Bremsstrahlung in transplanckian collisions

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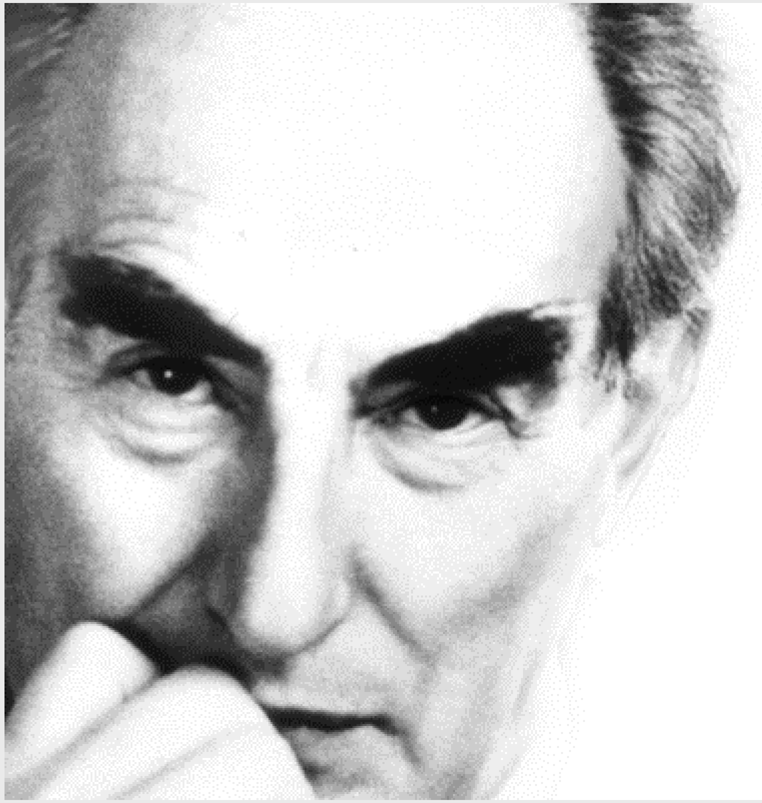


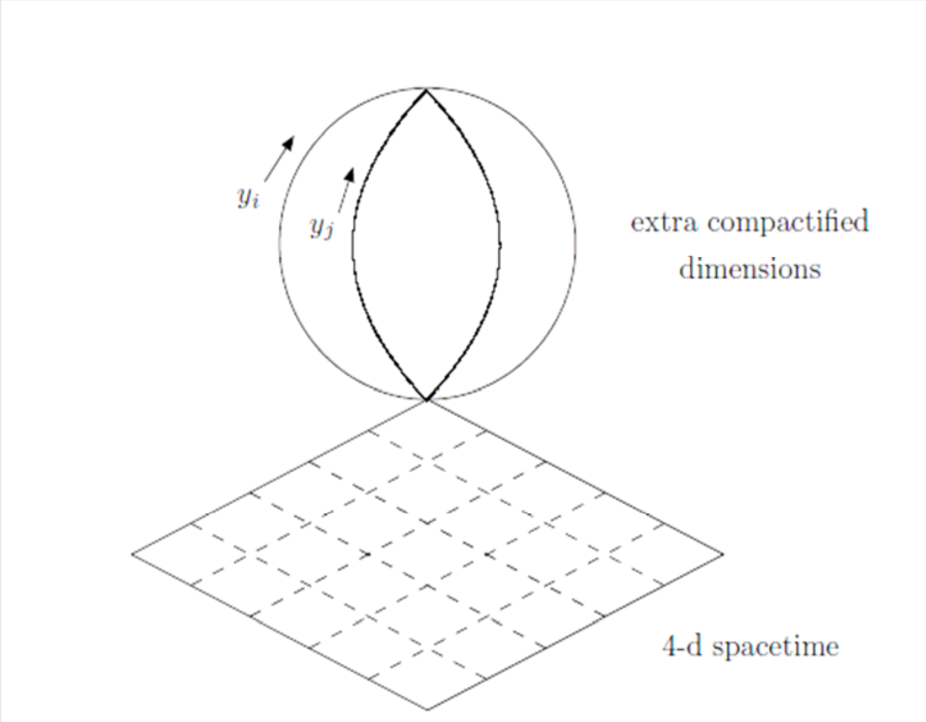
Gravity with extra dimensions

- Space-time as a domain wall
  (Akama, Rubakov…)

- String theory motivation: supersymmetry breaking via mesoscopic compactifications
  (Antoniadis, Bachas, Lewellen and Tomaras)

- Solution to the hierarchy problem

- Search for "new physics" at TeV scale



ADD Tev-scale gravity

- Linearized D-dimensional gravity, $g_{MN} = \eta_{MN} + \kappa_D h_{MN}$ matter on the brane

$$\frac{1}{G_D}\int R_D \sqrt{|g_D|}\, d^D x = \frac{V_d}{G_D}\int R_4 \sqrt{-g_4}\, d^4 x$$

$$D = 4 + d$$

implying

$$G_D = V_d G_4$$

$$M_{Pl}^2 = M_*^{d+2} V_d$$

$$V = (2\pi R)^d$$

For $M_* = 1\,\mathrm{TeV}$ size extra dimensions

$$l_* = (V_d)^{1/d} \cong 10^{30/d-17}\,\mathrm{cm}$$

n	R
1	1.5×10^{13} cm
2	$0.5\,\mathrm{mm} = 1/(10^{-4}\mathrm{eV})$
4	$3 \times 10^{-8}\,\mathrm{mm} = 3/(20\mathrm{KeV})$
6	$10^{-10}\,\mathrm{mm} = 1/(1\mathrm{MeV})$

Mode expansion

$$h_{MN} = \sum_{\vec{n}} \frac{1}{\sqrt{V_d}} h_{MN}^{(\vec{n})}(x) \exp\left(i \frac{\vec{n}\vec{y}}{R} \right)$$

$$(\partial_4^2 + m_{\vec{n}}^2) h_{MN}^{(n)} = 0$$

$$m_{\vec{n}}^2 = \left(\frac{\vec{n}}{R} \right)^2$$

Interaction

weak in each mode

$$-\frac{\kappa_4}{2} \int \sum_{\vec{n}} h_{MN}^{(\vec{n})} T^{MN}$$

Main effects

Astrophysically relevant:

- $\gamma\gamma \to g_{KK}$, Photon-photon annihilation;
- $e^- e^+ \to g_{KK}$, Electron-positron annihilation;
- $e^-\gamma \to e^- \; g_{KK}$, Gravi-Compton-Primakoff scattering;
- $e^-(Ze) \to e^-(Ze) \; g_{KK}$, Gravi-bremsstrahlung in a static electric field of the nuclei;
- $NN \to NN \; g_{KK}$, Nucleon-nucleon bremsstrahlung.

Colliders:

$$e^+ + e^- \to \gamma + \text{missing}, \quad e^+ + e^- \to Z + \text{missing}$$

at LEP and

$$p + \bar{p} \to \gamma + \text{missing}, \quad p + \bar{p} \to \text{jet} + \text{missing}$$

at Tevatron. The combined LEP limits are $M_* > 1.4\ TeV$ for $n = 2$, $M_* > .8\ TeV$ for $n = 3$, $M_* > .5\ TeV$ for $n = 4$, $M_* > .3\ TeV$ for $n = 5$ and $M_* > .2\ TeV$ for $n = 6$.

LHC:Transplanckian physics

- For $\sqrt{s} > M_*$ **CM energy exceeds the D-dimensional Planck mass**

Basic process : **creation of black holes**

P.C. Argyres, S. Dimopoulos, and J. March-Russell '98
Banks and Fischler '99
Aref'eva '99
Dimopoulos and Landsberg 2001
…............................

D-dimensional version of Thorne's hoop conjecture:
impact parameter *b* comparable to Schwarzschild radius of the CM energy of colliding particles

$$r_S = k_S \left(\frac{G_D \sqrt{s}}{c^4} \right)^{\frac{1}{d+1}}$$

$$k_S = \frac{1}{\sqrt{\pi}} \left(\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2} \right)^{\frac{1}{d+1}}$$

Shock wave as model of ultrarelativistic particle: Aichelburg-Sexl solution

Solution of the linearized gravity = exact solution (boosted Scwarzschild)

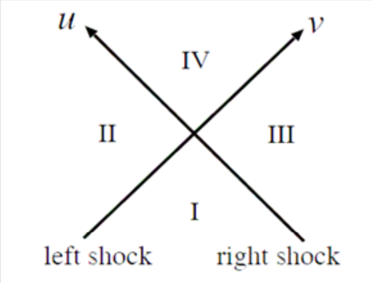
$$ds^2 = -dudv + d\rho^2 + \rho^2 d\Omega^2_{D-3} + \kappa\Phi(\rho)\delta(u)du^2$$

$$\Phi(\rho) = \begin{cases} -2\ln(\rho)\ , & D = 4 \\ \dfrac{2}{(D-4)\rho^{D-4}}\ , & D > 4 \end{cases}$$

$$\kappa \equiv 8\pi G_D \mu / \Omega_{D-3}$$

Non-vacuum: sourced by the particle energy-momentum tensor

Two waves can be superposed in the space-time region before collision



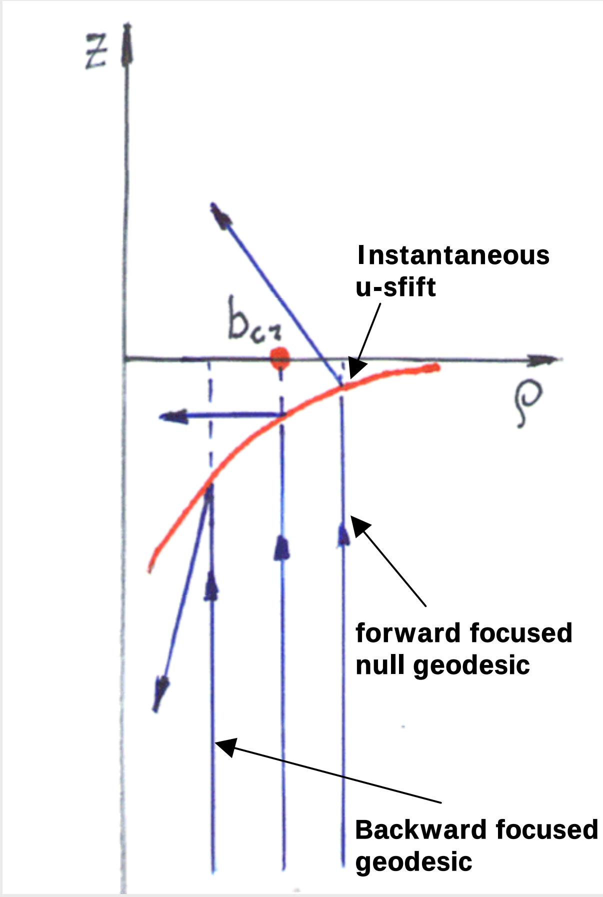
‘t Hooft picture of collision: particle scattered by shock wave

- Red line - instantaneous shift in u=t-z when crossing the wave front propagating in (–z) direction

- Geodesics impinging at impact parameters $b > b_{cr}$ are focused in the forward direction

- Geodesics falling at $b < b_{cr}$ are reflected

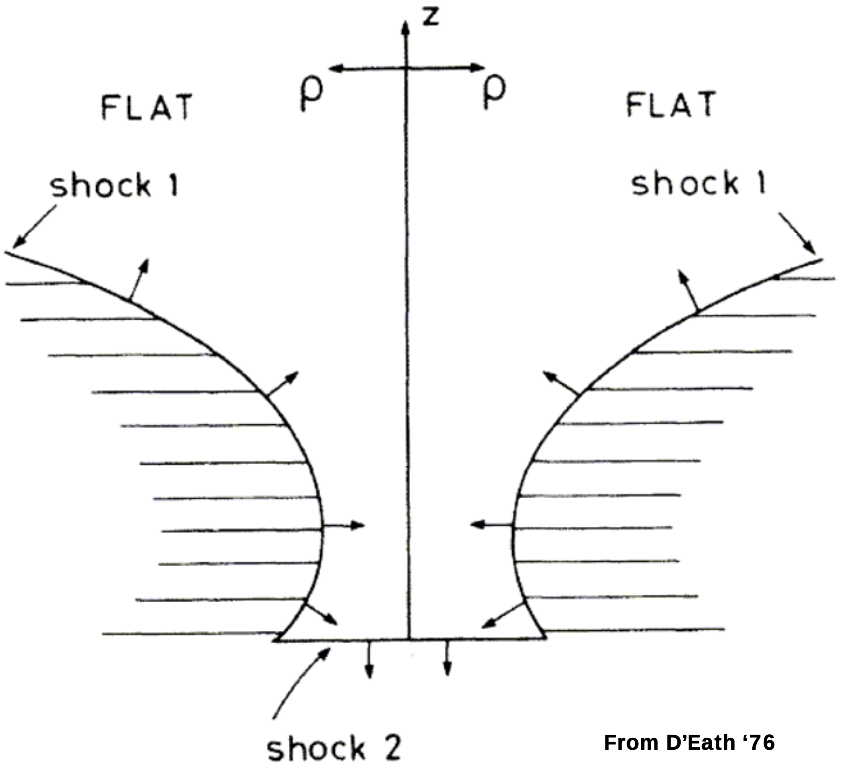
- Critical impact parameter b_{cr} marks position of the closed trapped surface in the forward collision of two shocks



Mutual focusing of shock waves due to gravitational attraction

- Deformation of the shock moving in z-direction in the flat region III. Different null generators are focused at different angles causing deformation of the front

- Later shock 2 meets z-axis at the caustic region which moves along the axis faster than light



From D'Eath '76

Formation of apparent horizon

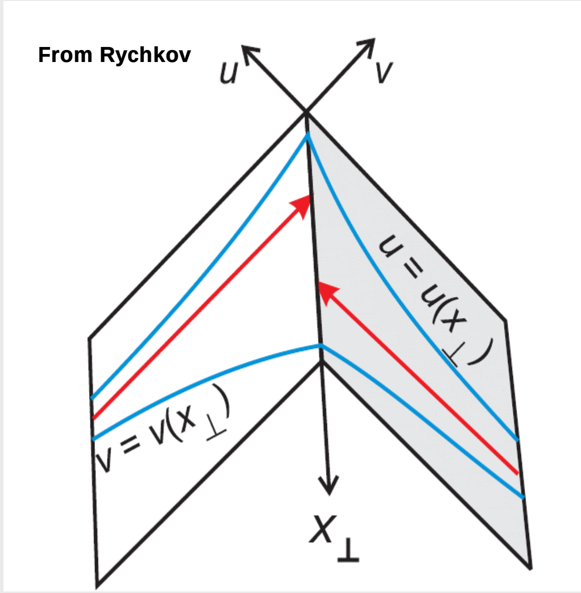
- **Conditions of formation of closed trapped surface**

$$\partial_\perp^2 u = \partial_\perp^2 v = 0$$

with matching on the boundary

$$u|_C = v|_C = 0$$
$$\nabla u \cdot \nabla v|_C = 4$$

Penrose '74,
Eardley and Giddings '02
Yoshino and Nambu '03
Nambu and Rychkov '05
....



Calculations show apparent horizon radius differs from r_S by the factor of the order of unity

At transplanckian energies gravity becomes not only dominant, but classical

- **The qualitative argument:**

$$\lambda_B = \hbar c/\sqrt{s}$$

$$r_S = \frac{1}{\sqrt{\pi}}\left[\frac{8\Gamma\left(\frac{d+3}{2}\right)}{d+2}\right]^{\frac{1}{d+1}}\left(\frac{G_D\sqrt{s}}{c^4}\right)^{\frac{1}{d+1}}$$

$$l_* = \left(\hbar G_D/c^3\right)^{1/(d+2)} = \hbar/M_* c$$

Classicality: $$\lambda_B \ll l_* \ll r_S$$

Achieved if $$s \gg G_D^{-2/(d+2)} = M_*^2$$

$$G_D = \text{fixed}$$

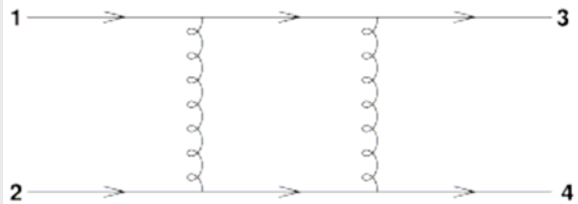
(Giudice, Rattazzi, Wells
Veneziano,…)

Elastic scattering: eikonalization

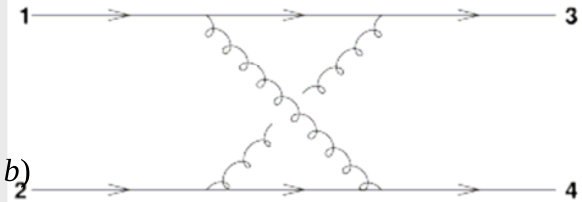
- One-graviton exchange amplitude diverges when summed up over KK massive states
- Two one-loop diagrams are finite in SUGRA-s (e.g. N=8)
- Summing up ladder and cross-ladder diagrams one obtains eikonal amplitude for s>>M* and –t/s<<1

$$M_{eik}(s,t) = 2is\int e^{i\mathbf{qb}}\left(1-e^{i\chi(s,t)}\right)d^2\mathbf{b}$$

$\Phi(b) = \lambda_B \cdot \chi(s,b)$



Two graviton exchange.

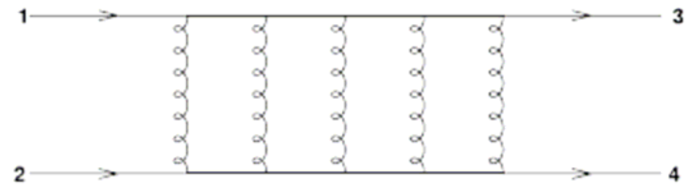


A two-graviton crossed ladder.

$$\chi(s,b) = \left(\frac{b_c}{b}\right)^d,$$

where

$$b_c \equiv \frac{1}{\sqrt{\pi}}\left(\frac{\varkappa_D^2\Gamma(d/2)\,s}{16\pi}\right)^{1/d}.$$



A ladder diagram with multiple graviton exchange.

For $b < r_S$ quantum description (Born), for $r_S < b < b_c$ – eikonal, for $b > b_c$ plane waves

Remarkably, the eikonal phase is equal to shock wave amplitude up to factor !

$$\Phi(b) = \lambda_B \cdot \chi(s,b)$$

‘t Hooft’s method versus shock wave description

- Equivalence of shock wave metric function and eikonal phase reflects classicalization of transplanckian region. Eikonal summation leads to Furry’s picture- type states in the classical shock wave field.

- Both the t’Hooft treatment of test particle (field) in a single shock wave generated by another particle and the analysis of two shock metric evolution are approximate: shock wave approximation does not account for matter sources of waves, test particle in a single shock wave does not account for non-linearity of Einstein gravity

- Predictions seems paradoxically different: ultrarelativistic test particle scattered with impact parameter less than radius of apparent horizon of future black hole is reflected by shock wave!

- Combination of both methods amounts to using Furry’s shock wave states in higher order quantum calculations (Lodone and Rychkov), but it is technically difficult problem

TP bremsstrahlung: methods of computing

Gravitational bremsstrahlung is the second important quasiclassical process inTP region. Various suggested methods include:

- **Estimates based on Hawking entropy** (Penrose, Eardley and Giddings…)

$$\epsilon_{\text{radiated}} \leq 1 - \frac{1}{2}\left(\frac{D-2}{2}\frac{\Omega_{D-2}}{\Omega_{D-3}}\right)^{\frac{1}{D-2}}$$

- **Classical calculations using shock waves** (d'Eath '92,…, Herdeiro et al '12)
- **BH perturbations: infall and scattering of test bodies** (too many!)
- **Classical post-linear formalism** (Thorne and Kovacs '77, DG,Grats and Matiukhin '78, DG, Kofinas, PS, Tomaras, 2010,…)
- **Imaginary part of eikonal in string theory** (Amati, Ciafaloni,Veneziano)
- **Furry's picture in shock wave, (quantum)** (Lodone and Rytchkov)
- **Numerical simulations** (Pretorius, Berti et al,…)

Bremsstrahlung via eikonal

In models with extra dimensions eikonal approximation is bound both sides: $r_s < b < b_c$

The real eikonal phase is found form Born amplitude:

$$\chi(s,b) = \frac{1}{2s}\int e^{-i\mathbf{q}\cdot\mathbf{b}}\mathcal{M}_{\rm Born}(s,t)\,\frac{d^2q}{(2\pi)^2}$$

Classical result (DG Kofinas Spirin Tomaras '09) corresponds to stationary phase point:

$$b_s = \left(\frac{db_c^d}{q}\right)^{1/(d+1)}$$

Imaginary part due to bremsstrahlung (ACV) is

$$\mathrm{Im}\chi \sim \left(\frac{b_r}{b}\right)^{3d+2}$$

where $\frac{b_r}{r_s} = \left(\frac{b_c}{r_s}\right)^{\frac{d}{3d+2}}$ so that $r_s \ll b \ll b_r$

If interpreted as number of emitted gravitons radiation would be large for b>>r_s

Only if frequencies are bound by $\omega_b = 1/b$ radiation is not catastrophic:

$$\epsilon = \frac{\Delta E}{E} \sim \left(\frac{r_s}{b}\right)^{\frac{d}{3d+3}}$$

(Giudice,Ratazzi and Wells)

But classical calculations show that bremsstrahlung spectrum at small angle scattering is dominated by $\omega \gg \omega_b$

Particles falling into black holes

- **D=4: Zerilli, Chranowski, Misner,**
- **Higher D: Cardoso,Lemos….**

- **Radiation is about 14% in radial infall D=4 increasing up to 40% in higher D**

- **Radiation grows with non-zero impact parameter being maximal in grazing collisions when particle make revolutions around an unstable photon orbit**

- **Constant radiation power of GSR implies possibility of large radiation (not fully explored yet)**

Continuation of colliding shock wave metrics (D'Eath)

- Metric in future sector of two superposed SW computed perturbatively in the frame where the energy of one wave is much less that another.

- In D=4 extensively studied by D'Eath and Payne '92 for b=0, recently generalized to higher D and b=0 (Herdeiro, Sampaio, Rebelo)

First order approximation gives bremsstrahlung loss varying from 25% in D=4 to 41,2% in D=10, consistent with entropy bounds. Second order gives about 2/3 of this

SW metric is continued as vacuum solution, no account for the matter source

Post-linear formalism

- **Based on expansion of the metric up to the second order and constructing metric and trajectories by iterations**

- **Valid for large b, applicability in D=4 restricted by small angle scattering** $\theta_s << 1/\gamma$
  (Thorne and Kovacs '77, DG, Grats,Matiukhin '78 also agree with Peters '70)

Energy loss in the rest frame of one mass $\Delta E \sim \frac{G^3 M^2 m^2 \gamma^3}{b^3 c^4}$

PLF valid for arbitrary masses, for $\gamma >> 1$ **gives zero efficiency at the limit of applicability! But precise limit on allowed b is not quite clear: no higher order available.**

Massless limit puzzling: in the CM frame $\Delta E \sim \frac{G^3 m^4 \gamma_{cm}^5}{b^3 c^4}$

In the limit m=0, $\gamma_{CM} >> 1$ **and finite** $m\gamma_{CM}$ **diverges for finite b, though goes to zero at the limit of applicability**

D-dimensional PLF setting (ADD and Minkowskian) (DG,Kofinas.Spirin.Tomaras)

$$S = -\frac{1}{\kappa_D^2}\int \sqrt{-g}\, R\, d^D x - \sum_a \frac{1}{2}\int \left(e_a\, g_{MN}(z_a)\, \dot{z}_a^M \dot{z}_a^N + \frac{m_a^2}{e_a} \right) d\tau$$

$$g_{MN} = \eta_{MN} + \kappa_D h_{MN}$$

Metric deviation (considered as Minkowski tensor) is further expanded in terms of gravitational coupling

$$h_{MN} = \overset{(1)}{h}_{MN} + \overset{(2)}{h}_{MN} + \ldots$$

Particles world lines are presented similarly

$$z^M(\tau) = \overset{(0)}{z}{}^M + \overset{(1)}{z}{}^M + \ldots$$

Perturbation expansions and iterations

$$G_{MN} = -\frac{\kappa_D}{2}\partial^2 \overset{(1)}{\psi}_{MN} - \frac{\kappa_D^2}{2} S_{MN} + \text{cubic terms}$$

harmonic gauge $\quad \partial^N \overset{(k)}{\psi}_{MN} = 0 \qquad \overset{(k)}{\psi}_{MN} \equiv \overset{(k)}{h}_{MN} - \frac{1}{2}\eta_{MN}\overset{(k)}{h}$

EOMs

$$\partial_D^2 \left(\overset{(k)}{h}_{MN} - \frac{1}{2}\eta_{MN} \overset{(k)}{h} \right) = -\kappa_D \overset{(k-1)}{\tau}_{MN}$$

$$\overset{(k)}{\ddot{z}}_M(\tau) = -\kappa_D \left(\overset{(k_1)}{h}_{MN,P} - \frac{1}{2} \overset{k_1}{h}_{NP,M} \right) \overset{(k_2)}{\dot{z}^N} \overset{(k_3)}{\dot{z}^P} \qquad k_1 + k_2 + k_3 = k$$

0-th order

$$\overset{(0)}{z^M} = u^M \tau + b^M \qquad \overset{(0)}{z'^M} = u'^M \tau \qquad \overset{(0)}{\tau}_{MN} = \overset{(0)}{T}_{MN} = \sum_a \int e_a \overset{(0)}{\dot{z}^a_M} \overset{(0)}{\dot{z}^a_N} \delta^D(x - \overset{(0)}{z}_a)\, d\tau$$

lab frame

1-st order

$$\overset{(1)}{h}_{MN} = \sum_a {}_a\overset{(1)}{h}_{MN} \qquad {}_a\overset{(1)}{\ddot{z}}_M(\tau) = -\kappa_D \left({}_{a'}\overset{(1)}{h}_{MN,P} - \frac{1}{2}\, {}_{a'}\overset{(1)}{h}_{NP,M} \right) {}_a\overset{(0)}{\dot{z}}^N \; {}_a\overset{(0)}{\dot{z}}^P$$

$$\overset{(1)}{\tau}_{MN} = \sum_{a=1,2} {}_a\overset{(1)}{T}_{MN} + S_{MN}$$

2-nd order (radiation)

$$\partial_D^2 \overset{(2)}{\psi}_{MN} = -\kappa_D \left(\overset{(1)}{T}_{MN} + S_{MN} \right)$$

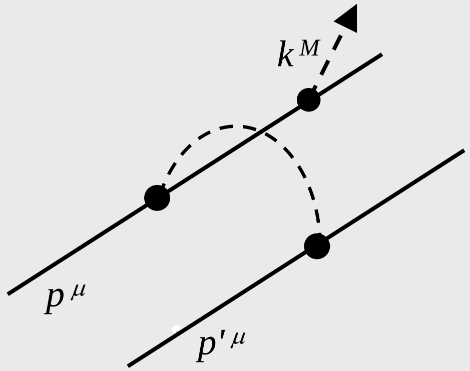
In coordinate space

$$\Delta P_M = -\frac{1}{2} \int \overset{(2)}{h}_{PQ,M}\, \partial^2 \overset{(2)}{\psi}{}^{PQ}\, d^D x$$

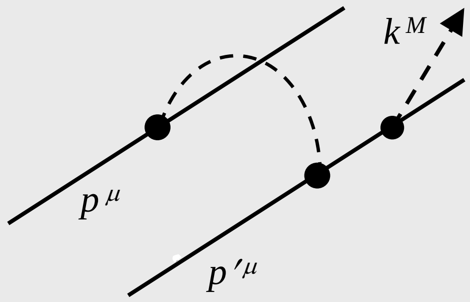
In momentum space

$$\Delta P^M = \frac{\kappa_D^2}{4(2\pi)^{D-1}} \sum_{\mathrm{pol}} \int |T_D^{(\mathrm{pol})}|^2 k^M \frac{d^{D-1}\mathbf{k}}{k^0}$$

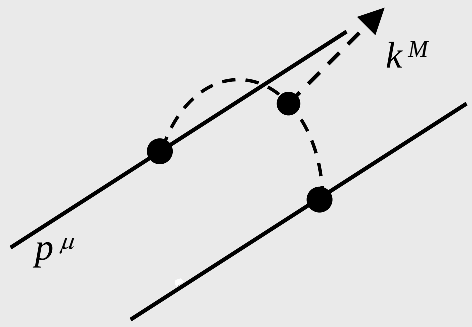
Radiation Amplitudes



$T(k)$



$T'(k)$



$S(k)$

Destructive Interference

$$\omega = \gamma / b \ldots \gamma^2 / b \qquad \vartheta < 1/\gamma$$

$$S = S^z + S^{z'} \qquad S^z \approx -T \qquad S^{z'} \approx -T'$$

ϑ \ ω	$\omega \sim 1/b$	$\omega \sim \gamma/b$	$\omega \sim \gamma^2/b$
γ^{-1}	no destructive interference $\tau \sim T \gg S$	no destructive interference $S^{[z]} \sim T \sim S^{[z']} \sim \gamma$	destructive interference: $T \approx -S^{[z]}$ $S^{[z']} \sim \exp(-\gamma)$, $\quad \tau = \mathcal{O}(T/\gamma^2) \sim 1/\gamma$
1	no destructive interference $\tau \sim T \sim S$	destructive interference: $S^{[z]} \approx T \sim \exp(-\gamma)$ $\tau = S = S^{[z']} \sim \gamma^{-1}$	destructive interference $T \sim S \sim \tau \sim \exp(-\gamma)$

$$\tau(\omega) \approx \frac{\tau(\omega_0)}{(\omega/\omega_0)^2} \qquad \omega_0 \cong \frac{\gamma}{b} \qquad \tau(\gamma^2/b) \approx \frac{\tau(\gamma/b)}{\gamma^2}$$

Due to destructive
interference at frequencies

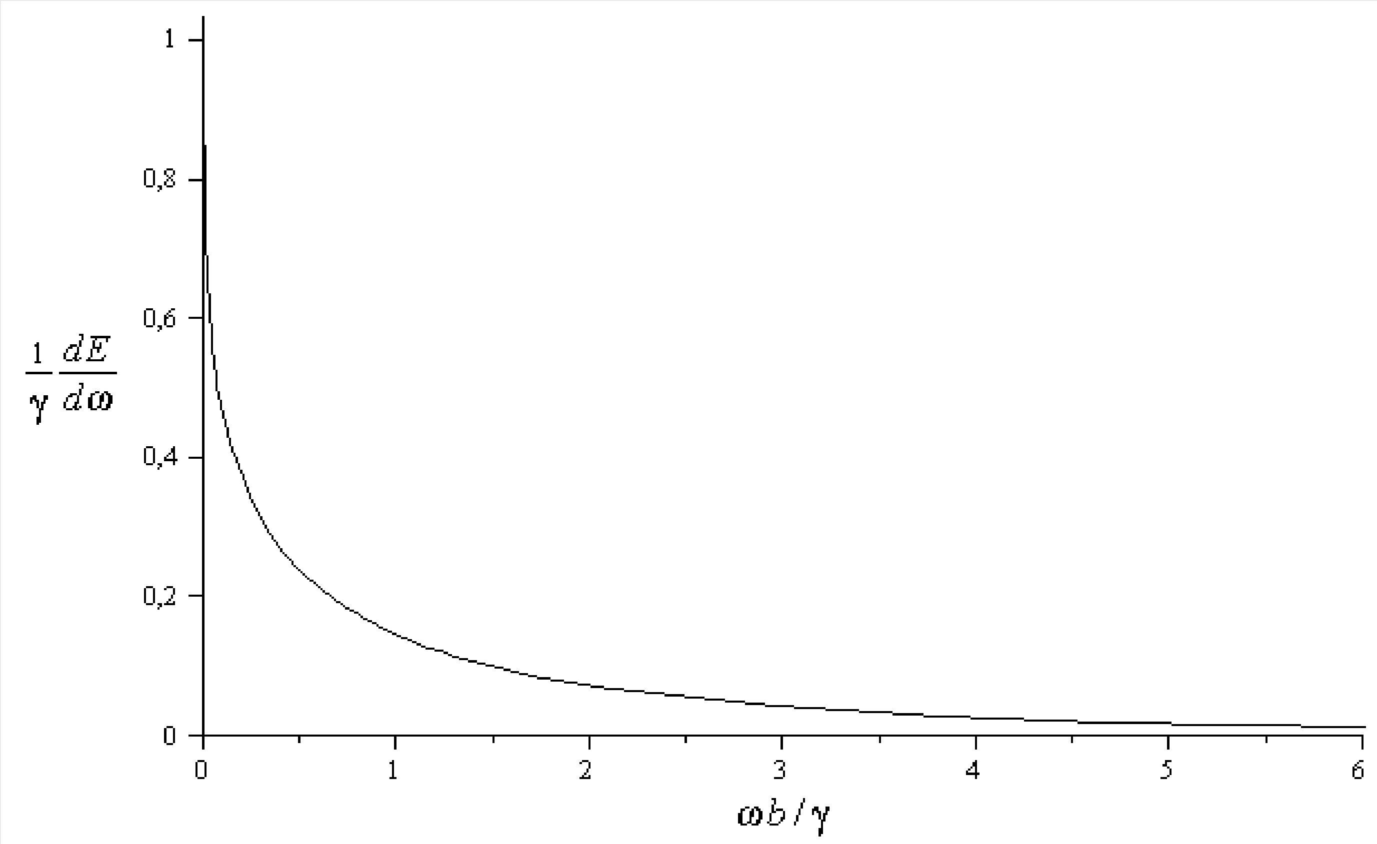
$$\omega = \gamma/b \ldots \gamma^2/b$$

$$\frac{dE_{\mathrm{rad}}}{d\omega} \cong \omega^{D-6}$$

Dominant frequency range depends on sign of D-6 !!

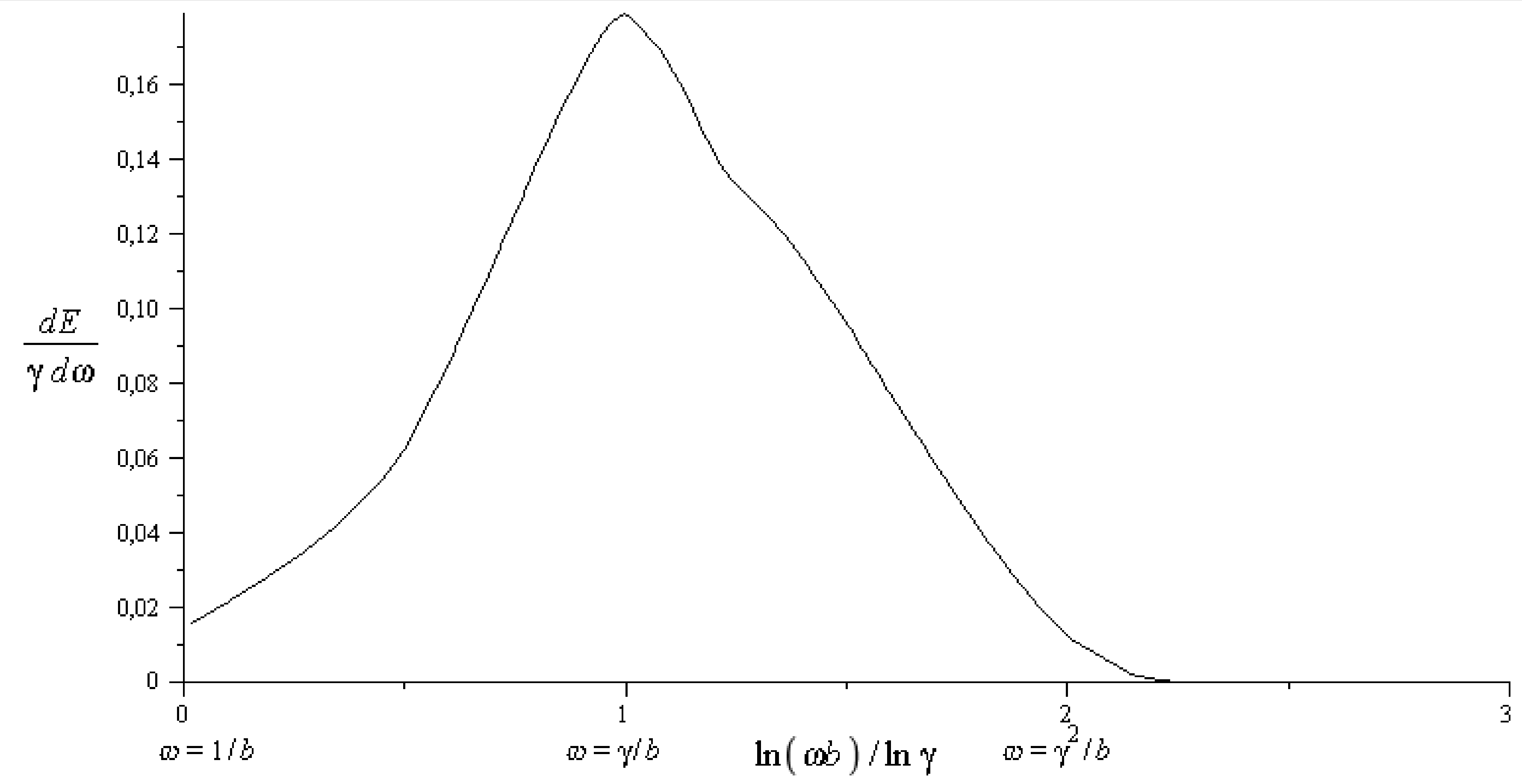
ϑ \ ω_D	$\omega_D \ll \gamma/b$	$\omega_D \sim \gamma/b$	$\omega_D \sim \gamma^2/b$	$\omega_D \gg \gamma^2/b$
γ^{-1}	negligible (phase space)	$E \sim \gamma^3$, from T and S	$E \sim \gamma^{d+2}$, from $T + S^{[z]}$	negligible radiation
1	negligible (phase space)	$E \sim \gamma^{d+1}$, from $S^{[z']}$	negligible radiation	negligible radiation

Frequency distribution in 4D



Angular distribution: beaming at angle $\theta < 1/\gamma$ (along fast-particle's motion direction) for all dimensions

Frequency distribution in 6D in logarithmic scale



Total PLF bremsstrahlung loss

$$E_{\rm rad} = C_D \frac{(\kappa_D^3 m m')^2}{b^{3d+3}} \begin{cases} \gamma^3, & D=4 \\ \gamma^3 \ln\gamma, & D=5, \\ \gamma^{D-2}, & D>5 \end{cases} \qquad C_D \cong 10^{-4}$$

Notice non-universal dependence of Lorentz factor in D<6

For D>5 radiation efficiency is (d=D-4):

$$\epsilon = \frac{E_{rad}}{m\gamma} \sim (r_S/b)^{3(d+1)}\, \gamma^{d-1/2}$$

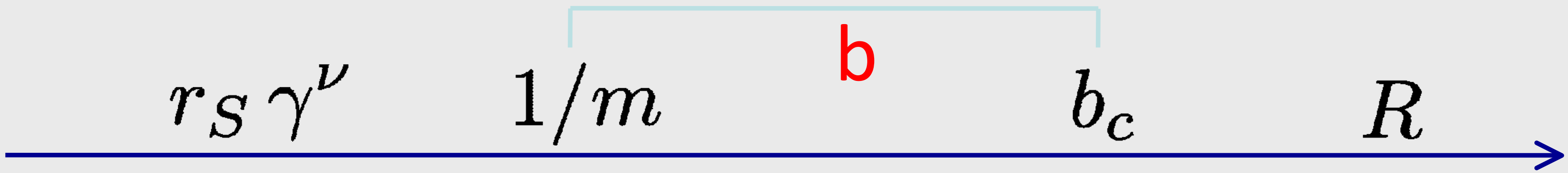
At minimal allowed impact parameter $b = r_S \gamma^{\frac{1}{2(d+1)}}$

one has $\epsilon \sim \gamma^{d-2}$

becoming catastrophic in dimensions higher than d>2 !!

APPLICABILITY WINDOW (including quantum bounds)

$$\omega \sim \gamma/b \, : \omega \ll m\gamma \rightarrow b \gg 1/m$$



$$r_S\,\gamma^{\nu} \ll 1/m \ll b_c$$

SATISFIED in a window depending on s, d, m, M_*

e.g. d=2 $M_* \sim 1\,TeV,\; m \sim 100\,GeV,\; \sqrt{s} \sim 10\,TeV$

Outlook

- **PFL calculation predicts strong bremsstrahlung within classical applicability window for d>2, mostly because of enhanced phase volume.**
- **Massless limit unclear, independent calculation needed.**
- **Matter source contribution in the SW calculations needed?**
- **Other techniques desirable, both classical and quantum**