Volume 104B, number 5 PHYSICS LETTERS 10 September 1981

RENORMALIZABLE ASYMPTOTICALLY FREE QUANTUM THEORY OF GRAVITY

E.S. FRADKIN and A.A. TSEYTLIN
Department of Theoretical Physics, P.N. Lebedev Physical Institute, USSR Academy of Sciences, Moscow 117924, USSR

Received 24 April 1981

We prove that quantum gravity with a quadratic lagrangian of a general type is asymptotically free in all essential coupling constants (including the effective Λ-term). One-loop counter terms are obtained and asymptotic freedom is established also in conformally-invariant theories. We discuss the possibility of a ultra-macroscopic radius of confinement of ghosts, the restoration of unitarity due to radiative corrections and propose also that interaction with gravity leads to asymptotic freedom for the effective masses and all coupling constants in grand unified gauge theories.

The well-known difficulties in the quantized Einstein theory have recently raised much interest in the theory of gravity with a quadratic lagrangian (see e.g. refs. [1–7]). The claims of the lack of unitarity (on the tree level) should not be taken too seriously: the question of unitarity is a dynamical one and must be discussed taking account of the radiative corrections and probably nonperturbatively [8,3–5]. We consider this theory as a possible alternative to the attempts of constructing a unified ultraviolet (UV)-*finite* theory on the basis of extended supergravity, leading (after the inclusion of matter, described by grand unified models) to a complete *renormalizable* theory of all interactions in nature.

Unified vector gauge theories solved the problem of gauge "charges" by indicating their dynamical origin (the meaning of asymptotic freedom (AF) is that the bare couplings are zero in the local limit) but left the question about the values of the bare masses and non-gauge (ϕ^4 and Yukawa) "charges" open. One can conjecture that gravity can provide AF-behaviour for all interactions but in the Einstein theory this is principally only possible in a nonperturbative approach [9]. We shall see that renormalizable gravity (treated in the context of perturbation theory) not only is AF in all its coupling constants but very likely solves the problem of masses and "charges" by establishing the AF-regime for them.

In this paper we obtain the one-loop counter terms first for free quantum gravity correcting the result of ref. [4]) and then in the presence of matter and discuss the UV-behaviour of the solutions of the corresponding renormalization group (RG) equations.

The theory under consideration has the following bare euclidean action [‡1]

$$I = \int_M \mathcal{L}\sqrt{g}\, d^4x - k_S^{-2}\Sigma + 32\pi^2 \alpha_S \chi_S ,$$

$$\Sigma \equiv \int_{\partial M} 2K\sqrt{\gamma}\, d^3x, \qquad (1)$$

$$\mathcal{L} = -k_V^{-2}(R - 2\Lambda) + aW + \tfrac{1}{3} bR^2 + \alpha_V R^* R^* , \qquad (2)$$

where Σ and χ_S are the boundary terms in the Einstein action [10,8,25,11] and in the Euler number [12] necessary (as Λ) for renormalizability and also (Σ) for correspondence with the Einstein theory on the level of the tree S-matrix. The essential coupling constants [‡2] are $a^{-1}, b^{-1}, \alpha_V^{-1}, \alpha_S^{-1}, \lambda \equiv \Lambda k_V^2$ and k_S (note that $R - 4\Lambda = -2bk_V^2 D^2 R$ on the classical

‡1 Our notations are: $R^\lambda{}_{\mu\nu\rho} = \partial_\nu \Gamma^\lambda{}_{\mu\rho} - ...,\ R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$, $W = R_{\mu\nu}^2 - \frac{1}{3}R^2,\ \chi = \chi_V + \chi_S = (32\pi^2)^{-1}(\int_M R^* R^* \times \sqrt{g}\, d^4x + \int_{\partial M} \Omega_S \sqrt{\gamma}\, d^3x)$.

‡2 Those on which the action depends being calculated on the general solution of classical field equations, thus having gauge independent β-functions (cf. refs. [13,14]).

field equations). To establish the correct physical meaning of the theory and consistency of the perturbative calculations one must put $k_V^2 = k_S^2 = k^2 = 16\pi G$ after renormalization. The action is positive when $a = f^{-2} > 0, b > 0, \Lambda > 3/8bk_V^2$ but we shall not assume that $b > 0$ because this apparently leads to the 0^+-tachyon on the tree level (if $\Lambda \approx 0$) [2].

Quantizing the theory (1) using the background field method [14] in the gauge $\chi_\mu[g, h]$ we have

$$Z[g] = (\det H)^{-1/2} \int \mathrm{d}h\, \mathrm{d}\bar{c}\, \mathrm{d}c$$

$$\times \exp\{-I[g+h] - \tfrac{1}{2}\chi H \chi - \bar{c}\widetilde{\Delta}_G c\}, \tag{3}$$

where $\widetilde{\Delta}_{G\mu\nu} = H_\mu{}^\lambda \Delta_{G\lambda\nu}$ and $\Delta_{G\mu\nu}$ are the "modified" and "ordinary" ghost operators and $H_{\mu\nu}[g]$ is an appropriate background covariant operator. In the one-loop approximation a convenient choice is

$$\chi_\mu = P_\mu + D_\mu D^{-2}(B - D_\lambda P^\lambda), \quad P_\mu \equiv D_\lambda \bar{h}^\lambda{}_\mu, \tag{4}$$

$$B = D_\mu P^\mu - \beta D^2\varphi - \xi_1 R_{\mu\nu}\bar{h}^{\mu\nu} - m^2\xi_2\varphi - \xi_3 R\varphi, \tag{5}$$

$$H_{\mu\nu} = -g_{\mu\nu}D^2 + D_\nu D_\mu - \gamma D_\mu D_\nu, \quad \gamma = \tfrac{2}{3}(1+\omega), \tag{6}$$

where $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{4}\varphi g_{\mu\nu}$, $\varphi = h^\mu{}_\mu$, $m^2 = 1/ak^2$, $\omega = -b/a$, $\beta = \frac{3}{4}\omega/(1+\omega)$ and ξ_i are gauge parameters. As a result

$$\widetilde{\Delta}_{G\mu\nu} = 2[(-g_{\mu\nu}D^2 + D_\lambda D_\mu)\delta P^\lambda/\delta h_{\alpha\beta}$$

$$- \gamma D_\mu \delta B/\delta h_{\alpha\beta}]\, g_{\nu(\alpha}D_{\beta)}, \tag{7}$$

what coincides with the expression obtained in ref. [4] from the requirement of BRS-invariance of (3). However, the $\chi H \chi$ structure of the gauge breaking term in (3) was not revealed in ref. [4] and therefore the essential factor $(\det H)^{-1/2}$ in (3) (which gives additional counter-terms) was missed. Finally we get (changing $\varphi \to \mathrm{i}\varphi$ when $b < 0$ as in the Einstein theory [15])

$$Z_1 = \mathrm{e}^{-I[g]} \det(\mu^{-4}\widetilde{\Delta}_G)[\det(\mu^{-2}H)\det(\mu^{-4}\hat{\Delta})]^{-1/2}, \tag{8}$$

where $\hat{\Delta}$ is the 4th order operator on $(\bar{h}\varphi)$ of the following (as well as $\widetilde{\Delta}_G$) type

$$\Delta_4 = D^4 + V^{\mu\nu}D_\mu D_\nu + 2N^\mu D_\mu + U, \quad V_{\mu\nu} = V_{\nu\mu}. \tag{9}$$

Using the well-known algorithm for the divergencies of the determinant of the 2nd order operator $\Delta_2 = -D^2 + X$ [16–19] one can find that (for simplicity we omit the boundary terms)

$$I_\infty = \tfrac{1}{2}\log\det(\mu^{-4}\Delta_4)$$

$$= -\tfrac{1}{2}[\tfrac{1}{2}B_0 L^4 + B_2 L^2 + B_4 \log(L^2/\mu^2)], \tag{10}$$

where

$$L \to \infty, \quad B_p = \int_M b_p \sqrt{g}\, \mathrm{d}^4x, \quad b_p = (4\pi)^{-2}\bar{b}_p,$$

and

$$\bar{b}_0 = \mathrm{tr}\,1, \quad b_2 = \mathrm{tr}(\tfrac{1}{3}1R + \tfrac{1}{4}V), \quad V = V^\mu{}_\mu, \tag{11}$$

$$\bar{b}_4 = \mathrm{tr}[\tfrac{1}{6}F_{\mu\nu}^2 + 2\cdot 1(\tfrac{1}{180}R^*R^* + \tfrac{1}{60}W + \tfrac{1}{12}R^2 + \tfrac{1}{30}D^2R)$$

$$+ \tfrac{1}{24}V_{\mu\nu}^2 + \tfrac{1}{48}V^2 - \tfrac{1}{6}V^{\mu\nu}R_{\mu\nu} + \tfrac{1}{12}VR$$

$$- U - \tfrac{1}{6}D_\mu D_\nu V^{\mu\nu} + \tfrac{1}{12}D^2V + D_\mu N^\mu]. \tag{12}$$

The method we have used [‡3] gives tje possibility to obtain also the boundary and divergence-like terms in I_∞ and b_p (as compared with the diagram method of refs. [17] and [4]).

The b_4-coefficient for all operators in (8) has the following general structure:

$$\bar{b}_4 = \beta_1 R^*R^* + \beta_2 W + \tfrac{1}{3}\beta_3 R^2 + \beta_4 R + \beta_5 + \beta_6 D^2R \tag{13}$$

[for example, for $H_{\mu\nu}$ (6) one finds using the "Δ_2-algorithm": $\beta_1 = -\frac{11}{180}, \beta_2 = \frac{7}{30}, \beta_3 = \frac{1}{12}, \beta_4 = \beta_5 = 0, \beta_6 = -\frac{1}{30}$]. The total result for the coefficients in the divergent part (10) of the effective action in (8) is given by [$b_p^{\mathrm{tot}} = b_p(\hat{\Delta}) + b_p(H) - 2b_p(\widetilde{\Delta}_G)$],

$$\bar{b}_0^{\mathrm{tot}} = N_{\mathrm{tot}} = 8,$$

$$\bar{b}_2^{\mathrm{tot}} = -(\tfrac{10}{3}\omega + 5)R - m^2(5 + 1/2\omega); \tag{14}$$

‡3 To find the coefficients in front of the invariants in (11), (12) it is sufficient to consider the two special cases of the representation $\Delta_4 = \Delta_2^{(1)}\Delta_2^{(2)}$: (i) $\Delta_2^{(1,2)} = -D^2 + X^{(1,2)}$; (ii) $\Delta_2^{(1,2)} = -D^2(A^{(1,2)}, A_\mu^{(1,2)} = A_\mu \pm Q_\mu$, Q_μ is an arbitrary matrix, and we use the known analog of (11) in the case of Δ_2.

$\bar{b}_4^{\rm tot}$: $\beta_1 = \frac{413}{180}$, $\beta_2 = \frac{133}{10}$,

$$\beta_3 = \tfrac{10}{3}\omega^2 + 5\omega - \tfrac{1}{12}, \quad \beta_4 = (\tfrac{10}{3}\omega - 1/4\omega - \tfrac{13}{6})m^2,$$

$$\beta_5 = 4\Lambda m^2(\tfrac{14}{3} + 1/6\omega) + m^4(\tfrac{5}{2} + 1/8\omega^2) \qquad (15)$$

[note that the L^4-divergencies are cancelled out [20] under the correct choice of the measure in (3)]. Our expression for the logarithmic counter terms differs from that of ref. [4] by the contribution of H.

The gauge-independent system of the RG-equations for the essential coupling constants has the form (L^2-terms are simply subtracted and do not contribute in it)

$$\dot{\alpha}_{\rm V} \equiv {\rm d}\alpha_{\rm V}/{\rm d}t = \beta_1, \quad \dot{a} = \beta_2, \quad \omega' = {\rm d}\omega/{\rm d}\tau = -\beta_2\omega - \beta_3, \qquad (16)$$

$$\bar{\lambda}' = (\beta_2 + 2m^{-2}\beta_4)\bar{\lambda} + \tfrac{1}{2}m^{-4}\beta_5, \quad \bar{\lambda} = a\lambda = a\Lambda k_{\rm V}^2, \qquad (17)$$

[where $t = (4\pi)^{-2}\log(\tilde{\mu}/\mu)$, ${\rm d}\tau = a^{-1}(t)\,{\rm d}t$] and can be completely integrated with the following consequences. The theory (1) is AF in $a^{-1} = f^2$ (and $\alpha_{\rm V}^{-1}$),

$$f^2(t) = f^2(0)[1 + \beta_2 f^2(0)t]^{-1}, \quad a(t) = a(0) + \beta_2 t; \qquad (18)$$

the solution for $\omega(t)$ has the UV-fixed point $\omega(\infty) = \omega_1$

$$\omega(t) = (\omega_1\eta - \omega_2)(\eta - 1)^{-1},$$

$$\eta(t) = |a(t)/a_0|^p, \quad a_0 = \text{const},$$

$$\omega_1 \approx 0.0046, \quad \omega_2 \approx -5.4946, \quad p \approx 1.36, \qquad (19)$$

implying AF behaviour for b^{-1} in (1) (under a natural assumption of $b < 0$ necessary for correspondence with the Einstein theory in the 0^+-sector). The conclusion about asymptotic freedom is valid also for λ due to the UV-fixed point for $\bar{\lambda}$, $\bar{\lambda}(\infty) = \bar{\lambda}_1$,

$$\bar{\lambda}(t)_{t\to\infty} \approx \bar{\lambda}_1 + ca^q, \quad \bar{\lambda}_1 \approx 166, \quad q \approx -1.1,$$

$$c = \text{const}. \qquad (20)$$

Eqs. (16), (17) also have the stable special fixed-point solution $\omega(t) = \omega_1$, $\bar{\lambda}(t) = \bar{\lambda}_1$, corresponding to $a_0 \to 0$ in (19) and $c \to 0$ in (20) [‡4].

Thus in the high-energy limit one may neglect R- and Λ-terms in (2) (this justifies the flat-space expansion and tells us that the space–time is *not* becoming more and more "foamy" with decrease of the scales, cf. refs. [10,23]) and use the perturbative expansion in a^{-1}, b^{-1}. A possible mechanism of restoring unitarity is the summation of the (one-loop) radiative corrections which may shift the ghost pole off the real axis [8,5,24,6]. One has for the inverse euclidean propagator (in the UV-limit)

$$2^+: \quad (p^2/4k^2)[1 + (k^2\beta_2/32\pi^2)p^2\log(p^2/\mu^2) + ...],$$

$$0^+: \quad -(p^2/2k^2)[1 - (k^2\beta_3/16\pi^2)p^2\log(p^2/\mu^2) + ...], \qquad (21)$$

and therefore our results $\beta_2 > 0$, $\beta_3(\omega(\infty)) < 0$ do imply absence of the real poles.

The large distance domain is the strong coupling region of the theory. In the vicinity of the infrared (IR) pole in (18) the effective masses of the 2^+-ghost $m = (ak^2)^{-1/2}$ and the 0^+-particle $m' = (-2bk^2)^{-1/2}$ are infinite [the W- and R^2-terms drop from (2)] which can be treated as a manifestation of "confinement" of the corresponding states [‡5]. This effect could provide a unitary quantum theory in the case of a *microscopic* value of the radius of confinement $r_{\rm c}$ if not for a difficulty connected with the apparent IR-growth of λ. That is why we suppose $r_{\rm c}$ to be *ultramacroscopic* (of the order of or greater than the size of the universe). Choosing the normalization point in (18) to be at the Planck mass $m_{\rm P}$ we have $f^2(0) \approx 0.05$ (cf. with the grand unification constant) and therefore $m(0) \approx 0.1\,m_{\rm P}$ what does not contradict the observations [2]. At the same time we can establish the needed small value for λ ($\approx 10^{-122}$) for distances smaller than $r_{\rm c}$ by a suitable choice of the integration

‡4 We have also found the renormalization of the boundary terms in (1), using the expressions for the boundary counterparts C_2 and C_4 of B_2 and B_4 in (10) (see, e.g. refs. [21, 22]). For example, for Δ_2; $\bar{C}_2 = \frac{1}{3}K\,{\rm tr}\,1$, $\bar{C}_4 = {\rm tr}\{1(\frac{1}{180}\Omega_{\rm S} + \frac{1}{15}R^{\mu\nu}K_{\mu\nu} + \frac{1}{30}RK + ...) - \frac{1}{3}XK + ...\}^2$ (the latter expression was derived by the method of "doubling" [21] and is probably new). As a result, $\alpha_{\rm V}(t) = \alpha_{\rm V}(0) + \frac{1}{180} \times N_{\rm tot}t$, $k_{\rm S}(t)_{t\to\infty} \approx k_{\rm S}(0)[a(0)/a(t)]^\nu$, $\nu \approx 25$ and we conclude about AF also for $\alpha_{\rm V}^{-1}$ and $k_{\rm S}$ in (1).

‡5 We stress that the IR-growth of m, m' shows in fact that the poles cannot occur in the IR-region. They could appear in the UV-region (where masses have a tendency to decrease) but here they are absent due to radiative corrections (21) (cf. the opposite proposals of the UV-growth of m and m' in refs. [3,4]).

constant c in (20). Thus a complementary property of the asymptotic freedom of the theory (1) may be the presence of some quantum-gravitational effects at ultra-large distances.

Let us now consider the case of the conformally-invariant theories,

$$\mathcal{L}_1 = aW + \alpha_V R^* R^*, \qquad (22)$$

$$\mathcal{L}_2 = -6\phi(-D^2 + \tfrac{1}{6}R)\phi + 2\lambda\phi^4 + aW + \alpha_V R^* R^*. \qquad (23)$$

It is important to stress that the expression for counter terms in the Weyl theory (22) cannot be obtained by putting $b = 0$ (and $k_V^{-2} = \Lambda = 0$) in (15): (i) $b \to 0$ changes the order of $\hat{\Delta}$ in (8) on φ which gives the implicit dependence of β_2 in (15) on b; (ii) one should deal properly with conformal invariance of the theory. Thus the value of β_2 for (22) in refs. [6,7] claimed to follow from the (independently incorrect) result of ref. [4] is erroneous. Our consistent calculation using the conformal gauge $\varphi = 0$ and (4), (5) with $\xi_2 = \xi_3 = 0$ and maintaining the background conformal invariance by the substitution $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu}\Phi^2(g)$, $R(\tilde{g}) = 0$ as in ref. [25], shows that for (22) $b_2 = 0$ and

$$\bar{b}_4: \quad \beta_1 = \tfrac{137}{60}, \quad \beta_2 = \tfrac{199}{15}, \quad \beta_3 = ... = \beta_6 = 0. \qquad (24)$$

Hence, according to (18) this theory is really AF as was proposed in ref. [6].

Calculating the divergencies in the case of $b = 0$ in (2) [in the gauge (4), (5) with $\xi_2 = -\frac{3}{8}$, $\xi_3 = 0$] we find $b_2 = -\frac{11}{2}R + \frac{4}{3}\Lambda - 5m^2$ and

$$\bar{b}_4: \quad \beta_1 = \tfrac{103}{45}, \quad \beta_2 = \tfrac{797}{60}, \quad \beta_3 = -\tfrac{11}{24}, \qquad (25)$$

$$\beta_4 = -\tfrac{13}{6}m^2 + \tfrac{2}{9}\Lambda, \quad \beta_5 = \tfrac{8}{9}\Lambda^2 + \tfrac{56}{3}\Lambda m^2 + \tfrac{5}{2}m^4$$

and hence after $g_{\mu\nu} \to \tilde{g}_{\mu\nu} = g_{\mu\nu}\phi^2 k^2$ (the conformal gauge is $\phi_{\text{quantum}} = 0$ [25]) the counter terms in the theory (23) [$\beta_3 \neq 0$ and therefore renormalizability takes place only on the field equations $R(\tilde{g}) = 4\Lambda$]. The RG-equation for the essential coupling constant λ [cf. (17)]

$$\bar{\lambda}' = -\tfrac{1}{3}\bar{\lambda}^2 + (5 + \beta_2)\bar{\lambda} + \tfrac{5}{4}, \qquad (26)$$

shows that (as well as for a^{-1}) there is AF-behaviour for λ [$\bar{\lambda}(\infty) = \bar{\lambda}_\infty \approx 54.9$, the solution is analogous to (10)].

Now we turn to the discussion of the interaction of gravity described by (1), (2) with matter. The corresponding total theory is renormalizable (in contrast with the Einstein theory [26]) if the $\sigma R\phi^2$-terms are added for interacting scalar fields. If all self-interactions of matter were absent, the expression for one-loop logarithmic divergencies would be given by the sum of three terms: the gravitational contribution (15), the contribution of matter fields in the background metric (which is easily found with the help of the "Δ_2-algorithm" [16–19], see e.g. refs. [19, 27])

$$\bar{b}_4^{\text{mat}}: \quad \tilde{\beta}_1 = -\tfrac{13}{180}N_1^{(0)} - \tfrac{1}{15}N_1 + \tfrac{7}{360}N_{1/2} + \tfrac{1}{180}N_0,$$

$$\tilde{\beta}_2 = \tfrac{1}{3}N_1^{(0)} + \tfrac{13}{60}N_1 + \tfrac{1}{10}N_{1/2} + \tfrac{1}{60}N_0,$$

$$\tilde{\beta}_3 = \tfrac{1}{24}N_1 + \tfrac{1}{24}N_0(1 - 6\sigma)^2,$$

$$\tilde{\beta}_4 = \tfrac{1}{2}m_1^2 N_1 - \tfrac{1}{3}m_{1/2}^2 N_{1/2} - \tfrac{1}{6}m_0^2 N_0(1-6\sigma),$$

$$\tilde{\beta}_5 = \tfrac{3}{4}m_1^4 N_1 - 2m_{1/2}^4 N_{1/2} + \tfrac{1}{2}m_0^4 N_0, \qquad (27)$$

(here N_s is the number of fields, with spin s and mass m_s and $N_1^{(0)}$ is the number of massless vector fields), and the contribution of the mixed "$h\phi$" term

$$\Delta\bar{b}_4 = \sum_s (\kappa_1^{(s)} J_1^{(s)} + \kappa_2^{(s)} J_2^{(s)}), \qquad (28)$$

where $J_1^{(s)}$ and $J_2^{(s)}$ are the kinetic and mass terms in the matter lagrangian $\mathcal{L}_m = \Sigma_s(J_1^{(s)} + J_2^{(s)})$. The direct calculation gives $\kappa_1 = (26\omega - 1)/12a\omega$, $\kappa_2 = (28\omega + 1)/3a\omega$ for the scalar and $\kappa_1 = \kappa_2 = 0$ for the massive vector field (the latter somewhat unexpected result shows that the interaction with gravity does not affect, in the one-loop approximation, the AF of the gauge fields). In the presence of gravity, masses become coupling constants but the *essential* couplings are not m_s but the dimensionless $M_s = k_V^2 m_s^2$ (the "effective masses"). The complete system of the RG-equations includes (16), (17) with $\beta_l \to \hat{\beta}_l \equiv \beta_l + \tilde{\beta}_l$ [see eqs. (15) and (27)] and the equations for M_s

$$\bar{M}_s' = [\hat{\beta}_2 + m^{-2}\hat{\beta}_4 + a(\kappa_2^{(s)} - \kappa_1^{(s)})]\bar{M}_s, \quad \bar{M}_s \equiv aM_s. \qquad (29)$$

As a consequence of (16), (27) the matter fields *support* the asymptotic freedom with respect to a^{-1} (see also ref. [5]). From the structure of (17) and (29) it follows that M_s and λ behave as a^{-1} in the UV-limit thus providing ***AF in the mass parameters*** if

$\bar{M}_s(\infty) > 0$. Really in the case of one scalar or one vector field we obtained, respectively: $\bar{M}_0(\infty) \approx 610$, and $\bar{M}_1(\infty) \approx 218$, $\bar{\lambda}(\infty) \approx 390$, and $\bar{\lambda}(\infty) \approx -648$.

When the self-interactions of matter ($g_1 \bar{\psi}\phi\psi$, $\frac{1}{4}g_2\phi^4, ...$) are taken into account (with the necessary $\alpha R\phi^2$-term) we get a system of interrelated RG-equations for g_i, $\bar{\sigma} \equiv a^{-1}\sigma, \lambda, M_{s,...}$. It seems very likely that the solution of this system will show *AF-behaviour for all couplings* in a relatic model. Two points are of principal importance here: (i) gravity gives the *essential* contribution to the RG-equations for masses and "charges" thus leading to the $a^{-1}(t)$ UV-behaviour for all dimensionless parameters; (ii) gravity is AF in $a^{-1}(t)$. For example, the flat space "ϕ^4" RG-equation, $\dot{g}_2 = 18g_2^2$ (with the "zero-charge" solution), will change in the presence of gravity (with the use of the equation for σ) into $\bar{g}_2' = A\bar{g}_2^2 + B\bar{g}_2 + C$, $\bar{g}_2 \equiv ag_2$, which has the AF-solution if $\bar{g}_2(\infty) > 0$. We have already seen the realization of the latter possibility for the example of the theory (27) [cf. eq. (26)].

We conclude that switching on the interaction of matter with renormalizable quantum gravity probably solves the problem of bare masses and (non-gauge) charges (and also possibly gives a mechanism of spontaneous symmetry breaking without the puzzle of the large Λ-term). Thus (renormalizable) gravity is not a "universal regularizator" but is called up to provide asymptotic freedom in all coupling constants (excluding the gauge ones which are already asymptotically free).

An extended version of this paper [28] will soon be published elsewhere.

One of the authors (A.T.) is very grateful to Dr. R.E. Kallosh for stimulating the check of the results of ref. [6].

References

[1] R. Utiyama and B.S. De Witt, J. Math. Phys. 3 (1962) 608;
S. Deser, in: Proc. Conf. on Gauge theories and modern field theories, eds. R. Arnowitt and P. Nath (MIT Press, Cambridge, 1975);
S. Weinberg, in: Proc. XVII Intern. Conf. on High energy physics, ed. J.R. Smith (Chilton, Didcot, 1974).
[2] K.S. Stelle, Phys. Rev. D16 (1977) 953; Gen. Rel. Grav. 9 (1978) 353.
[3] A. Salam and J. Strathdee, Phys. Rev. D18 (1978) 4480.
[4] J. Julve and M. Tonin, Nuovo Cimento 46B (1978) 137.
[5] E. Tomboulis, Phys. Lett. 70B (1977) 361; 97B (1980) 77.
[6] B. Hasslacher and E. Mottola, Phys. Lett. 99B (1981) 221.
[7] S.L. Adler, in: The high energy limit, the 1980 Erice Lectures, ed. A. Zichichi (Plenum, 1981).
[8] E.S. Fradkin and G.A. Vilkovisky, Bern preprint (1976); in: Proc. 18th Intern. Conf. on High energy physics (Tbilisi, 1976), Vol. 2, Section C, p. 28.
[9] E.S. Fradkin, Trieste preprint-IC/79/67 (1979).
[10] S.W. Hawking, in: General relativity, eds. S.W. Hawking and W. Israel (Cambridge U.P., 1979).
[11] R. Gastmans, R. Kallosh and C. Truffin, Nucl. Phys. B133 (1978) 417;
S.M. Christensen and M.J. Duff, Phys. Lett. 79B (1978) 213.
[12] P.B. Gilkey, Adv. Math. 15 (1975) 334.
[13] S. Weinberg, in: General relativity, eds. S.W. Hawking and W. Israel (Cambridge U.P., 1979).
[14] B.S. De Witt, in: Recent developments in gravitation, eds. M. Levy and S. Deser (Plenum, 1979); Santa Barbara preprint NSF-IIP-80-31 (1980);
R. Kallosh, Nucl. Phys. B78 (1974) 293.
[15] G.W. Gibbons, S.W. Hawking and M.J. Perry, Nucl. Phys. B138 (1978) 141.
[16] B.S. De Witt, Dynamical theory of groups and fields (Gordon and Breach, New York, 1965).
[17] G. 't Hooft and M. Veltman, Ann. Inst. H. Poincaré 20 (1974) 69.
[18] P.B. Gilkey, J. Diff. Geom. 10 (1975) 601.
[19] S.M. Christensen and M.J. Duff, Nucl. Phys. B154 (1979) 301.
[20] E.S. Fradkin and G.A. Vilkovisky, Phys. Rev. D8 (1973) 4241; CERN preprint TH-2332 (1977).
[21] H.P. McKean and I.M. Singer, J. Diff. Geom. 1 (1967) 43.
[22] G. Kennedy, R. Critchley and J.S. Dowker, Ann. Phys. 125 (1980) 346.
[23] P.K. Townsend, Phys. Rev. D15 (1977) 2795;
S.M. Christensen and M.J. Duff, Nucl. Phys. B170 [FS1] (1980) 480.
[24] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. 77B (1978) 262.
[25] E.S. Fradkin and G.A. Vilkovisky, Phys. Lett. 73B (1978) 209.
[26] S. Deser, in: Quantum gravity, eds. C.J. Isham, R. Penrose and D. Sciama (Oxford U.P., 1975).
[27] L. Parker, in: Recent developments in gravitation, eds. M. Levy and S. Deser (Plenum, 1979);
G.W. Gibbons, Lecture Notes Math. 676 (1978) 513.
[28] E.S. Fradkin and A.A. Tseytlin, Lebedev Phys. Inst. preprint (1981).