Ginzburg Conference on Physics, Moscow,
Lebedev Physical Institute

Soft and "hard" singularities in classical and quantum cosmology.

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May 28-June 2, 2012

Based on

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Introduction

- The cosmological singularities constitute one of the main problems of modern cosmology
- "Traditional" or "hard" singularities are associated with the zero volume of the universe (or of its scale factor), and with infinite values of the Hubble parameter, of the energy density and of the pressure –Big Bang and Big Crunch
- The discovery of the cosmic acceleration stimulated the development of "exotic" cosmological models of dark energy; some of these models possess the so called soft or sudden singularities characterized by the finite value of the radius of the universe and its Hubble parameter
- Tachyons (Born-Infeld fields) is a natural candidate for a dark energy

- The toy tachyon model, proposed in 2004 has two particular features:
  Tachyon field transforms itself into a pseudo-tachyon field,
  The evolution of the universe can encounter a new type of singularity - the Big Brake singularity.
- The Big Brake singularity is a particular type of the so called "soft" cosmological singularities - the radius of the universe is finite, the velocity of expansion is equal to zero, the deceleration is infinite.
- The predictions of the model do not contradict observational data on supenovae of the type Ia (2009,2010)
- The Big Brake singularity is a particular one - it is possible to cross it (2010)

- Open questions: other soft singularities - is it possible to cross them ?
- What can tell us the Quantum cosmology on the Big Brake singularity and other soft singularities ?

Description of the tachyon model

The flat Friedmann universe

$$ds^2 = dt^2 - a^2(t)dl^2$$

The tachyon Lagrange density

$$L = -V(T)\sqrt{1 - \dot{T}^2}$$

The energy density

$$\rho = \frac{V(T)}{\sqrt{1 - \dot{T}^2}}$$

The pressure

$$p = -V(T)\sqrt{1 - \dot{T}^2}$$

The Friedmann equation

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \rho$$

The equation of motion for the tachyon field

$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V_{,T}}{V} = 0$$

In our model

$$V(T) = \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]}$$
$$\times\sqrt{1-(1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]}\ ,$$

where k and $\Lambda > 0$ are the parameters of the model. The case $k > 0$ is more interesting.

Some trajectories (cosmological evolutions) finish in the infinite de Sitter expansion. In other trajectories the tachyon field transforms into the pseudotachyon field with the Lagrange density, energy density and positive pressure.

$$L = W(T)\sqrt{\dot{T}^2 - 1},$$
$$\rho = \frac{W(T)}{\sqrt{\dot{T}^2 - 1}},$$
$$p = W(T)\sqrt{\dot{T}^2 - 1},$$
$$W(T) = \frac{\Lambda}{\sin^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\ T\right]}$$
$$\times\sqrt{(1+k)\cos^2\left[\frac{3}{2}\sqrt{\Lambda(1+k)}\,T - 1\right]}$$

What happens with the Universe after the transformation of the tachyon into the pseudotachyon ?

It encounters the Big Brake cosmological singularity.

The Big Brake cosmological singularity and other soft singularities

$$t \to t_{BB} < \infty$$

$$a(t \to t_{BB}) \to a_{BB} < \infty$$

$$\dot{a}(t \to t_{BB}) \to 0$$

$$\ddot{a}(t \to t_{BB}) \to -\infty$$

$$R(t \to t_{BB}) \to +\infty$$

$$T(t \to t_{BB}) \to T_{BB}, \ |T_{BB}| < \infty$$

$$|\dot{T}(t \to t_{BB})| \to \infty$$

$$\rho(t \to t_{BB}) \to 0$$

$$p(t \to t_{BB}) \to +\infty$$

If $\dot{a}(t_{BB}) \neq 0$ it is more general soft singularity.

Crossing the Big Brake singularity and the future of the universe

At the Big Brake singularity the equations for geodesics are regular, because the Christoffel symbols are regular (moreover, they are equal to zero).

Is it possible to cross the Big Brake ?

Let us study the regime of approaching the Big Brake.

Analyzing the equations of motion we find that approaching the Big Brake singularity the tachyon field behaves as

$$T = T_{BB} + \left(\frac{4}{3W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

Its time derivative $s \equiv \dot{T}$ behaves as

$$s = -\left(\frac{4}{81W(T_{BB})}\right)^{1/3} (t_{BB} - t)^{-2/3},$$

the cosmological radius is

$$a = a_{BB} - \frac{3}{4}a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{4/3},$$

its time derivative is

$$\dot{a} = a_{BB}\left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}$$

and the Hubble variable is

$$H = \left(\frac{9W^2(T_{BB})}{2}\right)^{1/3} (t_{BB} - t)^{1/3}.$$

All these expressions can be continued in the region where $t > t_{BB}$,which amounts to crossing the Big Brake singularity. Only the expression for s is singular at $t = t_{BB}$ but this singularity is integrable and not dangerous.

Once reaching the Big Brake, it is impossible for the system to stay there because of the infinite deceleration, which eventually leads to the decrease of the scale factor. This is because after the Big Brake crossing the time derivative of the cosmological radius and Hubble variable change their signs. The expansion is then followed by a contraction, culminating in the Big Crunch singularity.

The classical and quantum dynamics in the scalar field model with a soft singularity

One of the simplest cosmological models revealing the Big Brake singularity is the model based on the anti-Chaplygin gas with an equation of state

$$p = \frac{A}{\rho}, \quad A > 0$$

Such an equation of state arises in the theory of wiggy strings (B. Carter, 1989, A. Vilenkin, 1990).

$$\rho(a) = \sqrt{\frac{B}{a^6} - A}$$

At $a = a_* = \left(\frac{B}{A}\right)^{1/6}$ the universe encounters the Big Brake singularity.

The scalar field model reproducing the cosmological evolution of the model based on the anti-Chaplygin gas has the potential

$$V(\varphi) = \pm\frac{\sqrt{A}}{2}\left(\sinh 3\varphi - \frac{1}{\sinh 3\varphi}\right).$$

We shall study the model with a more simple potential, which has basically the same qualitative behaviour

$$V = -\frac{V_0}{\varphi}, \quad V_0 > 0$$

We shall study first the classical dynamics of this model.

Here are the main results of our analysis.

1. The transitions between the positive and negative values of the scalar field are impossible.
2. All the trajectories (cosmological evolutions) with positive values of the scalar field begin in the Big Bang singularity, then achieve a point of maximal expansion, then contract and end their evolution in the Big Crunch singularity.
3. All the trajectories with positive values of the scalar field pass through the point where the value of the scalar field is equal to zero. After that the value of the scalar field begin growing. The point $\varphi = 0$ corresponds to a crossing of the soft singularity.
4. If the moment when the universe achieves the point of the maximal expansion coincides with the moment of the crossing of the soft singularity then the singularity is the Big Brake.

For completeness:
The evolutions with the negative values of the scalar field belong to two classes - first, an infinite expansion beginning from the Big Bang and second, the evolutions obtained by the time reversion of those of the first class, which are contracting and end in the Big Crunch singularity.

Quantum dynamics - the Wheeler - DeWitt equation

Applying the Hamiltonian formalism to the system Gravity + Scalar field, we obtain the super-Hamiltonian constraint

$$-\frac{p_a^2}{4a} + \frac{p_\varphi^2}{2a^3} + Va^3 = 0,$$

and the implementation of the Dirac quantization procedure gives the Wheeler-DeWitt equation

$$\left(-\frac{\hat{p}_a^2}{4a} + \frac{\hat{p}_\varphi^2}{2a^3} + Va^3\right)\psi(a,\varphi) = 0,$$

$$\left(\frac{a^2}{4}\frac{\partial^2}{\partial a^2} - \frac{1}{2}\frac{\partial^2}{\partial \varphi^2} - \frac{a^6 V_0}{\varphi}\right)\psi(a,\varphi) = 0.$$

Requiring the normalizability of the wave function of the universe we come to the conclusion that this wavefunction should vanish at $\varphi \to 0$.

This value classically corresponds to a soft singularity.

Does it indicate the presence of a quantum avoidance of the singularity ?

No !

The physical sense has the wave function depending on the physical degrees of freedom, obtained after the gauge fixing choice, which simultaneously introduces the time parameter (A.O. Barvinsky, 1990,1993).

If we choose the Hubble parameter as a new time parameter $\tau \equiv -H$, then its conjugated is a^3. The reduction of the initial set of variables to the smaller set of physical degrees of freedom implies the appearance of the Faddeev-Popov determinant which is equal to the Poisson bracket of the gauge-fixing condition and the constraint. This Faddeev-Popov determinant will be proportional to the potential, which is singular at $\varphi = 0$.

The “hard” Big Bang and Big Crunch singularities $a = 0$ correspond to $\varphi = \infty$. To provide the normalizability of the wave function one should have the integral on the values of the scalar field φ convergent, when $|\varphi| \to \infty$.
That means that, independently of details connected with the gauge choice, not only the wave function of the universe but also the probability density of scalar field values should decrease rather rapidly when the absolute value of the scalar field is increasing.
Thus, in this case, the effect of the quantum avoidance of the classical singularity is present.

The quantum cosmology of the tachyon model

The Wheeler-DeWitt equation for the system Gravity + Pseudotachyon is

$$\left(\sqrt{\hat{p}_T^2 - a^6 W^2} - \frac{a^2 \hat{p}_a^2}{4}\right)\psi(a, T) = 0.$$

Let us first consider the case of the constant potential $W(T) = W_0$. The Big Brake singularity corresponds to $|p_T| = a^3\sqrt{W_0}$.

It is convenient to work in the momenta representation $\psi(a, p_T)$.

Then requirement of the well-definiteness of

$$\sqrt{\hat{p}_T^2 - a^6 W_0^2}\psi(a, p_T)$$

becomes algebraic and implies

$$\psi(a, p_T)|_{|p_T| = a^3\sqrt{W_0}} = 0.$$

It does not mean that the probability of finding of the universe crossing the Big Brake is equal to zero because the corresponding Faddeev-Popov determinant contains a singular factor

$$\sim \frac{1}{\sqrt{p_T^2 - a^6 W_0^2}}.$$

One can see that in the case of the trigonometrical potential there is no need to require even the disappearance of $\psi(a, T)$ at the Big Brake.

Thus, there is no a quantum avoidance effect of the Big Brake singularity in the tachyon models.

There is the effect of quantum avoidance for the Big Bang and Big Crunch singularities, because these singularities correspond to

$$|p_T| \rightarrow \infty$$

and the corresponding probability density should tend to zero.

The paradox of the soft singularity crossing in the model with the anti-Chaplygin gas and dust

The energy density and the pressure are

$$\rho(a) = \sqrt{\frac{B}{a^6} - A} + \frac{M}{a^3}, \quad p(a) = \frac{A}{\sqrt{\frac{B}{a^6} - A}}.$$

Due to the dust component, the Hubble parameter has a non-zero value at the encounter with the singularity, therefore the dust implies further expansion. With continued expansion however, the energy density and the pressure of the anti-Chaplygin gas would become ill-defined.

We solve the paradox by redefining the anti-Chaplygin gas in a distributional sense. Then a contraction could follow the expansion phase at the singularity at the price of a jump in the Hubble parameter. Although such an abrupt change is not common in any cosmological evolution, we explicitly show that the set of Friedmann, Raychaudhuri and continuity equations are all obeyed both at the singularity and in its vicinity.
The jump in the Hubble parameter

$$H \to -H$$

leaves intact the first Friedmann equation $H^2 = \rho$, the continuity equations and the equations of state, however, it breaks the validity of the second Friedmann (Raychaudhuri) equation $\dot{H} = -\frac{3}{2}(\rho + p)$.

$$H(t) = H_S sgn(t_S - t) + \sqrt{\frac{3A}{2H_S a_S^4}} sgn(t_S - t)\sqrt{|t_S - t|},$$

$$\dot{H} = -2H_S\delta(t_S - t) - \sqrt{\frac{3A}{8H_S a_S^4}} \frac{sgn(t_S - t)}{\sqrt{|t_S - t|}}.$$

To restore the validity of the Raychaudhuri equation we add a singular δ -term to the pressure of the anti-Chaplygin gas

$$p = \sqrt{\frac{A}{6H_S|t_S - t|}} + \frac{4}{3}H_S\delta(t_S - t).$$

To preserve the equation of state we also modify the expression for its energy density:

$$\rho = \frac{A}{\sqrt{\frac{A}{6H_S|t_S-t|}} + \frac{4}{3}H_S\delta(t_S - t)}.$$

In order to prove that p and ρ represent a self-consistent solution of the system of cosmological equations, we used the following distributional identities:

$$\begin{aligned} \left[sgn\left(\tau\right)g\left(\left|\tau\right|\right)\right]\delta\left(\tau\right) &= 0 \ , \\ \left[f\left(\tau\right)+C\delta\left(\tau\right)\right]^{-1} &= f^{-1}\left(\tau\right) \ , \\ \frac{d}{d\tau}\left[f\left(\tau\right)+C\delta\left(\tau\right)\right]^{-1} &= \frac{d}{d\tau}f^{-1}\left(\tau\right) \ . \end{aligned}$$

Conclusions and discussion

- It is shown that the effect of quantum avoidance is absent for the soft singularities of the Big Brake type while it is present for the Big Bang and Big Crunch singularities.
- Thus, there is some kind of a classical - quantum correspondence, because soft singularities are traversable in classical cosmology, while the strong Big Bang and Big Crunch singularities are not traversable.
- Another type of soft singularity - Big Boost:

$$a(t_B) = a_B < \infty$$
$$\dot{a}(t_B) < \infty$$
$$\ddot{a}(t_B) = +\infty$$

It arises, for example, in the model based and on quantum and thermodynamical effects of presence of a big number of conformal fields.

- It would be interesting to find examples of the absence of the effect of the quantum avoidance of singularities, for the singularities of the Big Bang–Big Crunch type. The interest to the study of the possibility of crossing of such singularities is growing and some models treating this phenomenon have been elaborated during last few years (I. Bars, S.H. Chen, P.J. Steinhardt and N. Turok, 2010,2011).