Nonlocal ghost-free gravity theory and DE

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Introduction

Cosmic acceleration and modifications of gravity theory:

- explicit cosmological term
- quintessence type models
- f(R) models
- higher-dimensional and braneworld models
- massive gravity
- nonlocal gravity
- ……………

explicit DE scale encoded in the action

Nonlocal gravity and cosmology (Deser, Woodard, 2007)

$$S \sim \int dx\, g^{1/2}\, R\, f\left(\frac{1}{\Box} R\right)$$

Deffayet, Woodard, Esposito-Farese

Koivisto, Nojiri, Odintsov, Koshelev, Sasaki, Zhang, Bamba, …

Cosmic coincidence aspect of the CC and DE problem

$$M_P^2\,\Lambda_{\text{eff}} = \rho_{DE} \sim \rho_{\text{matter}}$$

Fine tuning concrete value of effective Λ to matter density

Alternative idea – the model that has a stable dS or AdS background with an arbitrary value of Λ (to be fixed separately by the analogue of symmetry breaking)

+

Realization of an old idea of a scale-dependent gravitational coupling – nonlocal Newton constant

$$G_{\text{Newton}} \;\Rightarrow\; G_{\text{eff}}(\Box)$$

Arbitrary Λ implies eqs. of motion: $$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 0 \;\Rightarrow\; R_{\mu\nu} = \Lambda\, g_{\mu\nu}$$

How to embed these equations of motion into a diffeomorphism invariant ghost-free action?

- A.Einstein, *Spielen die Gravitationsfelder im Aufbau der materiellen Elementarteilchen eine wesentliche Rolle?,* Sitzungsber. Preuss.Akad. Wiss., 1919, v.1, 349.

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \kappa T_{\mu\nu} \;\Rightarrow\; R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = \kappa\, T_{\mu\nu}$$

- Unimodular gravity: extra degrees of freedom = ghosts

- Conformal gravity (Maldacena, arXiv:1105:5632): exclusion of ghosts by boundary conditions (?)

$$S = a \int dx\, g^{1/2}\, C^2_{\mu\nu\alpha\beta} = a \int dx\, g^{1/2} \left\{ R^2_{\mu\nu} - \frac{1}{3} R^2 \right\}$$

In fact: $$S = \int dx\, g^{1/2} \left\{ a\, R^2_{\mu\nu} + b\, R^2 \right\} \quad \Rightarrow \quad \frac{\delta S}{\delta g^{\mu\nu}} \propto R_{\alpha\beta} - \frac{1}{4} g_{\alpha\beta} R$$

for any a and b

✦ **Our model:**

i) GR limit on flat space background
ii) Stable ghost-free (A)dS phase with arbitrary Λ
iii) Unexpected bonus – DM mechanism in this phase
iv) Nontrivial BH thermodynamics

A.Barvinsky
arXiv:1112.4340

S.Solodukhin,
arXiv:1203.2961

Realization of an old idea of a scale-dependent gravitational coupling – nonlocal Newton constant $G_{Newton} \Rightarrow G_{\text{eff}}(\Box)$

curvature

$$\int dx\, g^{1/2}\, R\, f\left(\frac{1}{\Box}R\right) \;\Rightarrow\; \int dx\, g^{1/2}\, R^{\mu\nu}\, \underbrace{f(\Box, R_{....})}_{\substack{\text{to be determined from}\\ \text{correspondence principle}\\ \text{with GR and stability}\\ \text{arguments}}} R_{\mu\nu}$$

Outline

Introduction

Flat-space background onset

Treatment of nonlocality: Schwinger-Keldysh technique vs Euclidean QFT

Nonlocal gravity with a stable (A)dS background

Conclusions

Flat-space background onset

Idea of scale dependent gravitational coupling (noncovariant)

$$M_P^2 \;\Rightarrow\; M^2_{\rm eff}(\Box)$$
$$G_{Newton} \;\Rightarrow\; G_{\rm eff}(\Box)$$
$$G_{\mu\nu} = 8\pi G_{\rm eff}(\Box)\, T_{\mu\nu}$$

Arkani-Hamed, Dimopoulos, Dvali and Gabadadze (2002)

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Einstein tensor

Einstein action on a flat-space background:

$$S_E = \frac{M_P^2}{2} \int dx\, g^{1/2} \left\{ -R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} + \mathrm{O}\,[R^3_{\mu\nu}] \right\},$$

A.Barvinsky (2003)

$$M_P^2\, R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} \;\Rightarrow\; R^{\mu\nu} \frac{M^2(\Box)}{\Box} G_{\mu\nu}$$

Correspondence principle with GR

$$R \;\Rightarrow\; R + R^{\mu\nu} F(\Box) G_{\mu\nu}$$

$$F(\Box) = \int d\mu^2\, \frac{\alpha(\mu^2)}{\mu^2 - \Box}$$

Trial choice

$$\alpha(\mu^2) \sim \delta(m^2 - \mu^2) \;\Rightarrow\; F(\Box) = \frac{\alpha}{m^2 - \Box}$$

Problem with $m^2 \neq 0$:

Structure of inverse propagator and characteristic equation for field modes

$$-\Box + \alpha \frac{\Box^2}{m^2 - \Box} = 0$$

$$\Downarrow$$

Massless and massive (ghost) graviton

$$\Box = m_\pm^2,\; m_-^2 = 0,\; m_+^2 = O(m^2)$$

ghost

$$\Downarrow$$

$$m^2 = 0$$

First step towards nonlocal gravity:

$$S = \frac{M^2}{2} \int dx\, g^{1/2} \left\{ -R + \alpha\, R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} \right\}$$

From post-Newtonian corrections $|\alpha| \ll 1$

Linearized theory

$$S = -\frac{M^2(1-\alpha)}{2} \int dx\, g^{1/2} R + \alpha\, O[\, h_{\mu\nu}^3]$$

Small renormalization of the Planck mass

$$M^2 = \frac{M_P^2}{1-\alpha}$$

Treatment of nonlocality: Schwinger-Keldysh technique vs Euclidean QFT

$$\frac{\delta S}{\delta g_{\mu\nu}(x)} \propto \nabla\nabla \int dy \underbrace{\Big[\, G(x,y) + G(y,x) \,\Big]}_{\text{not causal: } \neq 0 \text{ for } y^0 > x^0} R(y) + \ldots$$

Our nonlocal action is quantum effective action – generating functional of OPI diagrams with $g_{\mu\nu}$ – mean quantum field

Physical observables are always expectation values of some operators, i.e. transition probability:

$$\langle\, in \,|\, \hat{\mathcal{O}}(x) \,|\, in \,\rangle$$

Subject to Schwinger-Keldysh diagrammatic technique

$$P_{in\to fin} = \langle\, in \,|\, fin \,\rangle\langle\, fin \,|\, in \,\rangle = \langle\, in \,|\, \hat{P}_{fin} |\, in \,\rangle$$

$$\frac{\delta \langle\, in \,|\, \hat{\mathcal{O}}(x) \,|\, in \,\rangle}{\delta J(y)} = 0, \qquad x^0 < y^0$$

Generally non-manifest consequence of locality and unitarity achieved via a set of cancellations between nonlocal terms with chronological and anti-chronological boundary conditions

Schwinger-Keldysh technique and Euclidean QFT:

In-in mean field $$g_{\mu\nu} = \langle in|\, \hat{g}_{\mu\nu} |in\rangle$$

Quantum effective action of Euclidean QFT (nonlocal) $$S = S_{\text{Euclidean}}[\, g_{\mu\nu}\,]$$

Effective equations for *in-in* field

A.B. & G.A.Vilkovisky (1987)

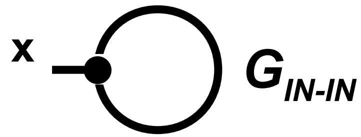
$$\left.\frac{\delta S_{\text{Euclidean}}}{\delta g_{\mu\nu}(x)}\right|^{\text{retarded}}_{++++\,\Rightarrow\,-+++} = 0.$$

Causal, diffeomorphism and gauge invariant !

One-loop effective equations for IN-IN mean field:

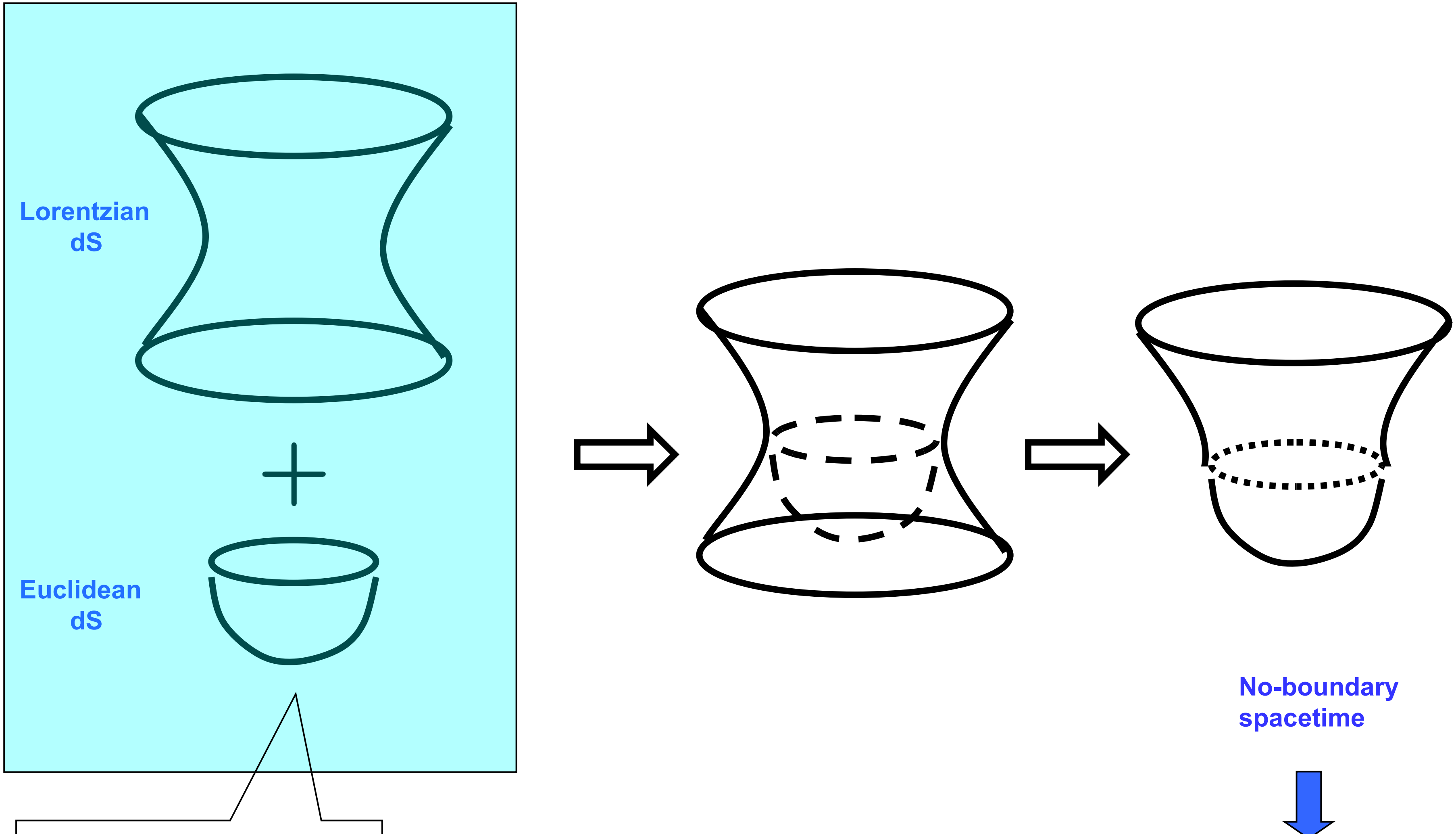
In-in Wightman Green's function in Poincare-invariant vacuum in asymptotically flat (AF) spacetime

$$G_{IN-IN}(x,y) = \langle in\,|\,\hat{g}(x)\,\hat{g}(y)\,|\,in\rangle$$



$$\frac{\delta S}{\delta g(x)} + \underbrace{\frac{i}{2}\int dy\,dz\,S_3(x,y,z)\,G_{IN-IN}(y,z)}_{=\ \left.\frac{\delta \Gamma_E^{1-loop}}{\delta g(x)}\right|^{\text{retarded}}_{++++\Rightarrow-+++}}$$

$$\left\{\begin{array}{l}\textbf{AF spacetime}\\ \textbf{Poincare vacuum}\end{array}\right. \quad \overset{?}{\Rightarrow} \quad \left\{\begin{array}{l}\textbf{(A)dS spacetime}\\ \textbf{Euclidean vacuum}\end{array}\right.$$



Tool for constructing Euclidean dS-invariant vacuum

No surface terms under integration by parts (integration by parts "trick" of Woodard)

Problem with AF $\Rightarrow$ (A)dS for the action

$$S = \frac{M^2}{2} \int dx\, g^{1/2} \left\{ -R + \alpha\, R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu} \right\}$$

IR divergence – presence of zero mode:

$$R_{\mu\nu}\Big|_{(A)dS} = \Lambda\, g_{\mu\nu}, \quad \Box g_{\mu\nu} = 0$$

$$R^{\mu\nu} \frac{1}{\Box} G_{\mu\nu}\Big|_{(A)dS} = \infty$$

No go!

Nonlocal gravity with a stable (A)dS background

$$S = \frac{M^2}{2} \int dx\, g^{1/2} \left\{ -R + \alpha\, R^{\mu\nu} \frac{1}{\Box + \hat{P}} G_{\mu\nu} \right\}$$

IR regulator

Operator $\Box + \hat{P} \equiv \Box\, \delta_{\alpha\beta}^{\;\;\mu\nu} + P_{\alpha\beta}^{\;\;\mu\nu}$

Action of its Green's function $\frac{1}{\Box + \hat{P}} G_{\mu\nu}(x) \equiv \int dy \left[\frac{1}{\Box + \hat{P}} \delta(x,y) \right]_{\mu\nu}^{\alpha\beta} G_{\alpha\beta}(y)$

$$\hat{P} \equiv P_{\alpha\beta}^{\;\;\mu\nu} = a R_{(\alpha\;\;\beta)}^{\;\,(\mu\;\;\nu)} + b\big(g_{\alpha\beta} R^{\mu\nu} + g^{\mu\nu} R_{\alpha\beta}\big) + c R_{(\alpha}^{(\mu} \delta_{\beta)}^{\nu)} + d R\, g_{\alpha\beta} g^{\mu\nu} + e R \delta_{\alpha\beta}^{\mu\nu}.$$

Generic potential term linear in the curvature

a, b, c, d, e **-- parameters to be restricted by the requirement of a stable (A)dS solution**

- **Exact (A)dS and Einstein metric background**

$$\hat{P}\, g_{\mu\nu} \equiv P_{\alpha\beta}{}^{\mu\nu}\, g_{\mu\nu} = (A + 4B)\, \Lambda\, g_{\mu\nu},$$

$$A \equiv a + 4\,b + c, \quad B \equiv b + 4\,d + e$$

Maximally symmetric background

$$\left.\frac{\delta S}{\delta g_{\mu\nu}}\right|_{\mathrm{(A)dS}} = -2M^2 \Lambda \left(1 + \frac{\alpha}{A + 4B}\right) g^{1/2}$$

(A)dS solution with arbitrary Λ exists under the following restriction:

$$\boxed{\alpha = -A - 4B}$$

Generalizes to generic Einstein metric (Solodukhin, 2012) :

$$E_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = 0$$

- **Vanishing on-shell action:** $\left. S \right|_{E_{\mu\nu}=0}$ **--- analogue of vacuum**

- **Scaling property:** $S[\,\lambda g_{\mu\nu}(x)\,] = \lambda\, S[\,g_{\mu\nu}(x)\,]$

- ## Stability of (A)dS background

$$g_{\mu\nu} \Rightarrow g_{\mu\nu}\Big|^{(A)dS} + h_{\mu\nu}, \quad S^{(2)} = ?$$

The hope for good *$S^{(2)}$* – why ???

DeWitt gauge $\chi^\mu \equiv \nabla_\nu h^{\mu\nu} - \frac{1}{2}\nabla^\mu h = 0$

Two nonlocal tensor structures $S^{(2)} \sim h^{\mu\nu} \times h_{\mu\nu} + h \times h$

Nonlocal parts of these structures to be canceled by the parameter choice

$$h^{\mu\nu}\frac{1}{\Box + \hat{P}}h_{\mu\nu} \qquad h\,\frac{1}{\Box - \alpha\Lambda}\,h$$

Quadratic part of the action:

traceless part

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{4} g_{\mu\nu} h$$

$$S_{(2)}\Big|_{E_{\mu\nu}=0} = -\frac{M_{\text{eff}}^2}{2} \int d^4x\, g^{1/2} \big(D_1 \bar{h}^{\mu\nu}\big) \frac{1}{D_2} \big(D_1 \bar{h}_{\mu\nu}\big).$$

Effective Planck mass

$$M_{\text{eff}}^2 = M^2 \frac{A^2 - \alpha^2}{\alpha}$$

$$D_1 \equiv \Box + 2\,\hat{W} - \frac{1}{6} R\,\hat{1},$$

$$D_2 \equiv \Box + a\,\hat{W} - \frac{C}{4} R\,\hat{1}, \quad C \equiv \frac{a}{3} - c - 4e$$

Weyl tensor acting on $h_{\mu\nu}$:

$$\hat{W} h \equiv W_{(\alpha\ \beta)}^{\ (\mu\ \nu)} h_{\mu\nu}$$

Absence of ghosts

$$D_1 = D_2$$

$$M_{\text{eff}}^2 > 0$$

$$\Downarrow$$

Restrictions on parameters:

$$C = \frac{2}{3}$$

$$a = 2$$

For Einstein (not maximally symmetric) space with $\hat{W} \neq 0$
(Solodukhin, 2012)

$$\frac{1}{8\pi M_{\text{eff}}^2} = G_{\text{eff}} > 0$$

$$\Downarrow$$

$$B < -\frac{\alpha}{2} \text{ and } B > 0, \quad \alpha > 0,$$

$$0 < B < -\frac{\alpha}{2}, \quad \alpha < 0$$

- **Free modes and gravitational potentials in the (A)dS phase and DM mechanism**

Linearized eqs. of motion

$$\frac{4}{M_{\rm eff}^2 g^{1/2}}\frac{\delta S_{(2)}}{\delta h^{\mu\nu}}\bigg|^{\rm retarded}_{++++\,\Rightarrow\,-+++} = \left(-\Box + \frac{2}{3}\Lambda\right) h_{\mu\nu} + \frac{1}{2}\, g_{\mu\nu}\left(\Box + \frac{2}{3}\Lambda\right) h$$
$$+\frac{1}{2}\, g_{\mu\nu} R_{(1)} + 2\,\nabla_{(\mu}\Phi_{\nu)} - g_{\mu\nu}\nabla_\alpha \Phi^\alpha = 0,$$

$$R_{(1)} = \nabla_\mu\nabla_\nu h^{\mu\nu} - \Box h - \Lambda h,$$
$$\Phi^\mu = \chi^\mu - \frac{1}{2}\nabla^\mu \frac{1}{\Box + 2\Lambda}\bigg|_{\rm ret} R_{(1)}$$

$$\Downarrow$$

Free waves in the DeWitt gauge:

$$\nabla^\nu h^{\rm phys}_{\mu\nu} = 0, \quad h^{\rm phys} = 0$$
$$\left(-\Box + \frac{2}{3}\Lambda\right) h^{\rm phys}_{\mu\nu} = 0$$

two physical transverse-traceless polarizations

Retarded gravitational potentials of matter sources:

$$h_{\mu\nu} = \frac{8\pi G_{\text{eff}}}{-\Box + \frac{2}{3}\Lambda}\left(T_{\mu\nu} + g_{\mu\nu}\frac{\Box - 2\Lambda}{\Box + 2\Lambda}\frac{\Lambda}{3\Box}T\right) + \text{gauge transform}$$

vs GR structure $T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T$

Effective gravitational coupling constant vs Newton constant

$$G_{\text{eff}} \equiv \frac{1}{8\pi M_{\text{eff}}^2} = \frac{\alpha(1-\alpha)}{8B(2B+\alpha)}G_N$$

DM simulation:

Attraction much stronger than in GR phase!

$$G_N \Rightarrow \boxed{G_{\text{eff}} \geq \frac{1-\alpha}{|\alpha|}G_N \gg G_N}, \quad \alpha < 0$$

$$-\frac{\sqrt{\alpha}+\alpha}{4} < B < -\frac{\alpha}{2}, \quad 0 < B < \frac{\sqrt{\alpha}-\alpha}{4}, \quad \alpha > 0$$

Another range of α:

$$\boxed{\frac{G_N}{G_{\text{eff}}} = O(1),} \quad |B| \simeq \sqrt{\alpha}/4$$

Desirable interpretation. Two phases:

This would seem to follow from the original action in **R/ → 0** limit:

$$\alpha\, R \frac{1}{\Box + P} R \ll R$$

GR phase (flat-space background, small scale phenomena – galactic, solar system, etc.)

$$\nabla\nabla \sim \Box \gg R$$

(A)dS phase

$$\nabla\nabla \ll R$$

Range of validity of (A)dS phase:

$$|\delta R^{\mu}_{\nu}| \sim |\nabla\nabla h^{\mu}_{\nu}| \ll \Lambda,$$

$$|h^{\mu}_{\nu}| \ll 1 \;\Leftrightarrow\; |T^{\mu}_{\nu}| \sim M^2_{\rm eff}\Lambda\, |h^{\mu}_{\nu}| \ll M^2_{\rm eff}\Lambda$$

Local energy density $|T_{\mu\nu}| \gg M_P^2 \Lambda$ ⇨ **GR regime**

However!

nonlocal operator

$$\frac{\delta S}{\delta g_{\mu\nu}} = g^{1/2}\, \Omega^{\mu\nu}{}_{\alpha\beta}(\nabla)\, E^{\alpha\beta},$$

$$E^{\alpha\beta} \equiv R^{\alpha\beta} - \frac{1}{4}\, g^{\alpha\beta} R$$

cf. A.Einstein, 1919

$$\Omega^{\mu\nu}{}_{\alpha\beta}(\nabla) = \Big(\Box\, \delta^{\mu\nu}{}_{\lambda\sigma} + g^{\mu\nu}\nabla_\lambda\nabla_\sigma - 2\nabla_{(\lambda}\nabla^{(\mu}\delta^{\nu)}_{\sigma)} + \frac{1}{2} R\, \delta^{\mu\nu}{}_{\lambda\sigma}\Big)\frac{\delta^{\lambda\sigma}{}_{\alpha\beta}}{\Box + \hat{P}} + O[\,E\,]$$

Actual UV limit $\nabla\nabla \gg R$

$$\frac{\delta S}{\delta g_{\mu\nu}} \simeq \frac{M^2_{\rm eff}}{2}\, g^{1/2}\Big(R_{\mu\nu} - \frac{1}{2}\nabla_\mu\nabla_\nu \frac{1}{\Box} R\Big)$$

vs GR limit $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$

No smooth crossover to GR in UV regime!

on a compact
Euclidean spacetime
without boundary

quadratic in $E_{\mu\nu}$!

$$S = \frac{M^2}{2}\int dx\, g^{1/2}\left\{-R + \alpha\, R^{\mu\nu}\frac{1}{\Box + \hat{P}}G_{\mu\nu}\right\} \equiv -\frac{M^2_{\rm eff}}{2}\int dx\, g^{1/2}\, E^{\mu\nu}\frac{1}{\Box + \hat{P}}E_{\mu\nu}$$

Based on the identity: $$\frac{1}{\Box + \alpha R}R \equiv \frac{1}{\alpha}$$

Non-analytic in *R* and *α* at *R*,*α* → 0 (because of the constant zero mode of the scalar on the no-boundary spacetime)

? **Possible remedy – conformal transform to another metric coupled to matter (Chameleon cosmology with the metric dependent nonlocal conformal factor)**

$$g_{\mu\nu} \to \tilde{g}_{\mu\nu} = \phi^2[\,g\,]\,g_{\mu\nu}, \quad \phi[\,g\,] \simeq \frac{1}{4\Box}R$$

$$R_{\mu\nu} - \frac{1}{2}\nabla_\mu\nabla_\nu\frac{1}{\Box}R \simeq \tilde{G}_{\mu\nu}, \quad \nabla\nabla \gg R$$

- **Generalization to black holes(S.Solodukhin, 2012)**

From maximally symmetric (A)dS background to generic Einstein space with a nonvanishing Weyl tensor -- a single additional constraint

$$\boxed{a = 2}$$

Schwarzschild-de Sitter BH as stable ghost-free backgrounds with zero Bekenstein-Hawking entropy!

Corollary of

$$S\Big|_{E_{\mu\nu}=0} = 0$$

Conclusions

i) GR limit on flat space background
ii) Stable ghost-free (A)dS phase with arbitrary Λ
iii) Unexpected bonus – DM mechanism in this phase

Problems for realistic cosmology and beyond:

i) PN corrections and effect of new type of nonlocality in the gravitational potentials;
ii) mechanism of crossover from GR to DE regime at a concrete scale;
iii) BH, AdS/CFT, etc. ramifications (zero entropy BH)