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Renormalized Set of Equations for the Green Functions and Its Asymptotical Solution in the Gauge Field Theory with Fermions

By

E. S. Fradkin and **O. K. Kalashnikov**

P. N. Lebedev Physical Institute of the U.S.S.R. Academy of Sciences, Moscow, U.S.S.R.

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Summary

The set of completely renormalized equations which is free of difficulties connected with the treating of the so-called overlapping divergencies is obtained in the gauge field theory with fermions. The asymptotically strict solution at this set is found in the region of the large transferred momentum. The ultraviolet asymptotic behaviour of all the Green and vertex functions is obtained and the connection between the bare and the experimental charge is discussed.

1. Introduction

Investigations of the last few years [1] have shown that there exists a definite class of quantum field theories without the known "zero-charge" difficulty [2]. Among these theories we would like to point out first of all Yang-Mills gauge field theory and that in which Yang-Mills fields interact with fermions. The latter theory is of particular interest, perhaps this is just the theory to be the basis of an asymptotically free theory of strong interactions. Here we bear in mind gluon models developed rather intensively in recent years by many authors [3]. Meanwhile a theory for such models has not yet been formulated at a due theoretical-field level. In particular, the mathematical technique of the Green functions has not yet been fully used here and all the known results have been obtained as a rule within perturbation theory only. At the same time the theory of gluon model is an example of a mathematically consistent and physically interesting theory. In this connection it would be natural to obtain for such a theory a set of exact equations for the Green functions and to carry out the renormalization program in a general form within. The next step is to investigate the qualitative features of the renormalized set of equations thus obtained beyond the scope of the perturbation theory.

The present paper is devoted to this problem. We shall proceed from the set of the dynamical equations describing this class of theories and reformulate them into the set of unrenormalized equations for the Green functions. The set of completely renormalized equations which is free of the known difficulty of treating the overlapping divergencies is then obtained following the method of one of the authors [4]. This set of equations is exact and closed and does not contain any divergencies and uncertainties. Therefore when solving these equations by perturbations as well as beyond the scope of the perturbation method no regularizations are needed. Here this set of equation is solved within the "three-gamma" approximation what provides an asymptotically precise result for all the Green and vertex functions in the limit of large momenta. The connection between the "bare" and experimental charges is established and the absence of the "zero charge" difficulty for this class of theories is discussed.

There is also another reason why we think it important to carry out the above mentioned program within dynamical equations. The point is that in our opinion a number of problems in this theory (such as the possibility that the quarks may form bound states when interacting through the Yang-Mills fields, as well as temperature and many-particle effects) can be solved consistently only when the closed set of renormalized dynamical equations is employed. In this connection the set of completely renormalized equations for the Green functions obtained in the present paper may turn out to be useful for a number of other applications, as well.

2. A Set of Unrenormalized Equations for the Green Functions

We shall proceed from the well known expression for the generating functional of the Yang-Mills field theory [5] extended by switching on the interaction of the Yang-Mills fields with fermions

$$\mathscr{Z} = \int \mathscr{D}[\ldots] \exp(iS),$$

$$\begin{aligned} S[\ldots] = &- \tfrac{1}{4}G_{\mu\nu}{}^a G_{\mu\nu}{}^a + \bar{C}^a(\nabla_\mu{}^{ab}\partial_\mu)C^b + \frac{\alpha}{2}(\partial_\mu W_\mu{}^a)^2 \\ &+ \bar{\psi}^k[\gamma_\mu(\partial_\mu\delta^{kp} + ig_0 t_{kp}{}^d W_\mu{}^d) + m_0\delta^{kp}]\psi^p \\ &+ [\bar{C}^a\eta_{\bar{C}}{}^a + \bar{\eta}_C{}^a C^a] + [\bar{\psi}^a\eta_{\bar{\psi}}{}^a + \bar{\eta}_\psi{}^a\psi^a] \\ &+ W_\mu{}^a[J_W]_\mu^a \end{aligned} \tag{2.1}$$

In Eq. (2.1) $t_{kp}{}^d$ are matrices of the representation with which ψ-fields are transformated; η and J are sources of the external fields. The other notations are usual. The summation or integration over the repeated labels is meant here.

The expression for S in Eq. (2.1) is reformulated in the next step identically to the following form:

$$
\begin{aligned}
S[\ldots] = & -\tfrac{1}{2}W(1)[D_{W^2}]_0^{-1}(1;2)W(2) \\
& + \frac{ig_0}{3!}\Gamma_{W^3}{}^{(0)}(1;2;3)W(1)W(2)W(3) \\
& + \frac{(ig_0)^2}{4!}\Gamma_{W^4}{}^{(0)}(1;2;3;4)W(1)W(2)W(3)W(4) \\
& - \bar{C}(1)[G_{C\bar{C}}]_0^{-1}(1;2)C(2) + ig_0\Gamma_{C\bar{C}W}{}^{(0)}(1;2|3)\bar{C}(1)C(2)W(3) \\
& - \bar{\psi}(1)[G_{\psi\bar{\psi}}]_0^{-1}(1;2)\psi(2) + ig_0\Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;2|3)\bar{\psi}(1)\psi(2)W(3) \\
& + [\bar{C}(1)\eta_{\bar{C}}(1) + \bar{\eta}_C(1)C(1)] + [\bar{\psi}(1)\eta_{\bar{\psi}}(1) + \bar{\eta}_\psi(1)\psi(1)] \\
& + W(1)J_W(1).
\end{aligned} \tag{2.2}
$$

Here we simplified the tensor structure of the initial expression (2.1) and used the usual form for the zero Green functions

$$
\begin{aligned}
([D_{W^2}]_0^{-1})_{\mu\nu}^{ab}(x;y) &= \delta(x-y)\delta^{ab}[-\delta_{\mu\nu}\Box + (1+\alpha)\partial_\mu\partial_\nu], \\
([G_{C\bar{C}}]_0^{-1})^{ab}(x;y) &= \delta(x-y)\delta^{ab}(-\Box), \\
([G_{\psi\bar{\psi}}]_0^{-1})^{ab}(x;y) &= \delta(x-y)\delta^{ab}(-\gamma_\mu\partial_\mu - m_0),
\end{aligned} \tag{2.3}
$$

and the vertex functions

$$
\begin{aligned}
&(\Gamma_{W^3}{}^{(0)})_{\eta\gamma\sigma}^{abc}(x;y;z) \\
&= if^{abc}\left\{\delta_{\eta\gamma}\left[-2\delta(x-z)\left(\frac{\partial}{\partial y_\sigma}\delta(y-x)\right) + \delta(x-y)\left(\frac{\partial}{\partial x_\sigma}\delta(x-z)\right)\right]\right. \\
&\quad + \delta_{\eta\sigma}\left[-2\delta(x-y)\left(\frac{\partial}{\partial x_\gamma}\delta(x-z)\right) + \delta(x-z)\left(\frac{\partial}{\partial y_\gamma}\delta(x-y)\right)\right] \\
&\quad \left. + \delta_{\gamma\sigma}\left[\delta(x-z)\left(\frac{\partial}{\partial y_\eta}\delta(y-x)\right) + \delta(x-y)\left(\frac{\partial}{\partial x_\eta}\delta(x-z)\right)\right]\right\}, \\
&(\Gamma_{C\bar{C}W}{}^{(0)})^{ab}|^c_\mu(x;y|z) = -if^{abc}\left(\frac{\partial}{\partial z_\mu}\delta(z-y)\right)\delta(x-z), \\
&(\Gamma_{\psi\bar{\psi}W}{}^{(0)})^{ab}_{\alpha\beta}|^c_\mu(x;y|z) = -\delta(x-z)\delta(z-y)\cdot[\gamma_\mu]_{\alpha\beta}\cdot t_{ab}{}^c, \\
&(\Gamma_{W^4}{}^{(0)})_{\mu\nu\delta\sigma}^{abcd}(z_1;z_2;z_3;z_4) \\
&= \delta(z_1-z_2)\delta(z_1-z_3)\delta(z_1-z_4)\cdot\{(f^{pab}f^{pcd})\cdot[\delta_{\delta\mu}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\delta}] \\
&\quad + (f^{pbc}f^{pad})\cdot[\delta_{\delta\sigma}\delta_{\mu\nu} - \delta_{\nu\delta}\delta_{\mu\sigma}]\}.
\end{aligned} \tag{2.4}
$$

The exact Green function

$$
D_{W^2}(1;2) = -i\frac{\delta^2 \ln \mathscr{Z}}{\delta J_W(1)\delta J_W(2)};
$$

$$
G_{C\bar{C}}(1;2) = -i\frac{\delta^2 \ln \mathscr{Z}}{\delta\bar{\eta}_C(1)\delta\eta_{\bar{C}}(2)}; \qquad G_{\psi\bar{\psi}}(1;2) = -i\frac{\delta^2 \ln \mathscr{Z}}{\delta\bar{\eta}_\psi(1)\delta\eta_{\bar{\psi}}(2)} \tag{2.5}
$$

and the exact vertex function of the theory under consideration

$$\Gamma_{W^3}(1;2;3) = -\frac{\delta D_{W^2}{}^{-1}(1;2)}{(ig_0)\delta\langle W(3)\rangle};$$

$$\Gamma_{W^4}(1;2;3;4) = -\frac{\delta^2 D_{W^2}{}^{-1}(1;2)}{(ig_0)^2\delta\langle W(3)\rangle\delta\langle W(4)\rangle};$$

$$\Gamma_{C\bar{C}W}(1;2|3) = -\frac{\delta G_{C\bar{C}}{}^{-1}(1;2)}{(ig_0)\delta\langle W(3)\rangle};$$

$$\Gamma_{\psi\bar{\psi}W}(1;2|3) = -\frac{\delta G_{\psi\bar{\psi}}{}^{-1}(1;2)}{(ig_0)\delta\langle W(3)\rangle} \tag{2.6}$$

are defined here in a usual way. Expression for S in the form (2.2) is convenient for treatment because the choice of the definite representation for the interacting fields can be made after all calculations. Thus the expressions to be obtained take place for every representation which offers the possibility to obtain here a number of general statements concerning the theory under consideration.

The set of functional equations is derived by a usual method and, according to (2.2), has the following simple form

$$\begin{aligned}
&[D_{W^2}]_0^{-1}(1;2)\langle W(2)\rangle - \frac{g_0}{2}\Gamma_{W^3}{}^{(0)}(1;2;3)\left[D_{W^2}(2;3) - \frac{1}{i}\langle W(2)\rangle\langle W(3)\rangle\right] \\
&\quad + \frac{(ig_0)^2}{6}\Gamma_{W^4}{}^{(0)}(1;2;3;4)\left[\frac{\delta D_{W^2}(2;3)}{\delta J_W(4)} - \frac{1}{i}\langle W(2)\rangle D_{W^2}(3;4)\right. \\
&\quad - \frac{1}{i}\langle W(3)\rangle D_{W^2}(2;4) - \frac{1}{i}\langle W(4)\rangle D_{W^2}(2;3) \\
&\quad \left. - \langle W(2)\rangle\langle W(3)\rangle\langle W(4)\rangle\right] \\
&\quad + g_0\Gamma_{WC\bar{C}}{}^{(0)}(1|2;3)\left[G_{C\bar{C}}(3;2) - \frac{1}{i}\langle C(3)\rangle\langle\bar{C}(2)\rangle\right] \\
&\quad + g_0\Gamma_{W\psi\bar{\psi}}{}^{(0)}(1|2;3)\left[G_{\psi\bar{\psi}}(3;2) - \frac{1}{i}\langle\psi(3)\rangle\langle\bar{\psi}(2)\rangle\right] = J_W(1), \\
&[G_{C\bar{C}}]_0^{-1}(1;2)\langle C(2)\rangle \\
&\quad - g_0\Gamma_{C\bar{C}W}{}^{(0)}(1;2|3)\left[\frac{\delta\langle C(2)\rangle}{\delta J_W(3)} - \frac{1}{i}\langle C(2)\rangle\langle W(3)\rangle\right] = \eta_{\bar{C}}(1), \\
&[G_{\psi\bar{\psi}}]_0^{-1}(1;2)\langle\psi(2)\rangle \\
&\quad - g_0\Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;2|3)\left[\frac{\delta\langle\psi(2)\rangle}{\delta J_W(3)} - \frac{1}{i}\langle\psi(2)\rangle\langle W(3)\rangle\right] = \eta_{\bar{\psi}}(1).
\end{aligned} \tag{2.7}$$

The set of unrenormalized equations for the corresponding Green functions is then obtained from (2.7) by differentiation of the latter with respect to the external sources. After some simple algebra it may be easily put into the form of the Schwinger-Dyson-type equations:

$$
\begin{aligned}
[D_{W^2}]^{-1}(1;2) &= [D_{W^2}]_0^{-1}(1;2) - (ig_0)\Gamma_{W^3}{}^{(0)}(1;2;3)\langle W(3)\rangle \\
&\quad - \frac{(ig_0)^2}{2}\Gamma_{W^4}{}^{(0)}(1;2;3;4)\langle W(3)\rangle\langle W(4)\rangle \\
&\quad - \Pi_{W^2}(1;2); \\
[G_{C\bar{C}}]^{-1}(1;2) &= [G_{C\bar{C}}]_0^{-1}(1;2) - \Sigma_{C\bar{C}}(1;2); \\
[G_{\psi\bar{\psi}}]^{-1}(1;2) &= [G_{\psi\bar{\psi}}]_0^{-1}(1;2) - \Sigma_{\psi\bar{\psi}}(1;2). \qquad (2.8)
\end{aligned}
$$

The sources of the C-field are switched off here and the self-energy operators are defined by the following integral representation:

$$
\begin{aligned}
\Pi_{W^2}(1;\bar{1}) &= -\frac{g_0^2}{2i}\Gamma_{W^4}{}^{(0)}(1;2;3;\bar{1})D_{W^2}(2;3) \\
&\quad - \frac{g_0^2}{2i}\Gamma_{W^3}{}^{(0)}(1;2;3)D_{W^2}(2;\bar{3})\Gamma_{W^3}(\bar{3};\bar{2};\bar{1})D_{W^2}(\bar{2};3) \\
&\quad - \frac{g_0^3}{2}\Gamma_{W^4}{}^{(0)}(1;2;3;4)\langle W(2)\rangle D_{W^2}(4;\bar{3})\Gamma_{W^3}(\bar{3};\bar{2};\bar{1})D_{W^2}(\bar{2};3) \\
&\quad - \frac{g_0^4}{2}\Gamma_{W^4}{}^{(0)}(1;2;3;4)D_{W^2}(2;\bar{4})\Gamma_{W^3}(\bar{4};5;6) \\
&\quad \cdot D_{W^2}(5;\bar{2})\Gamma_{W^3}(\bar{3};\bar{2};\bar{1})D_{W^2}(\bar{3};4)D_{W^2}(6;3) \\
&\quad - \frac{g_0^4}{6}\Gamma_{W^4}{}^{(0)}(1;2;3;4)D_{W^2}(2;\bar{2})\Gamma_{W^4}(\bar{4};\bar{3};\bar{2};\bar{1}) \\
&\quad \cdot D_{W^2}(3;\bar{3})D_{W^2}(\bar{4};4) \\
&\quad + \frac{g_0^2}{i}\Gamma_{WC\bar{C}}{}^{(0)}(1|2;3)G_{C\bar{C}}(3;\bar{3})\Gamma_{C\bar{C}W}(\bar{3};\bar{2}|\bar{1})G_{C\bar{C}}(\bar{2};2) \\
&\quad + \frac{g_0^2}{i}\Gamma_{W\psi\bar{\psi}}{}^{(0)}(1|2;3)G_{\psi\bar{\psi}}(3;\bar{3})\Gamma_{\psi\bar{\psi}W}(\bar{3};\bar{2}|\bar{1})G_{\psi\bar{\psi}}(\bar{2};2); \\
\Sigma_{C\bar{C}}(1;\bar{1}) &= -\frac{g_0^2}{i}\Gamma_{C\bar{C}W}{}^{(0)}(1;2|3)G_{C\bar{C}}(2;\bar{2})\Gamma_{WC\bar{C}}(\bar{3}|\bar{2};\bar{1})D_{W^2}(\bar{3};3); \\
\Sigma_{\psi\bar{\psi}}(1;\bar{1}) &= -\frac{g_0^2}{i}\Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;2|3)G_{\psi\bar{\psi}}(2;\bar{2})\Gamma_{W\psi\bar{\psi}}(\bar{3}|\bar{2};\bar{1})D_{W^2}(\bar{3};3).
\end{aligned}
$$

All the necessary vertex functions are to be found by direct differentiation of (2.8) according to their definition (2.6).

3. Renormalization of the Set of Equations for the Green Functions

Although the set of Eqs. (2.8) is closed and consistent, explicit calculations with the aid of it turn out to be extremely difficult. The point is that all the quantities involved appear to be infinite due to divergency of the corresponding integrals in the region of both small and large momenta. The divergencies in the region of small momenta are due to the masslessness of the Yang-Mills fields which introduce an actual difficulty to the theory in question. On the contrary, the ultraviolet divergencies are easily eliminated from the theory within a usual renormalization program. The latter can be carried out in a general form directly in the framework of the set of Eqs. (2.8).

For that purpose it is necessary first of all to introduce the Z-factors to the Green and vertex functions

$$\Gamma^R = Z_1 \cdot \Gamma,$$
$$G^R = Z_2^{-1} \cdot G; \qquad D^R = Z_3^{-1} \cdot D \tag{3.1}$$

and to go over in (2.8) to the renormalized quantities

$$\begin{aligned}
[D_{W^2}{}^R]^{-1}(1;2) &= Z_3{}^{W^2} \cdot [D_{W^2}]_0^{-1}(1;2) - (ig) Z_1{}^{W^3} \cdot \Gamma_{W^3}{}^{(0)}(1;2;3)\langle W^R(3)\rangle \\
&\quad - \frac{(ig)^2}{2} \cdot Z_1{}^{W^4} \Gamma_{W^4}{}^{(0)}(1;2;3;4)\langle W^R(3)\rangle\langle W^R(4)\rangle \\
&\quad - \Pi_{W^2}{}'(1;2); \\
[G_{C\bar{C}}{}^R]^{-1}(1;2) &= Z_2{}^{C\bar{C}} \cdot [G_{C\bar{C}}]_0^{-1}(1;2) - \Sigma_{C\bar{C}}{}'(1;2); \\
[G_{\psi\bar{\psi}}]^{-1}(1;2) &= Z_2{}^{\psi\bar{\psi}} \cdot [G_{\psi\bar{\psi}}]_0^{-1}(1;2) - \Sigma_{\psi\bar{\psi}}{}'(1;2)
\end{aligned} \tag{3.2}$$

with the account taken of the exact Ward identity [6]

$$Z_1{}^{W^3} \cdot [Z_3{}^{W^2}]^{-1} = Z_1{}^{WC\bar{C}} \cdot [Z_2{}^{C\bar{C}}]^{-1}; \qquad Z_1{}^{W^4} = [Z_1{}^{W^3}]^2 \cdot [Z_3{}^{W^2}]^{-1};$$
$$Z_1{}^{W^3} \cdot [Z_3{}^{W^2}]^{-1} = Z_1{}^{W\psi\bar{\psi}} \cdot [Z_2{}^{\psi\bar{\psi}}]^{-1} \tag{3.3}$$

as well as of the connection between the "bare" and experimental charges

$$g^2 = g_0{}^2 \cdot [Z_1{}^{W^3}] \cdot [Z_3{}^{W^2}]^3. \tag{3.4}$$

Then in view of the arbitrariness of the theory one can choose the Z-factors so that all the ultraviolet divergencies of the theory be eliminated. In particular, the $Z_{2,3}$-factors are determined from the requirement that the corresponding divergencies in the Green functions be eliminated

$$Z_3{}^{W^2} = 1 + \frac{\partial \Pi_{W^2}{}'(k_0{}^2)}{\partial k_0{}^2};$$
$$Z_2{}^{C\bar{C}} = 1 + \frac{\partial \Sigma_{C\bar{C}}{}'(k_0{}^2)}{\partial k_0{}^2}; \qquad Z_2{}^{\psi\bar{\psi}} = 1 + \frac{\partial \Sigma_{\psi\bar{\psi}}{}'(k_0{}^2)}{\partial k_0{}^2} \tag{3.5}$$

while the Z_1-factors serve for the elimination of ultraviolet divergencies in the vertex functions. The renormalized Green functions retain the structure of the Schwinger-Dyson-type equations

$$[D_{W^2}{}^R]^{-1}(1;2) = [D_{W^2}]_0^{-1}(1;2) - (ig)Z_1{}^{W^3}\cdot \Gamma_{W^3}{}^{(0)}(1;2;3)\langle W^R(3)\rangle$$
$$- \frac{(ig)^2}{2} Z_1{}^{W^4}\cdot \Gamma_{W^4}{}^{(0)}(1;2;3;4)\langle W^R(3)\rangle\langle W^R(4)\rangle$$
$$- \Pi_{W^2}{}^R(1;2);$$
$$[G_{C\bar{C}}{}^R]^{-1}(1;2) = [G_{C\bar{C}}]_0^{-1}(1;2) - \Sigma_{C\bar{C}}{}^R(1;2);$$
$$[G_{\psi\bar{\psi}}{}^R]^{-1}(1;2) = [G_{\psi\bar{\psi}}]_0^{-1}(1;2) - \Sigma_{\psi\bar{\psi}}{}^R(1;2), \tag{3.6}$$

however, self-energy operators in these equations

$$\tilde{\Pi}_{W^2}{}^R(k^2) = \tilde{\Pi}_{W^2}{}'(k^2) - k^2\frac{\partial\tilde{\Pi}_{W^2}{}'(k_0{}^2)}{\partial k_0{}^2};$$
$$\tilde{\Sigma}_{C\bar{C}}{}^R(k^2) = \tilde{\Sigma}_{C\bar{C}}{}'(k^2) - k^2\frac{\partial\tilde{\Sigma}_{C\bar{C}}{}'(k_0{}^2)}{\partial k_0{}^2};$$
$$\tilde{\Sigma}_{\psi\bar{\psi}}{}^R(k^2) = \tilde{\Sigma}_{\psi\bar{\psi}}{}'(k^2) - k^2\frac{\partial\tilde{\Sigma}_{\psi\bar{\psi}}{}'(k_0{}^2)}{\partial k_0{}^2}, \tag{3.7}$$

do not contain now ultraviolet divergencies. Here $k_0{}^2$ is an arbitrary point of renormalization; $\tilde{\Pi}'$ and $\tilde{\Sigma}'$ are those parts of the self-energy operators which are subject to renormalization. They have a tensor structure similar to that of the corresponding lowest order Green function.

At the next stage the "overlapping" divergencies connected with the resolution of the $0\cdot\infty$-type uncertainty should be eliminated from the theory. This uncertainty arises when one solves the set of renormalized Eqs. (3.5) due to the presence in it of the Z-factors and "bare" vertices. We shall apply here the same method as the one applied in paper [4] by one of the present authors and after some easy transformations we shall effectively eliminate the above mentioned difficulty. Thus a completely renormalized set of equations for the Green functions in the Yang-Mills field theory with fermion is obtained.

We begin with eliminating the $(Z_1{}^{WC\bar{C}}\cdot\Gamma_{WC\bar{C}}{}^{(0)})$ combination. For that purpose, instead of the four-point function $W_{C\bar{C}W^2}(1;2|3;4)$

$$W_{C\bar{C}W^2}(1;2|3;4) = \frac{\delta\Gamma_{C\bar{C}W}{}^R(1;2|4)}{(ig)\delta\langle W^R(3)\rangle}$$
$$+ \Gamma_{C\bar{C}W}{}^R(1;\bar{2}|3)G_{C\bar{C}}{}^R(\bar{2};\bar{1})\Gamma_{C\bar{C}W}{}^R(\bar{1};2|4)$$
$$+ \Gamma_{C\bar{C}W}{}^R(1;2|\bar{3})D_{W^2}{}^R(\bar{3};\bar{2})\Gamma_{W^3}{}^R(\bar{2};3;4), \tag{3.8}$$

defining the renormalized Γ-function

$$\Gamma_{C\bar{C}W}{}^R(1;2|3) = Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}(1;2|3)$$
$$- \frac{g^2}{i} Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}(1;\bar{2}|\bar{3})G_{C\bar{C}}{}^R(\bar{2};\bar{1})$$
$$\cdot W_{C\bar{C}W^2}(\bar{1};2|3;4)D_{W^2}{}^R(4;\bar{3}), \tag{3.9}$$

we introduce a new four-point function $P_{C\bar{C}W^2}(1;2|3;4)$

$$P_{C\bar{C}W^2}(1;2|3;4) = W_{C\bar{C}W^2}(1;2|3;4) + \frac{g^2}{i} W_{C\bar{C}W^2}(1;\bar{2}|\bar{3};4) D_{W^2}{}^R(\bar{4};\bar{3}) \cdot G_{C\bar{C}}{}^R(\bar{2};\bar{1}) P_{C\bar{C}W^2}(\bar{1};2|3;\bar{4}) \tag{3.10}$$

having used the block $W_{C\bar{C}W^2}(1;2|3;4)$ as an irreducible part. Eq. (3.9) defining the Γ-function is now equivalent to the following equation

$$\Gamma_{C\bar{C}W}{}^R(1;2|3) = Z_1{}^{C\bar{C}W} \cdot \Gamma_{C\bar{C}W}{}^{(0)}(1;2|3) - \frac{g^2}{i} \Gamma_{C\bar{C}W}{}^R(1;\bar{2}|\bar{3}) G_{C\bar{C}}{}^R(\bar{2};\bar{1}) \cdot P_{C\bar{C}W^2}(\bar{1};2|3;\bar{4}) D_{W^2}{}^R(\bar{4};\bar{3}) \tag{3.11}$$

which expresses $Z_1{}^{C\bar{C}W} \cdot \Gamma_{C\bar{C}W}{}^{(0)}$ only in terms of renormalized quantities. This is just the equation to be used for eliminating $Z_1{}^{C\bar{C}W} \cdot \Gamma_{C\bar{C}W}{}^{(0)}$ from all the other equations of the theory under consideration.

The procedure of eliminating $Z_1{}^{W\psi\bar{\psi}} \cdot \Gamma_{W\psi\bar{\psi}}{}^{(0)}$ combination is quite similar. The final expression takes the form of Eq. (3.11)

$$\Gamma_{\psi\bar{\psi}W}{}^R(1;2|3) = Z_1{}^{W\psi\bar{\psi}} \cdot \Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;2|3) - \frac{g^2}{i} \Gamma_{\psi\bar{\psi}W}{}^R(1;\bar{2}|\bar{3}) G_{\psi\bar{\psi}}{}^R(\bar{2};\bar{1}) \cdot P_{\psi\bar{\psi}W^2}(\bar{1};2|3;\bar{4}) D_{W^2}{}^R(\bar{4};\bar{3}) \tag{3.12}$$

where the block $P_{\psi\bar{\psi}W^2}(1;2|3;4)$ is defined now by the solution of the following equation

$$P_{\psi\bar{\psi}W^2}(1;2|3;4) = W_{\psi\bar{\psi}W^2}(1;2|3;4) + \frac{g^2}{i} W_{\psi\bar{\psi}W^2}(1;\bar{2}|\bar{3};4) D_{W^2}{}^R(\bar{4};\bar{3}) \cdot G_{\psi\bar{\psi}}{}^R(\bar{2};\bar{1}) P_{\psi\bar{\psi}W^2}(\bar{1};2|3;\bar{4}). \tag{3.13}$$

Here the block $W_{\psi\bar{\psi}W^2}(1;2|3;4)$ has rather a simple form

$$\begin{aligned} W_{\psi\bar{\psi}W^2}(1;2|3;4) = {} & \frac{\delta\Gamma_{\psi\bar{\psi}W}{}^R(1;2|4)}{(ig)\delta\langle W^R(3)\rangle} \\ & + \Gamma_{\psi\bar{\psi}W}{}^R(1;\bar{2}|3) G_{\psi\bar{\psi}}{}^R(\bar{2};\bar{1}) \Gamma_{\psi\bar{\psi}W}{}^R(\bar{1};2|4) \\ & + \Gamma_{\psi\bar{\psi}W}{}^R(1;2|3) D_{W^2}{}^R(\bar{3};\bar{2}) \Gamma_{W^3}{}^R(\bar{2};3;4) \end{aligned} \tag{3.14}$$

and is defined by the initial equation for $\Gamma_{\psi\bar{\psi}W}{}^R$ function

$$\Gamma_{\psi\bar{\psi}W}{}^R(1;2|3) = Z_1{}^{\psi\bar{\psi}W} \cdot \Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;2|3) - \frac{g^2}{i} Z_1{}^{\psi\bar{\psi}W} \cdot \Gamma_{\psi\bar{\psi}W}{}^{(0)}(1;\bar{2}|\bar{3}) \cdot G_{\psi\bar{\psi}}{}^R(\bar{2};\bar{1}) W_{\psi\bar{\psi}W^2}(\bar{1};2|3;4) D_{W^2}{}^R(4;\bar{3}) \tag{3.15}$$

which after eliminating uncertainties coincides with Eq. (3.12).

As to the elimination of $Z_1^{W^3}\cdot\Gamma_{W^3}{}^{(0)}$- and $Z_1^{W^4}\cdot\Gamma_{W^4}{}^{(0)}$-combinations, all the calculations are also analogous in many respects to those performed above. Here it is convenient, however, to unite equations for the corresponding vertex functions in the single matrix equation

$$
\begin{aligned}
&[\Gamma_{W^3}{}^R(1;2;3)\quad \Gamma_{W^4}{}^R(1;2;3;4)]\\
&= [Z_1^{W^3}\cdot\Gamma_{W^3}{}^{(0)}(1;2;3)\quad Z_1^{W^4}\cdot\Gamma_{W^4}{}^{(0)}(1;2;3;4)]\\
&\quad - [Z_1^{W^3}\cdot\Gamma_{W^3}{}^{(0)}(1;\bar{2};\bar{3})\quad Z_1^{W^4}\cdot\Gamma_{W^4}{}^{(0)}(1;\bar{2};\bar{3};\bar{4})]\\
&\quad\cdot\begin{bmatrix}
\dfrac{\delta K_{W^2W}(\bar{2};\bar{3}|2)}{(ig)\delta\langle W^R(3)\rangle} & \dfrac{\delta^2 K_{W^2W}(\bar{2};\bar{3}|2)}{(ig)^2\delta\langle W^R(3)\rangle\delta\langle W^R(4)\rangle}\\
\dfrac{\delta K_{W^3W}(\bar{2};\bar{3};\bar{4}|2)}{(ig)\delta\langle W^R(3)\rangle} & \dfrac{\delta^2 K_{W^3W}(\bar{2};\bar{3};\bar{4}|2)}{(ig)^2\delta\langle W^R(3)\rangle\delta\langle W^R(4)\rangle}
\end{bmatrix}\\
&\quad + \begin{bmatrix}
\dfrac{\delta M_{W^2}{}'(1;2)}{(ig)\delta\langle W^R(3)\rangle} & \dfrac{\delta^2 M_{W^2}{}'(1;2)}{(ig)^2\delta\langle W^R(3)\rangle\delta\langle W^R(4)\rangle}
\end{bmatrix}
\end{aligned}
\tag{3.16}
$$

because this simplifies considerably the consequent calculations. The new K; M'-functions in (3.16) are directly connected with the self-energy operators introduced above,

$$
\begin{aligned}
\Pi_{W^2}{}'(1;\bar{1}) = &- Z_1^{W^3}\Gamma_{W^3}{}^{(0)}(1;2;3)K_{W^2W}(3;2|\bar{1})\\
&- Z_1^{W^4}\cdot\Gamma_{W^4}{}^{(0)}(1;2;3;4)K_{W^3W}(4;3;2|\bar{1}) + M_{W^2}{}'(1;\bar{1}).
\end{aligned}
\tag{3.17}
$$

The four-point function F is defined by the corresponding matrix from Eq. (3.16)

$$
\begin{aligned}
&\begin{bmatrix}
F_{W^2W^2}(\bar{2};\bar{3}|2;3) & F_{W^2W^3}(\bar{2};\bar{3}|2;3;4)\\
F_{W^3W^2}(\bar{2};\bar{3};\bar{4}|2;3) & F_{W^3W^3}(\bar{2};\bar{3};\bar{4}|2;3;4)
\end{bmatrix}\\
&= \begin{bmatrix}
\dfrac{\delta K_{W^2W}(\bar{2};\bar{3}|2)}{(ig)\delta\langle W^R(3)\rangle} & \dfrac{\delta^2 K_{W^2W}(\bar{2};\bar{3}|2)}{(ig)^2\delta\langle W^R(3)\rangle\delta\langle W^R(4)\rangle}\\
\dfrac{\delta K_{W^3W}(\bar{2};\bar{3};\bar{4}|2)}{(ig)\delta\langle W^R(3)\rangle} & \dfrac{\delta K_{W^3W}(\bar{2};\bar{3};\bar{4}|2)}{(ig)^2\delta\langle W^R(3)\rangle\delta\langle W^R(4)\rangle}
\end{bmatrix}\\
&\quad + \begin{bmatrix}
\dfrac{\delta K_{W^2W}(\bar{2};\bar{3}|\bar{\bar{2}})}{(ig)\delta\langle W^R(\bar{\bar{3}})\rangle} & \dfrac{\delta^2 K_{W^2W}(\bar{2};\bar{3}|\bar{\bar{2}})}{(ig)^2\delta\langle W^R(\bar{\bar{3}})\rangle\delta\langle W^R(\bar{\bar{4}})\rangle}\\
\dfrac{\delta K_{W^3W}(\bar{2};\bar{3};\bar{4}|\bar{\bar{2}})}{(ig)\delta\langle W^R(\bar{\bar{3}})\rangle} & \dfrac{\delta^2 K_{W^3W}(\bar{2};\bar{3};\bar{4}|\bar{\bar{2}})}{(ig)^2\delta\langle W^R(\bar{\bar{3}})\rangle\delta\langle W^R(\bar{\bar{4}})\rangle}
\end{bmatrix}\\
&\quad\cdot\begin{bmatrix}
F_{W^2W^2}(\bar{\bar{2}};\bar{\bar{3}}|2;3) & F_{W^2W^3}(\bar{\bar{2}};\bar{\bar{3}}|2;3;4)\\
F_{W^3W^2}(\bar{\bar{2}};\bar{\bar{3}};\bar{\bar{4}}|2;3) & F_{W^3W^3}(\bar{\bar{2}};\bar{\bar{3}};\bar{\bar{4}}|2;3;4)
\end{bmatrix}
\end{aligned}
\tag{3.18}
$$

and makes it possible, some easy transformations carried out, to obtain a simple equation for eliminating $Z_1^{W^3}\cdot\Gamma_{W^3}{}^{(0)}$- and $Z_1^{W^4}\cdot\Gamma_{W^4}{}^{(0)}$-combinations from all the consequent calculations

$$\begin{aligned}
&[\Gamma_{W^3}{}^R(1;2;3) \quad \Gamma_{W^4}{}^R(1;2;3;4)] \\
&= [Z_1{}^{W^3}\cdot\Gamma_{W^3}{}^{(0)}(1;2;3) \quad Z_1{}^{W^4}\cdot\Gamma_{W^4}{}^{(0)}(1;2;3;4)] \\
&\quad - [\Gamma_{W^3}{}^R(1;\overline{2};\overline{3}) \quad \Gamma_{W^4}{}^R(1;\overline{2};\overline{3};\overline{4})] \\
&\quad \cdot \begin{bmatrix} F_{W^2W^2}(\overline{2};\overline{3}|2;3) & F_{W^2W^3}(\overline{2};\overline{3}|2;3;4) \\ F_{W^3W^2}(\overline{2};\overline{3};\overline{4}|2;3) & F_{W^3W^3}(\overline{2};\overline{3};\overline{4}|2;3;4)\end{bmatrix} \\
&\quad + \left[\frac{\delta M_{W^2}{}'(1;\overline{2})}{(ig)\delta\langle W^R(\overline{3})\rangle} \quad \frac{\delta^2 M_{W^2}{}'(1;\overline{2})}{(ig)^2\delta\langle W^R(\overline{3})\rangle\delta\langle W^R(\overline{4})\rangle}\right] \\
&\cdot\begin{bmatrix} \delta(2;\overline{2})\delta(3;\overline{3}) + F_{W^2W^2}(\overline{2};\overline{3}|2;3) & F_{W^2W^3}(\overline{2};\overline{3}|2;3;4) \\ F_{W^3W^2}(\overline{2};\overline{3};\overline{4}|2;3) & \delta(2;\overline{2})\delta(3;\overline{3})\delta(4;\overline{4}) + F_{W^3W^3}(\overline{2};\overline{3};\overline{4}|2;3;4)\end{bmatrix}.
\end{aligned} \tag{3.19}$$

Now the renormalization program may be considered to be over. The "overlapping divergencies" being excluded with the aid of (3.10) and (3.19), the self-energy operators (3.6) can now be subject for explicit calculations. The factors Z_1 are eliminated from (3.10) and (3.19) by usual methods (see for example [4]).

4. Asymptotic Behaviour of the Green Functions in the Region of Large Transferred Momenta

Asymptotic behaviour of the Green functions and of the corresponding vertex functions will be calculated in the framework of the above set of exact renormalized equations. We shall restrict ourselves to the so-called "three-gamma" approximation and carry out the investigation following the method of one of the present authors [7]. The results obtained will be nevertheless asymptotically exact since this approximation shows that the theory discussed here is asymptotically free. The latter, in its turn, is responsible for the small contribution of the higher corrections and thus the leading terms in the asymptotic behaviour of the Green functions and the corresponding vertex functions will be found here exactly.

It is convenient to start calculations with the derivation of a simplified set of equations for the Green functions and the corresponding vertex functions. Here we shall consider the equations for the vertex function ghosts only

$$\Gamma_{C\bar{C}W}(1;2;3) = Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}(1;2;3) - \frac{g^2}{i}\Gamma_{C\bar{C}W}{}^R(1;\overline{2}|\overline{3})G_{C\bar{C}}{}^R(\overline{2};\overline{1})$$
$$\cdot P_{C\bar{C}W^2}(\overline{1};2|3;\overline{4})D_{W^2}{}^R(\overline{4};\overline{3}) \tag{4.1}$$

and the set of equations for the derivatives of the corresponding Green functions

$$\frac{d[D_{W^2}{}^R]^{-1}(p^2)}{dp^2} = 1 - \frac{d\Pi_{W^2}{}^R(p^2)}{dp^2};$$

$$\frac{d[G_{C\bar{C}}{}^R]^{-1}(p^2)}{dp^2} = 1 - \frac{d\Sigma_{C\bar{C}}{}^R(p^2)}{dp^2};$$

$$\frac{d[G_{\psi\bar{\psi}}{}^R]^{-1}(p^2)}{dp^2} = 1 - \frac{d\Sigma_{\psi\bar{\psi}}{}^R(p^2)}{dp^2}. \tag{4.2}$$

We shall not need an equation for the Γ_{W^3} and $\Gamma_{\psi\bar{\psi}W}$ functions since the latters can be determined with the aid of the exact Ward identities; the Γ_{W^4} function appears not to be connected with these calculations because the corresponding part of the self-energy operator $\Pi_{W^2}{}^R$ does not contribute to the asymptotic region. The set of equations for the derivatives of the corresponding Green functions is as exact as the initial set of equations for the Green functions but enables us to avoid some difficulties in calculations in the study of asymptotic behaviour of the self-energy operators.

Further simplifications of the exact set of Eqs. (4.1), (4.2) within the "three-gamma" approximation are first of all due to an approximate change of $Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}$, $Z_1{}^{\psi\bar{\psi}W}\cdot\Gamma_{\psi\bar{\psi}W}{}^{(0)}$ and $Z_1{}^{W^3}\cdot\Gamma_{W^3}{}^{(0)}$ in (4.2). We shall restrict ourselves here to the leading terms of Eqs. (3.11), (3.12) and (3.19) obtained above

$$Z_1{}^{\psi\bar{\psi}W}\cdot\Gamma_{\psi\bar{\psi}W}{}^{(0)} \to \Gamma_{\psi\bar{\psi}W}{}^R, \qquad Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)} \to \Gamma_{C\bar{C}W}{}^R,$$
$$Z_1{}^{W^3}\cdot\Gamma_{W^3}{}^{(0)} \to \Gamma_{W^3}{}^R. \tag{4.3}$$

Besides, when solving Eq. (3.10) for the four-point function, one also should restrict oneself to the leading terms and simplify the expression for $W_{C\bar{C}W^2}$ within the same accuracy. As a result, instead of the exact Eq. (4.1) the vertex function of fictitious particles is to be determined from still simpler equation

$$\begin{aligned}
&\Gamma_{C\bar{C}W}{}^R(1;2|3)\\
&= Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}(1;2|3)\\
&\quad -\frac{g^2}{i}\Gamma_{C\bar{C}W}{}^R(1;\bar{2}|\bar{3})G_{C\bar{C}}{}^R(\bar{2};\bar{1})\Gamma_{C\bar{C}W}{}^R(\bar{1};2|\bar{\bar{3}})D_{W^2}{}^R(\bar{\bar{3}};\bar{\bar{2}})\Gamma_{W^3}{}^R(\bar{\bar{2}};3;4)\\
&\quad \cdot D_{W^2}{}^R(4;\bar{3}) - \frac{g^2}{i}\Gamma_{C\bar{C}W}{}^R(1;\bar{2}|\bar{3})G_{C\bar{C}}{}^R(\bar{2};\bar{1})\Gamma_{C\bar{C}W}{}^R(\bar{1};\bar{\bar{2}}|4)G_{C\bar{C}}{}^R(\bar{\bar{2}};\bar{\bar{1}})\\
&\quad \cdot\Gamma_{C\bar{C}W}{}^R(\bar{\bar{1}};2|3)D_{W^2}{}^R(4;\bar{3}).
\end{aligned} \tag{4.4}$$

The latter equation together with Eqs. (4.2) after the corresponding replacement of $Z_1{}^{C\bar{C}W}\cdot\Gamma_{C\bar{C}W}{}^{(0)}$; $Z_1{}^{\psi\bar{\psi}W}\cdot\Gamma_{\psi\bar{\psi}W}{}^{(0)}$ and $Z_1{}^{W^3}\cdot\Gamma_{W^3}{}^{(0)}$ combinations is the basis for our further calculations.

Now within these equations we must first of all separate the tensor structure. We shall deal with the Feynman gauge since it leads to the simplest tensor structure of the corresponding Green functions

$$[D_{W^2}{}^R]^{ab}_{\mu\nu}(p^2) = \delta^{ab}\cdot\delta_{\mu\nu}\cdot D_{W^2}{}^R(p^2),$$
$$[G_{C\bar{C}}{}^R]^{ab}(p^2) = \delta^{ab}\cdot G_{C\bar{C}}{}^R(p^2);\ [G_{\psi\bar{\psi}}{}^R]^{ab}_{\alpha\beta} \approx \delta^{ab}\cdot(-i\gamma_\mu)_{\alpha\beta}\cdot p_\mu\cdot G_{\psi\bar{\psi}}{}^R(p^2). \tag{4.5}$$

As concerns the tensor structure of the vertex functions, it is approximately taken as coinciding with that of the bare vertices

$$(\Gamma_{C\bar{C}W}{}^R)^{ab}|^c_\mu(p+k;p|k) = f^{abc}\cdot p_\mu\cdot\Gamma_{C\bar{C}W}{}^R(p+k;p|k),$$
$$(\Gamma_{W^3}{}^R)^{abc}_{\kappa\sigma\mu}(p+k;p;k) = f^{abc}\cdot[\delta_{\kappa\sigma}(2p+k)_\mu - \delta_{\kappa\mu}(2k+p)_\sigma - \delta_{\sigma\mu}(p-k)_\kappa]\cdot\Gamma_{W^3}{}^R(p+k;p;k),$$
$$(\Gamma_{\psi\bar{\psi}W}{}^R)^{ab}_{\alpha\beta}|^c_\mu(p+k;p|k) = -t_{ab}{}^c\cdot(\gamma_\mu)_{\alpha\beta}. \tag{4.6}$$

When substituted into the set of Eqs. (4.2), (4.4), the Green functions and the corresponding vertex functions in this form reproduce their tensor structure adopted in (4.6). Thus, the tensor structure is separated and we deal further with a set of scalar equations, which simplifies the problem.

The solution of the set thus obtained will be sought for, according to [7], in the following form:

$$D_{W^2}{}^R(p^2) = d_{W^2}(p^2)/p^2;$$
$$G_{C\bar{C}}{}^R(p^2) = h_{C\bar{C}}(p^2)/p^2;\quad G_{\psi\bar{\psi}}{}^R(p^2) = h_{\psi\bar{\psi}}(p^2)/p^2;$$
$$\Gamma_{W^3}{}^R(p+k;p;k) \to \Gamma_{W^3}(q^2);$$
$$\Gamma_{C\bar{C}W}{}^R(p+k;p|k) \to \Gamma_{C\bar{C}W}(q^2);\quad \Gamma_{\psi\bar{\psi}W}{}^R(p+k;p|k) \to \Gamma_{\psi\bar{\psi}W}(q^2). \tag{4.7}$$

Here q^2 is the largest 4-vector squared among the arguments of the Γ_{W^3}, $\Gamma_{C\bar{C}W}$ and $\Gamma_{\psi\bar{\psi}W}$ functions. The functions $d_{W^2}(p^2)$; $h_{C\bar{C}}(p^2)$; $h_{\psi\bar{\psi}}(p^2)$; $\Gamma_{W^3}(q^2)$; $\Gamma_{C\bar{C}W}(q^2)$ and $\Gamma_{\psi\bar{\psi}W}(q^2)$ are regarded to as rather slow functions of their arguments, i.e. the derivative of these functions is close to zero. With these designations the set of Eqs. (4.2), (4.6) takes the form of a set of rather simple integral equations

$$\frac{1}{d_{W^2}(\xi)} = 1 - \frac{19}{4}\frac{C_2(G)}{3}\frac{g^2}{16\pi^2}\int_\xi^0 dz\,\Gamma_{W^3}{}^2(z)d_{W^2}{}^2(z) - \frac{C_2(G)}{12}\frac{g^2}{16\pi^2}\int_\xi^0 dz\,\Gamma_{C\bar{C}W}{}^2(z)h_{C\bar{C}}{}^2(z) + \frac{8T(R)}{6}\frac{g^2}{16\pi^2}\int_\xi^0 dz\,\Gamma_{\psi\bar{\psi}W}{}^2(z)h_{\psi\bar{\psi}}{}^2(z);$$

$$\frac{1}{h_{C\bar{C}}(\xi)} = 1 - \frac{C_2(G)}{2}\frac{g^2}{16\pi^2}\int_{\xi}^{0} dz\,\Gamma_{C\bar{C}W}{}^2(z)h_{C\bar{C}}(z)d_{W^2}(z);$$

$$\frac{1}{h_{\psi\bar{\psi}}(\xi)} = 1 - \frac{g^2}{16\pi^2}\int_{\xi}^{0} dz\,\Gamma_{\psi\bar{\psi}W}{}^2(z)h_{\psi\bar{\psi}}\,d_{W^2}(z);$$

$$\Gamma_{C\bar{C}W}(\xi) = 1 + \frac{C_2(G)}{8}\frac{g^2}{16\pi^2}\int_{\xi}^{0} dz\,\Gamma_{C\bar{C}W}{}^3(z)h_{C\bar{C}}{}^2(z)d_{W^2}(z)$$

$$+\frac{3C_2(G)}{8}\frac{g^2}{16\pi^2}\int_{\xi}^{0} dz\,\Gamma_{C\bar{C}W}{}^2(z)\Gamma_{W^3}(z)d_{W^2}{}^2(z)h_{C\bar{C}}(z);$$

$$\Gamma_{W^3}(\xi)d_{W^2}(\xi) = \Gamma_{C\bar{C}W}(\xi)h_{C\bar{C}}(\xi); \qquad \Gamma_{W^3}(\xi)d_{W^2}(\xi) = \Gamma_{\psi\bar{\psi}W}h_{\psi\bar{\psi}}(\xi) \qquad (4.8)$$

where $\xi = \ln(p^2/m^2)$; $C_2(G)$ is the Casimir invariant; $T(R)$ is defined here as $Tr(t^a t^b) = T(R)\delta^{ab}$. The last one of the Eqs. (4.8) is a direct consequence of the Ward identity. Eqs. (4.8) are fully equivalent to the set of differential equations

$$\frac{1}{d_{W^2}}\frac{d}{d\xi}(d_{W^2}) = -\frac{19}{4}\frac{C_2(G)}{3}\frac{g^2}{16\pi^2}\Gamma_{W^3}{}^2 d_{W^2}{}^3$$

$$-\frac{C_2(G)}{12}\frac{g^2}{16\pi^2}\Gamma_{C\bar{C}W}{}^2 h_{C\bar{C}}{}^2 d_{W^2} + \frac{8T(R)}{6}\frac{g^2}{16\pi^2}\Gamma_{\psi\bar{\psi}W}{}^2 h_{\psi\bar{\psi}}{}^2 d_{W^2};$$

$$\frac{1}{\Gamma_{C\bar{C}W}}\frac{d\Gamma_{C\bar{C}W}}{d\xi} = -\frac{C_2(G)}{8}\frac{g^2}{16\pi^2}\Gamma_{C\bar{C}W}{}^2 h_{C\bar{C}}{}^2 d_{W^2}$$

$$-\frac{3C_2(G)}{8}\frac{g^2}{16\pi^2}\Gamma_{C\bar{C}W}\Gamma_{W^3}d_{W^2}{}^2 h_{C\bar{C}};$$

$$\frac{1}{h_{C\bar{C}}}\frac{dh_{C\bar{C}}}{d\xi} = -\frac{C_2(G)}{2}\frac{g^2}{16\pi^2}\Gamma_{C\bar{C}W}{}^2 h_{C\bar{C}}{}^2 d_{W^2};$$

$$\frac{1}{h_{\psi\bar{\psi}}}\frac{dh_{\psi\bar{\psi}}}{d\xi} = -\frac{g^2}{16\pi^2}\Gamma_{\psi\bar{\psi}W}{}^2 h_{\psi\bar{\psi}}{}^2 d_{W^2};$$

$$\Gamma_{W^3}d_{W^2} = \Gamma_{C\bar{C}W}h_{C\bar{C}}; \qquad \Gamma_{W^3}d_{W^2} = \Gamma_{\psi\bar{\psi}W}\cdot h_{\psi\bar{\psi}} \qquad (4.9)$$

with the boundary conditions

$$d_{W^2}(0) = h_{C\bar{C}} = h_{\psi\bar{\psi}} = \Gamma_{C\bar{C}W}(0) = 1.$$

The latter set of equations appears to be most convenient since it is easily solved in explicit form

$$\Gamma_{C\bar{C}W} = h_{C\bar{C}} = h_{\psi\bar{\psi}} = \Gamma; \qquad d_{W^2} = \Gamma^{\kappa-4}; \qquad \Gamma_{W^3} = \Gamma^{6-\kappa};$$

$$\Gamma = \left[1 + \kappa \frac{C_2(G)}{2} \frac{g^2}{16\pi^2} \xi\right]^{-1/\kappa}; \qquad \kappa = \frac{22}{3}\left(1 - \frac{4}{11} \frac{T(R)}{C_2(G)}\right). \quad (4.10)$$

Expressions (4.10) represent the final result of our calculations. Here κ is positive since $T(R) < (11/4)C_2(G)$. The latter inequality follows from the group identity $rT(R) = d(R)C_2(G)$ which relates $T(R)$ to the Casimir invariant $C_2(G)$. Here $d(R)$ is the dimensionality of the representation R and r is the dimensionality of the group G. The asymptotic behaviour of the Green functions and of the corresponding vertex functions is thus found in the explicit form.

The connection between the "bare" and experimental charges in this theory takes the following form

$$g_0^2 = \frac{g^2}{1 + \kappa \frac{C_2(G)}{2} \frac{g^2}{16\pi^2} \ln(\Lambda^2/m^2)} \quad (4.11)$$

which differs from the corresponding expressions of the Abelian theories by the opposite sign in the denominator. According to the expression the bare charge formally tends to zero after the cut-off parameter Λ^2 goes to infinity. Therefore at high energies the theory becomes asymptotically free. Expression (4.11) may be easily resolved with respect to the experimental charge

$$g^2 = \frac{g_0^2}{1 - \kappa \frac{C_2(G)}{2} \frac{g_0^2}{16\pi^2} \ln(\Lambda^2/m^2)}. \quad (4.12)$$

As is readily seen according to expression (4.12) this charge may have in this theory any necessary magnitude if in the local limit $p^2 = \Lambda^2 \to \infty$ the bare charge also tends to zero. This circumstance guarantees the absence of the "zero charge" difficulty from the nonabelian gauge theories and provides the smallness of the contribution coming from the ultraviolet integration domain of the approximations including higher number of vertices. Thus, the summation of all asymptotically essential terms is carried out effectively in the approximation obtained.

5. Brief Discussion of the Results

The main result obtained here is an exact and closed set of completely renormalized equations for the Green functions in the theory of Yang-Mills fields interacting with fermions. This set of equations as well as in the case of simpler models (see [4] and [8]) is free of divergencies and uncertainties and for its solution by perturbations or beyond the scope of perturbation theory no regularization is needed.

Efficiency of our set of equations is demonstrated by obtaining asymptotic behaviour of the Green and vertex functions at large transferred momenta. The simplest approximation, known in the literature as "three-gamma" approximation, leads here to the correct asymptotic behaviour of the Green and vertex functions. This approximation implies expressing the higher Green functions involved into the set of equations in terms of the exact two-point Green function and the simplest exact vertex function in the self-consistent manner. The approximation thus obtained is equivalent to summing up all the leading asymptotical terms of perturbation series. The results of the "three-gamma" approximation applied to the invariant charge are equivalent to its calculations by the renormalization group method in the so-called "one-loop" approximation. However, as distinct from usual calculations by the renormalization group method, we have obtained here not only the asymptotic behaviour for the invariant charge but also that of all the Green and vertex functions. The asymptotical behaviour obtained is exact since the theory under consideration is asymptotically free and thus the contribution from higher approximations into the asymptotical behaviour is negligibly small.

Our investigations are readily extended to the case when an additional interaction with a scalar field is present due to which vector particles can become massive.

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