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ULTRAVIOLET ASYMPTOTICS OF THE GREEN AND VERTEX FUNCTIONS IN SUPERGAUGE INVARIANT FIELD THEORY

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Ultraviolet asymptotics of the Green and vertex functions in supergauge invariant field theory for the SU(N) group is established. Renormalization of the gauge field coupling constant is calculated and the condition for asymptotic freedom is found.

1. Introduction

In recent years a number of authors (see, e.g. reviews quoted in ref. [1]) have built different versions of renormalizable unified models of weak, electromagnetic and strong interactions. These models of universal interactions have been built with the aid of Yang-Mills fields whose masses are induced through the interaction with scalar fields by the Higgs mechanism. Soon it became clear, however, that Yang-Mills field theory is not only renormalizable but also mathematically consistent [2], i.e. the "zero charge" difficulty [3] is absent from this theory and the theory becomes asymptotically free at large momenta. In this connection it became natural to require that asymptotic freedom in the unified theories take place for all the interacting fields and not only for the Yang-Mills fields. At the same time it became clear at once that the scalar field coupling necessary for realization of the Higgs mechanism suffers, as a rule, from the "zero charge" difficulty. If, on the contrary, one deliberately builts a model so as to keep the asymptotic freedom, one usually fails to provide mass for a sufficient number of vector fields [4]. Thus, in the usual models of universal interactions where it is rather easy to impart the necessary masses to all the particles, it appears rather difficult to guarantee that all the fields in such a theory (including scalar fields) be asymptotically free.

In this connection the appearance of theories with supersymmetry [5] has been of great interest in recent years. In these theories different super-multiplets, so necessary for the unified models, arise in a natural way. They unite Fermi and Bose fields with gauge fields and the interaction between these fields proceeds with the aid of one coupling constant. All this makes it possible to hope that the question connected with the asymptotic freedom of such a theory will be solved rather easily since the asymptotic freedom is gained here not for each field separately but simultaneously for the whole supermultiplet. Here however the cardinal problem is also how to formulate asymptotically free massive vector field theory, since up to now, nearly all the known examples of supersymmetric theories with asymptotic freedom [6] have been dealing with massless gauge fields only. There is also an attempt [7] to make gauge fields massive within supersymmetric theories. The gauge field mass is gained there due to the Higgs mechanism, for the realization of which a new coupling is introduced within the scalar supermultiplet. The retaining of asymptotic freedom in such a model has not, however, been investigated by the authors.

By the method of one of the authors [8] we shall obtain here the ultraviolet asymptotic behaviour of the Green and vertex functions in the supergauge invariant field theory for the SU(N) group. Calculations will be carried out in the "three-gamma" approximation for the exact set of dynamical equations. Renormalization of the gauge field coupling constant is found and the condition for asymptotic freedom of such a theory is pointed out. The case of a finite charge renormalization is investigated separately. In this case asymptotic behaviour of the Green and vertex functions is shown to become power like. The latter corresponds to the fact that a purely scaling solution with anomalous dimensionality is realized in the theory.

2. Theory without spontaneous symmetry breaking

We shall consider here the supersymmetric theory of massless gauge fields for the SU(N) group

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_0 C^{abc} W^b_\mu W^c_\nu)^2 \\ &+ \tfrac{1}{2}\bar{\lambda}^a \gamma_\mu (\partial_\mu \lambda^a + g_0 C^{abc} W^b_\mu \lambda^c) + \tfrac{1}{2} g_0^2 (t^d_{kp} A^{ka}_+ A^{pa}_-)(t^d_{st} A^{sb}_+ A^{tb}_-) \\ &- (\partial_\mu A^{ka}_- + i g_0 t^d_{kp} W^d_\mu A^{pa}_-)(\partial_\mu A^{ka}_+ - i g_0 \bar{t}^s_{kt} W^s_\mu A^{ta}_+) \\ &+ \tfrac{1}{2}\bar{\psi}^{ka}\gamma_\mu(\partial_\mu \psi^{ka} + i g_0 t^d_{kp} W^d_\mu \psi^{pa}) \\ &- \sqrt{2} i g_0 t^d_{kp} [A^{ka}_+ \bar{\lambda}^d \tfrac{1}{2}(1+\gamma_5)\psi^{pa} + A^{ka}_- \bar{\lambda}^d \tfrac{1}{2}(1-\gamma_5)\psi^{pa}] \, . \end{aligned} \tag{2.1}$$

The fields are united in one vector and n scalar supermultiplets; the interaction between them is carried out by one coupling constant. The gauge fields W_μ and λ are transformed with respect to the adjoint representation of the group G; the rest of the fields are transformed within some other representation of the group G, which is represented here by the matrices t. For the adjoint representation $it^d_{ab} = C^{adb}$. ψ and λ are the Majorana spinors, $A_\pm$ are zero-spin fields. We use here a pseudo-Euclidean metrics $g_{\mu\mu} = (-+++)$ and the γ matrices in the Schwinger representation $\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$. The Lagrangean of the model should be supplied by the transversality condition $\partial_\mu W_\mu = 0$.

2.1. Set of non-renormalized equations for the Green and vertex functions in the "three-gamma" approximation

The generating functional for this model is determined in the same way as in the usual gauge field theory [9]

$$\mathcal{Z} = \int \mathcal{D}[...] \exp(iS[...]) , \tag{2.2}$$

i.e. the action $S[...]$ besides usual terms corresponding to the Lagrangian (2.1), contains also the α term, which fixes the gauge, and the ghost terms:

$$\begin{aligned}
S[...] = &-\tfrac{1}{2}W(1)[D_{W^2}]_0^{-1}(1;2)W(2) \\
&+\frac{ig_0}{3!}\Gamma^{(0)}_{W^3}(1;2;3)W(1)W(2)W(3) + \frac{(ig_0)^2}{4!}\Gamma^{(0)}_{W^4}(1;2;3;4)W(1)W(2)W(3)W(4) \\
&-\bar{C}(1)[G_{C\bar{C}}]_0^{-1}(1;2)C(2) + (ig_0)\Gamma^{(0)}_{C\bar{C}W}(1;2|3)\bar{C}(1)C(2)W(3) \\
&-\tfrac{1}{2}\bar{\lambda}(1)[G_{\lambda\bar{\lambda}}]_0^{-1}(1;2)\lambda(2) + \tfrac{1}{2}g_0\Gamma^{(0)}_{\lambda\bar{\lambda}W}(1;2|3)\bar{\lambda}(1)\lambda(2)W(3) \\
&+\tfrac{1}{2}(ig_0)^2\widetilde{\Gamma}^{(0)}_{A_-^2A_+^2}(1;2;3;4)A_+(1)A_+(2)A_-(3)A_-(4) \\
&-A_+(1)[D_{A_-A_+}]_0^{-1}(1;2)A_-(2) + (ig_0)\Gamma^{(0)}_{A_-A_+W}(1;2|3)A_+(1)A_-(2)W(3) \\
&+\tfrac{1}{2}(ig_0)^2\Gamma^{(0)}_{A_-A_+W^2}(1;2;3;4)A_+(1)A_-(2)W(3)W(4) \\
&-\tfrac{1}{2}\bar{\psi}(1)[G_{\psi\bar{\psi}}]_0^{-1}(1;2)\psi(2) + \tfrac{1}{2}ig_0\Gamma^{(0)}_{\psi\bar{\psi}W}(1;2|3)\bar{\psi}(1)\psi(2)W(3) \\
&+(ig_0)[\Gamma^{(0)}_{A_-\lambda\bar{\psi}}(1|2;3)A_+(1)\bar{\lambda}(2)\psi(3) + \Gamma^{(0)}_{A_+\lambda\bar{\psi}}(1|2;3)A_-(1)\bar{\lambda}(2)\psi(3)] .
\end{aligned} \tag{2.3}$$

Such a form of the generating functional provides the unitarity of the theory and gauge-invariance of the S-matrix on the mass shell.

The set of non-renormalized equations for the Green functions is derived now by standard methods [10] and finally takes the form of the Schwinger-Dyson equations

$$D^{-1}(1;\bar{1}) = D_0^{-1}(1;\bar{1}) - \Pi(1;\bar{1}) , \qquad G^{-1}(1;\bar{1}) = G_0^{-1}(1;\bar{1}) - \Sigma(1;\bar{1}) . \tag{2.4}$$

In a general case, however, this set is rather cumbersome and therefore we shall write it down here only in the "three-gamma" approximation for the Green and vertex functions which we shall use hereafter

$$\Pi_{W^2}(1;\bar{1}) = -\frac{g_0^2}{2i}\Gamma^{(0)}_{W^3}(1;2;3)D_{W^2}(2;\bar{3})\Gamma_{W^3}(\bar{3};\bar{2};\bar{1})D_{W^2}(\bar{2};3)$$
$$+\frac{g_0^2}{i}\Gamma^{(0)}_{WC\bar{C}}(1|2;3)G_{C\bar{C}}(3;\bar{2})\Gamma_{C\bar{C}W}(\bar{2};\bar{3}|\bar{1})G_{C\bar{C}}(\bar{3};2)$$
$$-\frac{g_0^2}{i}\Gamma^{(0)}_{WA_-A_+}(1|2;3)D_{A_-A_+}(3;\bar{2})\Gamma_{A_-A_+W}(\bar{2};\bar{3}|\bar{1})D_{A_-A_+}(\bar{3};2)$$
$$+\frac{g_0^2}{2i}\Gamma^{(0)}_{W\lambda\bar{\lambda}}(1|2;3)G_{\lambda\bar{\lambda}}(3;\bar{2})\Gamma_{\lambda\bar{\lambda}W}(\bar{2};\bar{3}|\bar{1})G_{\lambda\bar{\lambda}}(\bar{3};2)$$
$$+\frac{g_0^2}{2i}\Gamma^{(0)}_{W\psi\bar{\psi}}(1|2;3)G_{\psi\bar{\psi}}(3;\bar{2})\Gamma_{\psi\bar{\psi}W}(\bar{2};\bar{3}|\bar{1})G_{\psi\bar{\psi}}(\bar{3};2)\,,$$

$$\Sigma_{C\bar{C}}(1;\bar{1}) = -\frac{g_0^2}{i}\Gamma^{(0)}_{C\bar{C}W}(1;2|3)G_{C\bar{C}}(2;\bar{2})\Gamma_{C\bar{C}W}(\bar{3}|\bar{2};1)D_{W^2}(\bar{3};3)\,,$$

$$\Gamma_{C\bar{C}W}(1;2|3) = \Gamma^{(0)}_{C\bar{C}W}(1;2|3)$$
$$-\tfrac{1}{2}g_0^2\Gamma^{(0)}_{C\bar{C}W}(1;\bar{\bar{2}}|\bar{\bar{3}})G_{C\bar{C}}(\bar{\bar{2}};5)\Gamma_{C\bar{C}W}(5;4|3)G_{C\bar{C}}(4;\bar{3})\Gamma_{WC\bar{C}}(\bar{2}|\bar{3};2)D_{W^2}(\bar{2};\bar{\bar{3}})$$
$$-\tfrac{1}{2}g_0^2\Gamma^{(0)}_{C\bar{C}W}(1;\bar{\bar{2}}|\bar{\bar{3}})G_{C\bar{C}}(\bar{\bar{2}};\bar{3})\Gamma_{WC\bar{C}}(\bar{2}|\bar{3};2)D_{W^2}(\bar{2};5)\Gamma_{W^3}(5;4;3)D_{W^2}(4;\bar{\bar{3}})\,. \quad (2.5)$$

Moreover, a number of terms which do not contribute to the ultraviolet asymptotic behaviour are omitted in (2.5). We shall not need here the rest of the equations for the Green and vertex functions either, since when solving the corresponding equations we shall use a number of exact Ward identities. Renormalization of the set of equations (2.4), (2.5), after the introduction of the necessary Z factors,

$$G^{\mathrm{R}} = Z_2^{-1}G\,, \qquad D^{\mathrm{R}} = Z_3^{-1}D\,, \qquad \Gamma^{\mathrm{R}} = Z_1\Gamma\,, \quad (2.6)$$

is carried out by the usual methods [11].

2.2. Asymptotic behaviour of the Green and vertex functions in the region of large transferred momenta

We shall find here a self-consistent solution of the set of renormalized Schwinger-Dyson equations for the Green and vertex functions in the asymptotic region of large transferred momenta. Direct calculations will be carried out on the Feynman gauge since it provides the simplest tensor structure of the D function of the gauge fields

$$[D^{\mathrm{R}}_{W^2}]^{ab}_{\mu\nu}(p^2) = \delta^{ab}\delta_{\mu\nu}D^{\mathrm{R}}(p^2)\,. \quad (2.7)$$

We identify here also the tensor structure of the rest of the Green functions and that of exact vertex functions in the asymptotic region with the tensor structure of "bare" functions. The set of completely renormalized equations for the Green and vertex functions is obtained here in the same way as [12]. In the "three-gamma" approximation it coincides with the set of equations (2.4), (2.5) if all the non-renormalized quantities in eqs. (2.4), (2.5) are replaced by renormalized ones and $Z_1 \Gamma_0$ is replaced by Γ^R.

We shall seek for the solution of this set in accordance with the method of one of the present authors [13]. First of all we shall obtain a set of corresponding equations for the scalar functions, i.e. in the initial equations we shall separate the tensor structure. The latter is easily separated, i.e. the tensor structure of the Green and vertex functions adopted above appears to be self-consistent within the asymptotic accuracy in the set of renormalized equations. The solution of the set of equations thus obtained for the scalar functions is now found as follows:

$$D^R(p^2) = \frac{d(p^2)}{p^2}, \qquad G^R(p^2) = \frac{h(p^2)}{p^2},$$

$$\Gamma^R_{(p+k;p;k)} \to \Gamma(q^2), \tag{2.8}$$

where q^2 is the largest four-vector squared of the arguments of the vertex function, the functions $d(p^2)$, $h(p^2)$ and $\Gamma(q^2)$ are weakly changing functions of their arguments. Then after some algebra we shall have a rather simple set of three integral equations for these functions

$$\frac{1}{d_{W^2}} = 1 - \frac{19}{4}\frac{C_2(G)}{3}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{W^3} \mathrm{d}^2_{W^2}$$

$$- \frac{C_2(G)}{12}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{WC\bar{C}} h^2_{C\bar{C}} + \frac{nT(R)}{3}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{A_-A_+W} d^2_{A_-A_+}$$

$$+ \frac{2C_2(G)}{3}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{W\lambda\bar{\lambda}} h^2_{\lambda\bar{\lambda}} + \frac{2nT(R)}{3}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{W\psi\bar{\psi}} h^2_{\psi\bar{\psi}},$$

$$\frac{1}{h_{C\bar{C}}} = 1 - \frac{C_2(G)}{2}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{C\bar{C}W} \mathrm{d}_{W^2} h_{C\bar{C}},$$

$$\Gamma_{C\bar{C}W} = 1 + \frac{C_2(G)}{8}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^3_{C\bar{C}W} d_{W^2} h^2_{C\bar{C}}$$

$$+ \frac{3C_2(G)}{8}\frac{g^2}{16\pi^2}\int_\xi^0 \mathrm{d}z \Gamma^2_{C\bar{C}W} \Gamma_{W^3} d_{W^2} h_{C\bar{C}}. \tag{2.9}$$

This set may be easily solved by the usual methods. To have a single-valued solution, however, we must add the following relations to it

$$\Gamma_{C\bar{C}W} h_{C\bar{C}} = \Gamma_{W^3} d_{W^2} = \Gamma_{A_+A_-W} d_{A_-A_+} = \Gamma_{\lambda\bar{\lambda}W} h_{\lambda\bar{\lambda}} = \Gamma_{\psi\bar{\psi}W} h_{\psi\bar{\psi}} , \tag{2.10}$$

which is a direct consequence of the exact Ward identities. Such an approach makes it possible to simplify our calculations since it is not necessary now to solve the complete set of equations for the Green and vertex functions. This approach does not lead to any loss of generality either, since the solution of the total set of equations for the Green and vertex functions in the asymptotic region is consistent with the exact Ward identities. In (2.9) $\xi = \ln(p^2/m^2)$, $G_2(G)$ – is the Casimir invariant of the G group, $T(R)$ is determined by the simple relation $t^d_{ab} t^d_{bc} = \delta_{ac} T(R)$. The solution of the set of eqs. (2.9) is most easily found after we have gone over to the set of equations in the differential form

$$\frac{1}{d_{W^2}} \frac{\mathrm{d}}{\mathrm{d}\xi} (d_{W^2}) = -\frac{19}{4} \frac{C_2(G)}{3} \frac{g^2}{16\pi^2} \Gamma^2_{W^3} d^3_{W^2}$$
$$- \frac{C_2(G)}{12} \frac{g^2}{16\pi^2} \Gamma^2_{WC\bar{C}} h^2_{C\bar{C}} d_{W^2} + \frac{nT(R)}{3} \frac{g^2}{16\pi^2} \Gamma^2_{A_-A_+W} d^3_{A_-A_+}$$
$$+ \frac{2C_2(G)}{3} \frac{g^2}{16\pi^2} \Gamma^2_{W\lambda\bar{\lambda}} h^2_{\lambda\bar{\lambda}} d_{W^2} + \frac{2nT(R)}{3} \frac{g^2}{16\pi^2} \Gamma^2_{W\psi\bar{\psi}} h^2_{\psi\bar{\psi}} d_{W^2} ,$$

$$\frac{1}{h_{C\bar{C}}} \frac{\mathrm{d}}{\mathrm{d}\xi} (h_{C\bar{C}}) = -\frac{C_2(G)}{2} \frac{g^2}{16\pi^2} \Gamma^2_{C\bar{C}W} d_{W^2} h^2_{C\bar{C}} ,$$

$$\frac{1}{\Gamma_{WC\bar{C}}} \frac{\mathrm{d}}{\mathrm{d}\xi} (\Gamma_{WC\bar{C}}) = -\frac{C_2(G)}{8} \frac{g^2}{16\pi^2} \Gamma^2_{C\bar{C}W} d_{W^2} h^2_{C\bar{C}}$$
$$- \frac{3C_2(G)}{8} \frac{g^2}{16\pi^2} \Gamma_{C\bar{C}W} \Gamma_{W^3} d_{W^2} h_{C\bar{C}} , \tag{2.11}$$

which is fully equivalent to the original one if we bear in mond the following boundary conditions $d_{W^2}(0) = h_{C\bar{C}}(0) = \Gamma_{C\bar{C}W}(0) = 1$. Finally we come down to the following results

$$\Gamma_{C\bar{C}W} = \Gamma , \qquad h_{C\bar{C}} = \Gamma , \qquad d_{W^2} = \Gamma^{\kappa-4} , \qquad \kappa = 2\left(3 - n\frac{T(R)}{C_2(G)}\right) ,$$
$$\Gamma = \left[1 + \kappa \frac{C_2(G)}{2} \frac{g^2}{16\pi^2} \xi\right]^{-1/\kappa} . \tag{2.12}$$

Now it is also easy to find the asymptotic behaviour of the remaining Green and vertex functions. We use here the results obtained above and the exact Ward identities. Its form is similar to (2.12)

$$h_{\lambda\bar{\lambda}} = \Gamma^{-2(6-\kappa)}\,, \qquad h_{\psi\bar{\psi}} = \Gamma^{-2(6-\kappa)/n}\,, \qquad \Gamma_{W^3} = \Gamma^{6-\kappa}\,,$$

$$\Gamma_{A_-A_+W} = \Gamma^2\,, \qquad \Gamma_{\psi\bar{\psi}W} = \Gamma^{2[1+(6-\kappa)/n]}\,, \qquad \Gamma_{\lambda\bar{\lambda}W} = \Gamma^{2(5-\kappa)}\,,$$

$$d_{A_-A_+} = 1\,, \qquad \Gamma_{A_-^2A_+} = \Gamma_{A_+^2A_-} = 1\,. \tag{2.13}$$

It is of interest that renormalization of the scalar field is absent from this model. This specific property turns out to be rather essential later on when the possibility is discussed of introducing the coupling into a scalar supermultiplet without violation of the asymptotic freedom of the model as a whole.

Renormalization of the charge is calculated here in accordance with (2.12) and has the form typical of the theory with non-Abelian fields

$$g_0^2 = g^2/1 + \kappa\frac{C_2(G)}{2}\,\frac{g^2}{16\pi^2}\ln\frac{\Lambda^2}{m^2}\,. \tag{2.14}$$

We see that if $\kappa > 0$, the "zero charge" difficulty is absent from the theory and the theory is asymptotically free in the region of large transferred momenta. Note that this limitation is rather strict for the given class of theories. Thus, in particular, in the class of theories under consideration no model satisfies this limitation if the groups of local and global symmetry coincide, and all the fields transform *via* the adjoint representation. The theory with $\kappa = 0$ is of particular interest. A purely scaling solution is realized in this case. The function Γ responsible for the asymptotic behaviour of all the Green and vertex functions according to (2.12) and (2.13) at $\kappa = 0$ takes the following form:

$$\Gamma = \left[\frac{p^2}{m^2}\right]^{-g^2/16\pi^2} \tag{2.15}$$

Unfortunately it is not quite clear what taking account of the higher-order approximations entails. Here, at least calculations in the next order of magnitude are necessary.

3. Brief discussion of the results

We have obtained here the ultraviolet asymptotic behaviour of the Green and vertex functions for supergauge invariant field theory with the SU(N) group. The connection between the "bare" and the experimental charges has been calculated and the conditions for the theory under consideration to be asymptotically free have been established. The finite charge renormalization is also discussed. All our calculations refer to the "three-gama" approximation, to the exact set of equations for the Green functions. This approximation implies the simplification of the exact set of dynamical equations due to self-consistent replacement of the highest Green functions and the highest verteces by the exact two-point function and the simplest exact vertex respectively. The calculational result following for the invariant charge within the present approach coincides with that of the "one-loop" approximation of the renormalization group equations.

Within the present approach, however, we obtain not only the ultraviolet asymptotic behaviour of the invariant charge but, at the same time, also the asymptotic behaviour of the Green and vertex functions in the same domain of the momentum transferred. For asymptotically free theories the results obtained here are exact since the contribution coming from the higher approximations is negligibly small in this case. For the case of finite charge renormalization i more detailed treatment is needed. The higher corrections are not small here [14] and it seems that one must not confine oneself to a finite number of them.

References

[1] E.S. Fradkin and I.V. Tyutin, Rivista Nuovo Cimento 4 (1974) 1;
S. Weinberg, Rev. Mod. Phys. 46 (1974) 255.

[2] D.J. Gross and F. Wilczek, Phys. Rev. Letters 30 (1973) 1343; Phys. Rev. D8 (1973) 3633;
J.D. Politzer, Phys. Rev. Letters 30 (1973) 1346.

[3] L.D. Landau and I.Y. Pomaranchuk, Dokl. Akad. Nauk (USSR) 102 (1955) 489;
E.S. Fradkin, ZhETF (USSR) 28 (1955) 750.

[4] T.P. Cheng, E. Eichten and L.-F. Li, Phys. Rev. D9 (1974) 2259.

[5] Yu. A. Gol'fand and E.P. Likthtman, ZhETF Pisma 13 (1971) 452;
J. Wess and B. Zumino, Phys. Letters B49 (1974) 52; Nucl. Phys. B70 (1974) 39;
J. Iliopoulos and B. Zumino, Nucl. Phys. B76 (1974) 310;
A. Salam and J. Strathdee, Nucl. Phys. B76 (1974) 477.
S. Ferrara, Nucl. Phys. B77 (1974) 73.

[6] A. Salam and J. Strathdee, Phys. Letters B51 (1974) 353;
S. Ferrara and B. Zumino, Nucl. Phys. B79 (1974) 413.

[7] A. Salam and J. Strathdee, Phys. Rev. D11 (1975) 1521.

[8] F.S. Fradkin, ZhETF (USSR) 28 (1955) 750; 29 (1955) 121, 377.

[9] D.S. De Witt, Phys. Rev. 162 (1967) 1195, 1239;
L.D. Faddeev and V.N. Popov, Phys. Letters B25 (1967) 30; preprint ITP (Kiev 1967);
S. Mandelstam, Phys. Rev. 175 (1968) 1580, 1604;
E.S. Fradkin and I.V. Tyutin, Phys. Rev. D2 (1970) 2842.

[10] E.S. Fradkin, ZhETF (USSR) 29 (1955) 121; Dissertation (Moscow, 1960), published in: Trudy Lebedev Fiz. Inst., 29 (1965) 7 (English translation, Consultant Bureau (New York, N.Y., 1967)).

[11] A.D. Galanin, B.L. Ioffe and E.Y. Pomeranchuk, Dokl. Akad. Nauk (USSR) 98 (1954) 361;
E.S. Fradkin, ZhETF (USSR) 26 (1954) 751.

[12] E.S. Fradkin and O.K. Kalashnikov, J. of Phys., to be published.

[13] E.S. Fradkin, ZhETF (USSR) 28 (1955) 750; 29 (1955) 377.

[14] T.R.D. Jones, Nucl. Phys. B87 (1975) 127.