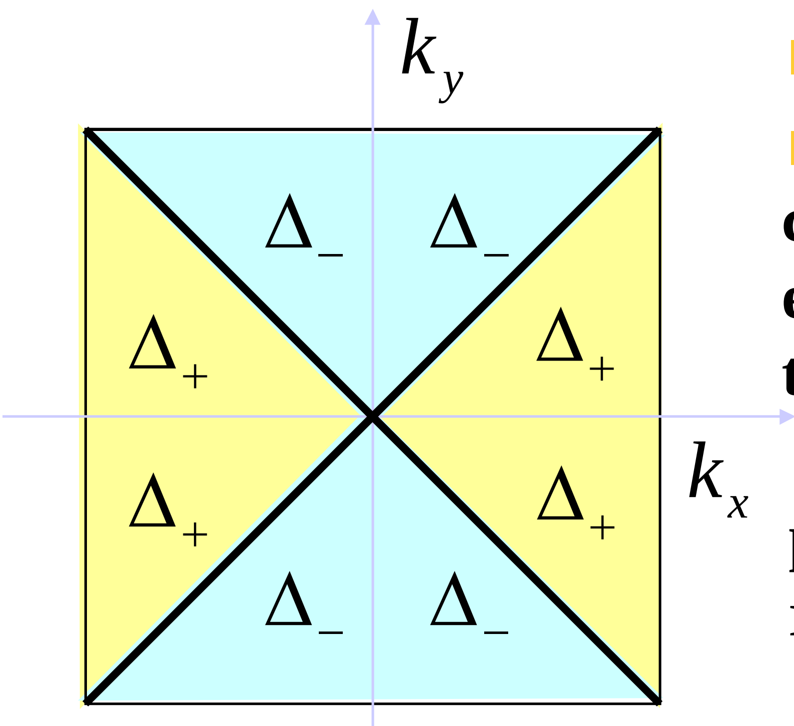
$$\Delta_+(k) = -\frac{1}{2}\int_{(+)} \frac{U(k-k')\Delta_+(k')d^2k'}{\sqrt{\xi^2(k')+\Delta_+^2(k')}}$$



- **Singlet pairing**
- **Attractive phonon-mediated pairing interaction of constant sign**
- **Momentum nearly independent order parameter without nodal lines crossing the Fermi contour**

d-wave superconductivity

$$\Delta_{\pm}(k) = -\frac{1}{2}\int_{(\pm)} \frac{U(k-k')\Delta_{\pm}(k')d^2k'}{\sqrt{\xi^2(k')+\Delta_{\pm}^2(k')}} - \frac{1}{2}\int_{(\mp)} \frac{U(k-k')\Delta_{\mp}(k')d^2k'}{\sqrt{\xi^2(k')+\Delta_{\mp}^2(k')}}$$



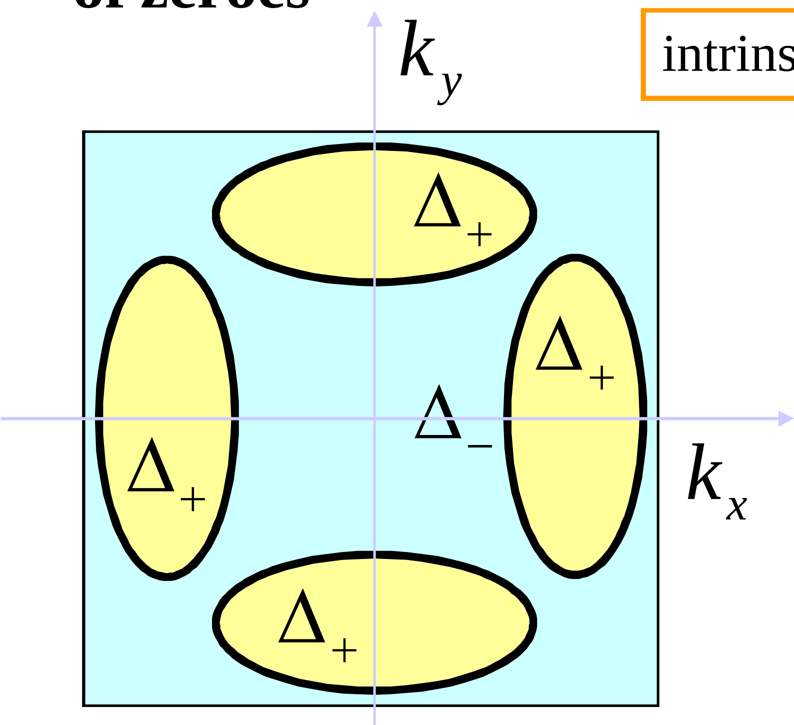
***d*-wave symmetry:** $\left(d_{x^2-y^2}\right)$

- **Singlet pairing**
- **Effective pairing interaction changes its sign: AF magnon exchange, RVB, Hubbard model, t-J-model**
- **Momentum dependent order parameter with nodal lines: rotational symmetry**

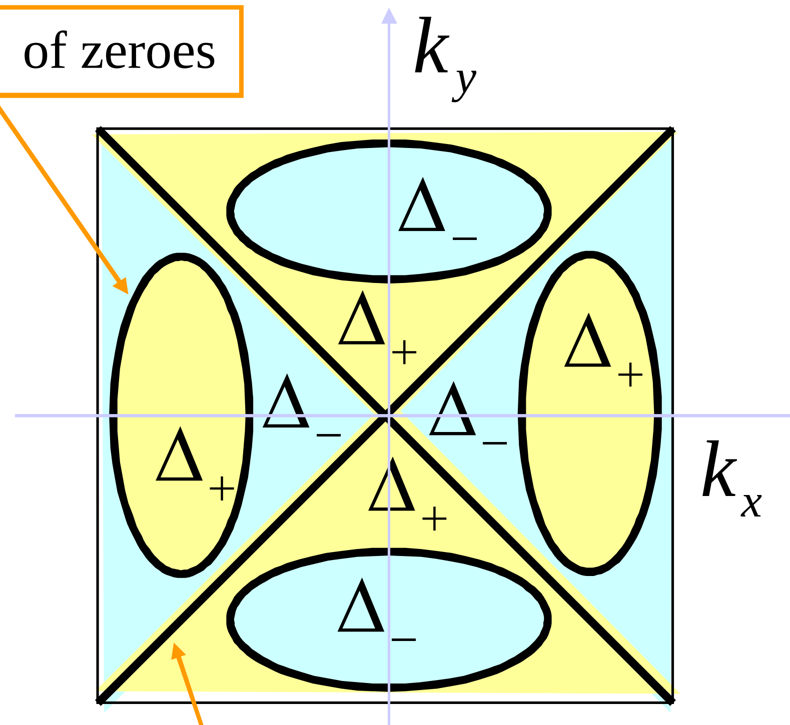
$$\Delta(k) = \Delta_0 \cdot (\cos k_x - \cos k_y)$$

Unconventional superconductivity under repulsive pairing interaction

- **Singlet pairing**
- **Effective pairing interaction changes its sign**
- **Momentum dependent order parameter with intrinsic lines of zeroes**



extended *s*-wave

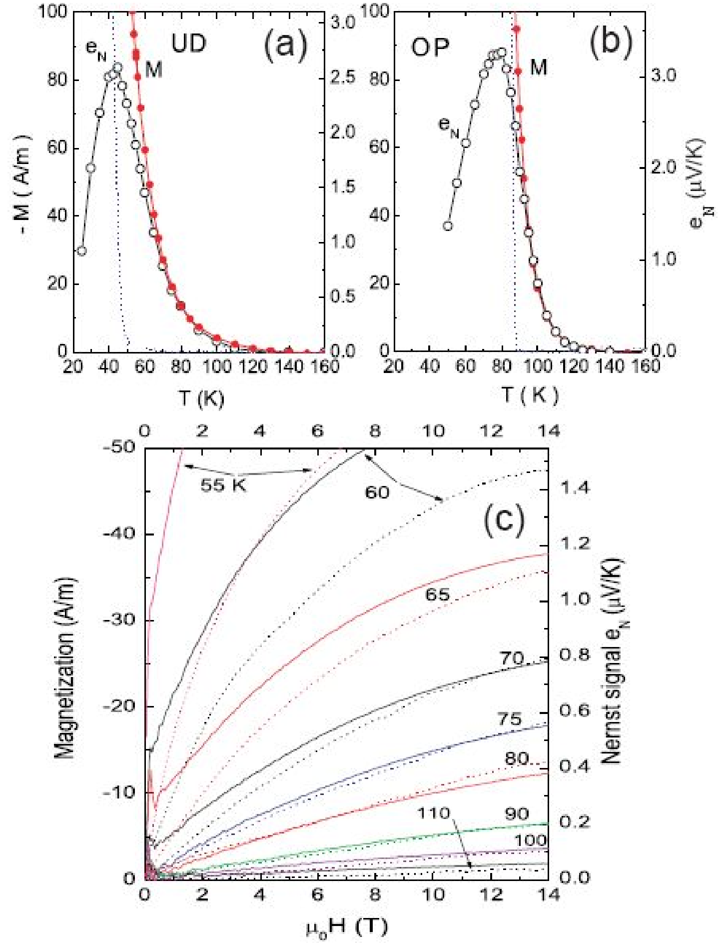


***d*-wave**

Fluctuations of the SC order in a broad region above T_c

Giant Nernst effect and enhanced diamagnetism

Z.A. Xu et al., *Nature* **406**, 486 (2000) $Bi_2Sr_{2-y}La_yCu_2O_6$, T_c=28 K



"In underdoped copper oxides, there is strong evidence that … the pseudogap opens up at a temperature significantly higher than the critical temperature (by 100–220 K)"

Nernst signal up to ≈ 100 K

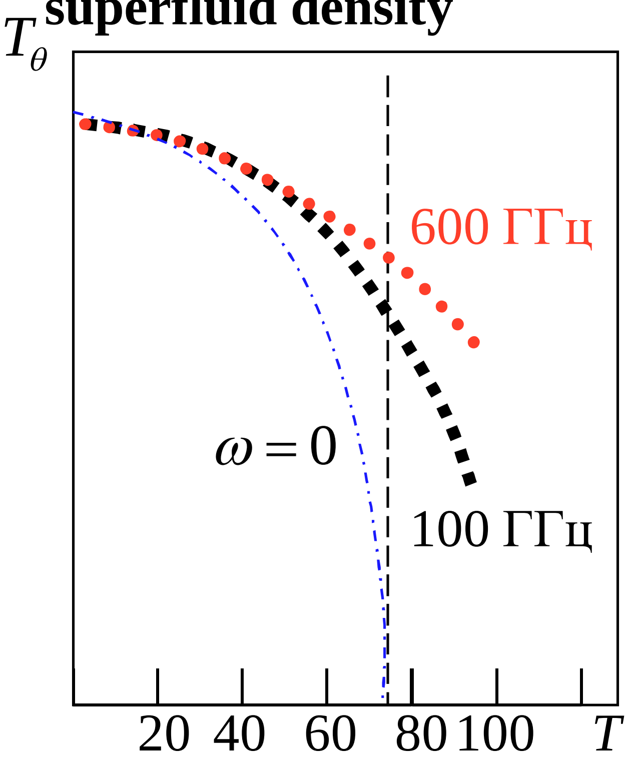
Anomalous diamagnetism above the loss of the phase coherence up to the crossover into the weak pseudogap

Fluctuations of the SC order in a broad region above T_c

High-frequency (~THz) conductivity

J. Corson, R. Mallozzi, J. Orenstein, J.N. Eckstein, I. Bozovic, Vanishing of phase coherence in underdoped $Bi_2Sr_2CaCu_2O_{8+\delta}$, *Nature* **398**, 221-223 (1999).

Frequency dependence of the superfluid density



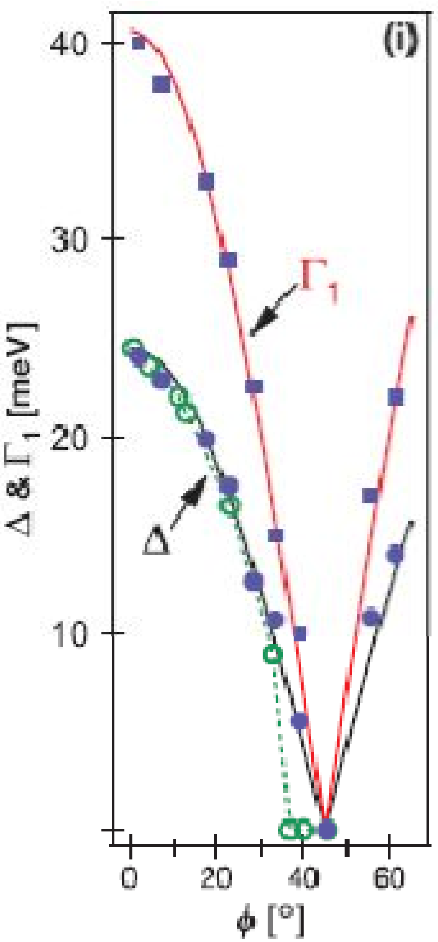
$$T_\theta = \frac{N_s \hbar^2}{m^*}$$

Direct evidence of the motion of thermally generated vortices

"Although the binding of electrons into Cooper pairs is essential in forming the superconducting state, its remarkable properties—zero resistance and perfect diamagnetism—require phase coherence among the pairs as well. When coherence is lost at the transition temperature T_c, pairing remains, together with phase correlations which are finite in space and time".

Fluctuations of the SC order in a broad region above T_c

ARPES



- **SC gap**
- **PG**
- **1/lifetime**

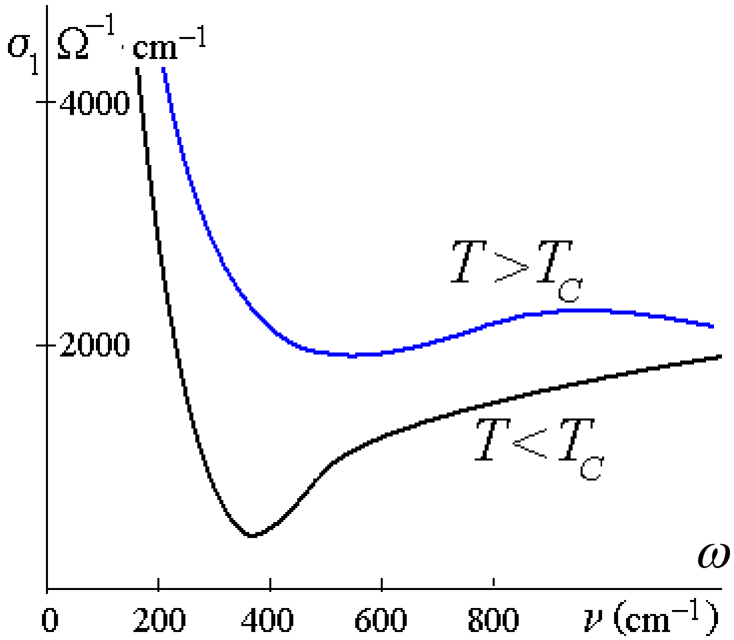
M. Shi et al., *E. Phys. Lett*. **88**, 27008 (2009)

"…direct evidence for preformed Cooper pairs, implying that the pseudogap phase is a precursor to superconductivity"

"…the superconducting and pseudogaps have the same d-wave form with the same amplitude"

"…experimental results support the idea that the pseudogap originates from preformed Cooper pairs for $T > T_c$. However, because the energy gap is larger than the phase stiffness in the pseudogap phase, the preformed pairs have a finite lifetime and can not travel in the crystal coherently"

Optical conductivity: high-energy problem



$$\sigma_1(T > T_c) - \sigma_1(T < T_c) \neq 0,\ \ \hbar\omega \sim 100\Delta$$

do not coincide up to $\hbar\omega \sim 100\Delta$

D.N. Basov, T. Timusk, *Rev. Mod. Phys.* **77**, 721 (2005)

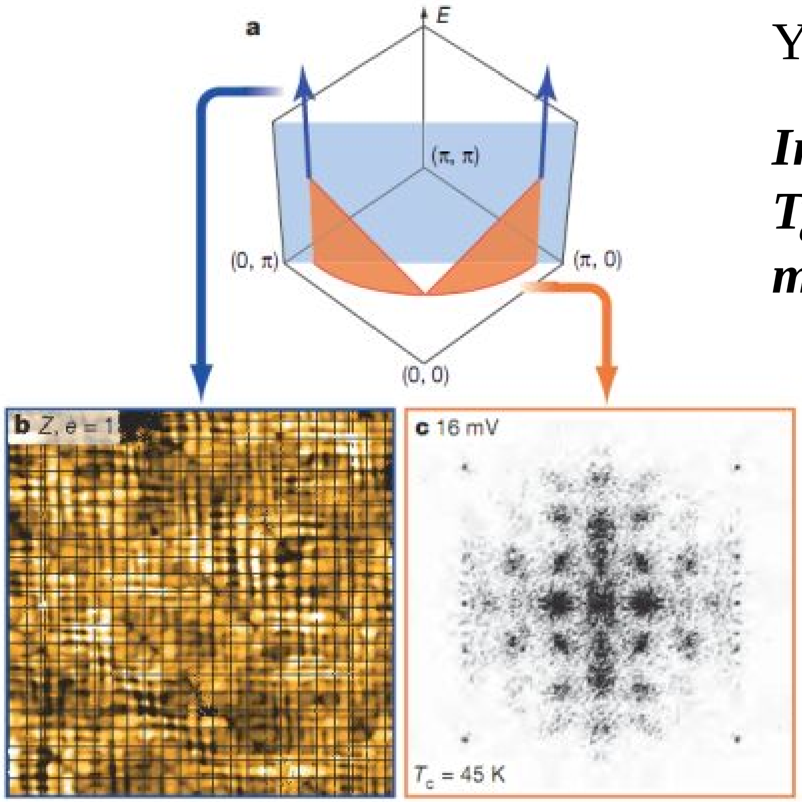
"… a complete theory of high-temperature superconductivity would still have to give an account of the high-energy eff ects of its onset."

A.J. Leggett, *Nature Physics* **2**, 134 (2006)

Real space **checkerboard** *pattern of the superconducting and pseudogap states*

Checkerboard in vortex cores: J.E. Hoffman et al., Science **295**, 466 (2002).

Checkerboard above T_c: C. Howald et al., PRB **67**, 014533 (2003);
K. McElroy et al., PRL 94, 197005 (2005).
Checkerboard above and below T_c: T. Hanaguri et al., Nature **430**, 1001 (2004).



Y. Kohsaka et al., *Nature* **454**, 1072 (2008)

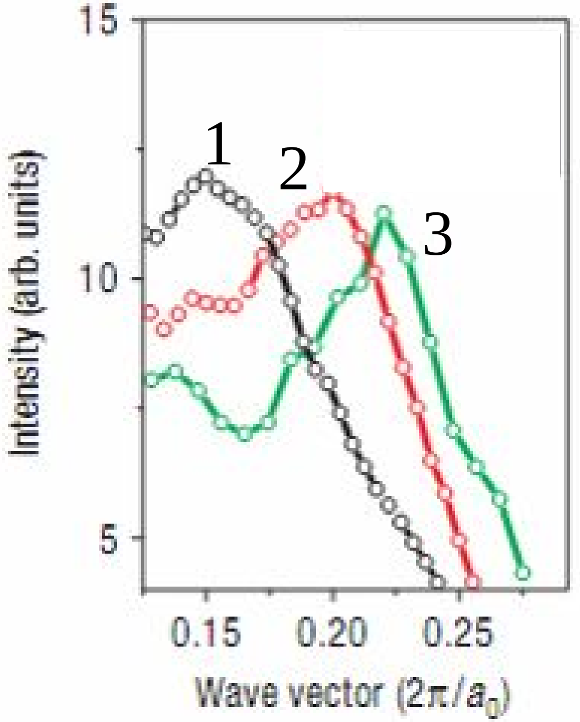
Imaging of the electron structure of high-T_c $Bi_2Sr_2CaCu_2O_{8+\delta}$ in real space and momentum space

- ***pseudogap and superconducting states attributed with different regions of the momentum space***
- ***reduction of the rotation symmetry during transition from the superconducting state into the pseudogap one:*** $C_4 \rightarrow C_2$

Doping dependence of the checkerboard

W.D.Wise et al. , ***Charge-density-wave origin of cuprate Checkerboard visualized by scanning tunnelling microscopy,*** *Nature Physics* **4**, 696 (2008)**.**

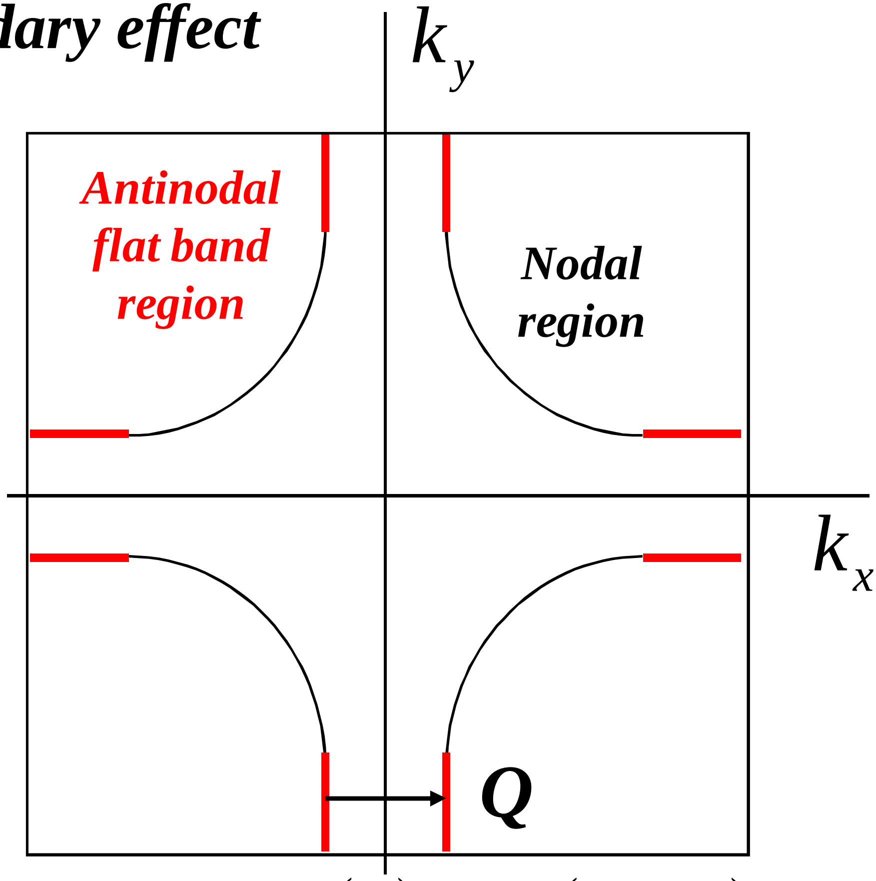
Pair-density wave (***PDW***) ***as a secondary effect***



$1: optimally\ doped\ (x = x_{opt}; T_c = 35\,K)$

$2: underdoped\ (x = x_2 < x_{opt}; T_c = 32\,K)$

$3: underdoped\ (x = x_3 < x_2; T_c = 25\,K)$



$Nesting: \varepsilon(\mathbf{p}) = -\varepsilon(\mathbf{Q} - \mathbf{p})$

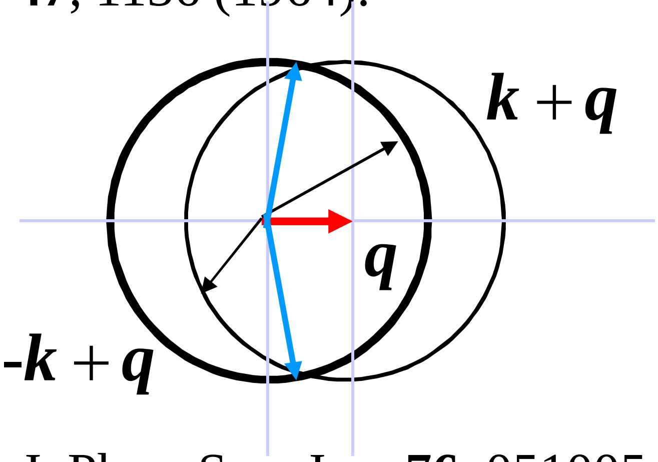
Fulde-Ferrell-Larkin-Ovchinnikov state

P. Fulde, R.A. Ferrell, Phys. Rev. **135**, A550 (1964).
A.I. Larkin, Yu.N. Ovchinnikov: Zh. Eksp. Teor. Fiz. **47**, 1136 (1964).

1D pair density waves

$\Delta \sim \exp(i\boldsymbol{qr})$: *current FF state*

$\Delta \sim \cos(\boldsymbol{qr})$: *currentless LO state*



$$q = 2\mu_B H / \hbar v_F << a^{-1}$$

2D FFLO patterns

Y. Matsuda, H. Shimahara, J. Phys. Soc. Jpn. **76**, 051005 (2007). H. Shimahara, Phys. Rev. B **50**, 12760 (1994).

triangular
$$\Delta(\boldsymbol{r}) = \Delta_1 \cdot \left(e^{i\boldsymbol{q}_1\boldsymbol{r}} + e^{i\boldsymbol{q}_2\boldsymbol{r}} + e^{i\boldsymbol{q}_3\boldsymbol{r}}\right)$$

square
checkerboard
$$\Delta(\boldsymbol{r}) = \Delta_1 \cdot [\cos(qx) + \cos(qy)]$$

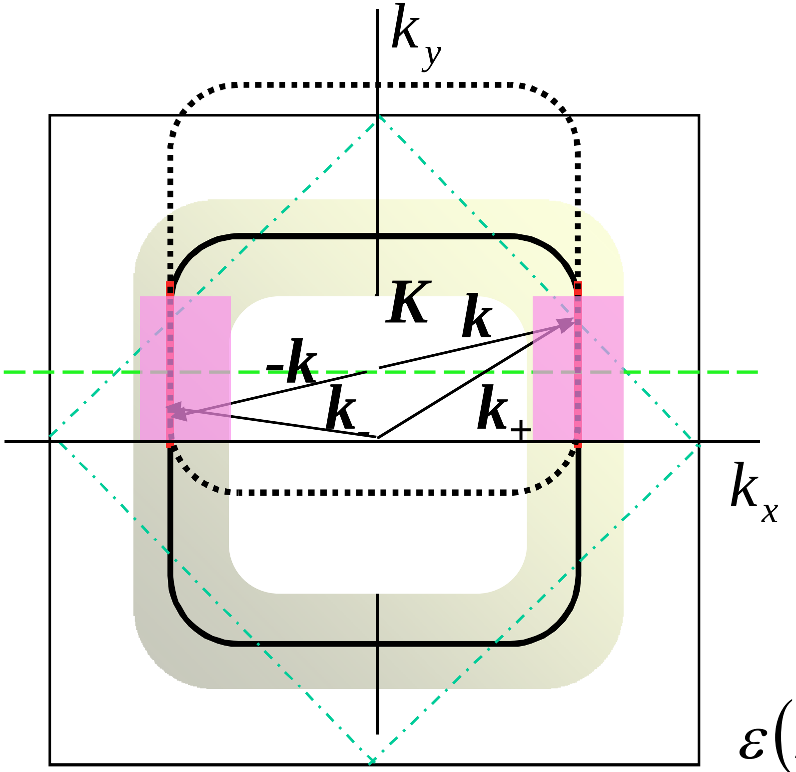
hexagonal
$$\Delta(\boldsymbol{r}) = \Delta_1 \cdot [\cos(\boldsymbol{q}_1\boldsymbol{r}) + \cos(\boldsymbol{q}_2\boldsymbol{r}) + \cos(\boldsymbol{q}_3\boldsymbol{r})]$$

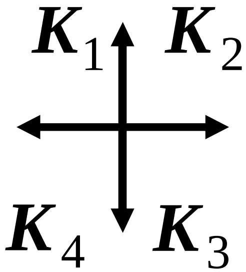
$$\boldsymbol{q}_1 = (q,0),\ \boldsymbol{q}_2 = \left(-q/2, \sqrt{3}q/2\right),\ \boldsymbol{q}_3 = \left(-q/2, -\sqrt{3}q/2\right)$$

Superconducting pairing with large momentum

- **Kinematic constraint: particles that form the SC pair must be both either inside or outside the Fermi contour**

- **Four crystal equivalent pair momenta: C_2 or C_4 current-less order parameter (checkerboard)**





$$\boldsymbol{K} \equiv \boldsymbol{K}_1$$

$$C_4: \ \Delta(\boldsymbol{r}) = \Delta_0 \cdot [\cos(Kx) + \cos(Ky)]$$

$$C_2: \ \Delta(\boldsymbol{r}) = \Delta_0 \cdot \cos(Kx)$$

$$\Delta(\boldsymbol{r}) = \Delta_0 \cdot \cos(Ky)$$

- **Mirror nesting**

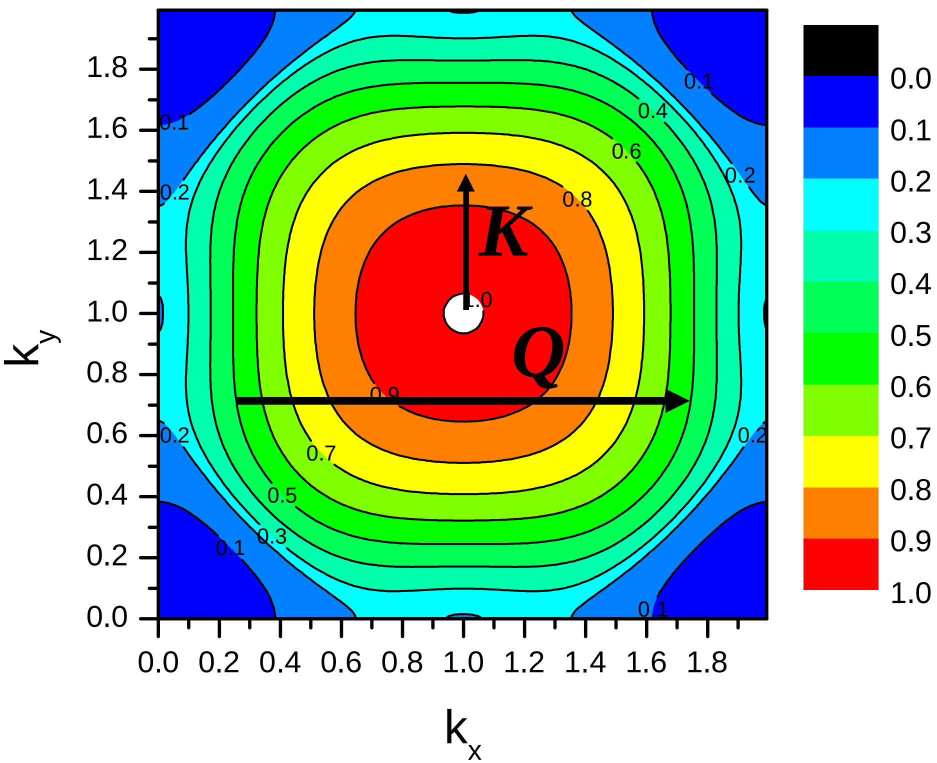
$$\varepsilon(\boldsymbol{K}/2 + \boldsymbol{k}) = \varepsilon(\boldsymbol{K}/2 - \boldsymbol{k}) \quad \boldsymbol{k}_\pm = \boldsymbol{K}/2 \pm \boldsymbol{k}$$

Nesting feature of the Fermi contour.

Deviation from perfect mirror nesting

$$\varepsilon(k_x, k_y) = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - 2t''(\cos 2k_x + \cos 2k_y)$$

$$t = 0.5eV,\ t'/t = -0.15,\ t''/t = 0.07$$



Q: nesting vector

K: total pair momentum

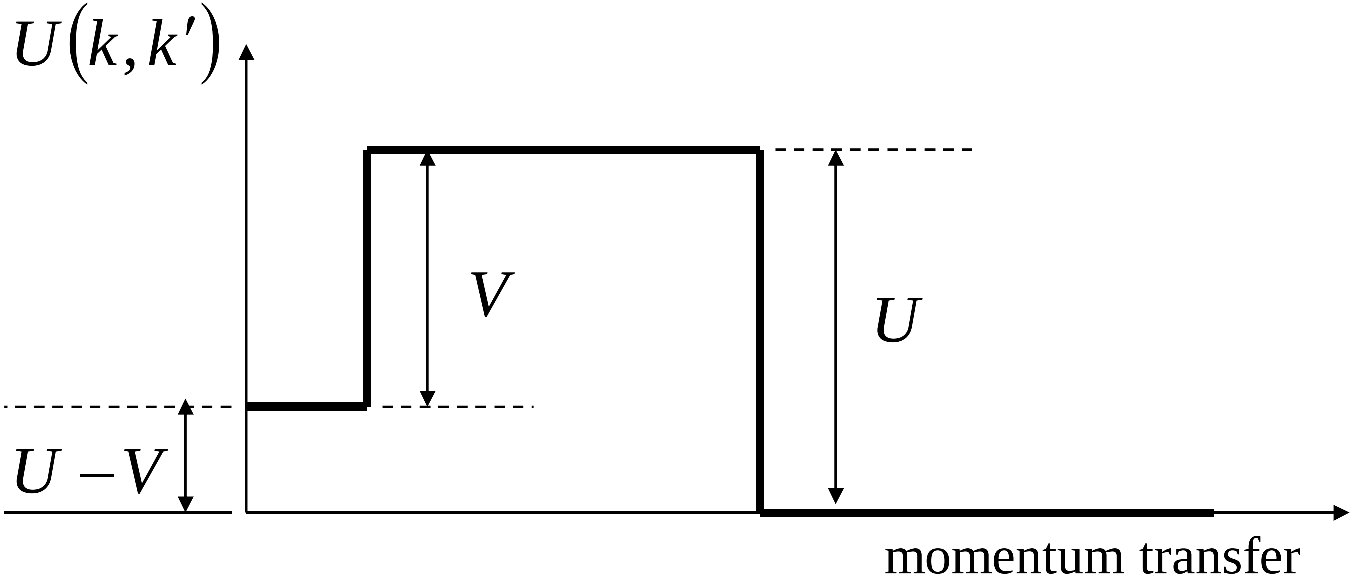
SC gap $\Delta = \sqrt{\Delta_0(\Delta_0 - \delta)}$

δ: **mean square deviation of the Fermi contour from perfect mirror nesting**

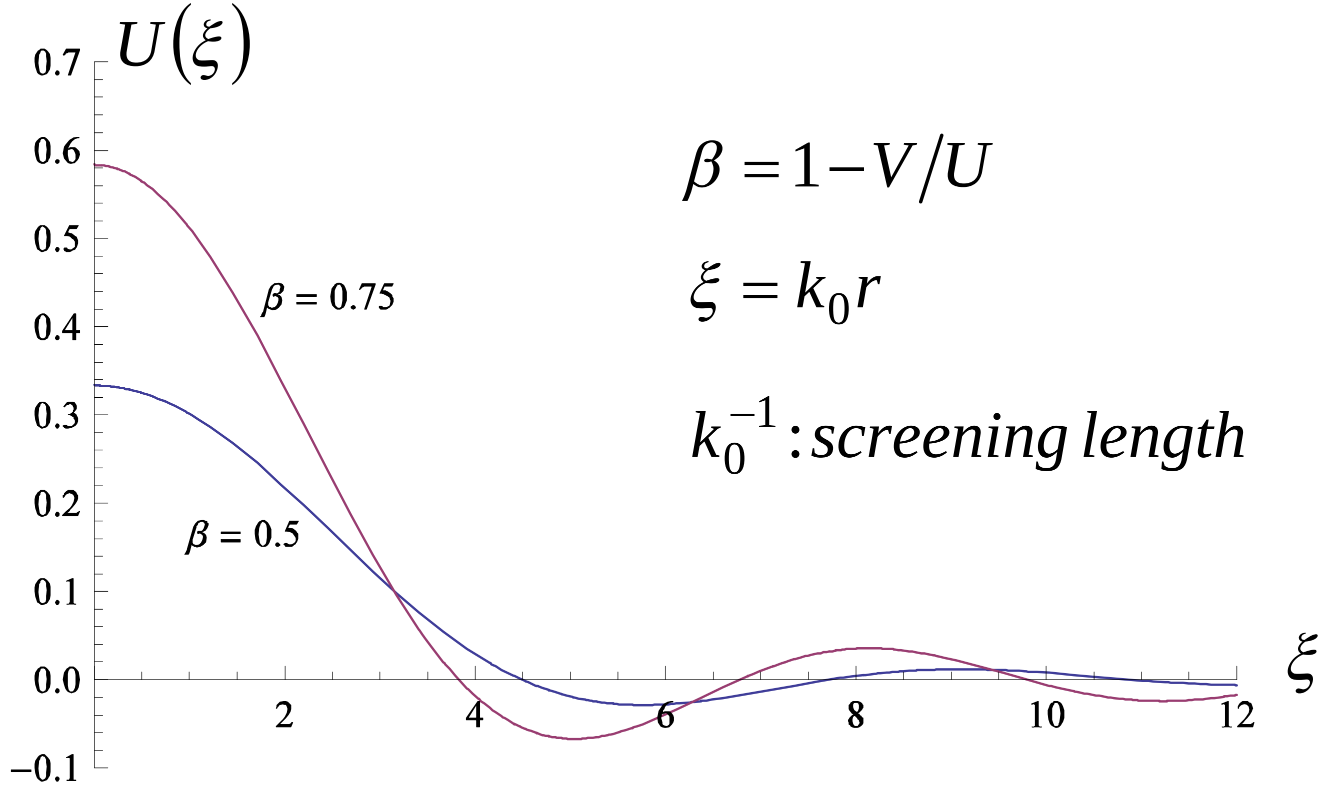
Δ_0: **SC gap in the case of perfect mirror nesting**

Model pairing potential

$$U(k,k') = U \text{ : inside the kinematic constraint}$$
$$U(k,k') = U - V \text{ : inside the phonon – mediated attraction}$$
$$U(k,k') = 0 \text{ : outside the kinematic constraint}$$



Pairing interaction: real space view



$$\beta = 1 - V/U$$

$$\xi = k_0 r$$

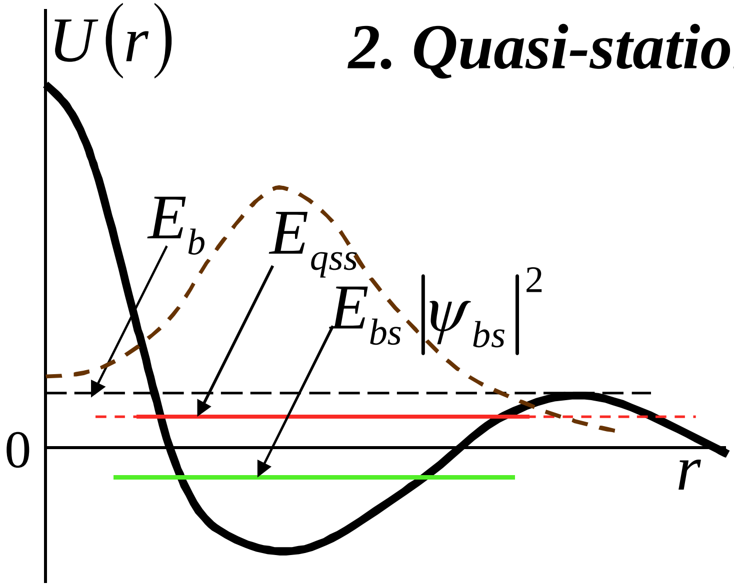
$$k_0^{-1} : \textit{screening length}$$

Cooper problem: a pair with large momentum

Wave function of the relative motion: two solutions

1. Bound state $E_{bs} \sim \exp\left(-1/gU_0\right)$

2. Quasi-stationary state $E \to E_{qss} - i\Gamma/2$



$$E_{qss} \sim \exp\left(-1/gU_0\right)$$

$$\Gamma \leq E_{qss}$$

U_0 coupling constant

g density of states

Non-diagonal short range order

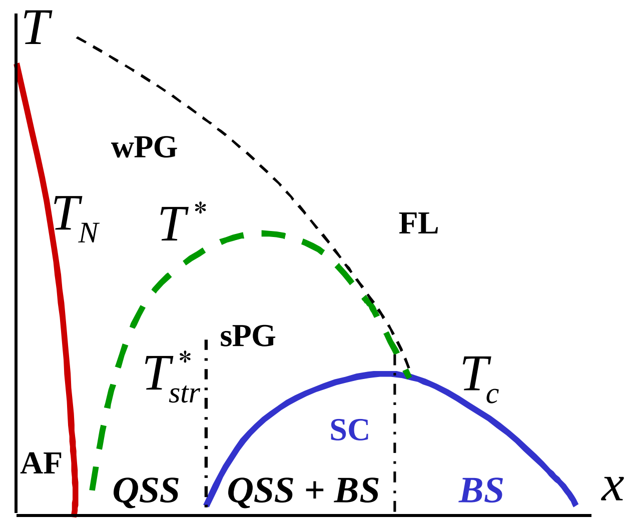
A rise of the coherence: transition into SC state

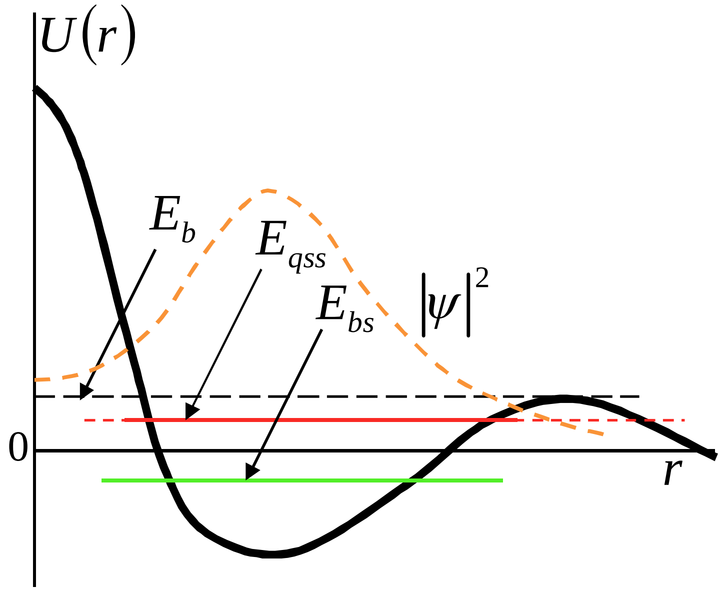
$$E_{qss} - E_{bs} \sim T_c \Rightarrow \Delta_K$$

$$T < T_c : \Delta_K \neq 0$$

Pair break energy

$$E_b - E_{bs} \sim T^*_{str} \Rightarrow \Delta \equiv \Delta_K + \Delta_{pg}$$





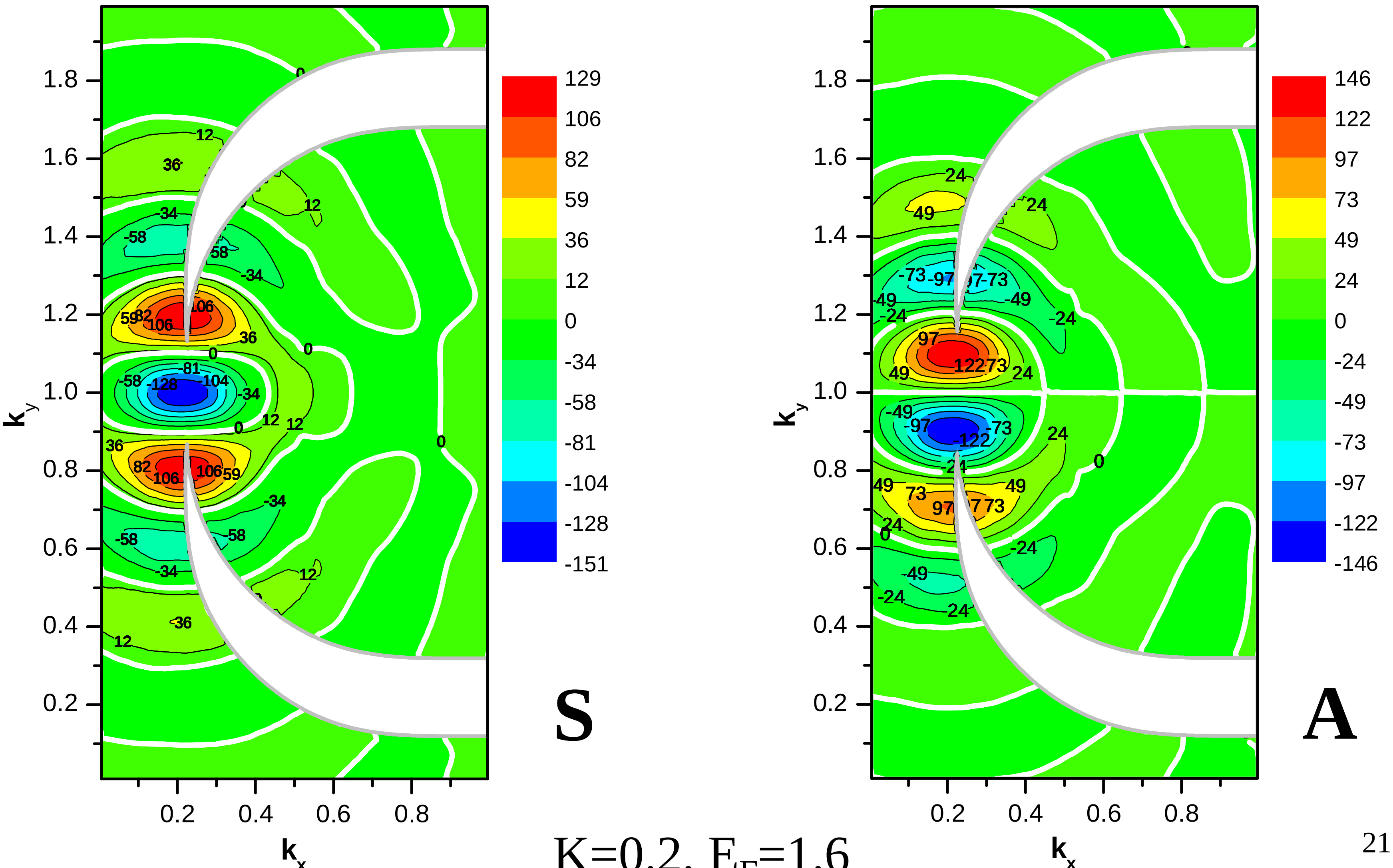
Strong PG: averaging over random phases

$$T_c < T < T^*_{str}: \; \Delta_K = 0, \left\langle \Delta_{pg} \right\rangle = 0, \left\langle \Delta^2_{pg} \right\rangle \neq 0$$

Broad fluctuation region above T_c

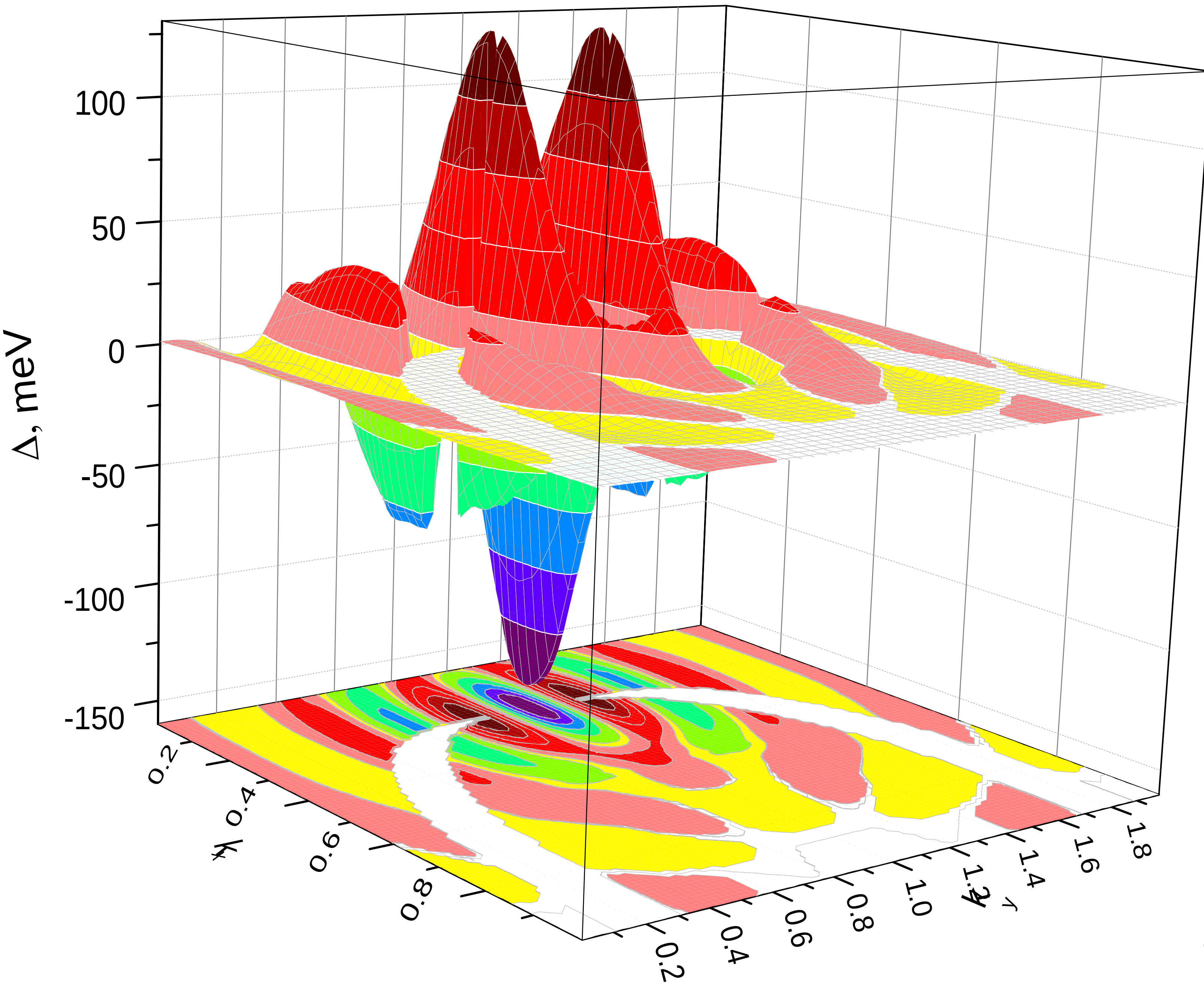
Topology of the SC order parameter

Symmetric and asymmetric solutions of the self-consistency equation

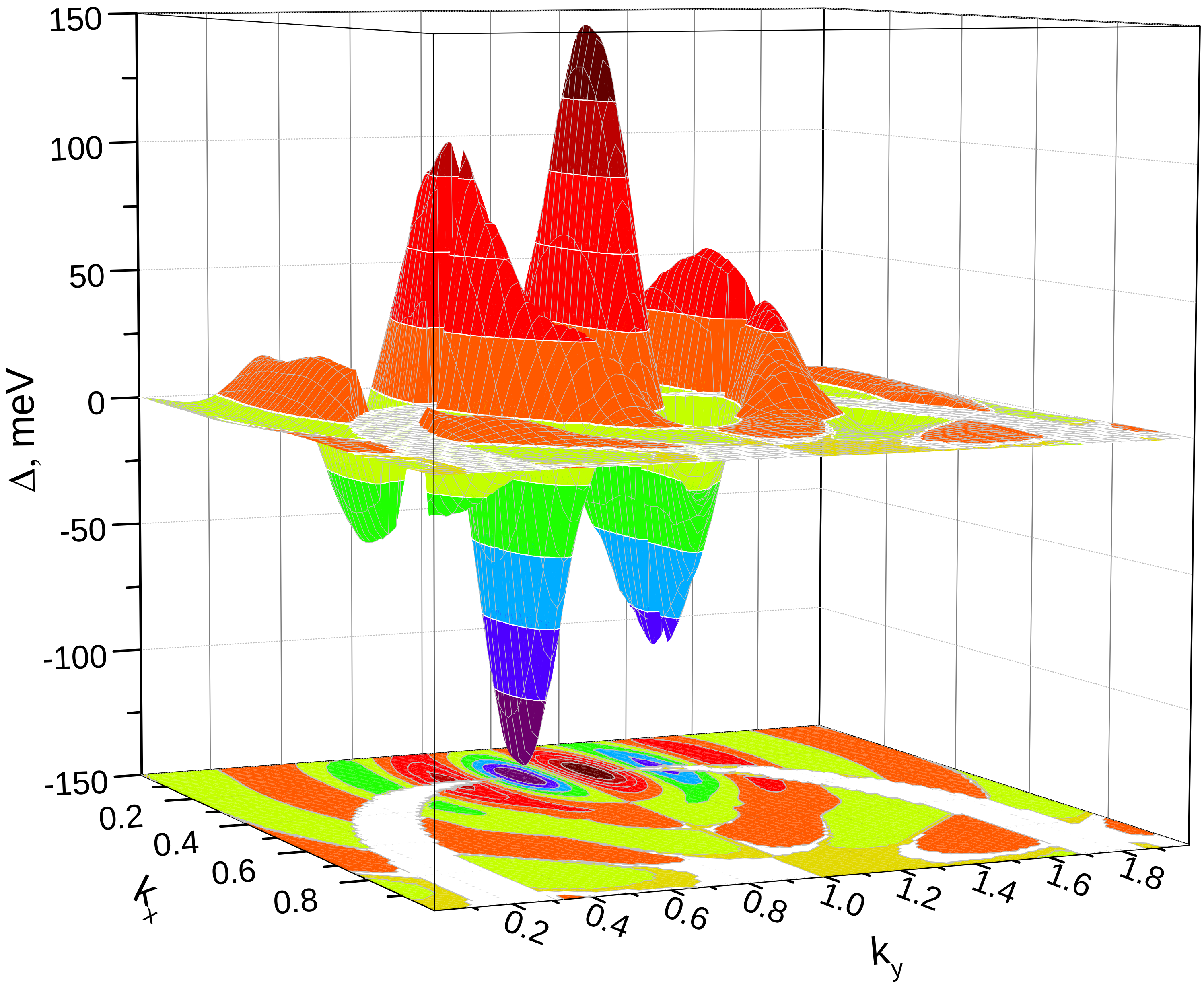


K=0.2, E_F=1.6

Symmetric solutions of the self-consistency equation

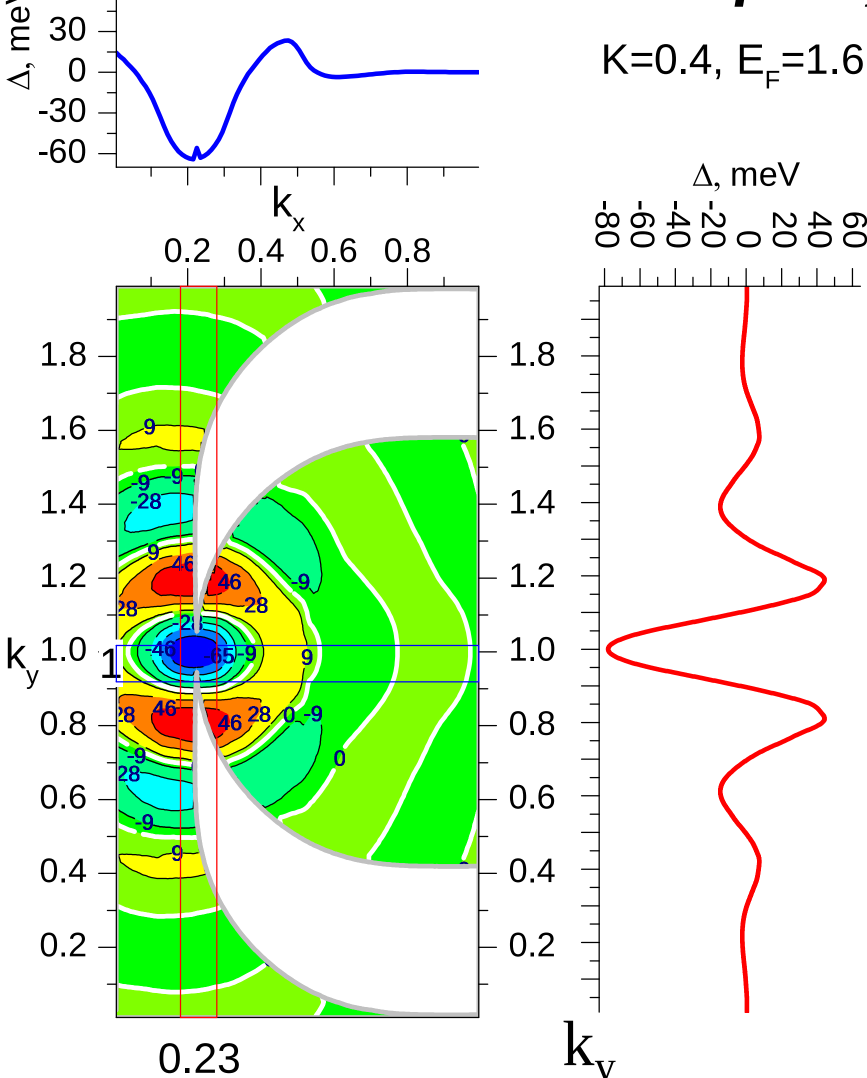


Asymmetric solutions of the self-consistency equation



SC order parameter in the antinodal region

Essential dependence of the SC order parameter on the relative motion momentum of the pair with large total momentum



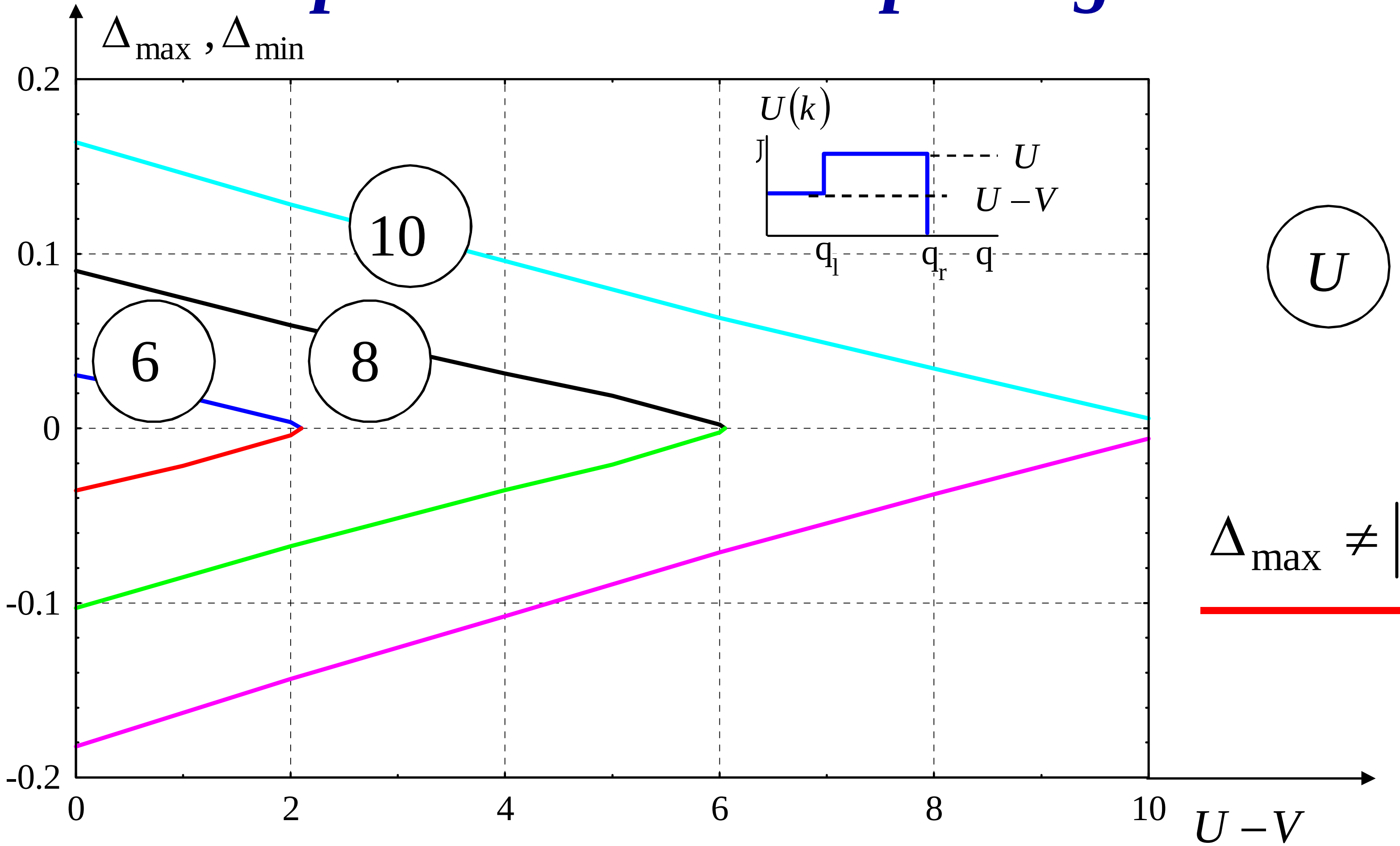
"High-energy" physics: Bogoliubov quasiparticle u-v asymmetry.

Superconductivity-induced chemical potential shift $\delta\mu \sim |\Delta|$ **instead of BCS-like** $\delta\mu \sim \Delta^2$

"… the experimental observation of a discontinuous change in slope at T_c of the temperature dependence of the work function of $YBa_2Cu_3O_{7-\delta}$. We interpret this effect as a superconductivity-induced temperature dependence of the chemical potential."

G. Rietveld, N.Y. Chen, D. van der Marel, PRL 69, 2578 (1992)

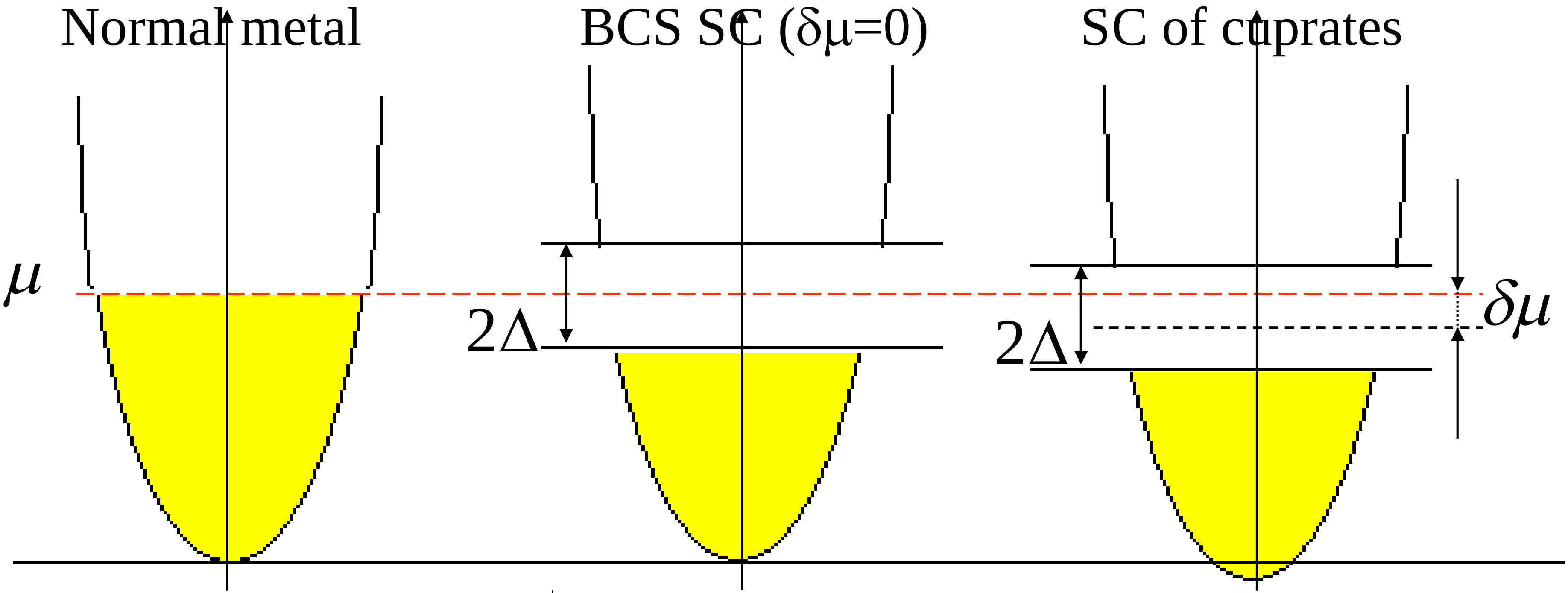
Symmetric solution: influence of phonon-mediated pairing

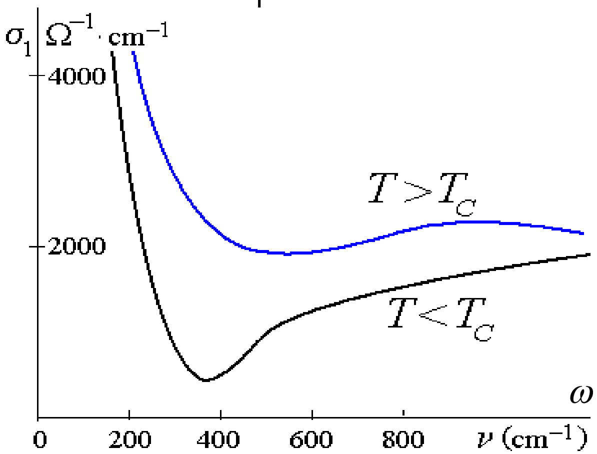


$\Delta_{max} \neq |\Delta_{min}|$

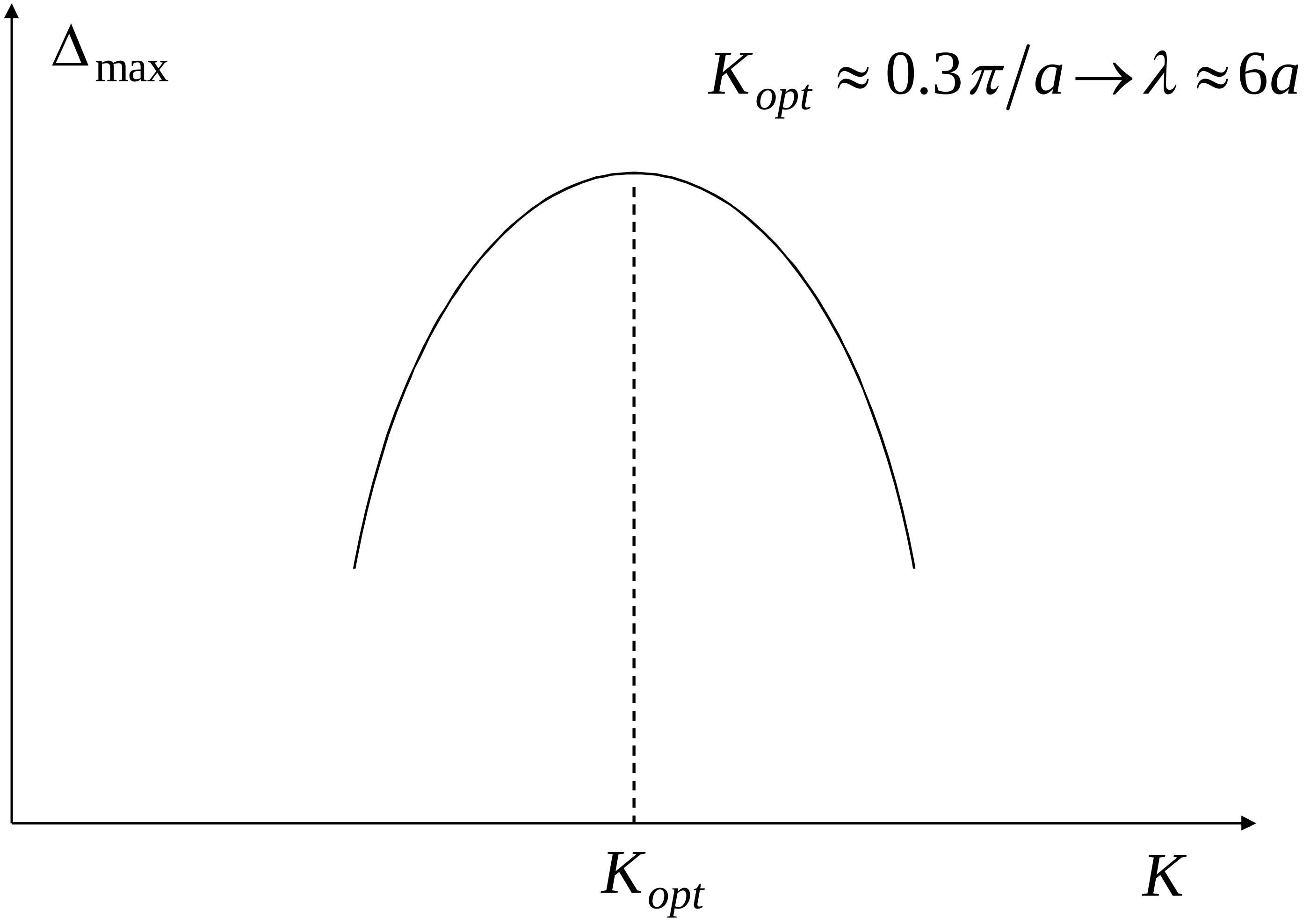
$U \geq 10$: *the solution exists even if* $V = 0$

Electron-hole asymmetry: "high-energy" physics





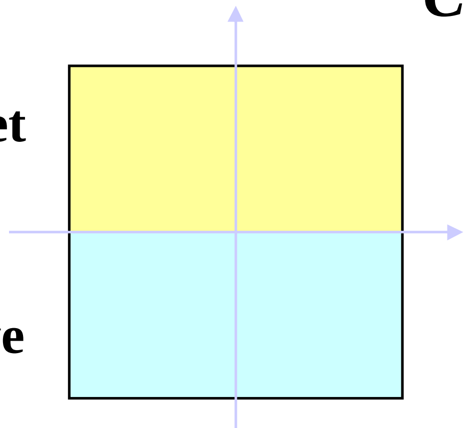
Checkerboard wave-vector



Orbital symmetry of the SC order parameter

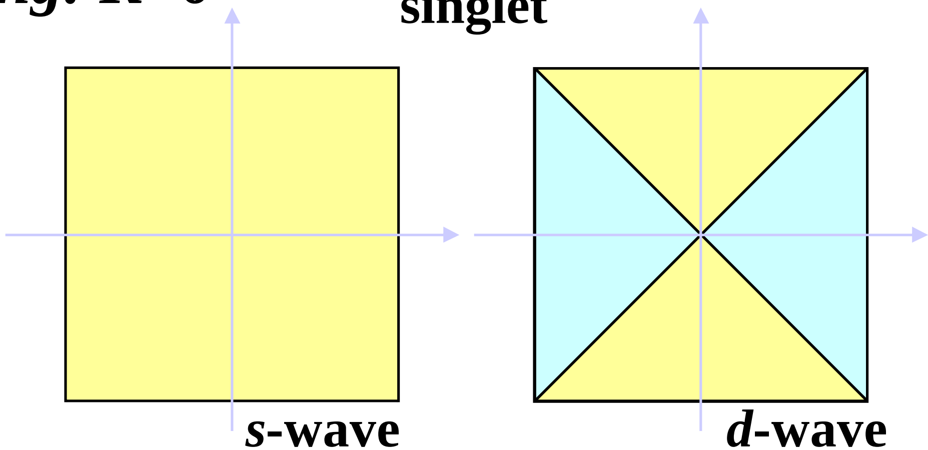
Cooper pairing: K=0

triplet



***p*-wave**

singlet

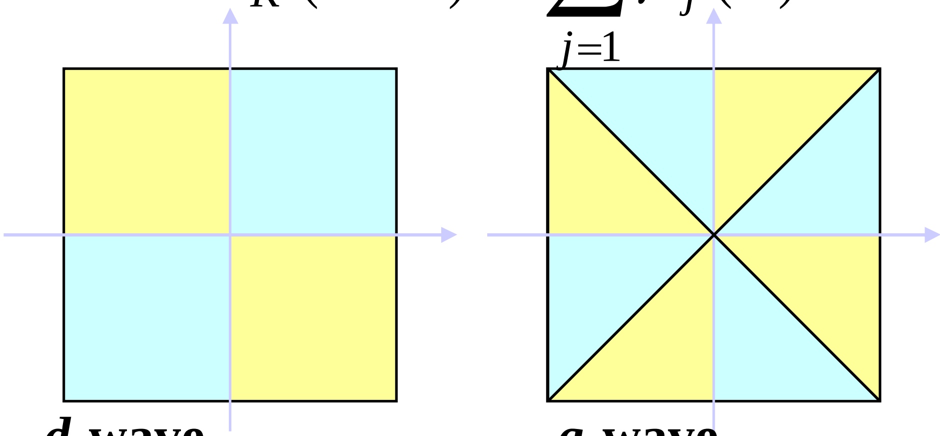


***s*-wave** ***d*-wave**

Singlet SC pairing with large momentum:

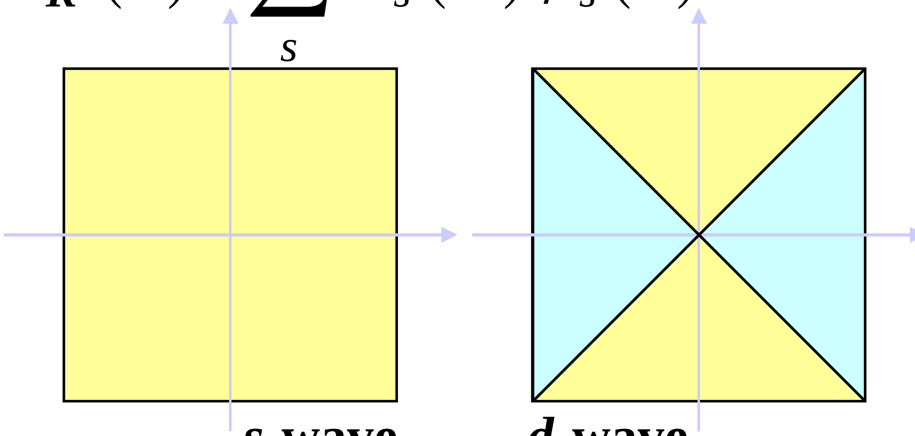
checkerboard order with intrinsic lines of zeroes (not shown)

$$\Delta_K(\boldsymbol{R},\boldsymbol{k}) = \sum_{j=1}^{4} \gamma_j(\boldsymbol{k})\, e^{iK_j \boldsymbol{R}}\, \Delta_{\boldsymbol{K}}^{(j)}(\boldsymbol{k}) = \sum_{s} \Delta_s(\boldsymbol{R})\varphi_s(\boldsymbol{k})$$



***d*-wave** ***g*-wave**

asymmetric solution



***s*-wave** ***d*-wave**

symmetric solution

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- V.I. Belyavsky, Yu.V. Kopaev, Biordered superconductivity and strong pseudogap state, *Phys. Rev*. B **76**, 214506 (2007)
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- V.I. Belyavsky, V.V. Kapaev, Yu.V. Kopaev, D.I. Mikhailyan, Symmetrization of superconducting Coulomb pairing potential due to electron-phonon interaction, *JETP* (in press, 2012)